

Every equation on this exam appears on the sheet in both its integral and differential form, so the graded skill is knowing which situation each one can actually solve – Gauss’s law is useless without symmetry, and Ampère’s law without a good loop. This booklet is that decision map. 公式表已给出积分与微分形式, 关键是判断何种对称性下哪个公式可用。

Electric field and Gauss’s law

Superposition first — For discrete charges, add fields as vectors by components: $E = kq/r^2$ each. For a continuous distribution, set up dE from $dq = \lambda dl$, σdA or ρdV and integrate, exploiting symmetry to cancel one component before integrating.

When Gauss’s law is usable — $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$ is always true but only USEFUL for spherical, cylindrical or planar symmetry, where E is constant over a surface you can choose. Without that symmetry it gives one equation and two unknowns.

The three standard results — Sphere: field outside is as if all charge sat at the centre; inside a uniform sphere it grows as r . Infinite line: $E = \lambda/2\pi\epsilon_0 r$. Infinite plane: $E = \sigma/2\epsilon_0$, independent of distance.

Conductors in equilibrium — The field inside is zero, all excess charge sits on the surface, and the surface field is perpendicular with magnitude σ/ϵ_0 . These three facts answer most conceptual parts outright.

Potential

Two directions — $V = -\int \vec{E} \cdot d\vec{l}$ going one way; $\vec{E} = -\nabla V$, in one dimension $E_x = -dV/dx$, coming back. The exam asks for both directions, often in the same question.

Scalar addition — Potential adds as a signed scalar, which makes it far easier than the field – compute V by summing kq/r , then differentiate to get E if needed.

Energy — $U = qV$ for a charge in a potential; for a pair of charges $U = kq_1q_2/r$. Assembling a configuration means summing over distinct pairs, counting each once.

Capacitance and circuits

Capacitance — $C = Q/V$, computed by putting charge $\pm Q$ on the plates, finding E by Gauss, integrating to get V , and dividing. That four-step derivation is a recurring free-response task.

Energy and dielectrics — $U = \frac{1}{2}CV^2 = Q^2/2C$; a dielectric multiplies C by κ . Whether V or Q is held constant determines whether the energy rises or falls – decide that before comparing.

RC circuits — Charging: $q = C\epsilon(1 - e^{-t/RC})$; discharging: $q = q_0e^{-t/RC}$. At $t = 0$ an uncharged capacitor behaves as a wire; after a long time, as a break. Those two limits solve most multiple-choice items without any exponential.

Magnetism and induction

Biot-Savart and Ampère — Biot-Savart $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$ works anywhere but needs an integral; Ampère’s law $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$ is quick only with a symmetric loop – long straight wire, solenoid, toroid.

Standard fields — Straight wire $B = \mu_0 I/2\pi r$; solenoid interior $B = \mu_0 nI$, uniform and independent of radius; outside an ideal solenoid, zero.

Faraday and Lenz — $\epsilon = -d\Phi_B/dt$ with $\Phi_B = \int \vec{B} \cdot d\vec{A}$. The minus sign is Lenz’s law: the induced current opposes the CHANGE. State which way the flux is changing, then the direction follows.

Motional e.m.f. and inductance — A bar of length L moving at v across B gives $\epsilon = BLv$. For an inductor, $\epsilon = -L dI/dt$ and $U = \frac{1}{2}LI^2$; in an LR circuit the current cannot change instantly, so at $t = 0$ an inductor behaves as a break and after a long time as a wire.

How the free-response is actually scored

Setting up an integral correctly – the right dq , the right limits, the right component – earns most of the available points even if you never evaluate it. Write the setup before attempting the algebra.

Justify the choice of law: “the charge distribution has cylindrical symmetry, so I choose a coaxial Gaussian cylinder” is itself a scored statement.

Check limiting behaviour at the end. Far from any finite distribution the field must look like a point charge; that check has caught more sign errors than any other habit.