Course Companion

AP Physics C: Electricity and Magnetism

Every topic handout in one volume

全部专题讲义合集

exercises.lol

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Electric Charges, Fields, and Gauss's Law

AP Physics C: Electricity and Magnetism

Electric Charge and Electric Force

Electric charge 电荷 comes in positive and negative; like charges repel, opposites attract. Charge is **quantized** 量子化–every charge is a whole-number multiple of the **elementary charge** 基本电荷 $e = 1.6 \times 10^{-19}$ C –and obeys **conservation of charge** 电荷守恒: charge is never created or destroyed, only moved. The force between two **point charges** 点电荷 is **Coulomb's law** 库仑定律:

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\,\hat{r},$$

an **inverse-square** 平方反比 force along the line joining them, with $\frac{1}{4\pi\varepsilon_0} = k = 9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}$. For several charges (AP uses four or fewer, unless symmetry helps), add the force *vectors* on the charge you care about –**superposition** 叠加.

Charge is a **fundamental property** 基本属性 of matter, and a **point charge** is a model that ignores the size of a charged object. The constant ε_0 is the **permittivity of free space** 真空介电常数–a material property of the vacuum. The **permittivity of matter** differs from ε_0, set by how easily the material polarizes, which is why a dielectric between charges weakens the force. Note too that although the electric force is far **stronger** than gravity, gravity dominates at astronomical scales because large bodies are nearly **neutral**.

Worked example. Two $+2.0\ \mu$C charges sit 0.30 m apart: $F = \frac{kq_1q_2}{r^2} = \frac{9.0 \times 10^9 (2.0 \times 10^{-6})^2}{(0.30)^2} = 0.40$ N, repulsive.

Worked example (vectors). Charges $+3.0\ \mu$C and $-3.0\ \mu$C sit at two corners of a right angle, each 0.30 m from a $+1.0\ \mu$C charge at the corner. Each exerts $F = \frac{(9.0 \times 10^9)(3.0 \times 10^{-6})(1.0 \times 10^{-6})}{0.09} = 0.30$ N –one a push, one a pull, at right angles to each other. The net force is $F_{\text{net}} = \sqrt{0.30^2 + 0.30^2} = 0.42$ N, pointing between them. Never add magnitudes blindly: components first.

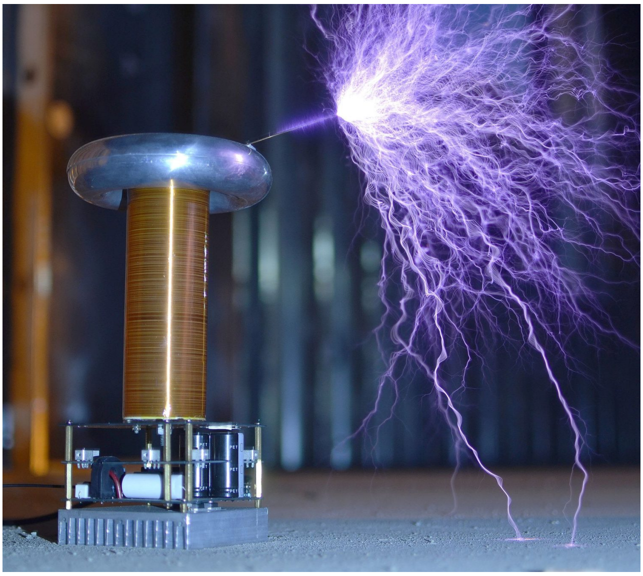
Electric Charge and the Process of Charging

A **conductor** 导体 lets charge move freely; an **insulator** 绝缘体 holds it in place. Objects gain net charge three ways:

- **Charging by friction** 摩擦起电: rubbing transfers electrons from one surface to the other.
- **Charging by conduction** 传导起电: touching a charged object shares charge of the *same* sign.
- **Charging by induction** 感应起电: a nearby charge pushes the conductor's charge apart; **ground** 接地 the far side, remove the ground wire *first*, and the conductor is left with the *opposite* sign –all without contact.

An insulator cannot pass charge, but it can develop **polarization** 极化: its molecules stretch or turn so one face is slightly positive and the other slightly negative. That is why a charged comb picks up neutral paper scraps. An **electroscope** 验电器 shows net charge by its leaves repelling; on any isolated conductor the excess charge sits entirely on the outer surface.

Electric Fields

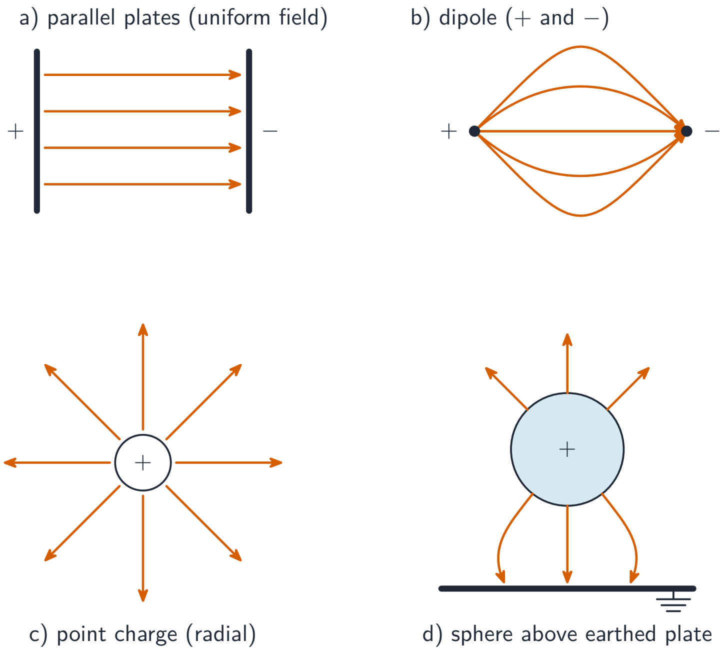


A Tesla coil throws sparks: it raises the voltage so high that the electric field ionizes the surrounding air, and charge leaps across as a visible discharge

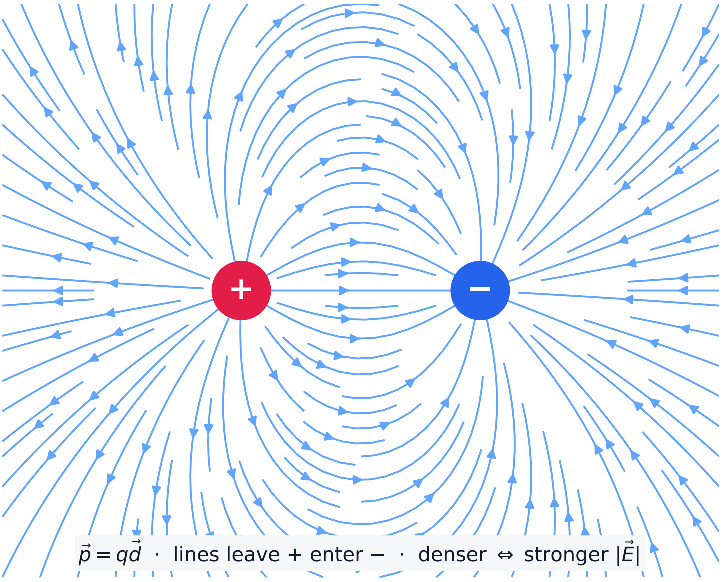
The **electric field** 电场 at a point is the force per unit charge that a small positive **test charge** 检验电荷 would feel there:

$$\vec{E} = \frac{\vec{F}}{q}, \qquad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,\hat{r} \text{ (point charge).}$$

A charge placed in a field feels $\vec{F} = q\vec{E}$ –positive charges along the field, negative charges against it. Fields from several sources add as vectors (superposition again). **Field lines** 电场线 make the field visible: they point away from positive charge and toward negative, their density shows the strength, and they never cross.

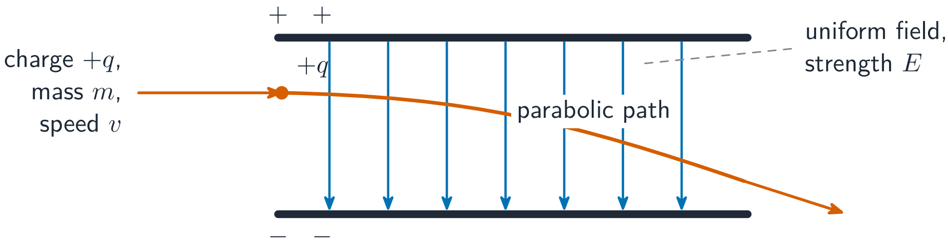


Electric field-line patterns for parallel plates, a dipole, and a point charge

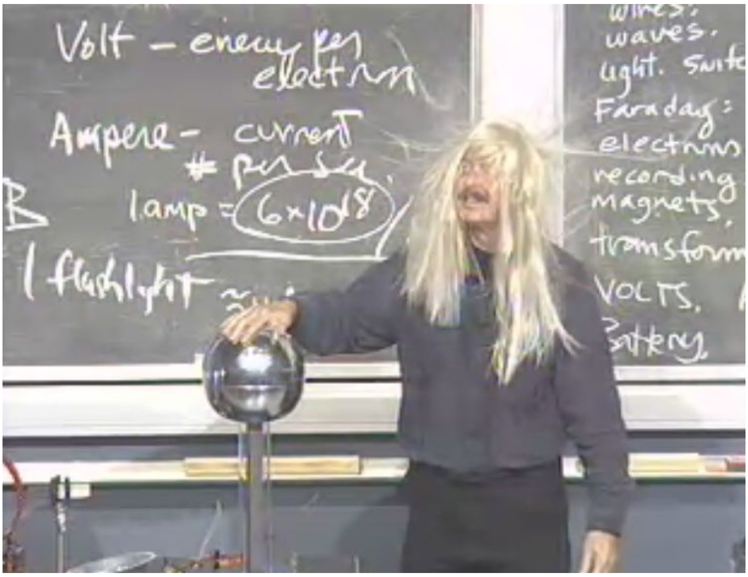


Electric field lines of a dipole point from the positive to the negative charge

In a **uniform 均匀** field a charge feels a constant force, so it accelerates uniformly – launched sideways, it follows a parabola, just like a projectile in gravity.



A charge crossing a uniform field follows a parabolic path



Charge spreads over the dome and onto the hair; like charges repel, so the strands push apart

Electric Fields of Charge Distributions

For a **continuous charge distribution** 连续电荷分布, cut the object into pieces dq and integrate their point-charge fields:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\,\hat{r}.$$

Write dq using the right density: $\lambda\,dl$ with the **linear charge density** 线电荷密度 (a line or ring), $\sigma\,dA$ with the **surface charge density** 面电荷密度, or $\rho\,dV$ with the **volume charge density** 体电荷密度. Then use **symmetry** 对称性 to cancel components *before* integrating –that sentence is usually a scored step. AP's calculus cases: a finite line (collinear point or perpendicular bisector), an infinite wire or cylinder, a ring on its axis, and an arc at its centre.

Worked example (ring, on axis). A ring of radius R carries charge Q. At a point z along the axis, each element is a distance $\sqrt{R^2+z^2}$ away, and by symmetry the sideways components cancel, leaving only the axial part ($\cos\alpha = z/\sqrt{R^2+z^2}$):

$$E_z = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{R^2+z^2}\cdot\frac{z}{\sqrt{R^2+z^2}} = \frac{1}{4\pi\varepsilon_0}\frac{Qz}{(R^2+z^2)^{3/2}}.$$

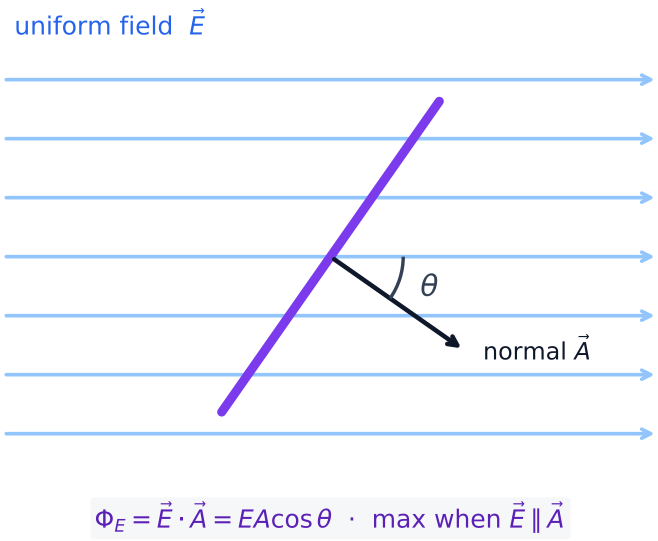
Check the limits: $E = 0$ at the centre ($z = 0$), and far away ($z \gg R$) it becomes kQ/z^2 –a point charge, as it must.

Electric Flux

Electric flux 电通量 measures how much field passes through a surface:

$$\Phi_E = \int \vec{E} \cdot d\vec{A}.$$

For a uniform field through a flat area, $\Phi_E = EA\cos\theta$, where the **area vector** 面积矢量 is perpendicular to the surface. Flux is positive where field lines *exit* a closed surface and negative where they *enter* –only the perpendicular component of $\vec{E}$ counts.



Electric flux through a surface depends on the angle between the field and the surface normal

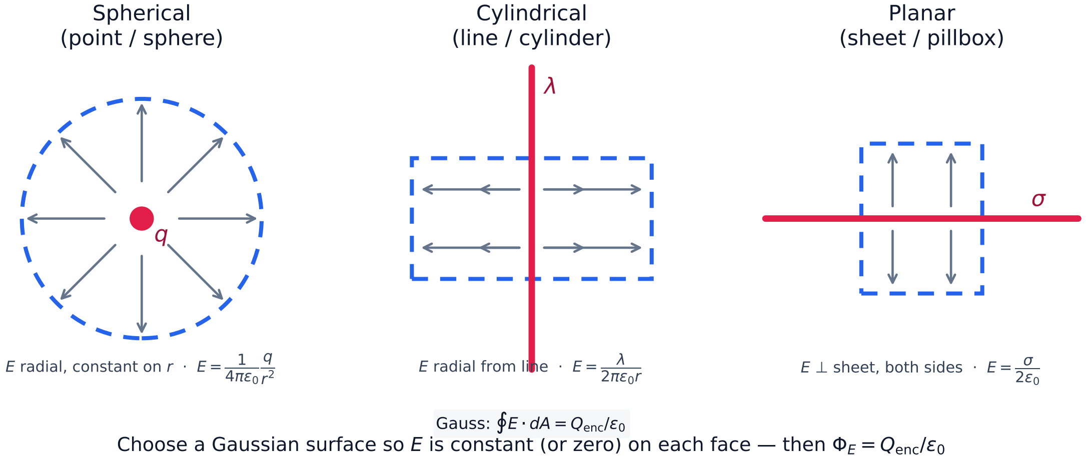
Gauss's Law

Gauss's law 高斯定律–the first of Maxwell's equations –relates the flux through any closed surface to the charge inside it:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\varepsilon_0}.$$

It is true for *any* closed surface, but it *solves* for E only when symmetry lets you choose a **Gaussian surface** 高斯面 on which E is constant and perpendicular (or parallel, contributing zero). The three symmetries AP tests: **spherical symmetry**

球对称 (concentric sphere), **cylindrical symmetry** 柱对称 (coaxial cylinder), and **planar symmetry** 平面对称 (a straddling "pillbox").



