Course Companion

AP Physics C: Electricity and Magnetism

Every topic handout in one volume

全部专题讲义合集

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Electric Charges, Fields, and Gauss's Law

AP Physics C: Electricity and Magnetism

Electric Charge and Electric Force

Peel a strip of sticky tape off a desk and it pulls hairs on your arm towards it from a few centimetres away. Nothing touches them. Somewhere a few trillion electrons have moved from one surface to the other, and that tiny imbalance is enough to lift a hair against the whole Earth's gravity. This unit builds the tools to describe that: the force between charges, the field around them, and Gauss's law, which finds the field in one line when the charge is arranged symmetrically.

Electric charge 电荷 comes in positive and negative; like charges repel, opposites attract. Charge is **quantized** 量子化 – every charge is a whole-number multiple of the **elementary charge** 基本电荷 $e = 1.6 \times 10^{-19}$ C – and obeys **conservation of charge** 电荷守恒: charge is never created or destroyed, only moved. The force between two **point charges** 点电荷 is **Coulomb's law** 库仑定律:

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\,\hat{r},$$

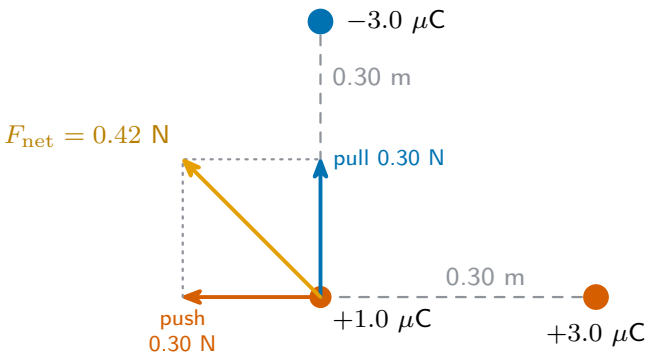
an **inverse-square** 平方反比 force along the line joining them, with $\dfrac{1}{4\pi\varepsilon_0} = k = 9.0 \times 10^9\ \mathrm{N\cdot m^2/C^2}$. For several charges (AP uses four or fewer, unless symmetry helps), add the force *vectors* on the charge you care about – **superposition** 叠加.

Charge is a **fundamental property** 基本属性 of matter, and a **point charge** is a model that ignores the size of a charged object. The constant ε_0 is the **permittivity of free space** 真空介电常数 – a material property of the vacuum. The **permittivity of matter** differs from ε_0, set by how easily the material polarizes, which is why a dielectric between charges weakens the force. Note too that although the electric force is far **stronger** than gravity, gravity dominates at astronomical scales because large bodies are nearly **neutral**.

> **Worked example.** Two $+2.0$ μC charges sit 0.30 m apart: $F = \dfrac{kq_1q_2}{r^2} = \dfrac{9.0\times10^9(2.0\times10^{-6})^2}{(0.30)^2} = 0.40$ N, repulsive.

> **Worked example (vectors).** Charges $+3.0$ μC and -3.0 μC sit at two corners of a right angle, each 0.30 m from a $+1.0$ μC charge at the corner. Each exerts $F = \dfrac{(9.0\times10^9)(3.0\times10^{-6})(1.0\times10^{-6})}{0.09} = 0.30$ N – one a push, one a pull, at right angles to each other. The net force is $F_{\text{net}} = \sqrt{0.30^2 + 0.30^2} = 0.42$ N, pointing between them. Never add magnitudes blindly: components first.

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Add force vectors, not magnitudes: two perpendicular 0.30 N forces give 0.42 N, pointing between them

Worked example (electric versus gravity). In a hydrogen atom the electron and proton are 5.3×10^{-11} m apart. The electric force is $F_E = \dfrac{ke^2}{r^2} = \dfrac{(9.0 \times 10^9)(1.6 \times 10^{-19})^2}{(5.3 \times 10^{-11})^2} = 8.2 \times 10^{-8}$ N. The gravitational force is $F_G = \dfrac{Gm_e m_p}{r^2} = \dfrac{(6.67 \times 10^{-11})(9.11 \times 10^{-31})(1.67 \times 10^{-27})}{(5.3 \times 10^{-11})^2} = 3.6 \times 10^{-47}$ N. The ratio is about 10^{39}, and it does not depend on r, because both forces fall off as $1/r^2$. Electric forces can attract *or* repel; gravity only attracts. A planet carries almost equal amounts of + and − charge, so the electric forces cancel and gravity is left.

Exam skill (hanging spheres). A common lab hangs a small charged sphere of mass m on a thread near a second charged sphere. At equilibrium the thread makes angle θ with the vertical, and the forces balance: $T\cos\theta = mg$ and $T\sin\theta = F_E$, so $F_E = mg\tan\theta$. Measure θ for several separations d and plot F_E against $1/d^2$: Coulomb's law predicts a straight line through the origin with slope kq_1q_2. If the spheres are *conductors*, the charges on them move. Like charges push each other to the far sides, so the centres of charge are further apart than d and the force is *smaller* than kq_1q_2/d^2. Opposite charges pull each other to the near sides, so the force is *larger*.

Electric Charge and the Process of Charging

A **conductor** 导体 lets charge move freely; an **insulator** 绝缘体 holds it in place. Objects gain net charge three ways:

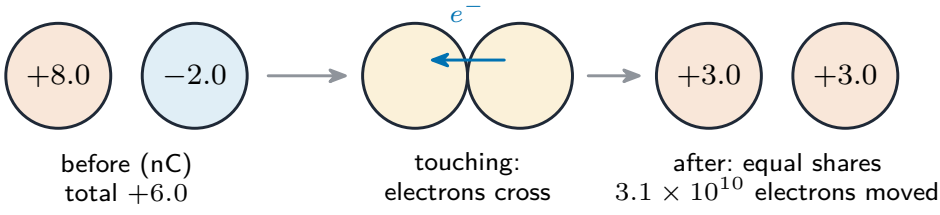
- **Charging by friction** 摩擦起电: rubbing transfers electrons from one surface to the other.
- **Charging by conduction** 传导起电: touching a charged object shares charge of the *same* sign.

- **Charging by induction** 感应起电: a nearby charge pushes the conductor's charge apart; **ground** 接地 the far side, remove the ground wire *first*, and the conductor is left with the *opposite* sign – all without contact.

An insulator cannot pass charge, but it can develop **polarization** 极化: its molecules stretch or turn so one face is slightly positive and the other slightly negative. That is why a charged comb picks up neutral paper scraps. An **electroscope** 验电器 shows net charge by its leaves repelling; in electrostatic equilibrium, excess charge lies on conducting surfaces; it lies entirely on the outer surface if cavities contain no charge.

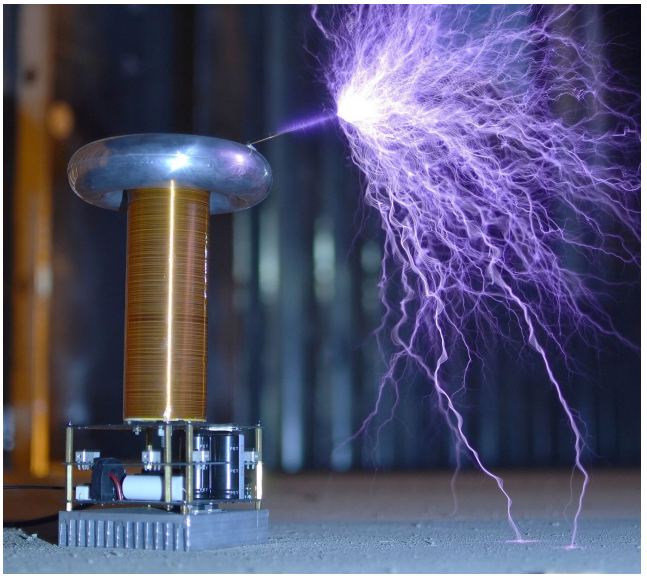
In every method, charge only *moves*; the total never changes. Charging almost always means moving electrons, because protons are locked inside nuclei. An object is positive because it has *lost* electrons, not because it gained protons. **Grounding** connects an object to the Earth, a body so large that it can give or take any amount of charge while staying neutral.

> **Worked example (sharing charge).** Two identical metal spheres carry $+8.0$ nC and -2.0 nC. They touch and are then separated. The total is $+6.0$ nC, and identical spheres share it equally: $+3.0$ nC each. The second sphere gained 5.0 nC of positive charge, which really means it *lost* $5.0 \times 10^{-9}/1.6 \times 10^{-19} = 3.1 \times 10^{10}$ electrons to the first sphere.



Identical spheres share the total equally: +6.0 nC becomes +3.0 nC each, by moving about 3.1 times 10 to the 10 electrons

Electric Fields

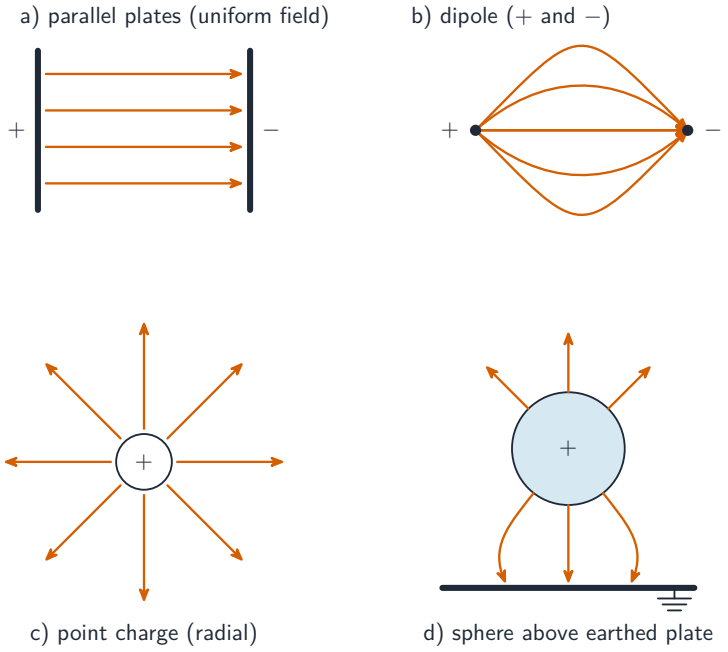


A Tesla coil throws sparks: it raises the voltage so high that the electric field ionizes the surrounding air, and charge leaps across as a visible discharge

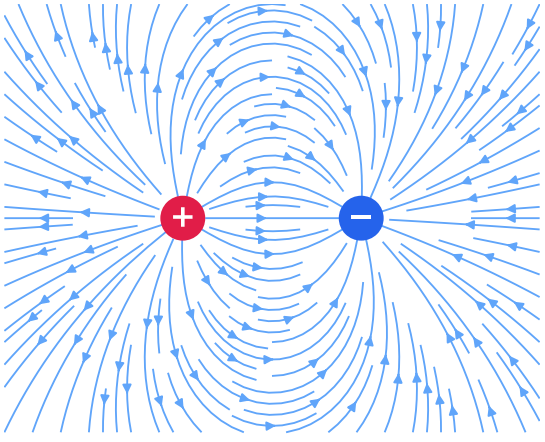
The **electric field** 电场 at a point is the force per unit charge that a small positive **test charge** 检验电荷 would feel there. "Small" matters: a test charge is too weak to push the source charges around, so it measures the field without changing it.

$$\vec{E} = \frac{\vec{F}}{q}, \qquad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,\hat{r} \text{ (point charge).}$$

A charge placed in a field feels $\vec{F} = q\vec{E}$ – positive charges along the field, negative charges against it. Fields from several sources add as vectors (superposition again). **Field lines** 电场线 make the field visible: they point away from positive charge and toward negative, their density shows the strength, and they never cross.



Electric field-line patterns for parallel plates, a dipole, and a point charge



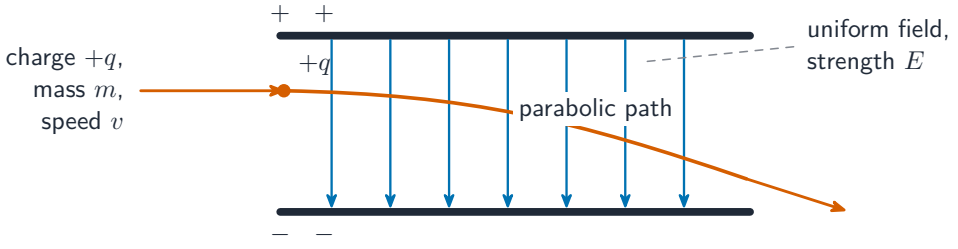
$\vec{p} = q\vec{d}$ · lines leave + enter − · denser ⇔ stronger $|\vec{E}|$

Electric field lines of a dipole point from the positive to the negative charge

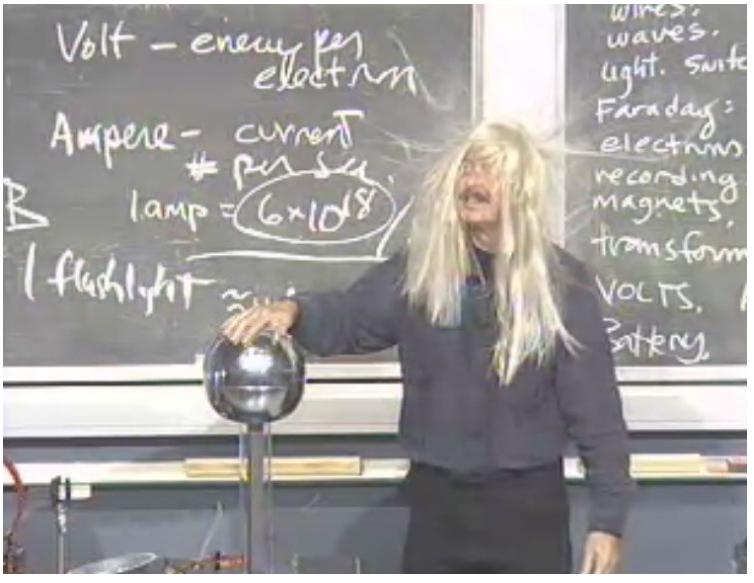
Conductors and insulators in equilibrium. In a conductor in electrostatic equilibrium, the excess charge sits on the surface, the field *within the conducting material* is zero, and the field just outside is perpendicular to the surface; a sideways part would push charge along the surface. In an insulator the charge cannot move away, so it can

stay *inside* the material as well as on the surface, and the field inside can be non-zero. A spherically symmetric charge distribution, seen from outside, has the field of a point charge at its centre.

In a **uniform** 均匀 field a charge feels a constant force, so it accelerates uniformly – launched sideways, it follows a parabola, just like a projectile in gravity.



A charge crossing a uniform field follows a parabolic path



Charge spreads over the dome and onto the hair; like charges repel, so the strands push apart

Electric Fields of Charge Distributions

For a **continuous charge distribution** 连续电荷分布, cut the object into pieces dq and integrate their point-charge fields:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\,\hat{r}.$$

Write dq using the right density: $\lambda\,dl$ with the **linear charge density** 线电荷密度 (a line or ring), $\sigma\,dA$ with the **surface charge density** 面电荷密度, or $\rho\,dV$

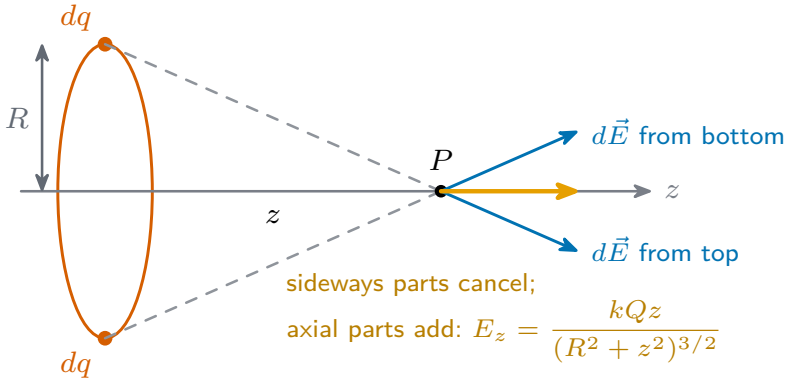
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with the **volume charge density** 体电荷密度. Then use **symmetry** 对称性 to cancel components *before* integrating – that sentence is usually a scored step. AP's calculus cases: a finite line (collinear point or perpendicular bisector), an infinite wire or cylinder, a ring on its axis, and an arc at its centre.

Worked example (ring, on axis). A ring of radius R carries charge Q uniformly. At a point z along the axis, each element is a distance $\sqrt{R^2+z^2}$ away, and by symmetry the sideways components cancel, leaving only the axial part ($\cos\alpha = z/\sqrt{R^2+z^2}$):

$$E_z = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{R^2+z^2}\cdot\frac{z}{\sqrt{R^2+z^2}} = \frac{1}{4\pi\varepsilon_0}\frac{Qz}{(R^2+z^2)^{3/2}}.$$

Check the limits: $E = 0$ at the centre ($z = 0$), and far away ($z \gg R$) it becomes kQ/z^2 – a point charge, as it must.



Symmetry first: opposite elements of the ring cancel sideways, so only the axial components are integrated

Worked example (beyond the end of a rod). A thin rod of length L carries charge Q spread evenly, $\lambda = Q/L$. Find E at a point on the rod's line, a distance a beyond one end. Put the point at the origin, so the rod runs from $x = a$ to $x = a + L$. Every element pushes along the same line, so no components cancel and the magnitudes simply add:

$$E = \frac{1}{4\pi\varepsilon_0}\int_a^{a+L}\frac{\lambda\,dx}{x^2} = \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{a+L}\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a(a+L)}.$$

Far away ($a \gg L$) this becomes kQ/a^2, a point charge again. With two rods, find each rod's field this way and add the two as vectors.

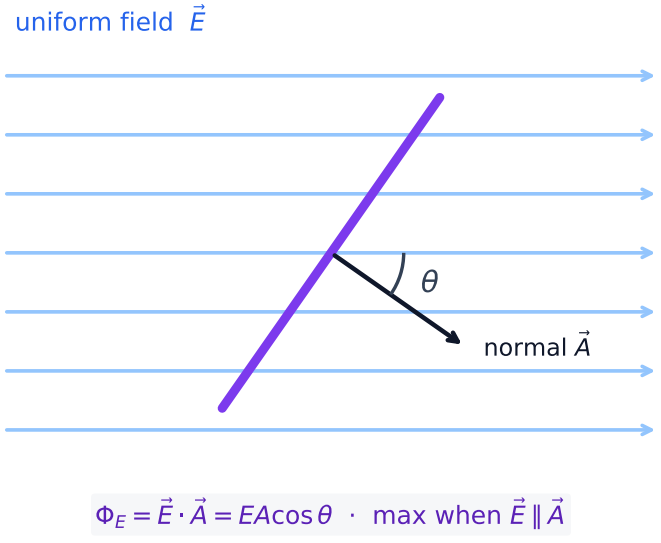
Electric Flux

Electric flux 电通量 measures how much field passes through a surface:

$$\Phi_E = \int \vec{E} \cdot d\vec{A}.$$

For a uniform field through a flat area, $\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta$, where the **area vector** 面积矢量 is perpendicular to the surface and, on a closed surface, points *outward*. Flux is positive where field lines *exit* a closed surface and negative where they *enter* – only the perpendicular component of $\vec{E}$ counts.

> **Worked example (flux through a flat card).** A uniform field of 400 N/C meets a 0.20 m × 0.30 m card. The angle between $\vec{E}$ and the area vector is 60°: $\Phi_E = EA\cos\theta = 400(0.060)(0.50) = 12\ \text{N}\cdot\text{m}^2/\text{C}$. Tilt the card until it lies along the field ($\theta = 90°$) and the flux is zero: no field lines pass through it. In a uniform field, a *closed* box has zero total flux: every line that enters on one face leaves through another.



