

*Every equation on this exam appears on the sheet in both its integral and differential form, so the graded skill is knowing which situation each one can actually solve – Gauss’s law is useless without symmetry, and Ampère’s law without a good loop. This booklet is that decision map. 公式表已给出积分与微分形式, 关键是判断何种对称性下哪个公式可用。*

## Electric field and Gauss’s law

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**Superposition first** — For discrete charges, add fields as vectors by components:  $E = kq/r^2$  each. For a continuous distribution, set up  $dE$  from  $dq = \lambda dl$ ,  $\sigma dA$  or  $\rho dV$  and integrate, exploiting symmetry to cancel one component before integrating.

**When Gauss’s law is usable** —  $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$  is always true but only USEFUL for spherical, cylindrical or planar symmetry, where  $E$  is constant over a surface you can choose. Without that symmetry it gives one equation and two unknowns.

**The three standard results** — Sphere: field outside is as if all charge sat at the centre; inside a uniform sphere it grows as  $r$ . Infinite line:  $E = \lambda/2\pi\epsilon_0 r$ . Infinite plane:  $E = \sigma/2\epsilon_0$ , independent of distance.

**Conductors in equilibrium** — The field inside is zero, all excess charge sits on the surface, and the surface field is perpendicular with magnitude  $\sigma/\epsilon_0$ . These three facts answer most conceptual parts outright.

## Potential

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**Two directions** —  $V = -\int \vec{E} \cdot d\vec{l}$  going one way;  $\vec{E} = -\nabla V$ , in one dimension  $E_x = -dV/dx$ , coming back. The exam asks for both directions, often in the same question.

**Scalar addition** — Potential adds as a signed scalar, which makes it far easier than the field – compute  $V$  by summing  $kq/r$ , then differentiate to get  $E$  if needed.

**Energy** —  $U = qV$  for a charge in a potential; for a pair of charges  $U = kq_1q_2/r$ . Assembling a configuration means summing over distinct pairs, counting each once.

## Capacitance and circuits

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**Capacitance** —  $C = Q/V$ , computed by putting charge  $\pm Q$  on the plates, finding  $E$  by Gauss, integrating to get  $V$ , and dividing. That four-step derivation is a recurring free-response task.

**Energy and dielectrics** —  $U = \frac{1}{2}CV^2 = Q^2/2C$ ; a dielectric multiplies  $C$  by  $\kappa$ . Whether  $V$  or  $Q$  is held constant determines whether the energy rises or falls – decide that before comparing.

**RC circuits** — Charging:  $q = C\epsilon(1 - e^{-t/RC})$ ; discharging:  $q = q_0e^{-t/RC}$ . At  $t = 0$  an uncharged capacitor behaves as a wire; after a long time, as a break. Those two limits solve most multiple-choice items without any exponential.

## Magnetism and induction

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**Biot-Savart and Ampère** — Biot-Savart  $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$  works anywhere but needs an integral; Ampère's law  $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$  is quick only with a symmetric loop – long straight wire, solenoid, toroid.

**Standard fields** — Straight wire  $B = \mu_0 I / 2\pi r$ ; solenoid interior  $B = \mu_0 n I$ , uniform and independent of radius; outside an ideal solenoid, zero.

**Faraday and Lenz** —  $\varepsilon = -d\Phi_B/dt$  with  $\Phi_B = \int \vec{B} \cdot d\vec{A}$ . The minus sign is Lenz's law: the induced current opposes the CHANGE. State which way the flux is changing, then the direction follows.

**Motional e.m.f. and inductance** — A bar of length  $L$  moving at  $v$  across  $B$  gives  $\varepsilon = BLv$ . For an inductor,  $\varepsilon = -L dI/dt$  and  $U = \frac{1}{2} LI^2$ ; in an LR circuit the current cannot change instantly, so at  $t = 0$  an inductor behaves as a break and after a long time as a wire.

## How the free-response is actually scored

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Setting up an integral correctly – the right  $dq$ , the right limits, the right component – earns most of the available points even if you never evaluate it. Write the setup before attempting the algebra.

Justify the choice of law: “the charge distribution has cylindrical symmetry, so I choose a coaxial Gaussian cylinder” is itself a scored statement.

Check limiting behaviour at the end. Far from any finite distribution the field must look like a point charge; that check has caught more sign errors than any other habit.