A Gaussian surface is chosen to match the symmetry of the charge

Worked example (sphere). Outside a sphere of total charge Q, a spherical Gaussian surface of radius r gives $E(4\pi r^2) = Q/\varepsilon_0$, so $E = \dfrac{kQ}{r^2}$ –identical to a point charge at the centre. *Inside* a uniformly charged solid sphere of radius R, the surface encloses only $Q_{\text{enc}} = Q\,r^3/R^3$, so $E = \dfrac{kQr}{R^3}$: zero at the centre, growing linearly to the surface.

Worked example (wire). For an infinite wire with charge per length λ, take a coaxial cylinder of radius r and length L. The ends contribute nothing ($\vec{E} \perp d\vec{A}$), so $E(2\pi rL) = \dfrac{\lambda L}{\varepsilon_0}$ and $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$. The same pillbox method gives an infinite sheet's field, $E = \dfrac{\sigma}{2\varepsilon_0}$, uniform on both sides.

Exam skill. A full-credit Gauss's-law answer has four parts: name the surface, state the symmetry argument (why E is constant and perpendicular on it), count Q_{enc}, then solve. Skipping the symmetry sentence loses the reasoning point –and remember Gauss's law also explains why $E = 0$ inside any conductor in equilibrium.

Exam tips

- Use **Coulomb's law** $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$ and superpose forces as **vectors** (components, not magnitudes).
- For a continuous charge, integrate $d\vec{E}$ over $dq = \lambda\, dl,\ \sigma\, dA,\ \rho\, dV$; use symmetry to cancel components.
- The field points **away from** positive and **toward** negative charge; draw field lines correctly.
- Distinguish the force on a charge ($\vec{F} = q\vec{E}$) from the field the charge creates.
- Keep the constant $k = \frac{1}{4\pi\varepsilon_0}$ straight and check units.

Electric Potential

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Electric Potential Energy

When you push two like charges together, you do **work 功** against the electric force. That work is stored by the pair as **electric potential energy 电势能**:

$$U_E = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = k\frac{q_1 q_2}{r}.$$

U_E is defined as the work an external force must do to bring the charges from infinitely far away to a distance r apart. The signs carry the physics:

- **Like charges:** $U_E > 0$. An outside push was needed to bring them together. If released, they fly apart, and the stored energy becomes **kinetic energy 动能**.
- **Opposite charges:** $U_E < 0$. The pair is bound. You must supply $|U_E|$ of work to pull them apart to infinity.

The electric force is a **conservative force 保守力**: the work it does depends only on the start and end points, not the path, and equals $-\Delta U_E$.

For a **system 系统** of several charges, add the potential energy of every distinct pair:

$$U_{\text{total}} = k\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right).$$

Worked example. Three $+2.0\ \mu$C charges sit at the corners of an equilateral triangle with sides 0.30 m. Each pair stores $U = \dfrac{kq^2}{r} = \dfrac{(9.0\times10^9)(2.0\times10^{-6})^2}{0.30} = 0.12$ J. There are three pairs, so $U_{\text{total}} = 3 \times 0.12\ \text{J} = 0.36$ J. This is the work needed to assemble the triangle from infinity –and the kinetic energy the charges would share if all three were released.

Electric Potential

Electric potential 电势 is electric potential energy per unit charge at a point in space. It is measured in volts (1 V = 1 J/C):

$$V = \frac{U_E}{q}, \qquad V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\ \text{(point charge)}.$$

Potential is a **scalar** 标量: it has a sign but no direction. This makes V far easier to work with than the field $\vec{E}$. For several **point charges** 点电荷, use scalar **superposition** 叠加–add the potentials with their signs:

$$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}.$$

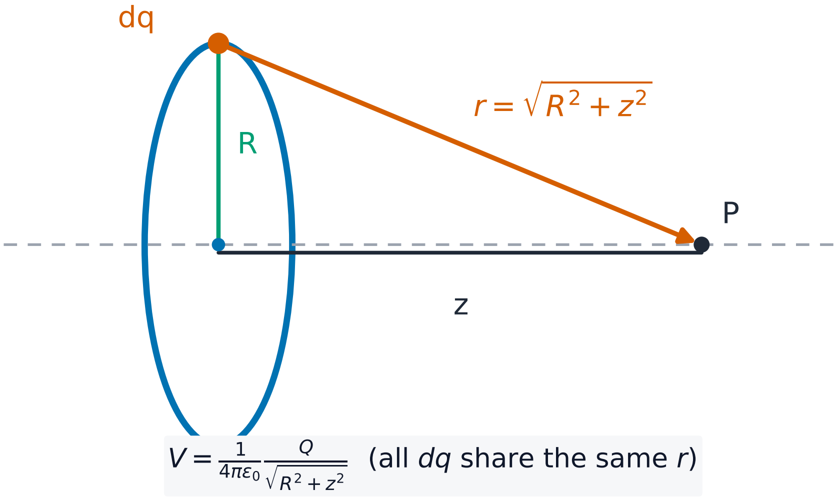
Worked example. The potential 0.10 m from a +3.0 nC point charge is $V = \frac{kq}{r} = \frac{9.0\times10^9(3.0\times10^{-9})}{0.10} = 270$ V. Now add a -3.0 nC charge 0.30 m from the same point: it contributes $\frac{9.0\times10^9(-3.0\times10^{-9})}{0.30} = -90$ V, so the total is $270-90 = 180$ V. Just a signed sum –no vector components to resolve.

Continuous charge distributions

For a **continuous charge distribution** 连续电荷分布, cut it into small pieces dq, treat each piece as a point charge, and **integrate** 积分:

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}.$$

This is a scalar integral, so it is usually much easier than the field integral. AP expects you to handle four shapes with calculus: a thin ring (at a point on its axis), an arc (at its centre), a finite line or wire (collinear, or on its perpendicular bisector), and an infinitely long wire or cylinder (at a distance from its axis).



A ring of charge: every element dq is the same distance from an axis point

Worked example (ring of charge). A thin ring of radius R carries total charge Q. For a point P on the axis a distance z from the centre, every element dq sits at the same distance $r = \sqrt{R^2 + z^2}$ from P. The distance is constant, so it comes out of the integral:

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{\sqrt{R^2+z^2}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+z^2}}.$$

On an FRQ, *say* that r is the same for every element –that statement is the marked step. The same idea gives $V = kQ/R$ at the centre of any arc, whatever its angle.

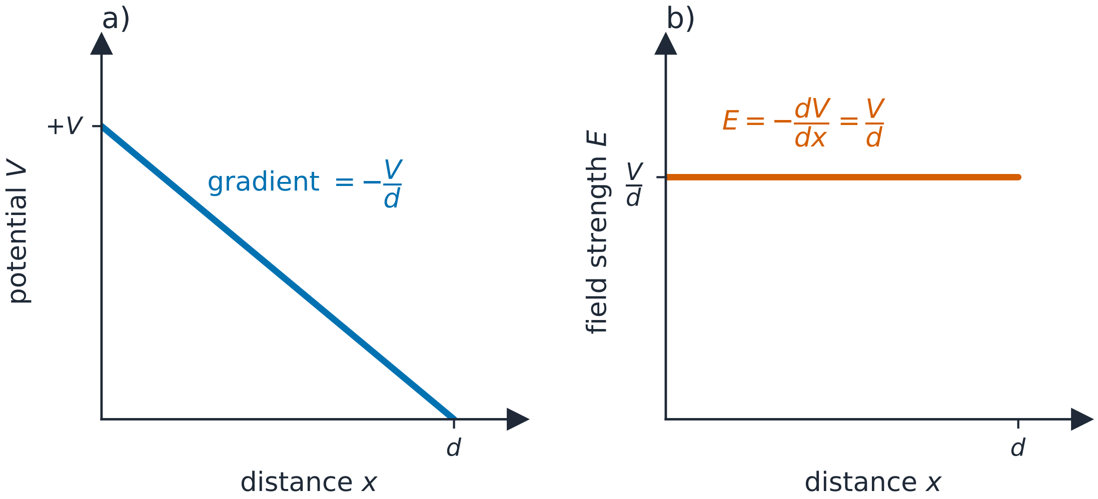
From potential to field

Potential and field contain the same information, linked by a derivative in each direction and by a path integral:

$$E_x = -\frac{dV}{dx}, \qquad \Delta V = V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{r}.$$

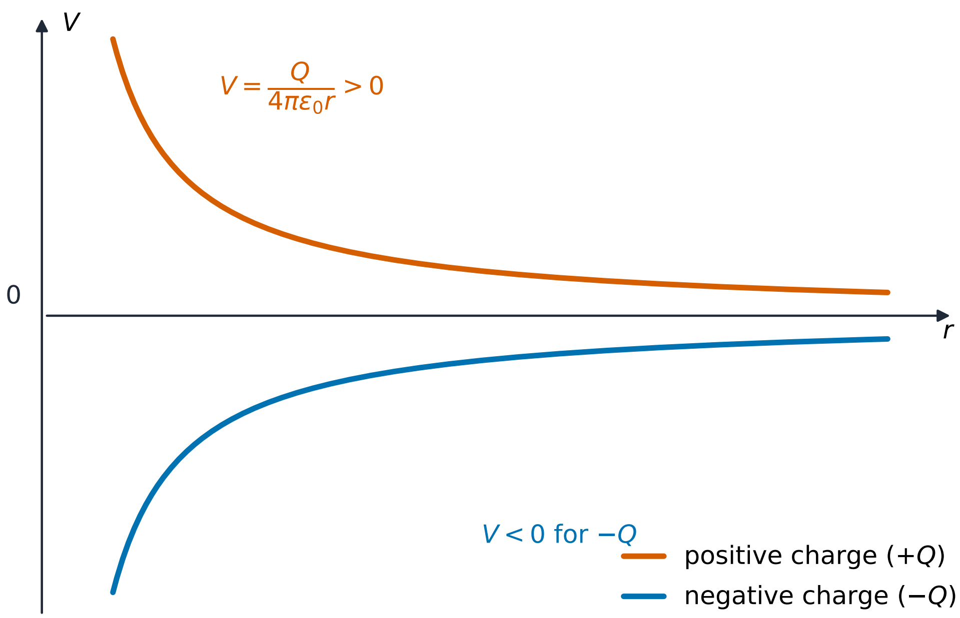
The field is the negative **gradient** 梯度 of the potential: $\vec{E}$ points from high potential to low potential, "downhill". Where V changes quickly, the field is strong.

Worked example. Along the x-axis, $V(x) = 3x^2 - 2x$ (volts, with x in metres). Then $E_x = -\dfrac{dV}{dx} = 2 - 6x$ V/m. At $x = 0.50$ m, $E_x = -1.0$ V/m: the field there points in the $-x$ direction.



In a uniform field the potential falls steadily with distance

The **potential difference** 电势差 between two points is the change in potential energy per unit charge moved between them: $\Delta V = \Delta U_E/q$. A battery makes a potential difference chemically: reactions inside separate positive from negative charge and hold the terminals a fixed ΔV apart.

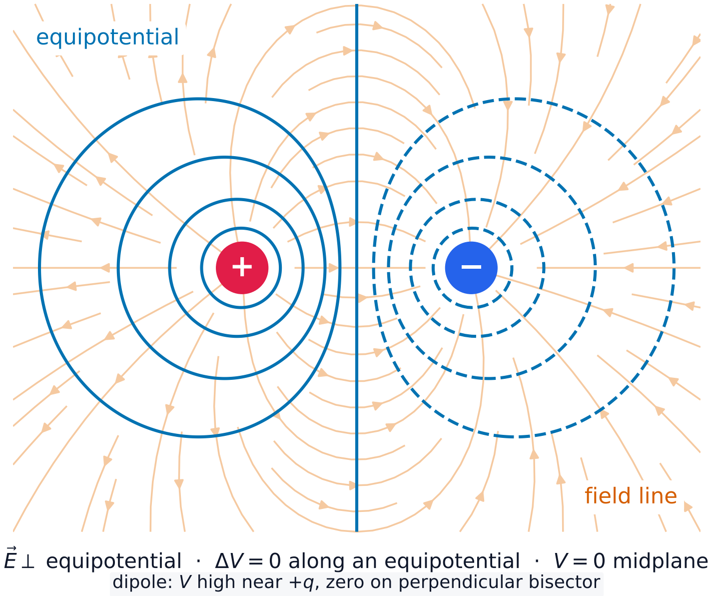


The potential near a point charge varies as 1/r

Equipotential maps

An **equipotential** 等势面 line (also called an **isoline** 等值线) joins points that share the same potential. Four rules let you read any map:

- Isolines are **perpendicular** 垂直 to field lines 电场线 everywhere they cross.
- $\vec{E}$ points from high V to low V –never along an isoline. Moving a charge along an isoline takes no work.
- Closely spaced isolines mean a strong field: $E \approx -\Delta V/\Delta x$.
- You can sketch the field map from an isoline map, and the isoline map from a field map.



Equipotential lines around a dipole cross the field lines at right angles

Exam skill. Given a map with isolines every 10 V spaced about 2 cm apart, estimate $E \approx \dfrac{10}{0.02} = 500$ V/m, pointing from the higher-value line to the lower one. The work an external agent does moving a charge q slowly from A to B is $W = q(V_B - V_A)$ –the path taken does not matter.

High-voltage power lines: electric potential energy is converted and transmitted as current at high voltage

Conservation of Electric Energy

When a charge q moves between two points that differ in potential by ΔV, the potential energy of the charge–field system changes by

$$\Delta U_E = q\,\Delta V.$$

If only the electric force acts, total energy is conserved, so the kinetic energy changes by the opposite amount:

$$\Delta K = -\Delta U_E = -q\,\Delta V.$$

Watch the two signs together: a positive charge speeds up when it moves to *lower* potential, while a negative charge speeds up when it moves to *higher* potential. Both are just the system trading potential energy for kinetic energy. This is how particle accelerators give charged particles their energy, and it underlies energy analysis in circuits.

Worked example. A proton ($q = 1.6 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) starts at rest and is accelerated through a potential drop of 500 V (it moves to lower potential, so $\Delta V = -500$ V and $\Delta K = -q\,\Delta V = +8.0 \times 10^{-17}$ J). Setting $\Delta K = \frac{1}{2}mv^2$:

$$v = \sqrt{\frac{2(8.0 \times 10^{-17})}{1.67 \times 10^{-27}}} = 3.1 \times 10^5 \text{ m/s}.$$

Exam skill. Choose energy methods, not kinematics, whenever the field is non-uniform or the path curves: only the end-point potentials matter. A typical FRQ chain is $q\,\Delta V \to \Delta K \to v$, with one line of justification: "the electric force is conservative, so energy is conserved."

Exam tips

- Relate field and potential by $V = -\int \vec{E} \cdot d\vec{l}$ and $\vec{E} = -\nabla V$ (in 1-D, $E_x = -\frac{dV}{dx}$).
- Potential is a **scalar** —add contributions with signs, no vector components needed.
- Potential energy of a pair is $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$; use energy conservation for a charge's speed.
- Know that no work is done moving along an **equipotential**, which is perpendicular to $\vec{E}$.
- Choose the zero of potential (usually infinity) and state it.

Conductors and Capacitors

AP Physics C: Electricity and Magnetism

Electrostatics with Conductors

In an ideal **conductor** 导体, electrons move freely. Put extra charge on one and the charges repel each other until, almost instantly, they settle into **electrostatic equilibrium** 静电平衡. In that state the conductor has four properties you must be able to state and use:

- The electric field **inside** the conductor is zero. If it were not, free electrons would still be moving.
- All excess charge sits **on the surface**. (Negative net charge = extra electrons on the surface; positive = a shortage of electrons there.)
- The field just outside is **perpendicular** 垂直 to the surface, with magnitude $E = \sigma/\varepsilon_0$. Any parallel component would push charge sideways along the surface.
- The whole conductor is one **equipotential surface** 等势面: every point, inside and on the surface, is at the same potential.

Charge density is largest where the surface curves sharply –at points and edges – so the outside field is strongest there. In an external field a conductor **polarizes**: charge shifts on its surface so that the interior stays field-free and the body stays an equipotential. Surrounding a region with a closed conducting shell keeps outside fields out entirely –**electrostatic shielding** 静电屏蔽, the idea behind the **Faraday cage** 法拉第笼.

Redistribution of Charge Between Conductors

When two conductors touch (or are wired together), charge flows between them until both surfaces reach the **same potential** –that is the stopping condition, not "equal charges". A larger sphere holds more charge at the same potential ($V = kQ/R$), so it takes the larger share.

Worked example. A small sphere of radius R carrying +6.0 μC touches a distant sphere of radius $2R$, then they separate. Equal potentials require $\dfrac{kq_1}{R} = \dfrac{kq_2}{2R}$, so $q_2 = 2q_1$. With $q_1 + q_2 = 6.0$ μC: $q_1 = 2.0$ μC and $q_2 = 4.0$ μC.

Ground 接地 is an idealized reference at zero potential that can absorb or supply any amount of charge. Grounding a conductor while an external charge is nearby leaves the conductor with a net **induced charge** 感应电荷: the external field pushes charge of one sign to ground, and cutting the ground wire before removing the external charge traps the rest.

Capacitors

A **capacitor** 电容器 stores charge on two conductors separated by a gap: $+Q$ on one plate, $-Q$ on the other. Its **capacitance** 电容 relates the stored charge to the potential difference between the plates:

$$C = \frac{Q}{\Delta V}.$$

Capacitance depends only on geometry and the material in the gap –never on Q or ΔV themselves.

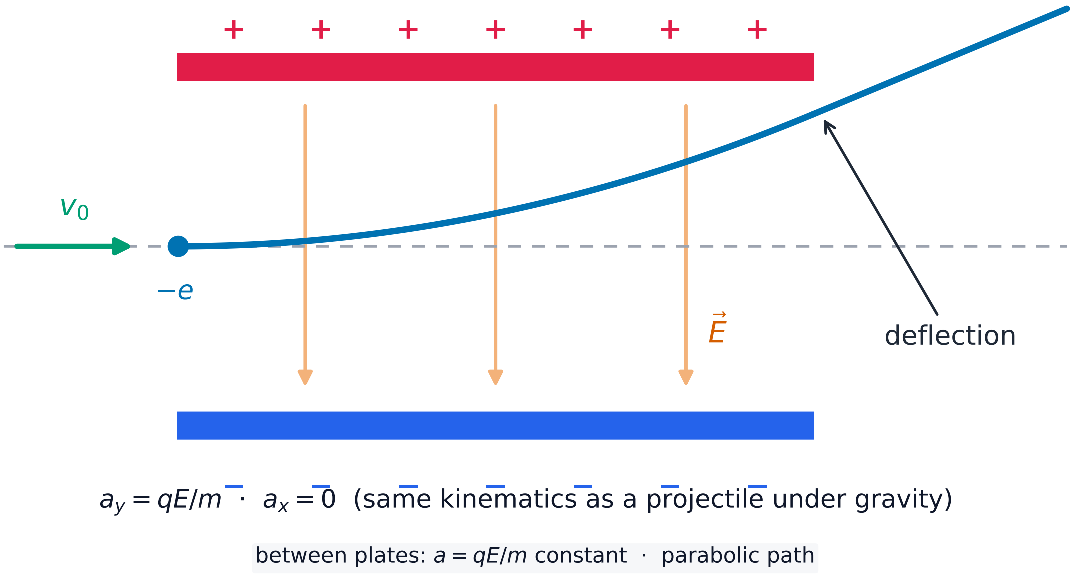
Worked derivation (parallel plates). For a **parallel-plate capacitor** 平行板电容器 with plate area A and small gap d: Gauss's law plus **superposition** 叠加 gives a **uniform** 均匀 field between the plates, $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$ (each plate alone contributes $\sigma/2\varepsilon_0$; between the plates the two add, outside they cancel). A uniform field means $\Delta V = Ed = \frac{Qd}{\varepsilon_0 A}$, so

$$C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}.$$

This $E \to \Delta V \to C$ chain is a standard FRQ derivation –learn it as three steps, and quote each one. The same method handles the other two shapes AP expects: concentric spheres ($C = 4\pi\varepsilon_0 \frac{ab}{b-a}$) and a coaxial cylinder of length L, where $E = \frac{\lambda}{2\pi\varepsilon_0 r}$ gives $\Delta V = \frac{\lambda}{2\pi\varepsilon_0} \ln\frac{b}{a}$ and so $C = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}$.

Worked example. Plates of area $A = 0.020\ \text{m}^2$ and gap $d = 1.0$ mm: $C = \frac{8.85 \times 10^{-12}(0.020)}{1.0 \times 10^{-3}} = 1.8 \times 10^{-10}$ F. Charged to 100 V it holds $Q = CV = 1.8 \times 10^{-8}$ C.

Because the field between the plates is uniform, a charged particle there feels a constant force, so it moves with constant acceleration –exactly like **projectile motion** 抛体运动 in gravity: constant speed across, uniform acceleration towards a plate.

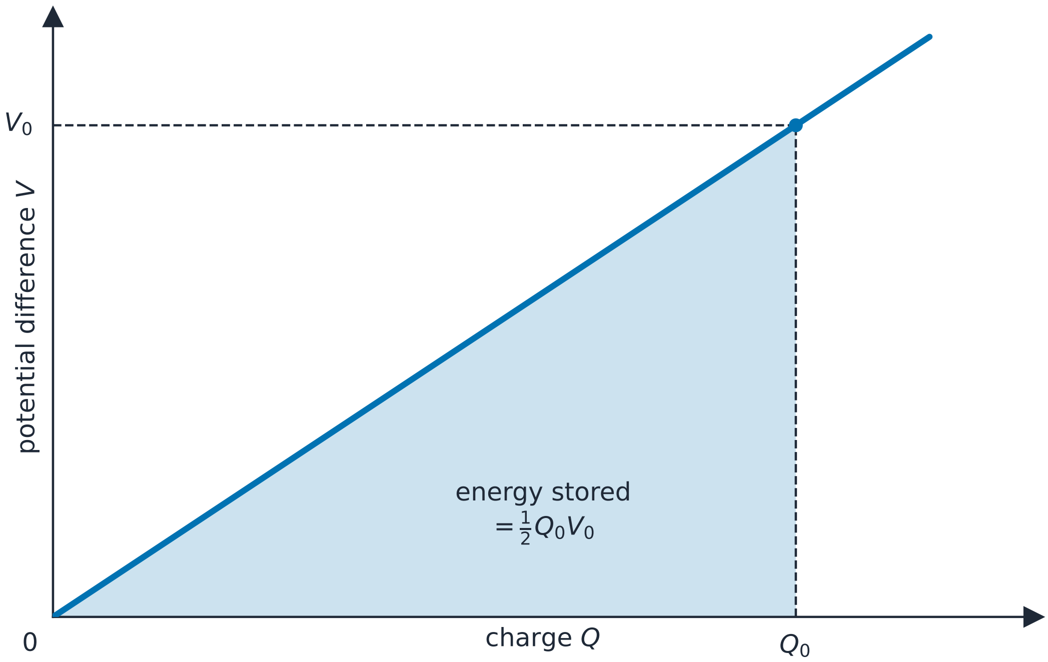


A charged particle between the plates follows a parabola, like a projectile

Worked example. An electron enters midway between plates at $v_0 = 2.0 \times 10^7$ m/s, parallel to them. The field is $E = 1.0 \times 10^3$ N/C and the plates are 4.0 cm long. Acceleration: $a = \dfrac{eE}{m} = \dfrac{(1.6 \times 10^{-19})(1.0 \times 10^3)}{9.11 \times 10^{-31}} = 1.8 \times 10^{14}$ m/s^2. Time between the plates: $t = \dfrac{0.040}{2.0 \times 10^7} = 2.0 \times 10^{-9}$ s. Deflection: $y = \frac{1}{2}at^2 = \frac{1}{2}(1.8 \times 10^{14})(2.0 \times 10^{-9})^2 \approx 3.5 \times 10^{-4}$ m –about 0.35 mm towards the positive plate.

Storing charge takes **work** 功: an external force must move each bit of charge against the field of the charge already there. The total work ends up as stored potential energy,

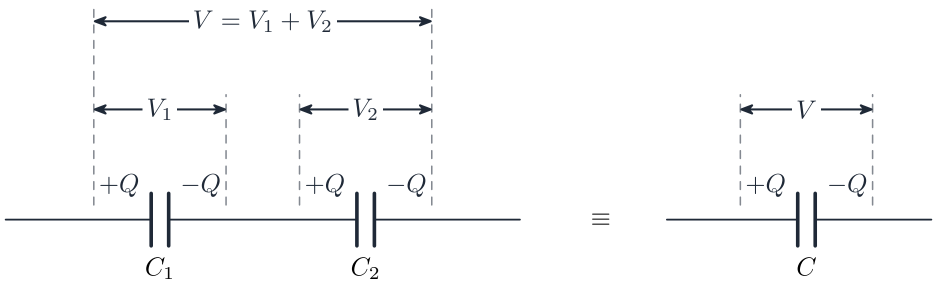
$$U_C = \tfrac{1}{2}Q\,\Delta V = \tfrac{1}{2}C(\Delta V)^2 = \frac{Q^2}{2C}.$$



The energy stored in a capacitor is the area under its charge-voltage line

The factor $\frac{1}{2}$ is the area of the triangle under the Q–ΔV line: the first charge moved across costs almost nothing, the last costs the full ΔV.

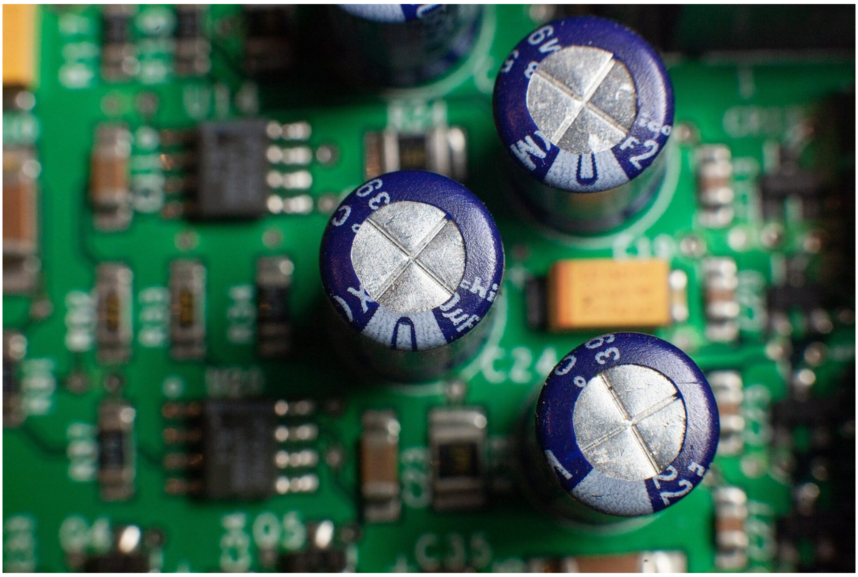
Capacitors combine oppositely to resistors: in **parallel** 并联 the capacitances add ($C_{\text{eq}} = C_1 + C_2$, same ΔV, charges add), while in **series** 串联 the reciprocals add ($\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$, same Q, voltages add).



Capacitors in series carry the same charge, and their p.d.s add

Worked example. A 2 μF and a 4 μF capacitor in series: $\frac{1}{C} = \frac{1}{2} + \frac{1}{4}$, so $C = \frac{4}{3}$ μF. The same pair in parallel: 6 μF.

Assorted capacitors: devices that store charge and energy in an electric field between conductors



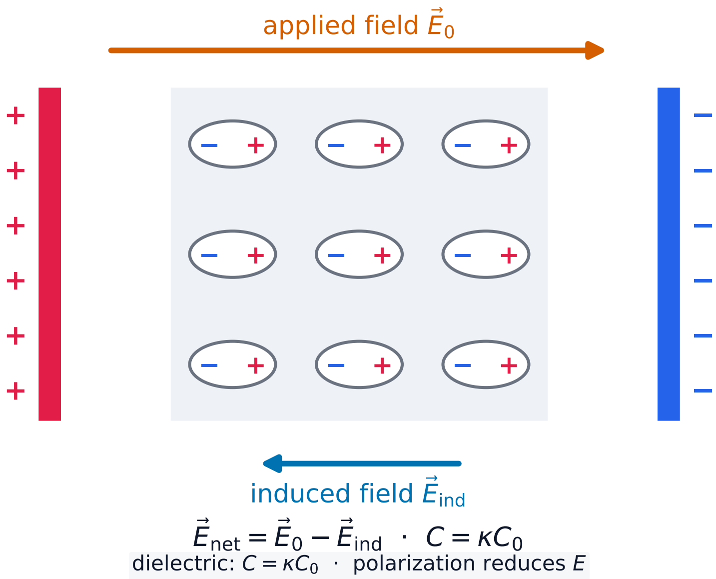
Electrolytic capacitors on a circuit board: capacitance stores energy as $U = ½CV^2$

Dielectrics

A **dielectric** 电介质 is an insulating material. Its charges cannot travel, but in an external field each molecule stretches or turns slightly –the material becomes **polarized** 极化. The lined-up molecules create their own small field *opposite* to the applied one, so the net field inside the material drops:

$$E = \frac{E_0}{\kappa}, \qquad \kappa = \frac{\varepsilon}{\varepsilon_0} \quad (\kappa > 1),$$

where κ is the **dielectric constant** 介电常数, the ratio of the material's permittivity to the **permittivity of free space** 真空介电常数.



A polarized dielectric creates an internal field opposing the applied field

Filling a capacitor with dielectric multiplies its capacitance:

$$C = \kappa\, C_0 \qquad \left(\text{parallel plates: } C = \frac{\kappa\varepsilon_0 A}{d}\right).$$

What happens next depends on what stays fixed –a favourite exam trap:

	battery stays connected (ΔV fixed)	**battery removed first (Q fixed)**
charge Q	rises to κQ_0	unchanged
voltage ΔV	unchanged	drops to $\Delta V_0/\kappa$
field E	unchanged	drops to E_0/κ
energy U	rises to κU_0	drops to U_0/κ

Worked example. A 100 pF capacitor is charged to 12 V, disconnected, then filled with a $\kappa = 3$ dielectric. Q is trapped, so ΔV falls to 4 V and the stored energy falls to one-third –the missing energy went into pulling the dielectric in. Reconnected to the 12 V battery instead, the capacitor would hold three times the charge and three times the energy.

Exam skill. Always begin dielectric questions by writing down which quantity is held fixed (Q or ΔV), then let $C = \kappa C_0$ drive everything else through $Q = C\Delta V$ and $U = \frac{1}{2}C(\Delta V)^2$.

Exam tips

- In electrostatic equilibrium a conductor has $\vec{E} = 0$ **inside** and all excess charge on the **surface**.
- Apply **Gauss's law** $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$ with a symmetry-matched Gaussian surface (sphere, cylinder, pillbox).
- The whole conductor is one **equipotential**, and the surface field is perpendicular to it.
- For a capacitor use $C = \frac{Q}{V}$, energy $U = \frac{1}{2}CV^2$, and how a **dielectric** raises C.
- Pick the Gaussian surface so $\vec{E}$ is constant and parallel (or zero) on each part.

Electric Circuits

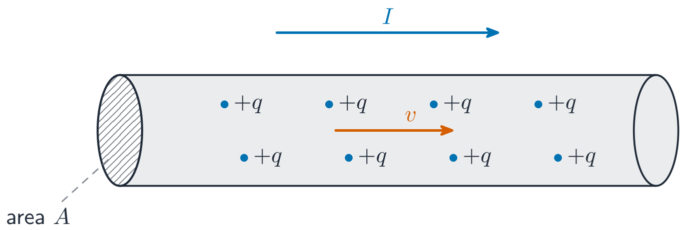
AP Physics C: Electricity and Magnetism

Electric Current

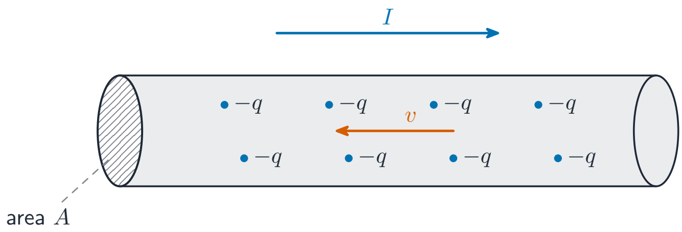
Electric current 电流 is the rate at which charge passes a cross-section of wire, $I = \frac{dq}{dt}$, measured in **amperes** 安培. Conventional current points the way *positive* charge would move. Microscopically, a current is a slow drift of many carriers:

$$I = nqv_dA,$$

with n the number of carriers per volume, q the charge each carries, v_d the **drift velocity** 漂移速度, and A the **cross-sectional area** 横截面积.



(a) positive carriers drift with I

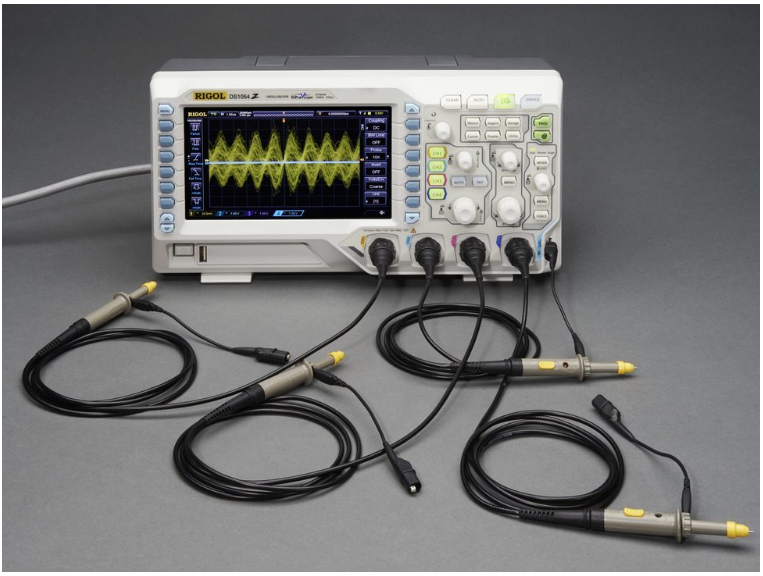


(b) electrons drift against I

Charge carriers drift slowly through a conductor to make a current

Worked example. A copper wire with $A = 1.0\times10^{-6}$ m^2 and $n = 8.5\times10^{28}$ m^{-3} carries 1.7 A. Then $v_d = \frac{I}{nqA} = \frac{1.7}{(8.5\times10^{28})(1.6\times10^{-19})(1.0\times10^{-6})} \approx 1.3\times10^{-4}$ m/s –the carriers drift slower than a snail, even though the signal travels near light speed.

Current density 电流密度 is charge flow per unit area, $\vec{J} = nq\vec{v}_d$, linked to the field driving it by $\vec{E} = \rho\vec{J}$. In general $I = \int \vec{J}\cdot d\vec{A}$; if $J(r)$ varies across the wire, integrate it over the cross-section to get the total current. One care point: current has a direction along its wire, but it is a **scalar** 标量–currents do not add as vectors, and there are no ”components of current”.



An oscilloscope: voltage against time reveals how current and charge evolve in a circuit

Electric Circuits

A circuit is a set of closed loops built from wires, batteries, resistors, lightbulbs, capacitors, inductors, switches, and meters; charge can flow only around a *closed* path. One element can belong to several loops at once –that is what makes multi-loop problems interesting. Every analysis starts by reading the **circuit diagram** 电路图: trace each loop and identify which elements share the same current (**series** 串联) and which share the same potential difference (**parallel** 并联).

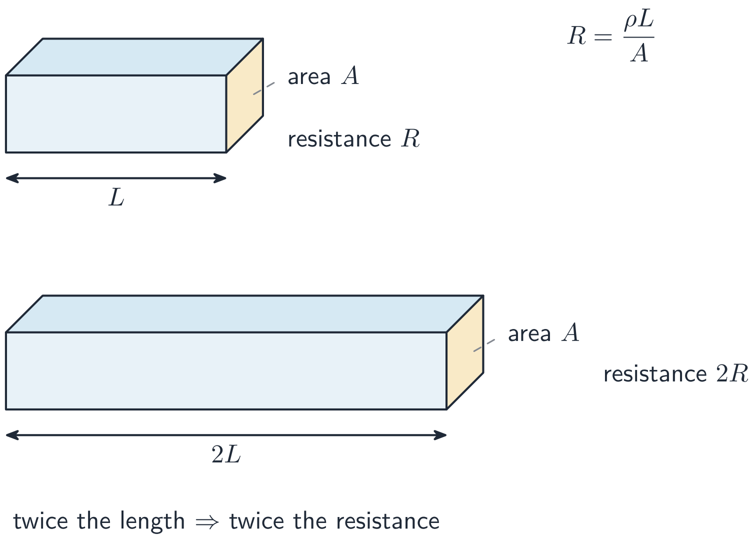
Resistance, Resistivity, and Ohm’s Law

Resistance 电阻 measures how strongly an object opposes charge flow. It grows with the material’s **resistivity** 电阻率 and the conductor’s length, and shrinks with its area:

$$R = \frac{\rho\,\ell}{A}.$$

Ohm’s law 欧姆定律 relates the current through an element to the potential difference across it:

$$I = \frac{\Delta V}{R}.$$



A longer conductor has more resistance; a wider one has less

Worked example. Stretch a wire to double its length: the volume is fixed, so the area halves, and $R = \rho\ell/A$ becomes $\rho(2\ell)/(A/2) = 4R$ –four times the resistance. An element is **ohmic 欧姆性** if R stays constant (a straight line through the origin on an I–ΔV graph); a lightbulb filament, which heats up, is not.

Electric Power

A charge q falling through a potential difference ΔV gives up energy $q\Delta V$, so the rate of energy transfer in an element is

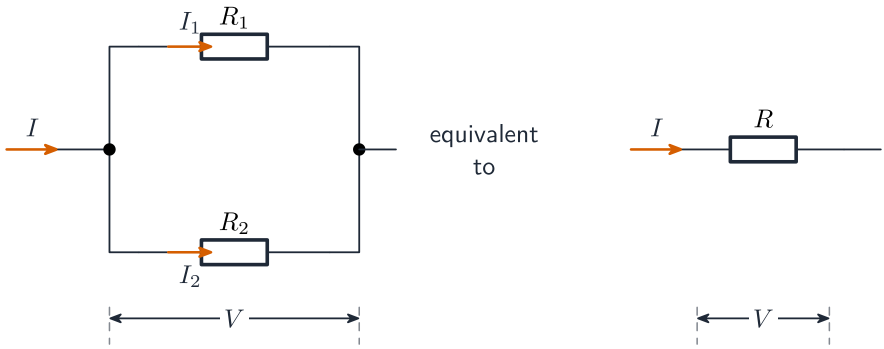
$$P = I\,\Delta V = I^2 R = \frac{(\Delta V)^2}{R}.$$

In a resistor all of it becomes heat. Use the form whose quantities you actually know –and use power to rank lightbulb brightness: brighter = more power, not necessarily more resistance. In series, the *larger* resistance is brighter ($P = I^2R$, same I); in parallel, the *smaller* one is ($P = \Delta V^2/R$, same ΔV).

Compound Direct Current Circuits

Reduce resistor networks to an **equivalent resistance** 等效电阻: **series** resistances add ($R_{eq} = R_1 + R_2 + \cdots$), while **parallel** resistances add as reciprocals ($\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$ –always less than the smallest branch). Collapse the network step by step to find the battery current, then expand back out to find each element's current and voltage.

Worked example. A 12 V battery drives a 4.0 Ω and a 2.0 Ω resistor in series: $R_{eq} = 6.0\ \Omega$, $I = 2.0$ A, the voltages split 8.0 V and 4.0 V, and the 4.0 Ω resistor dissipates $P = I^2R = 16$ W.



Resistors in parallel combine to a smaller equivalent resistance

Real batteries are not ideal. Model a battery as an **ideal battery** 理想电池 of emf ε in series with its own **internal resistance** 内阻 r. When current flows, some emf is used up inside, so the **terminal voltage** 端电压–what a voltmeter across the battery actually reads –drops:

$$\Delta V_{\text{terminal}} = \varepsilon - Ir.$$

Worked example. A battery with $\varepsilon = 12$ V and $r = 0.50\ \Omega$ supplies 2.0 A: the terminals sit at $\Delta V = 12 - 2.0(0.50) = 11$ V. With no current, a voltmeter reads the full 12 V.

Meters: an **ammeter** 电流表 goes *in series* at the point whose current you want (ideal ammeter: zero resistance); a **voltmeter** 电压表 goes *in parallel* across the element (ideal voltmeter: infinite resistance). Non-ideal meters disturb the circuit they measure –a real ammeter adds series resistance, a real voltmeter steals current.

Kirchhoff's Loop Rule

Charges moving through potential differences exchange energy ($\Delta U_E = q\Delta V$), and energy must balance around any closed path. That is **Kirchhoff's loop rule 基尔霍夫回路定则**:

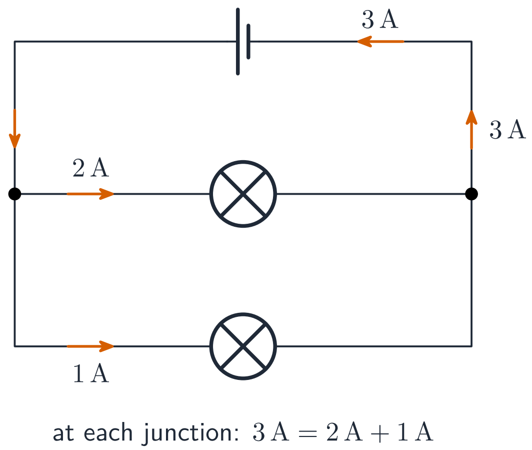
$$\sum \Delta V = 0 \text{ around any closed loop.}$$

Sign discipline wins these problems: crossing a battery from $-$ to $+$ is $+\varepsilon$; crossing a resistor *with* the assumed current is $-IR$ (against it, $+IR$). