# Electric Charges, Fields, & Gauss's Law

AP Physics C: E&M ▪ Unit 8

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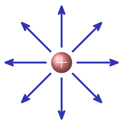
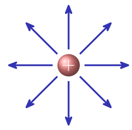




## What we will cover





# 1 Charge & Coulomb



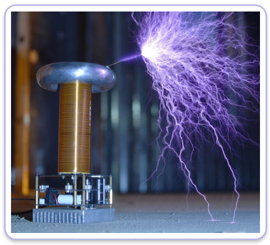
## Charge and Coulomb's law

**Charge** 电荷 is **quantized** 量子化 ($q = ne$) and **conserved** 守恒:

**A vector, inverse-square force**

$$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r}, \quad k = \frac{1}{4\pi\varepsilon_0}$$

Forces from many charges add by **superposition** 叠加原理 – the vector sum.



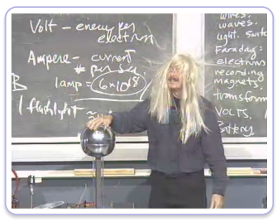
*Tesla coil discharge*

# 2 Charging



## The process of charging

- a **conductor** 导体 lets charge move; an **insulator** 绝缘体 holds it fixed
- **friction** 摩擦起电 – electrons rubbed across
- **conduction** 传导起电 – touch: same sign
- **induction** 感应起电 – near + **ground** 接地: opposite sign



*Static charge builds on a Van de Graaff generator*

# 3 Fields



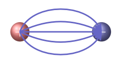
## Electric fields and field lines

The **electric field** 电场 is force per unit positive **test charge** 检验电荷:

**Field of a point charge**

$$\vec{E} = \frac{\vec{F}}{q_0}, \qquad \vec{E} = k\frac{q}{r^2}\hat{r}$$

- lines leave $+$, enter $-$; denser $=$ stronger
- lines never cross; $\vec{F} = q\vec{E}$ on any charge



dipole field

# 4 Distributions



## Fields of charge distributions

Split a spread-out charge into pieces $dq$ and integrate:

**Continuous superposition**

$$\vec{E} = \int k\frac{dq}{r^2}\hat{r}$$

**Densities** 电荷密度: $\lambda = \frac{dq}{d\ell}$, $\sigma = \frac{dq}{dA}$, $\rho = \frac{dq}{dV}$.
Use **symmetry** 对称性 to cancel components.

**Two standard results**

$$\text{ring (axis)}: \ E = \frac{kQx}{(x^2 + a^2)^{3/2}}$$

$$\text{infinite line}: \ E = \frac{2k\lambda}{r}$$

Point $\sim 1/r^2$, line $\sim 1/r$, sheet $\sim$ uniform.

# 5 Flux



## Electric flux

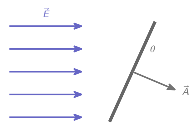
**Electric flux** 电通量 counts the field lines through a surface:

**Uniform, then general**

$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta$$

$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$

Largest when $\vec{E} \perp$ the surface ($\theta = 0$); zero when parallel.



# 6 Gauss



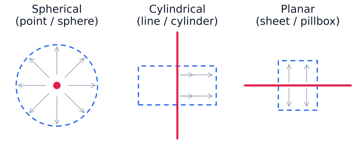
## Gauss's law

One of Maxwell's equations – flux out of a closed surface reveals the charge inside:

**Gauss's law**

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

Choose a **Gaussian surface** 高斯面 matching the symmetry so $E$ comes out of the integral.




*Sphere, cylinder, or pillbox – matched to the symmetry.*

# 7 Recap



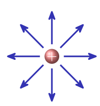
## Unit 8 – key takeaways

1. **Coulomb**
   $\vec{F} = kq_1q_2\hat{r}/r^2$; superpose vectors
2. **Field**
   $\vec{E} = kq\hat{r}/r^2$; integrate $\int k\,dq\,\hat{r}/r^2$ for distributions
3. **Flux**
   $\Phi_E = \oint \vec{E} \cdot d\vec{A}$; lines through a surface
4. **Gauss**
   $\oint \vec{E} \cdot d\vec{A} = q_{enc}/\varepsilon_0$; use symmetry



## Check yourself

1. Do charges outside a closed surface contribute net flux through it?
2. What integral gives the field of a continuous charge distribution?
3. When is the flux through a flat surface zero?
4. Where does all excess charge sit on a conductor?



**Answers** **1.** no (zero net) **2.** $\int k\,dq\,\hat{r}/r^2$ **3.** when $\vec{E}$ is parallel to it **4.** on its outer surface