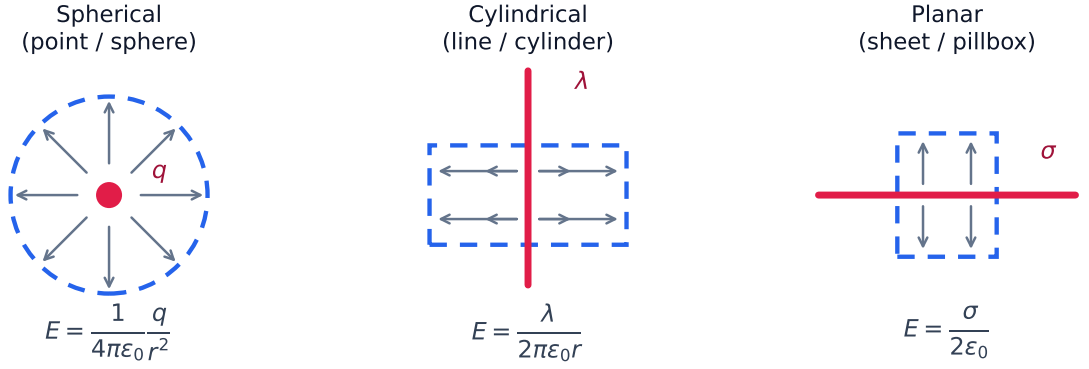
Electric flux through a surface depends on the angle between the field and the surface normal

Gauss's Law

Gauss's law 高斯定律 – the first of Maxwell's equations – relates the flux through any closed surface to the charge inside it:

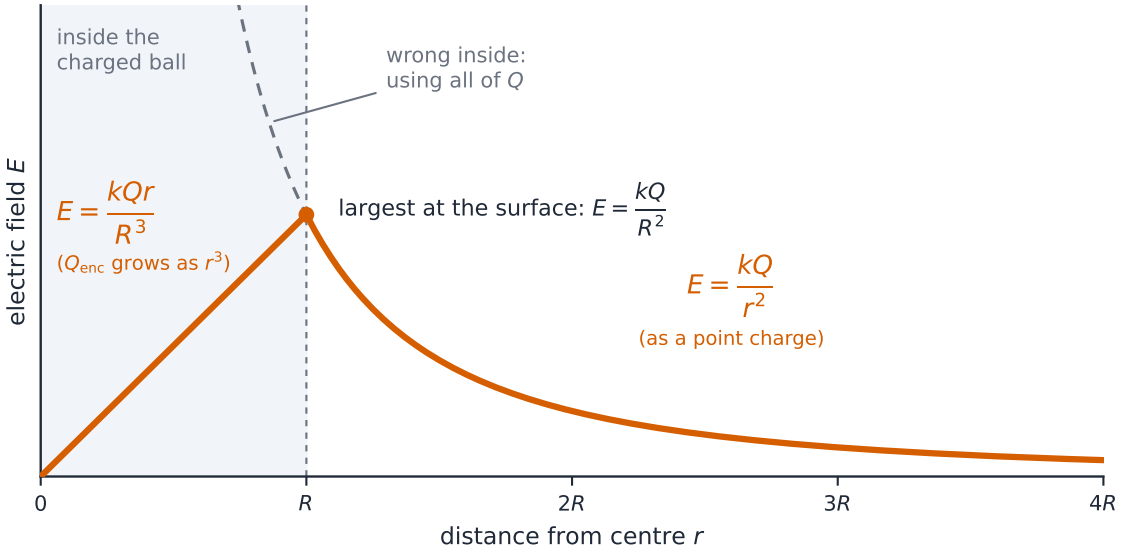
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\varepsilon_0}.$$

It is true for *any* closed surface, but it *solves* for E only when symmetry lets you choose a **Gaussian surface** 高斯面 on which E is constant and perpendicular (or parallel, contributing zero). The flux through a closed surface does not depend on its size or shape, only on the charge inside: a bigger sphere around a point charge has a weaker field over a bigger area, and the two changes cancel exactly. The three symmetries AP tests: **spherical symmetry** 球对称 (concentric sphere), **cylindrical symmetry** 柱对称 (coaxial cylinder), and **planar symmetry** 平面对称 (a straddling "pillbox").



A Gaussian surface is chosen to match the symmetry of the charge

Worked example (sphere). Outside a spherically symmetric charge distribution of total charge Q, a spherical Gaussian surface of radius r gives $E(4\pi r^2) = Q/\varepsilon_0$, so $E = \dfrac{kQ}{r^2}$ – identical to a point charge at the centre. *Inside* a uniformly charged solid sphere of radius R, the surface encloses only $Q_{\text{enc}} = Q\,r^3/R^3$, so $E = \dfrac{kQr}{R^3}$: zero at the centre, growing linearly to the surface.



The field of a uniformly charged insulating sphere grows linearly inside and falls as $1/r^2$ outside

Worked example (charge at the centre of a cube). A +4.0 nC charge sits at the centre of a cube. The total flux is $\Phi_E = \dfrac{q}{\varepsilon_0} = \dfrac{4.0 \times 10^{-9}}{8.85 \times 10^{-12}} = 450\ \text{N}\cdot\text{m}^2/\text{C}$. Gauss's law cannot give E on a cube face, because E is not constant there, but symmetry still splits the flux evenly: each of the six faces gets $75\ \text{N}\cdot\text{m}^2/\text{C}$.

Worked example (non-uniform charge density). A ball of radius R has charge density $\rho = ar$, where a is a constant, so it holds more charge near its surface. Integrate over thin shells of volume $4\pi r'^2\,dr'$: $Q_{\text{enc}} = \int_0^r ar'\,4\pi r'^2\,dr' = \pi a r^4$. Gauss's law gives $E(4\pi r^2) = \dfrac{\pi a r^4}{\varepsilon_0}$, so inside $E = \dfrac{ar^2}{4\varepsilon_0}$. Outside, the total charge is $\pi a R^4$, and $E = \dfrac{aR^4}{4\varepsilon_0 r^2}$. Check: the two answers agree at $r = R$.

Worked example (wire). For an infinite wire with charge per length λ, take a coaxial cylinder of radius r and length L. The ends contribute nothing ($\vec{E} \perp d\vec{A}$), so $E(2\pi r L) = \dfrac{\lambda L}{\varepsilon_0}$ and $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$. The same pillbox method gives an infinite sheet's field, $E = \dfrac{\sigma}{2\varepsilon_0}$, uniform on both sides.

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Worked example (inside a slab). A very large slab of thickness $2h$ has uniform charge density ρ. By symmetry the field points away from the middle plane and has the same size at equal distances on each side. Take a pillbox of face area A with its faces at $+z$ and $-z$ ($z < h$). Both faces carry outward flux EA, and the pillbox encloses $\rho(2zA)$, so $2EA = \dfrac{2\rho zA}{\varepsilon_0}$ and $E = \dfrac{\rho z}{\varepsilon_0}$. The field is zero on the middle plane and grows linearly to $\rho h/\varepsilon_0$ at the surface, and it stays at that value outside.

Exam skill. For a Gauss's-law field derivation: name the surface, state the symmetry argument (why E is constant and perpendicular on it), count Q_{enc}, then solve. State the symmetry argument explicitly. Electrostatic equilibrium requires $E = 0$ within conducting material; Gauss's law alone does not establish this from zero enclosed charge.

Practice

Answer each before reading the answers below.

1. Find the force between charges of $+2.0$ µC and -3.0 µC that are 0.30 m apart. Is it attractive or repulsive?
2. Two charges of $+4.0$ µC are 0.20 m apart. Find the electric field at the midpoint.
3. Use Gauss's law to find the field a distance r from an infinite line of charge with density λ.
4. A solid insulating sphere of radius R carries total charge Q spread uniformly. Find E inside and outside.
5. A closed surface encloses $+2.0$ nC and -5.0 nC. Find the electric flux through it.
6. Find the acceleration of an electron in a uniform field of 1000 N/C.

Answers

1. $F = \dfrac{(9.0 \times 10^9)(2.0 \times 10^{-6})(3.0 \times 10^{-6})}{(0.30)^2} = 0.60$ N, attractive.
2. Zero: the two fields are equal in size and opposite in direction.
3. A cylinder of radius r and length ℓ: $E(2\pi r\ell) = \dfrac{\lambda\ell}{\varepsilon_0}$, so $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$.
4. Inside: $E(4\pi r^2) = \dfrac{Qr^3/R^3}{\varepsilon_0}$, so $E = \dfrac{kQr}{R^3}$. Outside: $E = \dfrac{kQ}{r^2}$.
5. $\Phi = \dfrac{q_{\text{enc}}}{\varepsilon_0} = \dfrac{-3.0 \times 10^{-9}}{8.85 \times 10^{-12}} = -339$ N · m^2/C.
6. $a = \dfrac{eE}{m} = \dfrac{(1.6 \times 10^{-19})(1000)}{9.11 \times 10^{-31}} = 1.8 \times 10^{14}$ m/s^2, opposite to the field.

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Exam tips

- Use **Coulomb's law** $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$ and superpose forces as **vectors** (components, not magnitudes).
- For a continuous charge, integrate $d\vec{E}$ over $dq = \lambda\, dl,\ \sigma\, dA,\ \rho\, dV$; use symmetry to cancel components.
- The field points **away from** positive and **toward** negative charge; draw field lines correctly.
- Distinguish the force on a charge ($\vec{F} = q\vec{E}$) from the field the charge creates.
- Keep the constant $k = \frac{1}{4\pi\varepsilon_0}$ straight and check units.

Common mistakes

- Adding field or force magnitudes from several charges as numbers. Resolve into components and add as vectors.
- Using Gauss's law to find E without symmetry. The law is always true, but it gives E only when E is constant and perpendicular on the chosen surface.
- Counting charge outside the Gaussian surface. Only Q_{enc} enters $\oint \vec{E}\cdot d\vec{A} = Q_{\text{enc}}/\varepsilon_0$.
- Putting the full charge Q in for a point inside a uniformly charged sphere. Inside, $Q_{\text{enc}} = Qr^3/R^3$.
- In charging by induction, removing the charged rod before the ground wire. The induced charge then flows straight back and the conductor ends neutral.
- Saying a positive object "gained protons". It lost electrons; protons do not move.
- Getting the sign of flux wrong. The area vector of a closed surface points outward, so field lines going *in* give negative flux.
- Saying "the flux is zero, so the field is zero". Zero net flux means zero net charge inside; the field on the surface can still be large.

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Electric Potential

AP Physics C: Electricity and Magnetism

Electric Potential Energy

Shuffle across a carpet and touch a doorknob: the spark jumps across a potential difference of several thousand volts, yet it does you no harm. A lightning bolt crosses a hundred million volts and can kill. The difference is not only the voltage but the **energy**, which depends on how much charge moves. This unit separates those ideas: potential energy belongs to a system of charges, potential is energy per unit charge, and energy conservation turns both into speeds. When you push two like charges together, you do **work** 功 against the electric force. That work is stored by the pair as **electric potential energy** 电势能:

$$U_E = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = k\frac{q_1 q_2}{r}.$$

U_E is defined as the work an external force must do to bring the charges slowly from infinitely far away to a distance r apart, with no net change in kinetic energy. The signs carry the physics:

- **Like charges:** $U_E > 0$. An outside push was needed to bring them together. If released, they fly apart, and the stored energy becomes **kinetic energy** 动能.
- **Opposite charges:** $U_E < 0$. Starting from rest at this separation, you must supply $|U_E|$ of work to pull them apart to infinity.

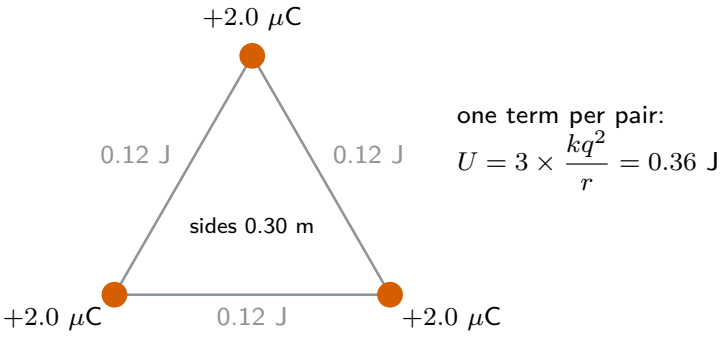
The electric force is a **conservative force** 保守力: the work it does depends only on the start and end points, not the path, and equals $-\Delta U_E$.

For a **system** 系统 of several charges, add the potential energy of every distinct pair:

$$U_{\text{total}} = k\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right).$$

Worked example. Three $+2.0\ \mu$C charges sit at the corners of an equilateral triangle with sides 0.30 m. Each pair stores $U = \dfrac{kq^2}{r} = \dfrac{(9.0\times10^9)(2.0\times10^{-6})^2}{0.30} = 0.12$ J. There are three pairs, so $U_{\text{total}} = 3 \times 0.12$ J $= 0.36$ J. This is the work needed to assemble the triangle from infinity – and the kinetic energy the charges would share if all three were released.

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One potential-energy term per pair: three pairs of 0.12 J store 0.36 J

Worked example (a bound pair). In a hydrogen atom the electron and proton are about 5.3×10^{-11} m apart:

$$U = k\frac{(+e)(-e)}{r} = -\frac{(9.0 \times 10^9)(1.6 \times 10^{-19})^2}{5.3 \times 10^{-11}} = -4.3 \times 10^{-18}\ \text{J}.$$

The negative sign places the potential energy below its zero at infinity; binding depends on the total energy, including kinetic energy. Atomic energies are easier to read in **electron volts** 电子伏特: $1\ \text{eV} = 1.6 \times 10^{-19}$ J, the energy one electron gains crossing 1 V. So $U \approx -27$ eV.

Electric Potential

Electric potential 电势 is electric potential energy per unit charge at a point in space. It is measured in volts (1 V = 1 J/C):

$$V = \frac{U_E}{q}, \qquad V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\ \text{(point charge)}.$$

Potential is a **scalar** 标量: it has a sign but no direction. This makes V far easier to work with than the field $\vec{E}$. For several **point charges** 点电荷, use scalar **superposition** 叠加 – add the potentials with their signs:

$$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}.$$

Worked example. The potential 0.10 m from a +3.0 nC point charge is $V = \frac{kq}{r} = \frac{9.0 \times 10^9(3.0 \times 10^{-9})}{0.10} = 270$ V. Now add a −3.0 nC charge 0.30 m from the same point: it contributes $\frac{9.0 \times 10^9(-3.0 \times 10^{-9})}{0.30} = -90$ V, so the total is $270 - 90 = 180$ V. Just a signed sum – no vector components to resolve.

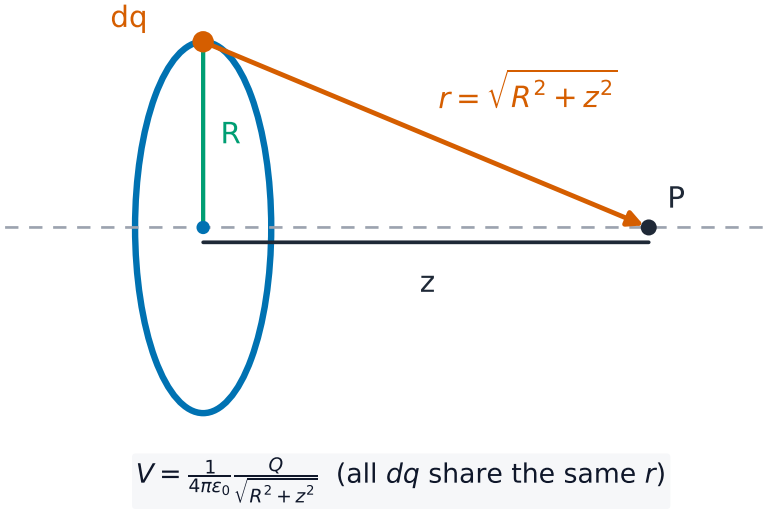
The potential of a point charge is graphed further down this section. Close to a positive charge V is large and positive; it falls as $1/r$ and approaches zero at infinity. Near a negative charge it is large and negative.

■ Continuous charge distributions

For a **continuous charge distribution** 连续电荷分布, cut it into small pieces dq, treat each piece as a point charge, and **integrate** 积分:

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}.$$

This is a scalar integral, so it is usually much easier than the field integral. AP expects you to handle four shapes with calculus: a thin ring (at a point on its axis), an arc (at its centre), a finite line or wire (collinear, or on its perpendicular bisector), and an infinitely long wire or cylinder (at a distance from its axis).



A ring of charge: every element dq is the same distance from an axis point

> **Worked example (ring of charge).** A thin ring of radius R carries total charge Q. For a point P on the axis a distance z from the centre, every element dq sits at the same distance $r = \sqrt{R^2 + z^2}$ from P. The distance is constant, so it comes out of the integral:
>
> $$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{\sqrt{R^2+z^2}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+z^2}}.$$
>
> In a derivation, *say* that r is the same for every element to justify taking it outside the integral. The same idea gives $V = kQ/R$ at the centre of any arc, whatever its angle.

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Worked example (a rod, end-on). A thin rod of length L carries charge Q spread evenly, so $\lambda = Q/L$. Find V at point P on the rod's line, a distance d beyond one end. Put P at the origin, so a piece dx of rod at distance x from P has $dq = \lambda\,dx$. The rod runs from $x = d$ to $x = d + L$:

$$V = \frac{1}{4\pi\varepsilon_0}\int_d^{d+L}\frac{\lambda\,dx}{x} = \frac{\lambda}{4\pi\varepsilon_0}\ln\frac{d+L}{d}.$$

Check the limit: far away ($d \gg L$), $\ln(1+L/d) \approx L/d$, so $V \approx kQ/d$, the point-charge result. A good check on any integral is that it matches a simple case.

Worked example (an infinite wire). For a very long wire with charge λ per metre, the field from Gauss's law is $E = \lambda/(2\pi\varepsilon_0 r)$, pointing away from the wire. Integrating from r_a to r_b:

$$V_b - V_a = -\int_{r_a}^{r_b}\frac{\lambda}{2\pi\varepsilon_0 r}\,dr = -\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_b}{r_a}.$$

For $\lambda = 2.0\times10^{-8}$ C/m, going from 1.0 cm to 4.0 cm away gives $\Delta V = -(2)(9.0\times10^{9})(2.0\times10^{-8})\ln 4 = -500$ V. Notice that for an infinite wire you **cannot** put $V = 0$ at infinity: the logarithm grows without limit. Choose a reference point at a finite distance instead.

■ From potential to field

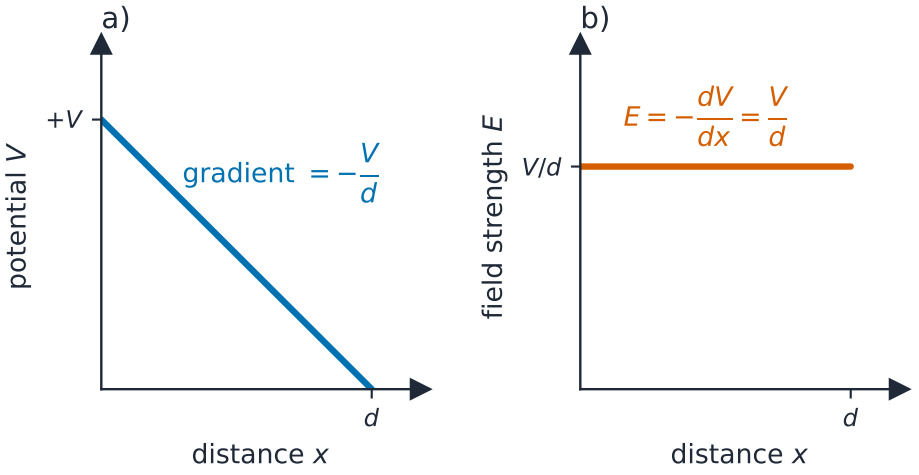
Potential and field contain the same information, linked by a derivative in each direction and by a path integral:

$$E_x = -\frac{dV}{dx}, \qquad \Delta V = V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{r}.$$

The field is the negative **gradient** 梯度 of the potential: $\vec{E}$ points from high potential to low potential, "downhill". Where V changes quickly, the field is strong.

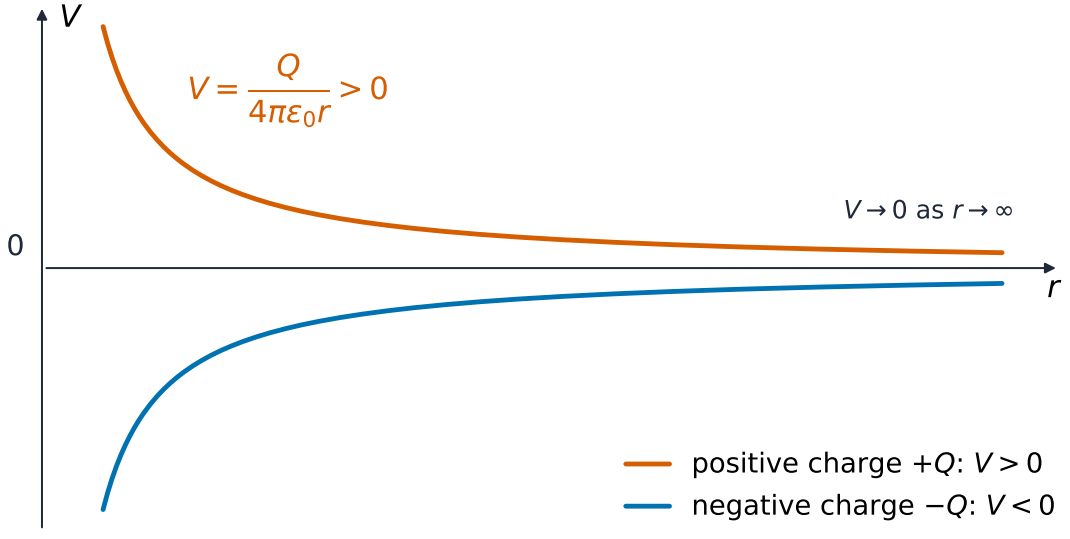
Worked example. Along the x-axis, $V(x) = 3x^2 - 2x$ (volts, with x in metres). Then $E_x = -\dfrac{dV}{dx} = 2 - 6x$ V/m. At $x = 0.50$ m, $E_x = -1.0$ V/m: the field there points in the $-x$ direction.

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In a uniform field the potential falls steadily with distance

The **potential difference** 电势差 between two points is the change in potential energy per unit charge moved between them: $\Delta V = \Delta U_E/q$. A battery makes a potential difference chemically: reactions inside separate positive from negative charge and hold the terminals a fixed ΔV apart.



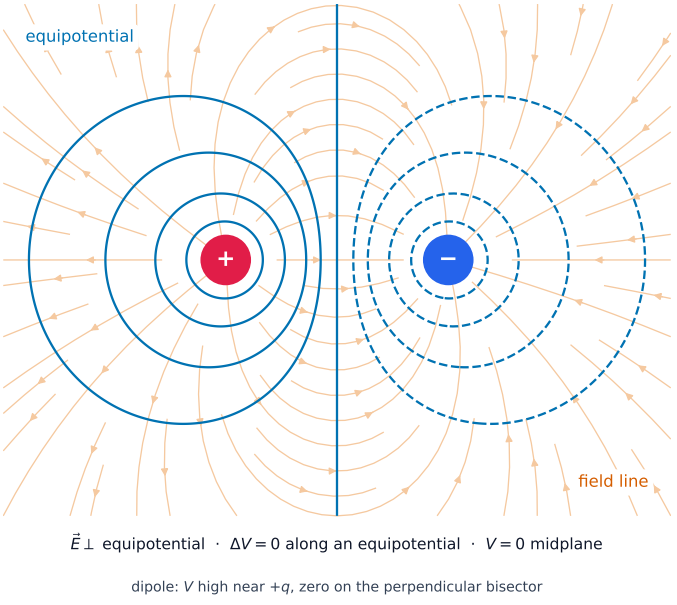
The potential near a point charge varies as 1/r

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■ Equipotential maps

An **equipotential** 等势面 surface contains points that share the same potential; its intersection with a map appears as an **isoline** 等值线. Four rules let you read any map:

- Isolines are **perpendicular** 垂直 to field lines 电场线 everywhere they cross.
- $\vec{E}$ points from high V to low V – never along an isoline. Moving a charge along an isoline takes no work.
- Closely spaced isolines mean a strong field: $E \approx -\Delta V/\Delta x$.
- You can sketch the field map from an isoline map, and the isoline map from a field map.



Equipotential lines around a dipole cross the field lines at right angles

Exam skill. Given a map with isolines every 10 V spaced about 2 cm apart, estimate $E \approx \dfrac{10}{0.02} = 500$ V/m, pointing from the higher-value line to the lower one. The work an external agent does moving a charge q slowly from A to B is $W = q(V_B - V_A)$ – the path taken does not matter.

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High-voltage power lines: electric potential energy is converted and transmitted as current at high voltage

Conservation of Electric Energy

When a charge q moves between two points that differ in potential by ΔV, the potential energy of the charge–field system changes by

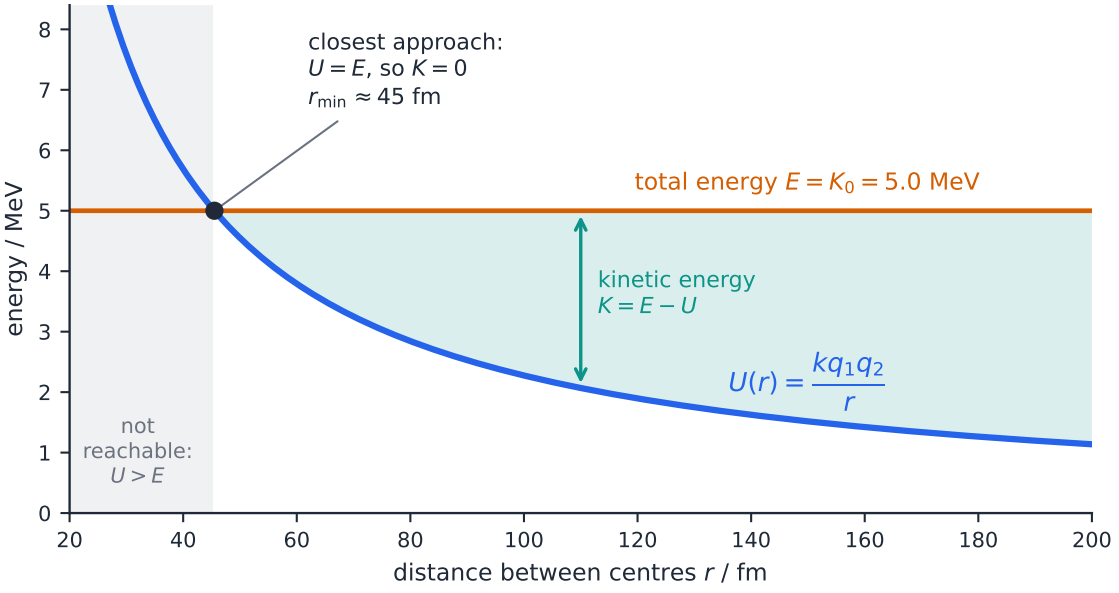
$$\Delta U_E = q\,\Delta V.$$

If only the electric force acts, total energy is conserved, so the kinetic energy changes by the opposite amount:

$$\Delta K = -\Delta U_E = -q\,\Delta V.$$

Watch the two signs together: a positive charge speeds up when it moves to *lower* potential, while a negative charge speeds up when it moves to *higher* potential. Both are just the system trading potential energy for kinetic energy. This is how particle accelerators give charged particles their energy, and it underlies energy analysis in circuits.

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The gap between the total-energy line and $U(r)$ is the kinetic energy; the particle turns back where the two meet

Worked example (closest approach). In Rutherford's experiment an alpha particle ($q = +2e$) with kinetic energy 5.0 MeV heads straight for a gold nucleus ($Q = +79e$), which is so heavy it barely moves. The alpha particle slows as it climbs the potential "hill" and stops where all its kinetic energy has become potential energy:

$$K_0 = \frac{kqQ}{r_{\min}}, \qquad r_{\min} = \frac{(9.0\times10^{9})(2)(79)(1.6\times10^{-19})^{2}}{(5.0\times10^{6})(1.6\times10^{-19})} = 4.5\times10^{-14}\ \text{m}.$$

That is about 45 femtometres, which showed that the positive charge of an atom is packed into a tiny nucleus. The same picture as the potential-energy curves in mechanics applies: the particle can only be where $U \le E$.

Worked example. A proton ($q = 1.6\times10^{-19}$ C, $m = 1.67\times10^{-27}$ kg) starts at rest and is accelerated through a potential drop of 500 V (it moves to lower potential, so $\Delta V = -500$ V and $\Delta K = -q\,\Delta V = +8.0\times10^{-17}$ J). Setting $\Delta K = \tfrac{1}{2}mv^2$:

$$v = \sqrt{\frac{2(8.0\times10^{-17})}{1.67\times10^{-27}}} = 3.1\times10^{5}\ \text{m/s}.$$

Exam skill. Choose energy methods, not kinematics, whenever the field is non-uniform or the path curves: only the end-point potentials matter. A typical FRQ chain is $q\,\Delta V \to \Delta K \to v$, with one line of justification: "the electric force is conservative, so energy is conserved."

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Practice

Answer each before reading the answers below.

1. Find the potential 0.20 m from a 5.0 nC point charge.
2. How much work is needed to bring a +2.0 µC charge from infinity to that point?
3. The potential along the x-axis is $V(x) = 3x^2 - 2x$ (V, m). Find E_x.
4. A proton is accelerated from rest through 500 V. Find its kinetic energy and speed.
5. Explain why no work is done moving a charge along an equipotential surface.
6. Find the potential on the axis of a ring of radius R and charge Q, a distance x from its center.

Answers

1. $V = \dfrac{(9.0 \times 10^9)(5.0 \times 10^{-9})}{0.20} = 225$ V.
2. $W = qV = (2.0 \times 10^{-6})(225) = 4.5 \times 10^{-4}$ J.
3. $E_x = -\dfrac{dV}{dx} = -6x + 2$ V/m.
4. $K = eV = 8.0 \times 10^{-17}$ J; $v = \sqrt{2K/m} = \sqrt{\dfrac{1.6 \times 10^{-16}}{1.67 \times 10^{-27}}} = 3.1 \times 10^5$ m/s.
5. $W = q\Delta V$, and $\Delta V = 0$ along an equipotential; the field is perpendicular to the surface, so it has no component along the motion.
6. Every piece of the ring is the same distance $\sqrt{R^2 + x^2}$ away, so $V = \dfrac{kQ}{\sqrt{R^2 + x^2}}$.

Exam tips

- Relate field and potential by $V = -\int \vec{E} \cdot d\vec{l}$ and $\vec{E} = -\nabla V$ (in 1-D, $E_x = -\frac{dV}{dx}$).
- Potential is a **scalar** – add contributions with signs, no vector components needed.
- Potential energy of a pair is $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$; use energy conservation for a charge's speed.
- Know that no work is done moving along an **equipotential**, which is perpendicular to $\vec{E}$.
- Choose the zero of potential (usually infinity) and state it.

■ Common mistakes

- Treating potential as a vector. Potentials from several charges add as signed numbers.
- Losing the sign in $E_x = -dV/dx$: the field points towards *decreasing* potential.
- Getting the sign wrong for electrons. $\Delta U = q\,\Delta V$ with q negative: an electron gains kinetic energy moving to *higher* potential.

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- Saying that $V = 0$ means $E = 0$ (or the reverse). Midway between equal and opposite charges $V = 0$ but $E \neq 0$.
- Putting $V = 0$ at infinity for an infinitely long wire or sheet. Its potential grows without limit, so choose a finite reference point.
- Counting a pair twice in a system's potential energy. Three charges have three pairs, four charges have six.

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Conductors and Capacitors

AP Physics C: Electricity and Magnetism

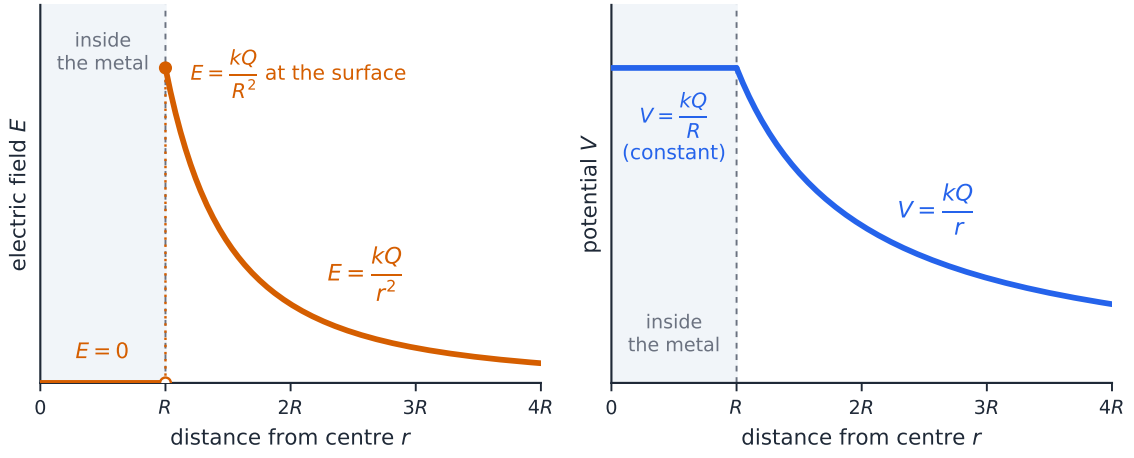
Electrostatics with Conductors

When a person's heart stops beating in a regular rhythm, a defibrillator can restart it. It charges a capacitor for a few seconds, then dumps about 200 joules through the chest in a few milliseconds. A small battery alone cannot supply this brief high-power pulse; a capacitor can, because it stores energy in the electric field between two conductors. This unit starts with how charge behaves on conductors and builds up to capacitors. In an ideal **conductor** 导体, electrons move freely. Put extra charge on one and the charges repel each other until, almost instantly, they settle into **electrostatic equilibrium** 静电平衡. In that state the conductor has four properties you must be able to state and use:

- The electric field **within the conducting material** is zero. If it were not, free electrons would still be moving.
- All excess charge sits **on the surface**. (Negative net charge = extra electrons on the surface; positive = a shortage of electrons there.)
- The field just outside is **perpendicular** 垂直 to the surface, with magnitude $E = |\sigma|/\varepsilon_0$. Any parallel component would push charge sideways along the surface.
- The whole conductor is one **equipotential surface** 等势面: every point, inside and on the surface, is at the same potential.

Charge density is largest where the surface curves sharply – at points and edges – so the outside field is strongest there. In an external field a conductor **polarizes**: charge shifts on its surface so that the interior stays field-free and the body stays an equipotential. Surrounding a region with a closed conducting shell keeps outside fields out entirely – **electrostatic shielding** 静电屏蔽, the idea behind the **Faraday cage** 法拉第笼. This is why the metal body of a car protects the people inside from lightning. Conducting enclosures can also attenuate radio waves, so a phone may lose signal inside a lift.

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Inside the metal $E = 0$ and V is constant; outside, both match a point charge at the centre. E jumps at the surface, but V does not

The graphs show all four properties at once for a sphere of radius R carrying $+Q$. Outside, Gauss's law gives the point-charge field $E = kQ/r^2$, so $V = kQ/r$. Inside, $E = 0$, so V cannot change: it keeps its surface value kQ/R all the way to the centre. A common exam error is to draw V falling to zero inside.

> **Worked example (a sphere inside a shell).** A solid metal sphere of radius a carries $+Q$. Around it, with the same centre, is a neutral metal shell with inner radius b and outer radius c. Use Gauss's law in each region:

- $r < a$ (inside the sphere's metal): $E = 0$.
- $a < r < b$ (the gap): the Gaussian sphere encloses $+Q$, so $E = kQ/r^2$.
- $b < r < c$ (inside the shell's metal): $E = 0$, so the Gaussian sphere must enclose zero net charge. The shell's *inner* surface therefore carries $-Q$. The shell is neutral, so its *outer* surface carries $+Q$.
- $r > c$: the total enclosed is $+Q$, so $E = kQ/r^2$ again.

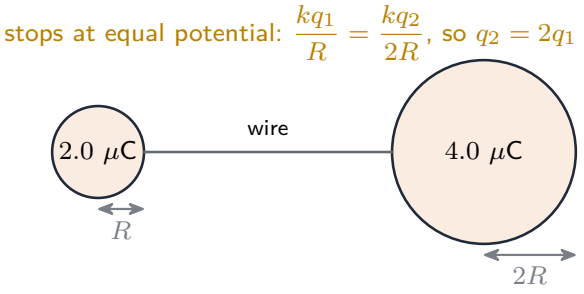
If the shell is grounded, the $+Q$ on its outer surface flows away to Earth, and the field outside becomes zero. The $-Q$ on the inner surface stays, held by the attraction of the sphere.

Redistribution of Charge Between Conductors

When two conductors touch (or are wired together), charge flows between them until both surfaces reach the **same potential** – that is the stopping condition, not "equal charges". A larger sphere holds more charge at the same potential ($V = kQ/R$), so it takes the larger share.

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Worked example. A small sphere of radius R carrying $+6.0\ \mu\text{C}$ is connected by a thin wire to a distant, initially neutral sphere of radius $2R$, then the wire is removed. Equal potentials require $\dfrac{kq_1}{R} = \dfrac{kq_2}{2R}$, so $q_2 = 2q_1$. With $q_1 + q_2 = 6.0\ \mu\text{C}$: $q_1 = 2.0\ \mu\text{C}$ and $q_2 = 4.0\ \mu\text{C}$.



Charge flows until the potentials are equal, not the charges: the sphere twice as large holds twice the charge

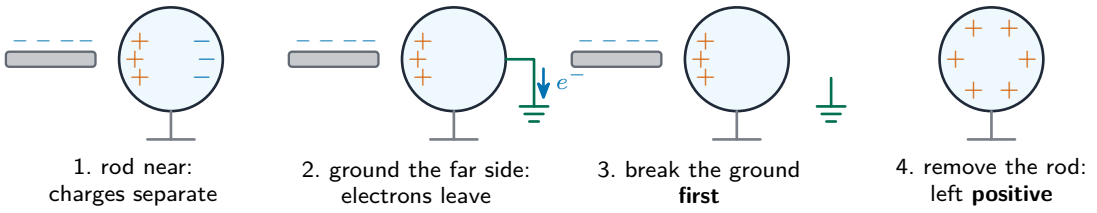
Ground 接地 is an idealized reference at zero potential that can absorb or supply any amount of charge. Grounding a conductor while an external charge is nearby leaves the conductor with a net **induced charge** 感应电荷: the external field pushes charge of one sign to ground, and cutting the ground wire before removing the external charge traps the rest.

Worked example (charging by induction). A neutral metal sphere stands on an insulating base.

1. Bring a negatively charged rod near, without touching. The sphere's electrons are pushed to the far side; the near side is left positive. The sphere is still neutral overall.
2. Touch the far side to ground. The repelled electrons flow away to the ground.
3. Remove the ground connection **first**, while the rod is still near.
4. Now take the rod away. The sphere is left with a net **positive** charge, which spreads evenly over its surface.

The induced charge always has the opposite sign to the nearby charge. If you removed the rod before breaking the ground connection, electrons would flow back and the sphere would end up neutral.

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Charging by induction: break the ground connection before removing the rod, and the sphere keeps the opposite sign to the rod

Capacitors

A **capacitor** 电容器 stores charge on two conductors separated by a gap: $+Q$ on one plate, $-Q$ on the other. Its **capacitance** 电容 relates the stored charge to the potential difference between the plates:

$$C = \frac{Q}{\Delta V}.$$

Capacitance depends only on geometry and the material in the gap – never on Q or ΔV themselves.

Worked derivation (parallel plates). For a **parallel-plate capacitor** 平行板电容器 with plate area A and small gap d: Gauss's law plus **superposition** 叠加 gives a **uniform** 均匀 field between the plates, $E = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q}{\varepsilon_0 A}$ (each plate alone contributes $\sigma/2\varepsilon_0$; between the plates the two add, outside they cancel). A uniform field means $\Delta V = Ed = \dfrac{Qd}{\varepsilon_0 A}$, so

$$C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}.$$

This $E \to \Delta V \to C$ chain is a standard FRQ derivation – learn it as three steps, and quote each one.

Worked derivation (concentric spheres). An inner sphere of radius a carries $+Q$ and an outer shell of radius b carries $-Q$. Between them, a spherical Gaussian surface encloses only $+Q$, so $E = kQ/r^2$. Then

$$\Delta V = \int_a^b \frac{kQ}{r^2}\,dr = kQ\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{kQ(b-a)}{ab}, \qquad C = \frac{Q}{\Delta V} = \frac{ab}{k(b-a)} = 4\pi\varepsilon_0\frac{ab}{b-a}.$$

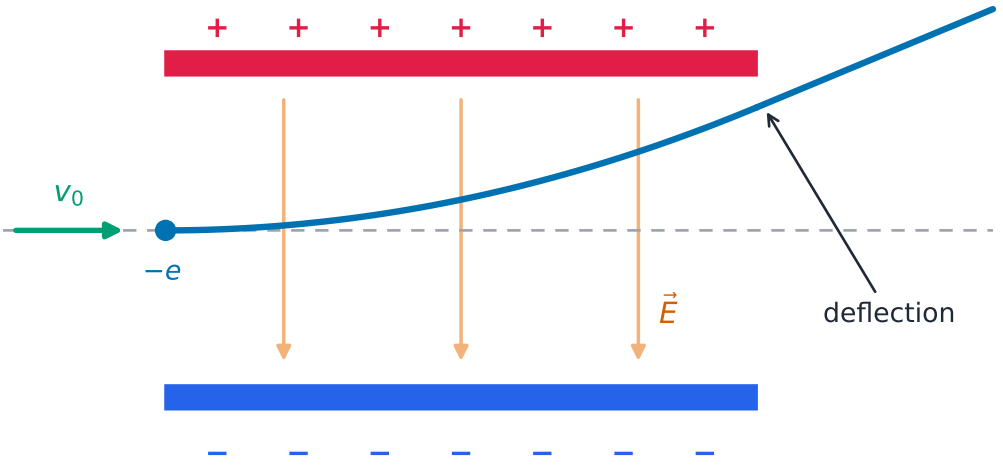
When the gap $b-a$ is small compared with a, the area is about $4\pi a^2$ and this becomes $\varepsilon_0 A/d$ again, which is a good check. The same method handles the other two shapes

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AP expects: concentric spheres ($C = 4\pi\varepsilon_0 \dfrac{ab}{b-a}$) and a coaxial cylinder of length L, where $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$ gives $\Delta V = \dfrac{\lambda}{2\pi\varepsilon_0}\ln\dfrac{b}{a}$ and so $C = \dfrac{2\pi\varepsilon_0 L}{\ln(b/a)}$.

Worked example. Plates of area $A = 0.020\ \text{m}^2$ and gap $d = 1.0\ \text{mm}$: $C = \dfrac{8.85\times10^{-12}(0.020)}{1.0\times10^{-3}} = 1.8\times10^{-10}\ \text{F}$. Charged to 100 V it holds $Q = CV = 1.8\times10^{-8}\ \text{C}$.

Because the field between the plates is uniform, a charged particle there feels a constant force, so it moves with constant acceleration – exactly like **projectile motion** 抛体运动 in gravity: constant speed across, uniform acceleration towards a plate.

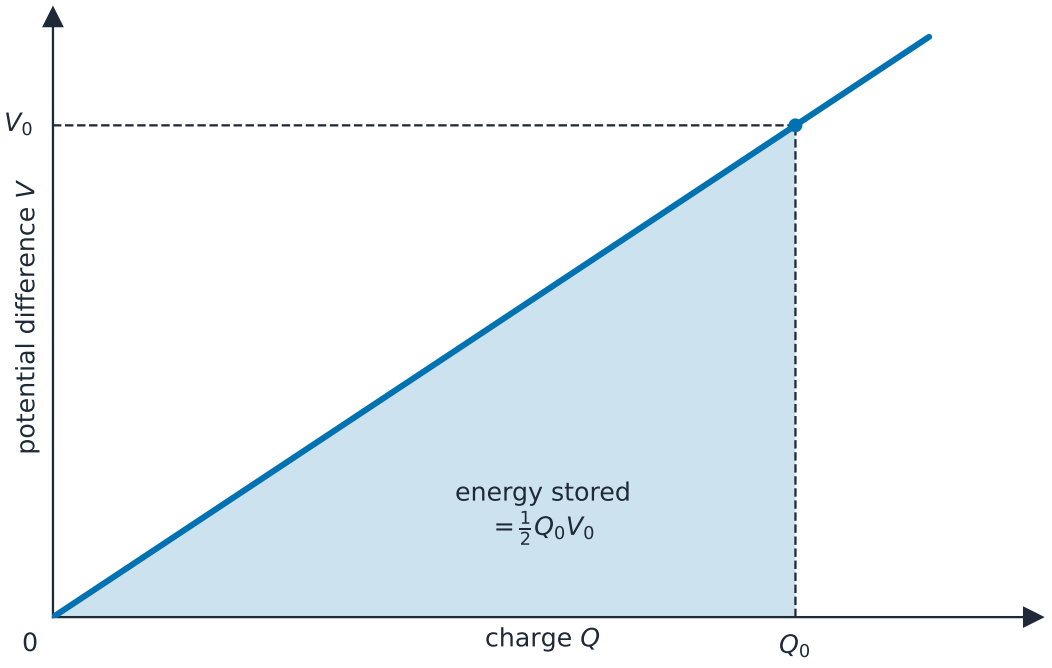


A charged particle between the plates follows a parabola, like a projectile

Worked example. An electron enters midway between plates at $v_0 = 2.0\times10^7\ \text{m/s}$, parallel to them. The field is $E = 1.0\times10^3\ \text{N/C}$ and the plates are 4.0 cm long. Acceleration: $a = \dfrac{eE}{m} = \dfrac{(1.6\times10^{-19})(1.0\times10^3)}{9.11\times10^{-31}} = 1.8\times10^{14}\ \text{m/s}^2$. Time between the plates: $t = \dfrac{0.040}{2.0\times10^7} = 2.0\times10^{-9}\ \text{s}$. Deflection: $y = \tfrac12 at^2 = \tfrac12(1.8\times10^{14})(2.0\times10^{-9})^2 \approx 3.5\times10^{-4}\ \text{m}$ – about 0.35 mm towards the positive plate.

Storing charge takes **work** 功: an external force must move each bit of charge against the field of the charge already there. The total work ends up as stored potential energy,

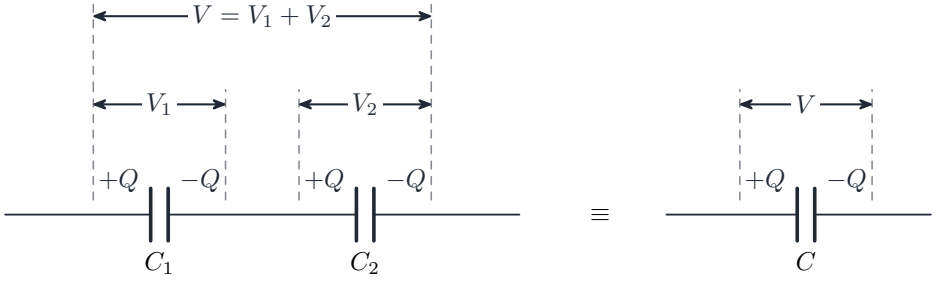
$$U_C = \tfrac12 Q\,\Delta V = \tfrac12 C(\Delta V)^2 = \frac{Q^2}{2C}.$$



The energy stored in a capacitor is the area under its charge-voltage line

The factor $\frac{1}{2}$ is the area of the triangle under the Q–ΔV line: the first charge moved across costs almost nothing, the last costs the full ΔV.

Capacitors combine oppositely to resistors: in **parallel** 并联 the capacitances add ($C_{\text{eq}} = C_1 + C_2$, same ΔV, charges add), while in **series** 串联 the reciprocals add ($\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$, same Q, voltages add).



Capacitors in series carry the same charge, and their p.d.s add

> **Worked example.** A 2 μF and a 4 μF capacitor in series: $\frac{1}{C} = \frac{1}{2} + \frac{1}{4}$, so $C = \frac{4}{3}$ μF. The same pair in parallel: 6 μF.

Exam skill (measuring a capacitance). For a parallel-plate capacitor $C = \kappa\varepsilon_0 A/d$, so a graph of C against $1/d$ is a straight line through the origin with slope $\kappa\varepsilon_0 A$. To find the dielectric constant of paper, put sheets of paper between two metal plates,

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measure C for different numbers of sheets, and plot C against $1/d$: then $\kappa = \dfrac{\text{slope}}{\varepsilon_0 A}$. Larger plates make a steeper line.

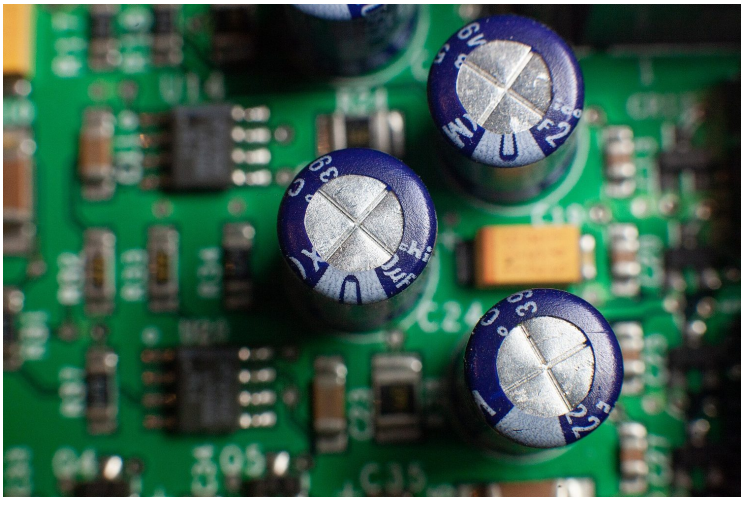
Worked example (the defibrillator). A defibrillator must store 200 J at 2000 V. The capacitance needed is

$$C = \frac{2U}{(\Delta V)^2} = \frac{2(200)}{(2000)^2} = 1.0 \times 10^{-4}\ \text{F} = 100\ \mu\text{F},$$

holding $Q = C\Delta V = 0.20$ C. If the pulse lasts 5 ms, the average power is $200/0.005 = 40{,}000$ W, far more than a battery of the same size could supply.



Assorted capacitors: devices that store charge and energy in an electric field between conductors



Electrolytic capacitors on a circuit board: capacitance stores energy as $U = \tfrac{1}{2}CV^2$

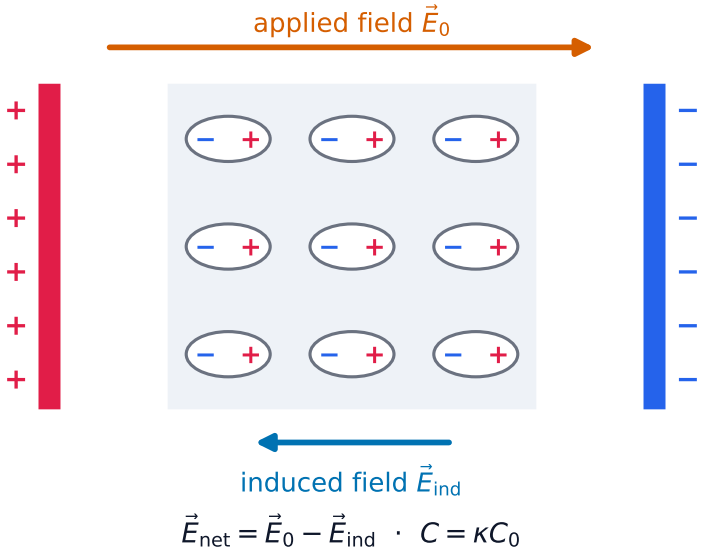
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Dielectrics

A **dielectric** 电介质 is an insulating material. Its charges cannot travel, but in an external field each molecule stretches or turns slightly – the material becomes **polarized** 极化. The lined-up molecules create their own small field *opposite* to the applied one, so, for fixed free charge on the plates of a fully filled parallel-plate capacitor, the net field drops:

$$E = \frac{E_0}{\kappa}, \qquad \kappa = \frac{\varepsilon}{\varepsilon_0} \quad (\kappa > 1),$$

where κ is the **dielectric constant** 介电常数, the ratio of the material's permittivity to the **permittivity of free space** 真空介电常数.



A polarized dielectric creates an internal field opposing the applied field

Filling a capacitor with dielectric multiplies its capacitance:

$$C = \kappa\, C_0 \qquad \left(\text{parallel plates: } C = \frac{\kappa \varepsilon_0 A}{d}\right).$$

What happens next depends on what stays fixed – a favourite exam trap:

	battery stays connected (ΔV fixed)	**battery removed first (Q fixed)**
charge Q	rises to κQ_0	unchanged
voltage ΔV	unchanged	drops to $\Delta V_0/\kappa$
field E	unchanged	drops to E_0/κ
energy U	rises to κU_0	drops to U_0/κ

Worked example. A 100 pF capacitor is charged to 12 V, disconnected, then filled with a $\kappa = 3$ dielectric. Q is trapped, so ΔV falls to 4 V and the stored energy falls to one-third – the missing energy went into pulling the dielectric in. Reconnected to the 12 V battery instead, the capacitor would hold three times the charge and three times the energy.

Worked example (a half-filled gap). A slab of dielectric κ fills the **lower half** of the gap d of a parallel-plate capacitor of area A. The field passes through the slab and then the air, one after the other, so treat them as two capacitors in **series**, each with gap $d/2$:

$$C_1 = \frac{\kappa\varepsilon_0 A}{d/2}, \qquad C_2 = \frac{\varepsilon_0 A}{d/2}, \qquad C = \frac{C_1 C_2}{C_1 + C_2} = \frac{2\kappa}{\kappa+1}\cdot\frac{\varepsilon_0 A}{d}.$$

For $\kappa = 3$, $C = 1.5\,C_0$, less than the $3C_0$ of a full slab. If instead the slab filled the **left half** of the plate area across the whole gap, the two parts would share the same ΔV: they are in **parallel**, and $C = \frac{\kappa+1}{2}C_0 = 2C_0$.

Exam skill. Always begin dielectric questions by writing down which quantity is held fixed (Q or ΔV), then let $C = \kappa C_0$ drive everything else through $Q = C\Delta V$ and $U = \frac{1}{2}C(\Delta V)^2$.

Worked example (a dielectric that changes across the gap). A parallel-plate capacitor with area A and gap d is filled with a material whose dielectric constant rises steadily from 1 at one plate to 2 at the other: $\kappa(x) = 1 + x/d$. Cut the gap into thin slices of thickness dx. Each slice is a capacitor $\dfrac{\kappa\varepsilon_0 A}{dx}$, and the slices are in series, so their reciprocals add:

$$\frac{1}{C} = \int_0^d \frac{dx}{\kappa(x)\,\varepsilon_0 A} = \frac{1}{\varepsilon_0 A}\int_0^d \frac{dx}{1 + x/d} = \frac{d\ln 2}{\varepsilon_0 A}, \qquad C = \frac{\varepsilon_0 A}{d\ln 2} \approx 1.44\,\frac{\varepsilon_0 A}{d}.$$

That lies between the empty value ($\kappa = 1$) and the value for $\kappa = 2$ throughout, as it must.

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Practice

Answer each before reading the answers below.

1. State the electric field inside a conductor in electrostatic equilibrium, and where any excess charge sits.
2. Two identical conducting spheres carry $+6.0$ µC and -2.0 µC. They touch and are separated. Find the charge on each.
3. A parallel-plate capacitor has $A = 0.010$ m^2 and $d = 1.0$ mm. Find C, and the charge and energy at 12 V.
4. While connected to a battery, a capacitor is filled with a dielectric of $\kappa = 3$. What happens to C, V and Q?
5. Find the equivalent capacitance of 2.0 µF and 3.0 µF in series and in parallel.
6. An isolated charged capacitor's plates are pulled farther apart. What happens to Q, C, V and the stored energy?

Answers

1. $E = 0$ within the conducting material; excess charge sits on its surfaces (entirely on the outer surface if cavities contain no charge).
2. Total $+4.0$ µC is shared equally: $+2.0$ µC each.
3. $C = \dfrac{\varepsilon_0 A}{d} = 8.85 \times 10^{-11}$ F; $Q = CV = 1.1 \times 10^{-9}$ C; $U = \tfrac{1}{2}CV^2 = 6.4 \times 10^{-9}$ J.
4. C triples; V stays equal to the battery voltage; $Q = CV$ triples.
5. Series: $\left(\tfrac{1}{2} + \tfrac{1}{3}\right)^{-1} = 1.2$ µF. Parallel: 5.0 µF.
6. Q stays constant; C decreases; $V = Q/C$ increases; $U = Q^2/2C$ increases, from the work done pulling the plates apart.

Exam tips

- In electrostatic equilibrium a conductor has $\vec{E} = 0$ **inside** and all excess charge on the **surface**.
- Apply **Gauss's law** $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$ with a symmetry-matched Gaussian surface (sphere, cylinder, pillbox).
- The whole conductor is one **equipotential**, and the surface field is perpendicular to it.
- For a capacitor use $C = \frac{Q}{V}$, energy $U = \frac{1}{2}CV^2$, and how a **dielectric** raises C.
- Pick the Gaussian surface so $\vec{E}$ is normal and constant in magnitude on flux-carrying parts, and tangent on zero-flux parts.

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■ Common mistakes

- Not stating what stays fixed when a dielectric goes in. Isolated: Q is fixed and V falls. Connected to a battery: V is fixed and Q rises.
- Using the resistor rules for capacitors. Capacitors in parallel add; in series the reciprocals add.
- Saying the field inside a conductor in equilibrium is small. It is exactly zero, and all excess charge sits on the surface.
- Using $U = QV$ for a charged capacitor. The stored energy is $\frac{1}{2}QV = \frac{1}{2}CV^2$.
- Drawing the potential inside a charged conductor as zero. It is constant and equal to the surface value.
- Saying charge spreads so that touching spheres hold equal charges. They reach equal **potentials**; the larger sphere holds more charge.

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Electric Circuits

AP Physics C: Electricity and Magnetism

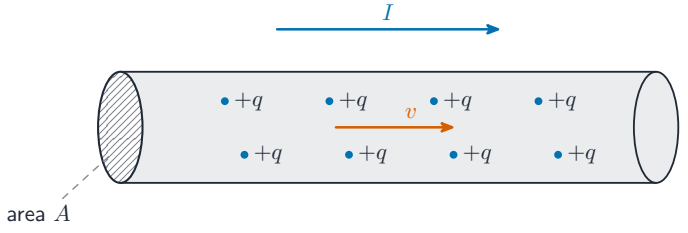
Electric Current

Sit in a car at night with the headlights on and turn the key. As the starter motor turns, the headlights dim for a moment. Nothing is wrong with the lights. The starter draws a current of more than 100 A, and the battery's own internal resistance uses up part of its emf, so less potential difference is left for everything else. This unit gives you the tools to predict effects like that: current, resistance, power, Kirchhoff's two rules, and circuits in which a capacitor charges and discharges over time.

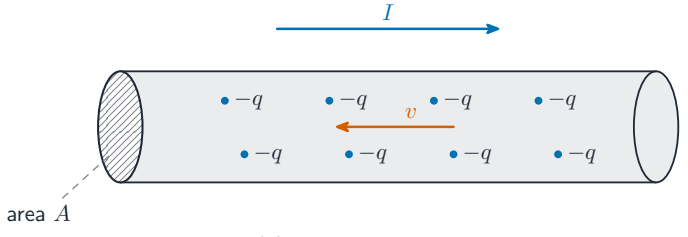
Electric current 电流 is the rate at which charge passes a cross-section of wire, $I = \dfrac{dq}{dt}$, measured in **amperes** 安培. Conventional current points the way *positive* charge would move. Microscopically, a current is a slow drift of many carriers:

$$I = nqv_dA,$$

with n the number of carriers per volume, q the charge each carries, v_d the **drift velocity** 漂移速度, and A the **cross-sectional area** 横截面积.



(a) positive carriers drift with I



(b) electrons drift against I

Charge carriers drift slowly through a conductor to make a current

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Worked example. A copper wire with $A = 1.0 \times 10^{-6}$ m^2 and $n = 8.5 \times 10^{28}$ m^{-3} carries 1.7 A. Then $v_d = \dfrac{I}{nqA} = \dfrac{1.7}{(8.5 \times 10^{28})(1.6 \times 10^{-19})(1.0 \times 10^{-6})} \approx 1.3 \times 10^{-4}$ m/s – the carriers drift slower than a snail, even though the signal travels near light speed.

In metal wires the carriers are actually electrons, which drift *against* the conventional current. Circuit diagrams still use conventional current. And "zero current" does not mean the electrons are still: they move fast in random directions, but with no *net* drift, so no net charge crosses the wire.

Current density 电流密度 is charge flow per unit area, $\vec{J} = nq\vec{v}_d$, linked to the field driving it by $\vec{E} = \rho\vec{J}$. In general $I = \int \vec{J}\cdot d\vec{A}$; if $J(r)$ varies across the wire, integrate it over the cross-section to get the total current. One care point: current has a direction along its wire, but it is a **scalar** 标量 – currents do not add as vectors, and there are no "components of current".

Worked example (non-uniform current density). In a wire of radius a, the current density is $J = J_0\, r/a$: zero on the axis, largest at the surface. Cut the cross-section into thin rings of area $dA = 2\pi r\, dr$: $I = \int_0^a J_0 \dfrac{r}{a}\, 2\pi r\, dr = \dfrac{2\pi J_0}{a}\cdot\dfrac{a^3}{3} = \dfrac{2}{3}\pi a^2 J_0$. That is two-thirds of what a uniform J_0 would give.



An oscilloscope: voltage against time reveals how current and charge evolve in a circuit

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Electric Circuits

A circuit is a set of closed loops built from wires, batteries, resistors, lightbulbs, capacitors, inductors, switches, and meters; a sustained direct current needs a *closed* conducting path. One element can belong to several loops at once – that is what makes multi-loop problems interesting. Every analysis starts by reading the **circuit diagram** 电路图: trace each loop and identify which elements share the same current (**series** 串联) and which share the same potential difference (**parallel** 并联).

Three words describe a path. In a **closed circuit** charge can flow all the way round. In an **open circuit** there is a gap, such as an open switch, so no steady current flows across the gap. In a **short circuit** a wire with no resistance joins two points, so charge flows between them with no potential difference. A diagonal arrow drawn across a symbol means the element is variable, such as a variable resistor.

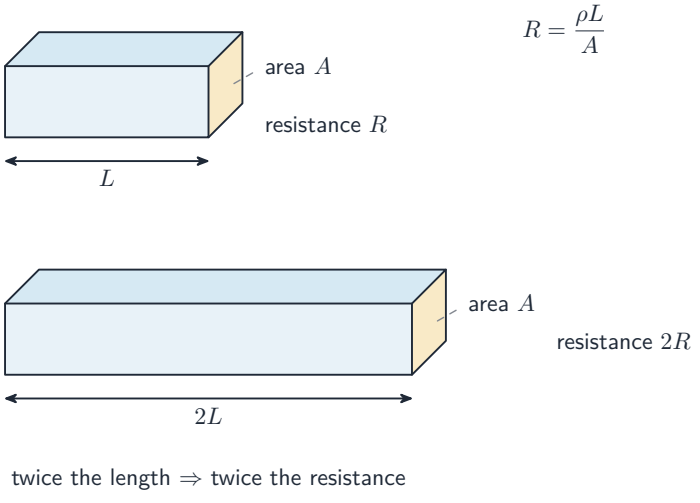
Resistance, Resistivity, and Ohm's Law

Resistance 电阻 measures how strongly an object opposes charge flow. It grows with the material's **resistivity** 电阻率 and the conductor's length, and shrinks with its area:

$$R = \frac{\rho\,\ell}{A}.$$

Ohm's law 欧姆定律 relates the current through an element to the potential difference across it:

$$I = \frac{\Delta V}{R}.$$



A longer conductor has more resistance; a wider one has less

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Worked example. Stretch a wire to double its length: the volume is fixed, so the area halves, and $R = \rho\ell/A$ becomes $\rho(2\ell)/(A/2) = 4R$ – four times the resistance. An element is **ohmic** 欧姆性 if R stays constant (a straight line through the origin on an I–ΔV graph); a lightbulb filament, which heats up, is not. For an ohmic element, the slope of an I-against-ΔV graph is $1/R$.

Resistivity is a property of the material. For most metals it *increases* with temperature: hotter atoms vibrate more and get in the way of the drifting electrons. Electrical energy turned into thermal energy in a resistor warms it and its surroundings. If the resistivity changes along a wire, add up thin slices: $R = \displaystyle\int \frac{\rho(\ell)\,d\ell}{A}$.

Worked example (resistivity that changes along a rod). A rod of length L and area A has $\rho(x) = \rho_0(1 + x/L)$. Then $R = \dfrac{1}{A}\displaystyle\int_0^L \rho_0\Big(1 + \frac{x}{L}\Big)dx = \frac{\rho_0}{A}\Big(L + \frac{L}{2}\Big) = \dfrac{3\rho_0 L}{2A}$, between the two values for uniform ρ_0 and uniform $2\rho_0$, as it should be.

Electric Power

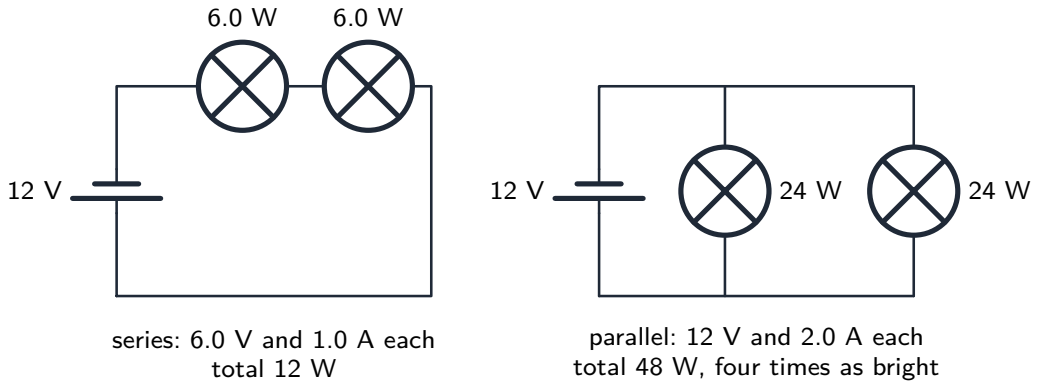
A charge q falling through a potential difference ΔV gives up energy $q\Delta V$, so the rate of energy transfer in an element is $P = I\,\Delta V$. For a resistor, this becomes

$$P = I\,\Delta V = I^2R = \frac{(\Delta V)^2}{R}.$$

In a resistor all of it becomes heat. Use the form whose quantities you actually know – and use power to rank lightbulb brightness: brighter = more power, not necessarily more resistance. In series, the *larger* resistance is brighter ($P = I^2R$, same I); in parallel, the *smaller* one is ($P = \Delta V^2/R$, same ΔV).

Worked example (bulb brightness). Two identical bulbs, each 6.0 Ω, are joined to a 12 V ideal battery. In series, each gets 6.0 V and 1.0 A, so each uses 6.0 W. In parallel, each gets the full 12 V and 2.0 A, so each uses 24 W: four times the electrical power, and the battery supplies 48 W in all, so it also runs down four times as fast as the series pair (whose total is 12 W). Unscrew one bulb: in series the other goes out; in parallel the other is unchanged.

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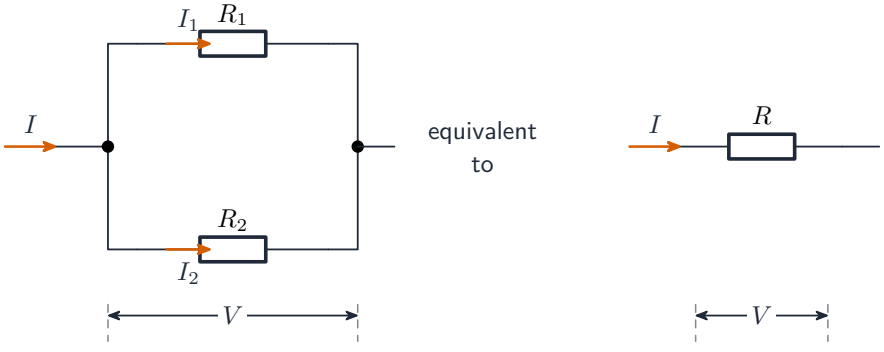


The same two bulbs: 6.0 W each in series, 24 W each in parallel, so the parallel pair is brighter and drains the battery four times as fast

Compound Direct Current Circuits

Reduce resistor networks to an **equivalent resistance** 等效电阻: **series** resistances add ($R_{\text{eq}} = R_1 + R_2 + \cdots$), while **parallel** resistances add as reciprocals ($\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$ – always less than the smallest branch). Collapse the network step by step to find the battery current, then expand back out to find each element's current and voltage.

> **Worked example.** A 12 V battery drives a 4.0 Ω and a 2.0 Ω resistor in series: $R_{\text{eq}} = 6.0\ \Omega$, $I = 2.0$ A, the voltages split 8.0 V and 4.0 V, and the 4.0 Ω resistor dissipates $P = I^2R = 16$ W.



Resistors in parallel combine to a smaller equivalent resistance

Real batteries are not ideal. Model a battery as an **ideal battery** 理想电池 of emf ε in series with its own **internal resistance** 内阻 r. When current flows, some emf is used up inside, so the **terminal voltage** 端电压 – what a voltmeter across the battery actually reads – drops:

$$\Delta V_{\text{terminal}} = \varepsilon - Ir.$$

Worked example. A battery with $\varepsilon = 12$ V and $r = 0.50\ \Omega$ supplies 2.0 A: the terminals sit at $\Delta V = 12 - 2.0(0.50) = 11$ V. With no current, a voltmeter reads the full 12 V.

Worked example (parallel branch). A 6.0 Ω and a 3.0 Ω resistor in parallel sit in series with a 4.0 Ω resistor across an ideal 12 V battery. The parallel pair is $\dfrac{6.0 \times 3.0}{6.0 + 3.0} = 2.0\ \Omega$, so $R_{\text{eq}} = 6.0\ \Omega$ and $I = 2.0$ A. The 4.0 Ω resistor takes 8.0 V, leaving 4.0 V across the pair: 0.67 A in the 6.0 Ω and 1.33 A in the 3.0 Ω branch. Check: $0.67 + 1.33 = 2.0$ A.

Wires are treated as having no resistance because theirs is tiny next to the other elements. That fails when nothing else in the loop has resistance: a battery shorted by a bare wire drives a huge current, limited only by its internal resistance and the wire.

Meters: an **ammeter** 电流表 goes *in series* at the point whose current you want (ideal ammeter: zero resistance); a **voltmeter** 电压表 goes *in parallel* across the element (ideal voltmeter: infinite resistance). Non-ideal meters disturb the circuit they measure – a real ammeter adds series resistance, a real voltmeter steals current.

Exam skill (making a straight line). Free-response lab questions often give a table and ask you to graph two quantities that give a straight line, then read a value from its slope or intercept. To find a battery's ε and r, connect it to a variable resistor R and measure V across R. Since $I = \dfrac{\varepsilon}{R + r}$ and $V = IR$, $V = \dfrac{\varepsilon R}{R + r}$. Take reciprocals:

$$\frac{1}{V} = \frac{1}{\varepsilon} + \frac{r}{\varepsilon} \cdot \frac{1}{R}.$$

Plot $1/V$ against $1/R$: the intercept is $1/\varepsilon$ and the slope is r/ε. If the best-fit line has intercept $0.111\ \text{V}^{-1}$ and slope $0.0556\ \Omega/\text{V}$, then $\varepsilon = 1/0.111 = 9.0$ V and $r = 0.0556 \times 9.0 = 0.50\ \Omega$. Use the line, not two data points. The same idea finds a resistivity: plot R against length ℓ for wires of one thickness, and the slope is ρ/A.

Kirchhoff's Loop Rule

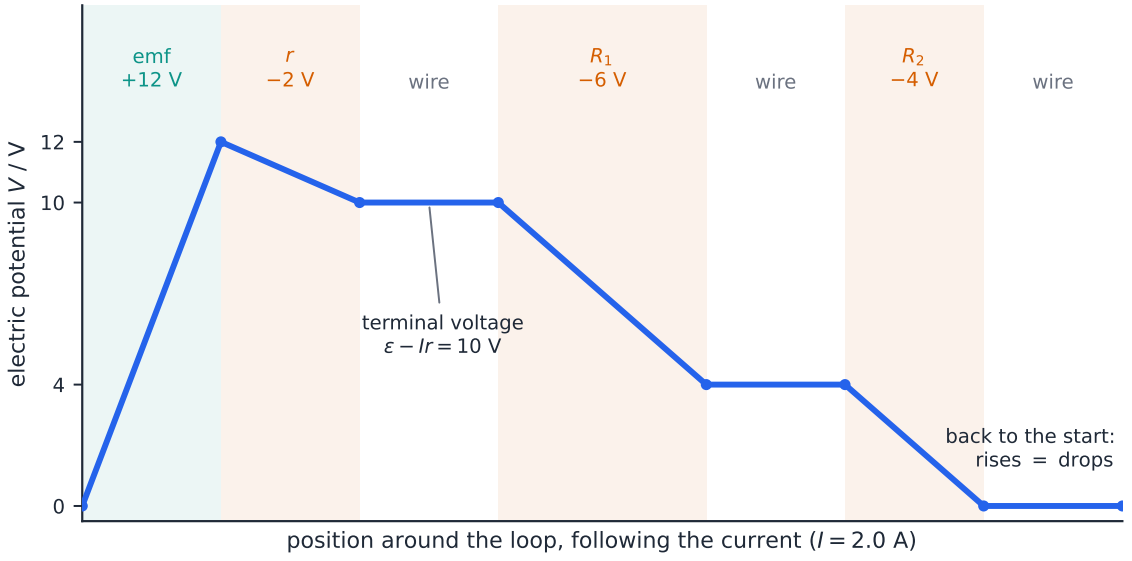
Charges moving through potential differences exchange energy ($\Delta U_E = q\Delta V$), and energy must balance around any closed path. That is **Kirchhoff's loop rule** 基尔霍夫回路定则:

$$\sum \Delta V = 0 \text{ around any closed loop.}$$

Sign discipline wins these problems: crossing a battery from − to + is $+\varepsilon$; crossing a resistor *with* the assumed current is $-IR$ (against it, $+IR$). Write one equation per

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independent loop.



Electric potential around a single loop: the emf raises it, every resistor lowers it, and the loop ends where it started

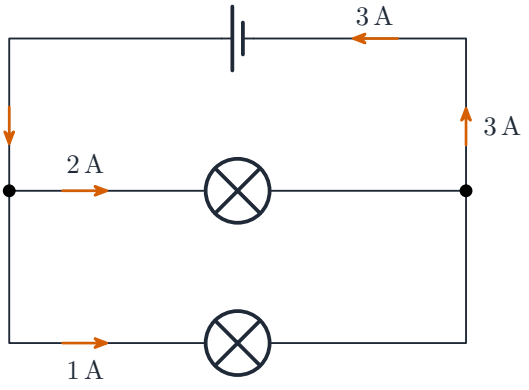
A graph of potential against position, as above, shows the loop rule at a glance. Here a 12 V battery with $r = 1.0\ \Omega$ drives $R_1 = 3.0\ \Omega$ and $R_2 = 2.0\ \Omega$, so $I = 12/6.0 = 2.0$ A. Ideal wires are flat lines: no resistance, no drop. The rise of 12 V equals the drops $2 + 6 + 4 = 12$ V.

Kirchhoff's Junction Rule

Kirchhoff's junction rule 基尔霍夫节点定则 is conservation of charge at a **junction** 节点:

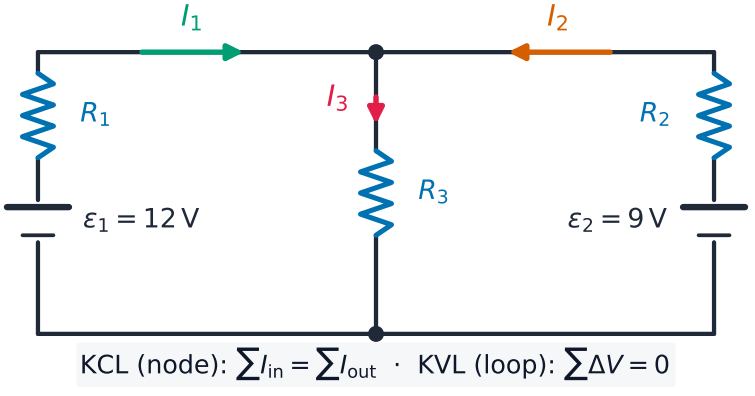
$$\sum I_{\text{in}} = \sum I_{\text{out}}.$$

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Current divides at a junction: what flows in equals what flows out

Together the two rules solve any multi-loop circuit: assign a current to each branch, write junction equations, then loop equations, and solve. A negative answer just means that current flows opposite to your assumed direction.



Two loop equations and one junction equation solve this two-battery circuit

Worked example. In the circuit above, $\varepsilon_1 = 12$ V with $R_1 = 1.0\ \Omega$ on the left, $\varepsilon_2 = 9.0$ V with $R_2 = 1.0\ \Omega$ on the right, and a shared middle resistor $R_3 = 2.0\ \Omega$ carrying $I_3 = I_1 + I_2$ (junction rule). The two loop equations are

$$12 = I_1 + 2(I_1 + I_2) = 3I_1 + 2I_2, \qquad 9 = I_2 + 2(I_1 + I_2) = 2I_1 + 3I_2.$$

Solving: $I_1 = 3.6$ A, $I_2 = 0.60$ A, so $I_3 = 4.2$ A through the middle. Check with the second loop: $2(3.6) + 3(0.60) = 9.0$ ✓.

Resistor-Capacitor (RC) Circuits

Capacitor networks reduce like resistors but with the rules swapped: parallel capacitances add ($C_{\text{eq}} = C_1 + C_2$), series add as reciprocals – and initially uncharged capacitors in series carry equal charge magnitudes if the intermediate conductors remain isolated and neutral, by conservation of charge. Use the **equivalent capacitance** 等效电容 to analyse the network, then expand back.

In an **RC circuit** RC 电路, Kirchhoff's loop rule gives the differential equation

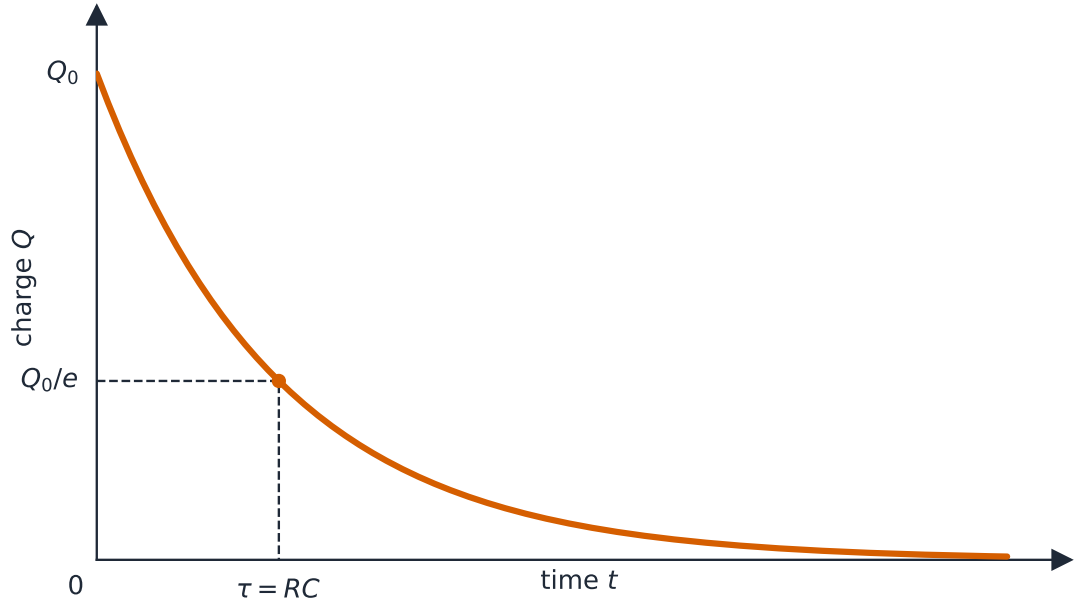
$$\varepsilon = R\frac{dq}{dt} + \frac{q}{C},$$

whose solutions are exponentials with **time constant** 时间常数 $\tau = RC$:

$$q(t) = Q\big(1 - e^{-t/RC}\big) \text{ (charging)}, \qquad q(t) = Q\,e^{-t/RC} \text{ (discharging)}, \qquad i(t) = \frac{\varepsilon}{R}e^{-t/RC} \text{ (charging)}.$$

For discharge, with $i = dq/dt$, $i(t) = -Qe^{-t/RC}/(RC)$; it flows opposite to the charging direction.

The current magnitude is largest at the first instant and decays – it never "waits" for the capacitor. Learn the two limits: at $t = 0$ an uncharged capacitor acts like a plain wire (maximum current); after a long time it is fully charged, no current flows in its branch, and it acts like a break. In a DC **steady state** 稳态, cover the capacitor branch with your finger, solve the resistor circuit, then read the capacitor's voltage from the element it sits across.



The charge on a capacitor decays exponentially as it discharges

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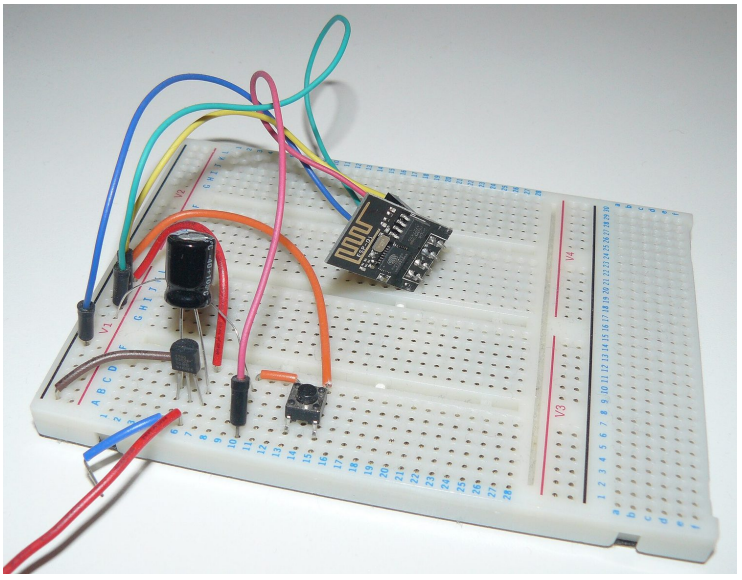
Worked example. With $R = 5.0\ \text{k}\Omega$, $C = 200\ \mu\text{F}$, and a 10 V battery: $\tau = RC = 1.0$ s; initial current $\dfrac{\varepsilon}{R} = 2.0$ mA; after one time constant the charge is $q = CV(1 - e^{-1}) \approx 1.3 \times 10^{-3}$ C, about 63% of full charge, and the current has fallen to 37% of its initial value.

Exam skill (RC data). To find RC from a discharge, record ΔV_C at regular times and plot $\ln \Delta V_C$ against t. Since $\ln \Delta V_C = \ln V_0 - \dfrac{t}{RC}$, the graph is a straight line of slope $-1/RC$. A capacitor that takes longer than expected to reach half its final voltage has a larger RC than you assumed. Charging and discharging to 50% take the same time, $RC \ln 2$.

Where the exponential comes from. For discharging, there is no battery: $0 = R\dfrac{dq}{dt} + \dfrac{q}{C}$. Separate the variables: $\dfrac{dq}{q} = -\dfrac{dt}{RC}$. Integrate from Q at $t = 0$: $\ln \dfrac{q}{Q} = -\dfrac{t}{RC}$, so $q = Qe^{-t/RC}$. A free-response question may ask for exactly these steps.

Worked example (steady state with two branches). A 12 V ideal battery drives $R_1 = 2.0\ \Omega$ in series with $R_2 = 4.0\ \Omega$. A 50 μF capacitor, in series with $R_3 = 10\ \Omega$, is connected across R_2. *Just after* the switch closes, the empty capacitor acts as a wire, so R_3 is in parallel with R_2: $\dfrac{4.0 \times 10}{14} = 2.86\ \Omega$, total 4.86 Ω, battery current 2.47 A. *After a long time*, no current flows in the capacitor branch: $I = 12/6.0 = 2.0$ A, the capacitor has the same p.d. as R_2, $\Delta V = 8.0$ V, and it stores $U = \frac{1}{2}CV^2 = \frac{1}{2}(50 \times 10^{-6})(8.0)^2 = 1.6$ mJ. Now *open* the battery switch. The capacitor discharges through R_3 and R_2 in series, with $\tau = (14)(50 \times 10^{-6}) = 0.70$ ms. Its initial current is $8.0/14 = 0.57$ A, and the current in R_2 drops at once from 2.0 A to 0.57 A. It keeps its direction, because the capacitor's positive plate is joined to the high side of R_2, and then it decays to zero. All 1.6 mJ stored ends up as thermal energy in the two resistors, shared in the ratio of their resistances, since they carry the same current.

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A real circuit on a breadboard: a resistor and capacitor together set how fast the voltage rises and falls

Practice

Answer each before reading the answers below.

1. Find the resistance of a copper wire 10 m long with cross-section 1.0×10^{-6} m^2 ($\rho = 1.7 \times 10^{-8}\ \Omega \cdot$ m).
2. A 12 V battery drives a 4.0 Ω resistor in series with a parallel pair of 6.0 Ω and 3.0 Ω. Find the total current and the current in each parallel branch.
3. Find the power dissipated in the 4.0 Ω resistor of question 2.
4. A 10 kΩ resistor charges a 100 µF capacitor from a battery. Find the time constant and the fraction of the final voltage reached after one time constant.
5. In an RC circuit, how does an uncharged capacitor behave just after the switch closes, and a long time later?
6. Which conservation law underlies Kirchhoff's junction rule, and which underlies the loop rule?

Answers

1. $R = \rho L/A = 0.17\ \Omega$.
2. Parallel pair $= 2.0\ \Omega$; total $6.0\ \Omega$; $I = 2.0$ A. Voltage across the pair $= 4.0$ V: 0.67 A in 6.0 Ω, 1.33 A in 3.0 Ω.
3. $P = I^2R = (2.0)^2(4.0) = 16$ W.
4. $\tau = RC = 1.0$ s; $V_C = \varepsilon(1 - e^{-1}) = 63\%$ of the final value.
5. Just after: like a wire (zero voltage across it). Long after: like a break (no current through its branch).

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6. Junction rule: conservation of charge. Loop rule: conservation of energy.

Exam tips

- Relate current to charge flow $I = \frac{dQ}{dt}$ and use $J = \sigma E$, $R = \frac{\rho L}{A}$ for resistance.
- Apply **Kirchhoff's laws** (junction: charge; loop: energy) with consistent sign conventions.
- Analyse **RC circuits** with calculus: charging/discharging give exponentials with time constant $\tau = RC$.
- Combine resistors (series add, parallel reciprocal) and track power $P = IV = I^2R$.
- At $t = 0$ an initially uncharged capacitor acts like a wire; after a long time ($t \to \infty$) it acts like an open branch.

Common mistakes

- Choosing one direction for every current and then forgetting the signs. A negative answer just means the current flows the other way; keep it.
- Getting the loop-rule sign of a resistor wrong. Crossing a resistor in the direction of the current is a *drop* of IR.
- Saying a fully charged capacitor still carries current. In the steady state it acts as a break in the circuit.
- Using RC as the half-life. After one time constant the current has fallen to 37%; halving takes $RC \ln 2$.
- Ignoring the internal resistance when the question gives one: terminal voltage $= \varepsilon - Ir$.
- Saying the current is "used up" in a resistor. The current is the same on both sides of a series resistor; it is *energy* that is transferred.
- Saying that a larger resistance always makes a bulb brighter. That is true in series only; in parallel the smaller resistance is brighter.
- In a straight-line graph question, calculating with two raw data points. Use points read from your best-fit line.
- Adding currents as vectors at a junction. Current is a scalar: only "in" and "out" matter, not the angles of the wires.

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Magnetic Fields and Electromagnetism

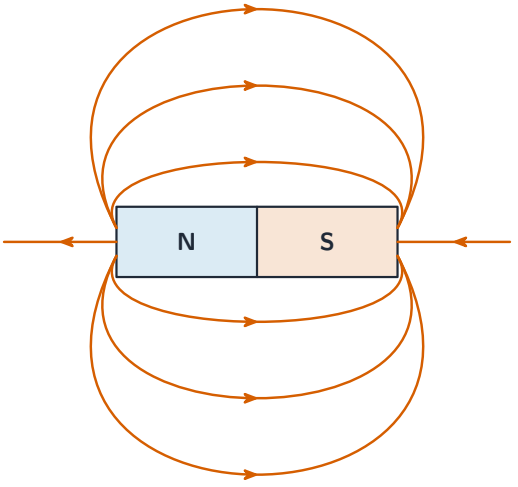
AP Physics C: Electricity and Magnetism

Magnetic Fields

A hospital MRI scanner holds a field of 1.5 to 3 T, about 50 000 times Earth's field at the surface. No permanent magnet is involved: the field comes from a long coil of superconducting wire carrying a few hundred amperes, which never switches off. A loose steel chair in the room can be pulled in fast enough to injure someone. This unit explains where such fields come from (moving charges and currents), how they push on moving charges and wires, and how to calculate them with the Biot–Savart law and Ampère's law.

A **magnetic field** 磁场 $\vec{B}$ is a **vector field** 矢量场 that surrounds magnets, moving charges, and currents; it determines the magnetic force on any moving charge placed in it. **Magnetic field lines** 磁感线 have no starting or ending points; for a bar magnet they form loops: they leave the north pole, return to the south pole, and continue through the magnet. Denser lines mean a stronger field.

field lines run from **N** to **S** outside the magnet



Field lines run from N to S outside a bar magnet

The absence of magnetic sources and sinks is expressed by Gauss's law for magnetism, the second of **Maxwell's equations** 麦克斯韦方程组:

$$\oint \vec{B} \cdot d\vec{A} = 0.$$

The net magnetic flux through *any* closed surface is zero – as many field lines leave as enter. That is exactly the statement that isolated **magnetic monopoles** 磁单极子 do not exist. A bar magnet is a **magnetic dipole** 磁偶极子 (a north–south pair). Magnetism in materials arises from electrons' intrinsic magnetic moments and orbital motion; moving charges and currents also create magnetic fields. Break a bar magnet in half and you get two smaller dipoles, never a lone pole. Like poles repel, opposite poles attract, and a free dipole – a **compass** 指南针 – rotates to line up with the local field. Earth's own field is approximately a dipole, which is why a compass works. Dipole fields weaken with distance. Like an electric field, $\vec{B}$ can be drawn as a **vector field map**: an arrow at each point shows the field's direction and, by its length, its size.

How a material responds to an external field depends on how its internal dipoles behave:

- **Ferromagnetic** 铁磁性 (iron, nickel, cobalt): an external field aligns whole **magnetic domains** 磁畴, and the alignment survives after the field is removed – a permanent magnet.
- **Paramagnetic** 顺磁性 (aluminum, titanium): dipoles align weakly with the field but relax when it is removed.
- **Diamagnetic** 抗磁性 (all materials): the electronic structure produces a usually weak alignment *opposite* to the field.

The strength of a material's response is its **magnetic permeability** 磁导率. Free space has the constant value μ_0 (the vacuum permeability); the permeability of matter differs from μ_0 and is not even constant – it varies with temperature, orientation, and field strength.



An aurora: charged particles from the Sun are steered by Earth's magnetic field toward the poles

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Magnetism and Moving Charges

A moving charge does two things: it *creates* a magnetic field, and it *feels* a force in an external one. The field it creates at a point is **perpendicular** 垂直 to both its velocity and the position vector from charge to point (right-hand rule again), and is largest where those two are perpendicular – zero directly ahead of or behind the motion. Its size grows with the charge's speed and falls with distance; for a slow point charge, $B = \dfrac{\mu_0}{4\pi}\dfrac{|q|v\sin\theta}{r^2}$, the same shape as the Biot–Savart law below.

The force on a charge moving in a field $\vec{B}$ is the cross product

$$\vec{F}_B = q\,\vec{v}\times\vec{B}, \qquad F = |q|vB\sin\theta,$$

perpendicular to both $\vec{v}$ and $\vec{B}$ – point the fingers of your right hand along $\vec{v}$, curl toward $\vec{B}$, and the thumb gives the force on a *positive* charge (reverse it for negative). Because $\vec{F}_B \perp \vec{v}$, the magnetic force does no work on the charge: it changes direction, never speed.

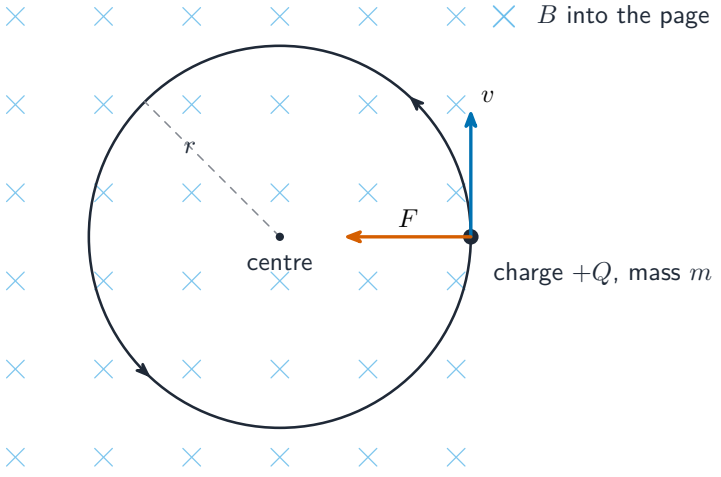
A charge moving perpendicular to a uniform field therefore travels in a circle, the magnetic force supplying the **centripetal force** 向心力:

$$|q|vB = \frac{mv^2}{r} \quad\Rightarrow\quad r = \frac{mv}{|q|B}, \qquad T = \frac{2\pi m}{|q|B}.$$

Here magnitudes use $|q|$; the vector force law uses signed q. A velocity component parallel to a uniform field stays constant, giving a helical path when both parallel and perpendicular components are present.

Notice the period T does not depend on the speed – faster particles ride larger circles in the same time.

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$$BQv = \frac{mv^2}{r} \;\Rightarrow\; r = \frac{mv}{BQ}$$

A charged particle moving across a magnetic field follows a circular path

Worked example. A proton ($q = 1.6 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) enters a 0.50 T field at 2.0×10^5 m/s, perpendicular to it. The magnetic force $F = |q|vB = 1.6 \times 10^{-14}$ N bends it into a circle of radius $r = \dfrac{mv}{|q|B} = \dfrac{1.67 \times 10^{-27}(2.0 \times 10^5)}{1.6 \times 10^{-19}(0.50)} = 4.2 \times 10^{-3}$ m.

In a region with both an electric and a magnetic field, the two forces act independently and add as vectors. Balancing them makes a **velocity selector** 速度选择器: with $\vec{E}$ and $\vec{B}$ crossed, only charges with $|q|E = |q|vB$, i.e. $v = E/B$, pass straight through. The **Hall effect** 霍尔效应 is the same physics inside a conductor: a field with a component perpendicular to the current pushes the moving carriers sideways, charge builds up on one face, and a measurable potential difference appears across the conductor – its sign reveals whether the carriers are positive or negative. The sideways push stops when the electric force from that build-up balances the magnetic force, $qE = qv_dB$. Across a strip of width w this gives the **Hall voltage** $\Delta V_H = v_dBw$, which is why a Hall probe can measure B.

Worked example (measuring e/m). Electrons are accelerated from rest through a potential difference ΔV, then enter a uniform field B at right angles. Energy gives $e\Delta V = \frac{1}{2}mv^2$, and the circle gives $r = \dfrac{mv}{eB}$. Eliminate v:

$$r^2 = \frac{2m}{eB^2}\,\Delta V.$$

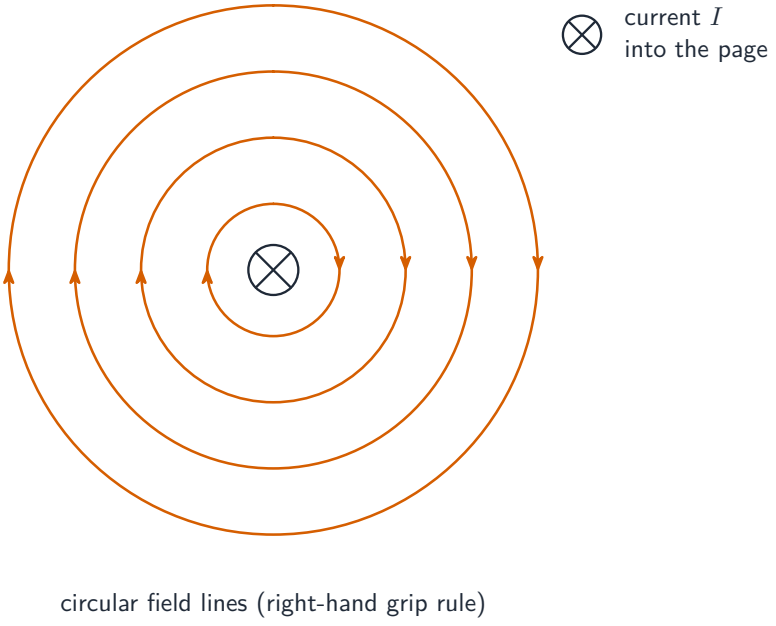
So a graph of r^2 against ΔV is a straight line through the origin with slope $\dfrac{2m}{eB^2}$. With $B = 1.0$ mT and a measured slope of 1.14×10^{-5} m^2/V, $\dfrac{m}{e} = \dfrac{B^2 \times \text{slope}}{2} = 5.7 \times 10^{-12}$ kg/C, the accepted value for the electron. At $\Delta V = 200$ V the beam circles with $r = \sqrt{(1.14 \times 10^{-5})(200)} = 4.8$ cm.

Magnetic Fields of Current-Carrying Wires

A current is a stream of moving charges, so it creates a magnetic field. The **Biot–Savart law** 毕奥-萨伐尔定律 adds up the field of each current element:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2}.$$

Around any straight segment the field vectors are **tangent** 相切 to **concentric circles** 同心圆 centred on the wire – no component toward, away from, or along the wire. Curl your right hand around the wire with the thumb along the current: your fingers give the field direction.



Concentric circular field lines surround a straight current-carrying wire

The Biot–Savart integrals AP expects: a long straight wire ($B = \dfrac{\mu_0 I}{2\pi r}$, or a point on the perpendicular bisector of a finite wire), and the centre of a circular loop,

$$B_{\text{centre of loop}} = \frac{\mu_0 I}{2R},$$

where every element $d\vec{l}$ is perpendicular to $\hat{r}$ and equally distant R – say that in an FRQ derivation. An arc that is a fraction of a full circle contributes that same fraction of $\mu_0 I/2R$ at its centre. Radial straight segments joining an arc point straight at its centre, so they add nothing there ($d\vec{l} \parallel \hat{r}$).

For a straight wire of length $2L$, at a distance a from its middle, the Biot–Savart integral gives

$$B = \frac{\mu_0 I}{4\pi a} \cdot \frac{2L}{\sqrt{L^2 + a^2}}.$$

When the wire is very long ($L \gg a$) the last fraction becomes 2, giving $B = \dfrac{\mu_0 I}{2\pi a}$: a quick check of any finite-wire answer.

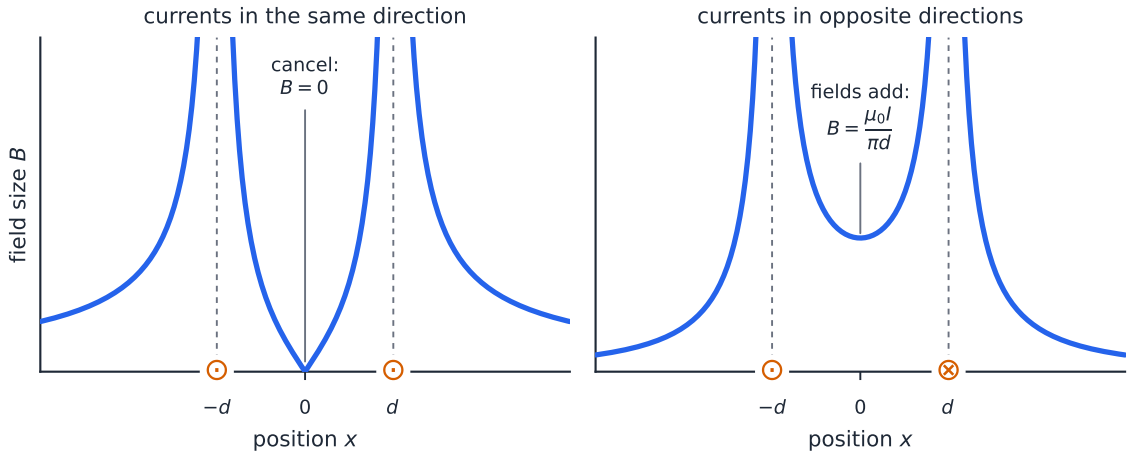
A field also pushes on a current-carrying wire, element by element:

$$\vec{F}_B = \int I\, d\vec{l} \times \vec{B} \qquad (\vec{F} = I\vec{L} \times \vec{B} \text{ for a straight wire in a uniform field}).$$

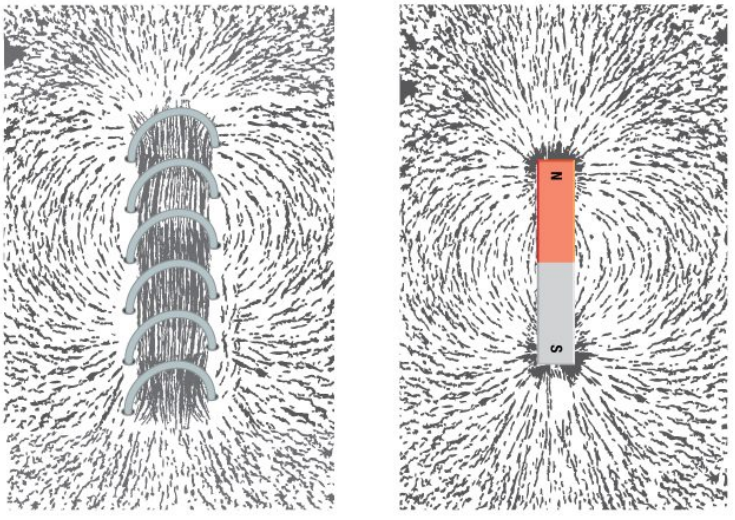
Worked example. Two long parallel wires a distance $d = 0.10$ m apart each carry 5.0 A in the same direction. Wire 1's field at wire 2 is $B = \dfrac{\mu_0 I}{2\pi d} = 1.0 \times 10^{-5}$ T, so wire 2 feels $\dfrac{F}{L} = IB = 5.0 \times 10^{-5}$ N/m, pulled *toward* wire 1. Same-direction currents attract; opposite currents repel.

Worked example (net field from two wires). Two long parallel wires are 0.20 m apart; each carries 10 A. At the midpoint each wire is 0.10 m away and makes $B = \dfrac{\mu_0 I}{2\pi r} = 2.0 \times 10^{-5}$ T. If the currents are in the *same* direction, the right-hand rule gives opposite field directions at the midpoint, so $B_{\text{net}} = 0$. If they are *opposite*, the fields point the same way and $B_{\text{net}} = 4.0 \times 10^{-5}$ T. At a point off the line between the wires, find each field (perpendicular to the line from *its* wire to the point), then add components, just as for electric fields.

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The field along the line through two parallel wires: same-direction currents cancel midway, opposite currents add



A current-carrying coil (solenoid) makes a magnetic field shaped just like a bar magnet's

Ampère's Law

For steady, symmetric current distributions, **Ampère's law** 安培定律 finds the field far faster than Biot–Savart:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}.$$

Choose an **Amperian loop** 安培环路 that matches the symmetry, so that B is constant along the loop (and parallel to it) and comes out of the integral. AP applies

it to long straight wires, long solenoids, and conductors carrying a **current density** 电流密度 (slabs and cylinders); combinations are handled by **superposition** 叠加.

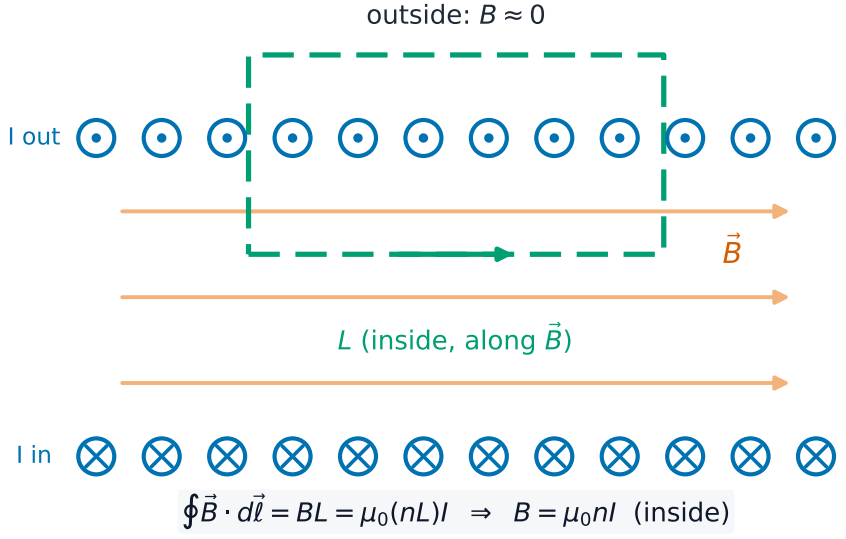
> **Worked example.** Find the field 0.10 m from a long wire carrying 5.0 A. A circular loop of radius r shares the field's symmetry, so $B(2\pi r) = \mu_0 I$ and
>
> $$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7})(5.0)}{2\pi(0.10)} = 1.0 \times 10^{-5}\ \text{T}.$$

A long **solenoid** 螺线管 (assumed: uniform field inside, negligible field outside) is the other classic. Take a rectangular loop with one side of length L inside, parallel to the axis: only that side contributes to the integral, so $BL = \mu_0(nL)I$ and

$$B_{\text{sol}} = \mu_0 n I,$$

with n the turns per metre.

> **Worked example (measuring μ_0).** Students measure B at the centre of a long solenoid with $n = 2000$ turns per metre for several currents, and plot B against I. Since $B = \mu_0 n I$, the slope is $\mu_0 n$. A best-fit slope of 2.48×10^{-3} T/A gives $\mu_0 = \dfrac{2.48 \times 10^{-3}}{2000} = 1.24 \times 10^{-6}$ T · m/A, a percent error of $\dfrac{|1.24 - 1.257|}{1.257} \times 100\% \approx 1.4\%$. If the line does not pass through the origin, something else adds a constant field, such as Earth's field along the solenoid's axis ($\sim 10^{-5}$ T). A constant offset changes the intercept but not the slope, so μ_0 is still good. Choose a solenoid with many turns per metre, so its field is large compared with Earth's.



A rectangular Amperian loop derives the uniform field inside a solenoid

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Worked example (cylinder). A solid cylindrical conductor of radius R carries current I spread uniformly over its cross-section. Inside ($r < R$), a circular loop encloses $I_{\text{enc}} = I\dfrac{r^2}{R^2}$, so $B = \dfrac{\mu_0 I r}{2\pi R^2}$ – the field grows linearly to the surface, then falls off as $1/r$ outside.

Exam skill. Every Ampère's-law answer earns its marks in the setup: name the loop, state *why* B is constant and parallel to it (symmetry), and count I_{enc} carefully before you solve. Maxwell's fourth equation adds one more idea you should know qualitatively: a *changing electric field* also creates a magnetic field – but AP will not ask you to compute with that term.

Practice

Answer each before reading the answers below.

1. A proton moves at 2.0×10^6 m/s perpendicular to a 0.50 T field. Find the force on it and the radius of its path.
2. Find the field 5.0 cm from a long straight wire carrying 10 A.
3. Two parallel wires carry currents in the same direction. Do they attract or repel? Explain.
4. Use Ampère's law to find B inside a long solid wire of radius R carrying current I spread uniformly, a distance $r < R$ from its axis.
5. Find the field inside a solenoid with 1000 turns per metre carrying 2.0 A.
6. Explain why a magnetic field cannot change a charged particle's speed.

Answers

1. $F = |q|vB = 1.6 \times 10^{-13}$ N; $r = \dfrac{mv}{|q|B} = \dfrac{(1.67 \times 10^{-27})(2.0 \times 10^6)}{(1.6 \times 10^{-19})(0.50)} = 0.042$ m.
2. $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{(4\pi \times 10^{-7})(10)}{2\pi(0.050)} = 4.0 \times 10^{-5}$ T.
3. They attract: each wire sits in the other's field, and $\vec{F} = I\vec{L} \times \vec{B}$ points toward the other wire.
4. $B(2\pi r) = \mu_0 I\dfrac{r^2}{R^2}$, so $B = \dfrac{\mu_0 I r}{2\pi R^2}$.
5. $B = \mu_0 n I = (4\pi \times 10^{-7})(1000)(2.0) = 2.5 \times 10^{-3}$ T.
6. The magnetic force is always perpendicular to the velocity, so it does no work; it changes only the direction of motion.

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Exam tips

- The magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ is **perpendicular** to velocity – use the right-hand rule and note it does **no work**.
- A charge moving perpendicular to a uniform field moves in a circle with $r = \frac{mv}{|q|B}$.
- Find the field of currents with **Biot–Savart** $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$ or **Ampère's law** $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ when there is symmetry.
- Force on a wire is $\vec{F} = I\vec{L} \times \vec{B}$; keep the cross-product direction straight.
- Distinguish the force **on** a charge from the field the current **creates**.

Common mistakes

- Using the right-hand rule unchanged for a negative charge. $\vec{F} = q\vec{v} \times \vec{B}$: for an electron the force is opposite to the right-hand result.
- Saying a magnetic field changes a charge's speed. The force is perpendicular to $\vec{v}$, so it does no work.
- Using Ampère's law where the field is not symmetric. Like Gauss's law, it gives B only for a loop on which B is constant and tangent.
- Using the total number of turns for n in $B = \mu_0 nI$. n is turns per metre.
- Doubling the field midway between two same-direction currents. There the two fields point opposite ways and cancel.
- Drawing a wire's field pointing toward or away from the wire. The field of a straight wire is always tangent to a circle around it.
- Confusing the force rule with the field rule. Use the curled-fingers rule for the field a current *makes*; use $\vec{F} = q\vec{v} \times \vec{B}$ or $I\vec{L} \times \vec{B}$ for the force it *feels*.

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Electromagnetic Induction

AP Physics C: Electricity and Magnetism

Magnetic Flux

Put an iron pan on an induction hob and it is hot within seconds, while the glass top heats indirectly from the hot pan. A glass bowl placed there would not heat directly by induction. Under the glass is a coil carrying a rapidly alternating current. Its changing magnetic field induces currents in the metal base of the pan, and those currents heat the pan directly. Electric guitars, wireless phone chargers, transformers and every power-station generator use the same idea: a *changing* magnetic flux makes an emf. This unit is about how to calculate that emf, its direction, and the forces and energy that come with it.

Magnetic flux 磁通量 measures how much magnetic field passes through a surface. For a uniform field through a flat loop it is the dot product $\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta$; in general it is the **surface integral** 面积分

$$\Phi_B = \int \vec{B}\cdot d\vec{A}.$$

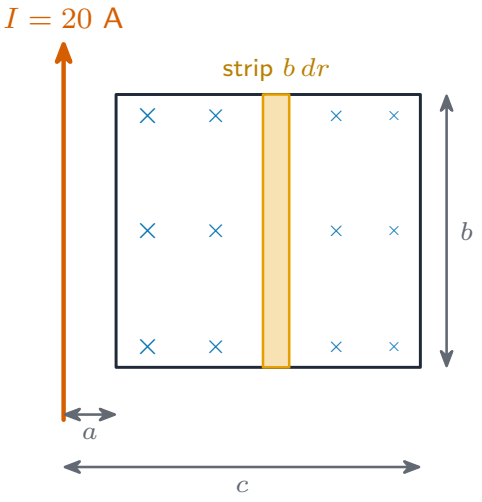
The **area vector** 面积矢量 is perpendicular to the surface (outward from a closed one), and the sign of the flux comes from the dot product. Flux changes if B changes, the area changes, or the loop turns – keep all three routes in mind, because each one is an exam question.

> **Worked example (flux from a long wire).** A long straight wire carries $I = 20$ A. A rectangular loop lies in the same plane, with its sides of length $b = 0.30$ m parallel to the wire, its near side $a = 0.010$ m away and its far side $c = 0.060$ m away. The field $B = \dfrac{\mu_0 I}{2\pi r}$ changes across the loop, so cut it into strips of width dr and area $b\,dr$:

$$\Phi_B = \int_a^c \frac{\mu_0 I}{2\pi r}\, b\, dr = \frac{\mu_0 I b}{2\pi}\ln\frac{c}{a} = (2.0\times10^{-7})(20)(0.30)\ln 6 = 2.2\times10^{-6}\ \text{Wb}.$$

If the current falls steadily to zero in 0.10 s, the average emf in the loop is 2.2×10^{-5} V.

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$$B = \frac{\mu_0 I}{2\pi r} \text{ weakens with } r$$

$$\Phi_B = \frac{\mu_0 I b}{2\pi} \ln\frac{c}{a} = 2.2 \times 10^{-6} \text{ Wb}$$

$a = 0.010$ m, $c = 0.060$ m, $b = 0.30$ m

The field weakens with distance, so the flux is summed strip by strip: mu0 I b over 2 pi times ln(c/a) = 2.2 times 10 to the -6 Wb

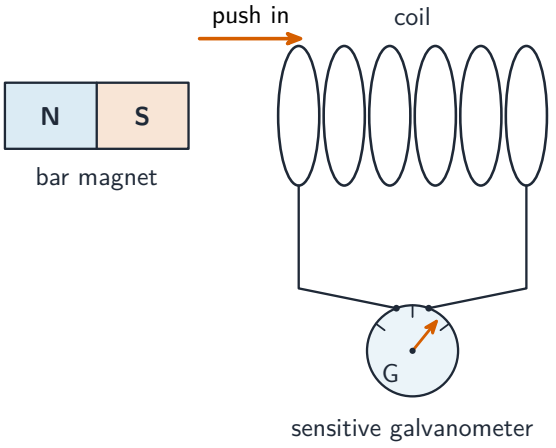
Electromagnetic Induction

A **changing** flux induces an **emf** 电动势 – **Faraday's law** 法拉第定律:

$$\varepsilon = -\frac{d\Phi_B}{dt}.$$

With constant area, $\varepsilon = -A\dfrac{dB_\perp}{dt}$; with constant field, $\varepsilon = -B\dfrac{dA_\perp}{dt}$. A coil of N turns multiplies the single-loop emf: $|\varepsilon_{\text{sol}}| = N\left|\dfrac{d\Phi_B}{dt}\right|$.

Lenz's law 楞次定律 is the minus sign: the **induced current** 感应电流 flows so that its own magnetic field *opposes the change* in flux that created it. Push a magnet toward a loop and the loop pushes back; pull it away and the loop pulls it in. Use the **right-hand rule** 右手定则 to turn "oppose the change" into a current direction.



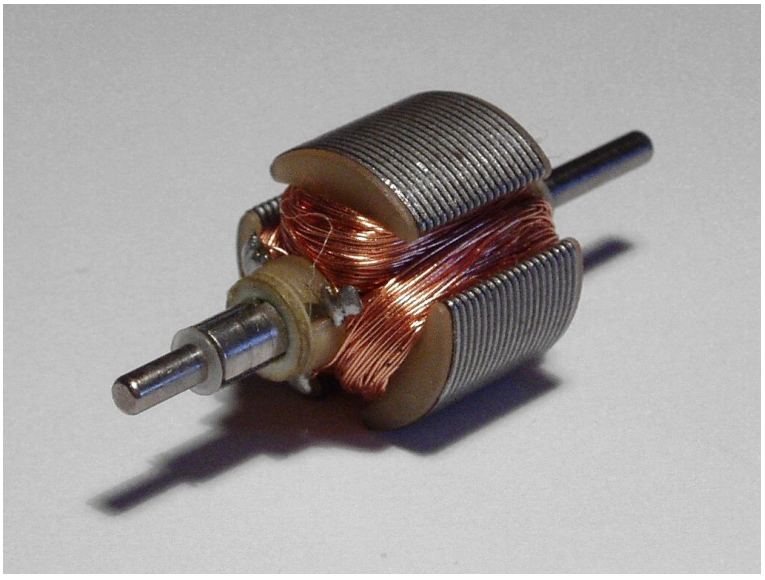
Moving a magnet into a coil induces an e.m.f. that drives a current

A rod of length L sliding at speed v, with the rod, velocity and magnetic field mutually perpendicular, is the special case worth memorising – the **motional emf** 动生电动势 $\varepsilon = BLv$.

> **Worked example.** A 0.20 m rod slides at 3.0 m/s across a 0.50 T field: $\varepsilon = BLv = 0.50(0.20)(3.0) = 0.30$ V. Separately, if a single loop's flux drops from 0.020 Wb to 0.008 Wb in 0.030 s, the average emf is $\varepsilon = \dfrac{0.012}{0.030} = 0.40$ V.

Faraday's law is also the third of **Maxwell's equations** 麦克斯韦方程组, in a deeper form: a changing magnetic flux creates a circulating *electric field*, $\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$ – that field is what pushes the charges around the loop. Together, Maxwell's equations predict **electromagnetic waves** 电磁波 travelling at $c = 1/\sqrt{\varepsilon_0 \mu_0}$ (you should know this connection, but AP will not ask you to derive it).

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Coils of wire spinning in a magnetic field: moving them past the field induces a current, the basis of a generator



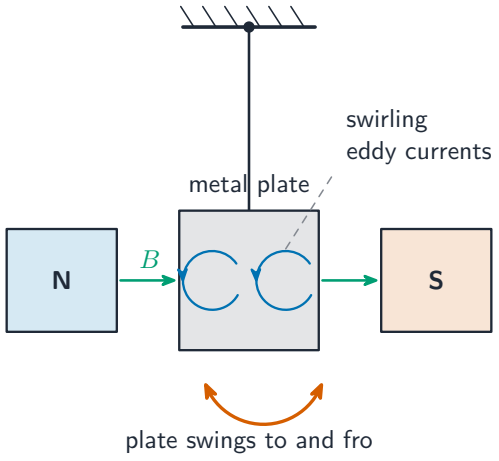
A substation transformer: changing magnetic flux in coils induces the voltages that power the grid

Induced Currents and Magnetic Forces

Once an induced current flows, the external field exerts forces on it ($\vec{F} = \int I\, d\vec{l} \times \vec{B}$) – and for an unpowered loop moving through a static external field, the induced forces resist the motion that changes its flux. Only the segments of the loop actually inside the field feel a force, which can make a loop accelerate, rotate, or brake. This is the origin of **eddy currents** 涡电流 braking, and you can apply Newton's second law to

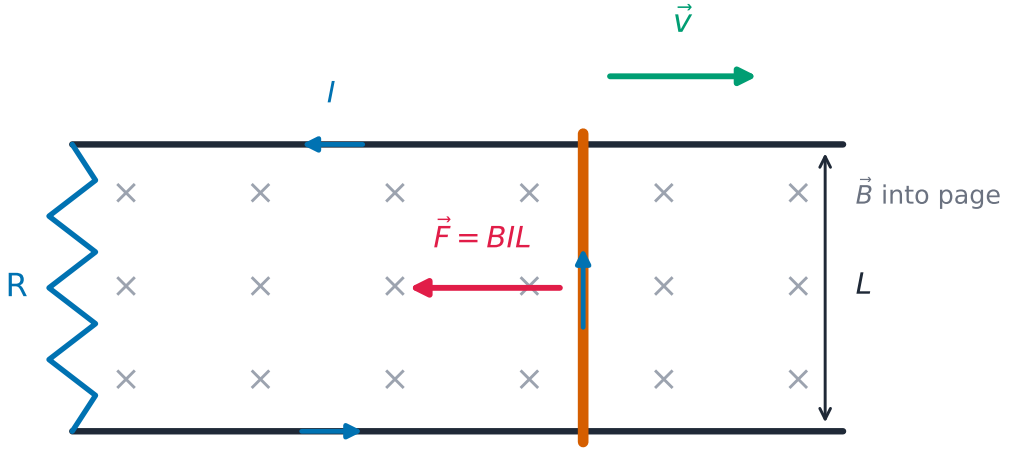
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a moving loop or rod like any other mechanics problem.



Induced eddy currents oppose motion, quickly damping a metal plate swinging in a field

The classic setup is a rod sliding on conducting rails connected by a resistor:



A rod sliding on rails: the induced current feels a force opposing the motion

Worked example (the full chain). Rails $L = 0.20$ m apart with resistance $R = 0.60\ \Omega$ sit in a 0.50 T field into the page. The rod is pushed at a constant 3.0 m/s. Then: $\varepsilon = BLv = 0.30$ V; $I = \varepsilon/R = 0.50$ A; the field pushes back on the rod with $F = BIL = 0.50(0.50)(0.20) = 0.050$ N. The pushing force does work at $P = Fv = 0.15$ W – exactly the $P = I^2R = 0.15$ W dissipated in the resistor. Mechanical work becomes electrical energy: that is a **generator** 发电机, and energy is conserved. Released with no push, the rod slows exponentially: $ma = -\dfrac{B^2L^2}{R}v$.

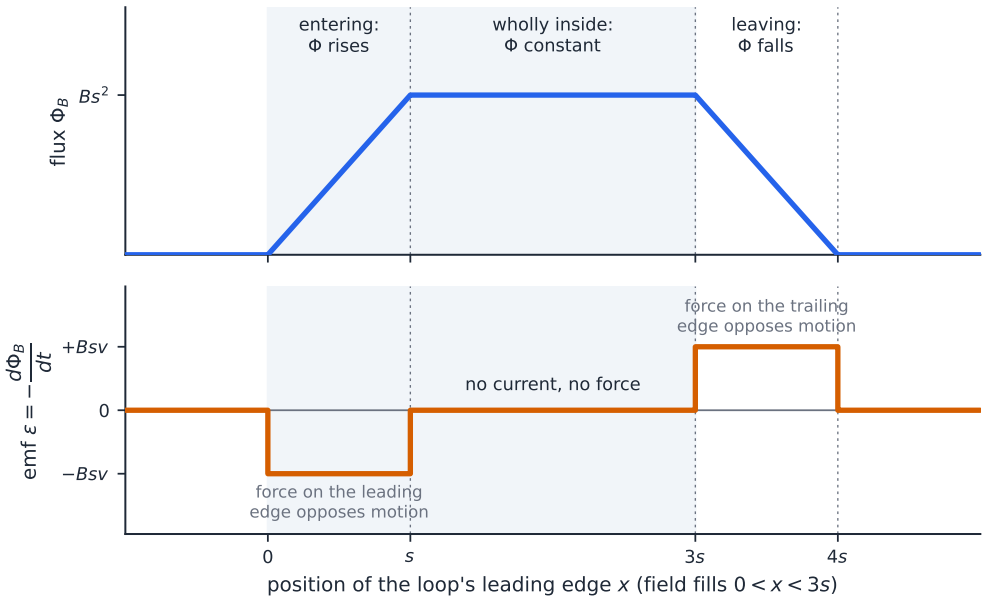
Worked example (falling bar, terminal velocity). A bar of mass $m = 0.020$ kg and length $L = 0.25$ m falls from rest down vertical frictionless rails joined at the top by $R = 0.50\ \Omega$, in a horizontal field perpendicular to the plane of the rails, $B = 0.40$ T. At speed v the current is $\dfrac{BLv}{R}$, so the upward magnetic force is $\dfrac{B^2L^2v}{R}$. Newton's second law:

$$m\frac{dv}{dt} = mg - \frac{B^2L^2}{R}v.$$

At terminal velocity the two forces balance: $v_T = \dfrac{mgR}{B^2L^2} = \dfrac{(0.020)(9.8)(0.50)}{(0.40)^2(0.25)^2} = 9.8$ m/s. Separating variables gives $v(t) = v_T\big(1 - e^{-t/\tau}\big)$ with $\tau = \dfrac{mR}{B^2L^2} = 1.0$ s. At terminal velocity gravity does work at $mgv_T = 1.9$ W, which equals I^2R with $I = 1.96$ A: all the lost gravitational energy heats the resistor.

Loops moving through a field region. Consider a loop translating through a static field region, neglecting self-inductance. The emf depends on the *change* in flux, not on the flux itself. When a loop enters the field region, the flux grows and a current flows; while the loop is wholly inside a uniform field, the flux is constant, so there is no current and no magnetic force, even though the loop is full of field. For a rectangular loop crossing a straight field boundary, forces on the side segments cancel transversely; the edge inside the field supplies the net drag.

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A loop crossing a field region: flux rises, stays constant, then falls, so the emf is non-zero only while the loop enters or leaves

Worked example (rotating ring). A ring of area $A = 0.50\ \text{m}^2$ and resistance $R = 2.0\ \Omega$ spins at $\omega = 10$ rad/s about a diameter perpendicular to a uniform field $B = 0.50$ T. Then $\Phi_B = BA\cos\omega t$ and $\varepsilon = -\dfrac{d\Phi_B}{dt} = BA\omega\sin\omega t$, so $\varepsilon_{\max} = BA\omega = 2.5$ V and $I_{\max} = 1.25$ A. The emf is largest when the ring's plane is *parallel* to the field, where the flux is zero but changing fastest. Spin it twice as fast and $\varepsilon_{\max}$ doubles, so the power ε^2/R is four times as large.

Exam skill. FRQs walk this exact chain: flux → emf → current → force → Newton's second law. Write each link separately and check the direction with Lenz's law at the end.

Inductance

Inductance 电感 is a conductor's tendency to oppose a change in its own current: changing current changes its own flux, which self-induces an emf. From Faraday's law,

$$\varepsilon = -L\frac{dI}{dt}.$$

An **inductor** 电感器 is a circuit element built to have large inductance – usually a **solenoid** 螺线管, where geometry gives

$$L_{\text{sol}} = \frac{\mu_{\text{core}} N^2 A}{\ell},$$

with N total turns, area A, length ℓ, and the **magnetic permeability** 磁导率 of the core. (Straight wires are modelled as having zero inductance.) An inductor carrying current I stores energy in its magnetic field:

$$U_L = \tfrac{1}{2} L I^2,$$

which can later be dissipated in a resistor or moved into a capacitor – conservation of energy applies as usual.

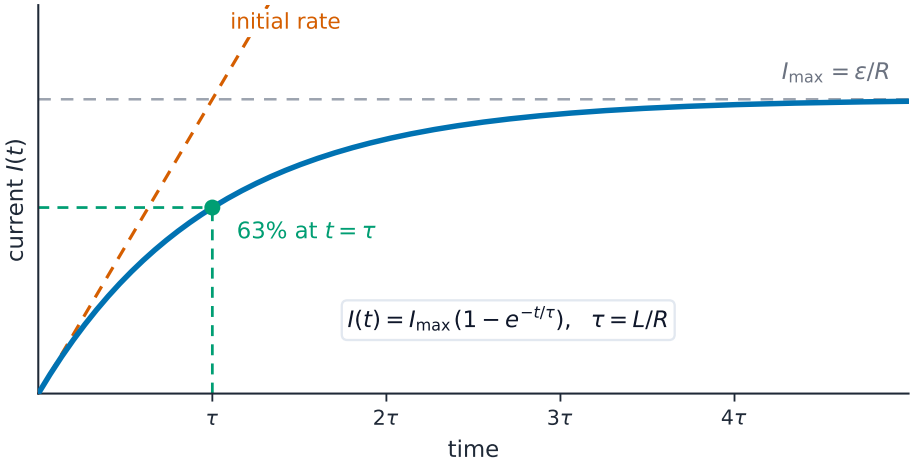
Circuits with Resistors and Inductors

In an **RL circuit** RL 电路 starting with zero current, Kirchhoff's loop rule for a battery ε, resistor R, and inductor L in series gives a **differential equation** 微分方程:

$$\varepsilon = IR + L\frac{dI}{dt} \quad \Rightarrow \quad I(t) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right), \qquad \tau = \frac{L}{R}.$$

The **time constant** 时间常数 τ sets the pace: after one τ a rising current reaches about 63% of its final value (a decaying one falls to 37%); it is also how long the change *would* take at the initial rate. Learn the two limits – they answer most conceptual questions:

- **Just after the switch closes** ($t = 0$): current cannot jump, so the inductor momentarily blocks it, self-inducing an emf equal and opposite to the applied potential difference.
- **Long after** ($t \gg \tau$): the current is steady, $dI/dt = 0$, and the inductor behaves as a plain wire – the exact opposite of a capacitor.



RL rise: I is within 1% of $I_{max} = \varepsilon/R$ after 5τ

The current in an RL circuit rises exponentially, reaching 63% at one time constant

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Worked example. $\varepsilon = 12$ V, $R = 6.0\ \Omega$, $L = 3.0$ H: final current $\varepsilon/R = 2.0$ A, $\tau = L/R = 0.50$ s. At $t = 0.50$ s the current is $2.0(1 - e^{-1}) \approx 1.3$ A. At $t = 0$ the inductor's potential difference is the full 12 V; as $t \to \infty$ it falls to zero. The current approaches its final value exponentially, and the inductor voltage decays exponentially. Stored magnetic energy follows $U_B = \frac{1}{2}L[I(t)]^2$, so it is not a single exponential during this current growth.

Worked example (opening the switch). In the circuit above, a 30 Ω resistor is also connected in parallel with the inductor. Long after closing the switch, the inductor carries 2.0 A and acts as a wire, so no current flows in the 30 Ω resistor. Now open the switch. The inductor's current cannot change instantly, so the full 2.0 A now flows round the inductor–30 Ω loop. The potential difference across that resistor is $(2.0)(30) = 60$ V, five times the battery's emf. The current then decays with $\tau = L/R = 3.0/30 = 0.10$ s, and the resistor dissipates all of the stored $\frac{1}{2}LI^2 = \frac{1}{2}(3.0)(2.0)^2 = 6.0$ J. This is why opening a switch on a large inductor can make a spark.

Exam skill (measuring L). Past FRQs ask for a straight-line graph that gives an inductance. Measure the time constant for several series resistances R: since $\tau = L/R$, a graph of τ against $1/R$ is a straight line through the origin with slope L. From one current–time curve, plot $\ln\!\left(1 - I/I_{\max}\right)$ against t instead: the slope is $-R/L$.

Circuits with Capacitors and Inductors

An **LC circuit** LC 电路 has no resistance, so nothing dissipates energy: it **oscillates** 振荡, sloshing energy between the capacitor's electric field and the inductor's magnetic field. The loop rule gives

$$\frac{d^2q}{dt^2} = -\frac{1}{LC}\,q,$$

the same equation as a mass on a spring – **simple harmonic motion** 简谐运动 with charge playing the role of displacement and

$$\omega = \frac{1}{\sqrt{LC}}.$$

The total energy $\dfrac{q^2}{2C} + \frac{1}{2}LI^2$ stays constant: all in the capacitor at maximum charge, all in the inductor at maximum current.

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Worked example. $L = 2.0$ H, $C = 8.0\ \mu$F: $\omega = \dfrac{1}{\sqrt{2.0(8.0\times10^{-6})}} = 250$ rad/s, period $T = 2\pi/\omega \approx 0.025$ s. If the capacitor starts charged to 12 V, then $U = \frac{1}{2}CV^2 = 5.8\times10^{-4}$ J, and the maximum current follows from $\frac{1}{2}LI_{\max}^2 = U$: $I_{\max} = \sqrt{2U/L} = 0.024$ A – **conservation of energy** 能量守恒, no calculus needed.

Practice

Answer each before reading the answers below.

1. A 0.50 T field passes through a loop of area 0.020 m^2 at 60° to the loop's normal. Find the flux.
2. A 100-turn coil of area 0.020 m^2 is in a 0.50 T field along its axis. The field falls to zero in 0.10 s. Find the average induced emf.
3. A 0.50 m rod slides at 4.0 m/s along rails in a 0.20 T field, in a circuit of resistance 2.0 Ω. Find the emf, the current and the force needed to keep it moving at constant speed.
4. A bar magnet's north pole is pushed toward a loop. State the direction of the induced field in the loop and the law you used.
5. An RL circuit has $L = 2.0$ H, $R = 10\ \Omega$ and a 20 V battery. Find the time constant and the final current. How does the inductor behave at $t = 0$?
6. Find the angular frequency of an LC circuit with $L = 0.10$ H and $C = 10$ µF.

Answers

1. $\Phi = BA\cos 60° = 5.0\times10^{-3}$ Wb.
2. $\varepsilon = N\dfrac{\Delta\Phi}{\Delta t} = \dfrac{100(0.50)(0.020)}{0.10} = 10$ V.
3. $\varepsilon = BLv = 0.40$ V; $I = 0.20$ A; $F = ILB = 0.020$ N.
4. Lenz's law: the induced field opposes the increase in flux, so it points back toward the magnet (the loop's face toward the magnet acts as a north pole).
5. $\tau = L/R = 0.20$ s; $I_{\text{final}} = 20/10 = 2.0$ A. At $t = 0$ the current is zero: the inductor acts like a break.
6. $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.10)(10\times10^{-6})}} = 1000$ rad/s.

Exam tips

- Compute **flux** $\Phi_B = \int \vec{B}\cdot d\vec{A}$ and get the induced EMF from **Faraday's law** $\varepsilon = -\frac{d\Phi_B}{dt}$.
- Use **Lenz's law** (the minus sign) to fix the direction: the induced current opposes the change in flux.

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- Flux changes three ways – changing B, changing area, or changing angle – identify which and differentiate.
- For a rod of length L moving at speed v with the rod, motion and field mutually perpendicular, the motional EMF is BLv.
- An inductor stores energy $\frac{1}{2}LI^2$ and resists **changes** in current (RL time constant $\tau = L/R$).

■ Common mistakes

- Using the full B in the flux when the field is at an angle. $\Phi_B = BA\cos\theta$, with θ between $\vec{B}$ and the normal to the loop.
- Saying the induced current opposes the field. It opposes the *change* in flux: if the flux falls, the induced field points the same way as the original.
- Treating an inductor like a capacitor at $t = 0$. Just after switching on from zero current, an inductor carries no current (like a break); after a long time it acts as a plain wire.
- Forgetting energy in an LC circuit swaps between $\frac{1}{2}CV^2$ and $\frac{1}{2}LI^2$ with the total constant.
- Saying a loop wholly inside a uniform field has a current because it is "in a field". Only a *changing* flux induces an emf.
- Using B at one distance for the flux from a long wire. The field changes across the loop, so integrate strips: the answer has a ln.
- Thinking an inductor's current stops the moment a switch opens. It keeps flowing, through whatever path is left, and decays with $\tau = L/R$ of that path.

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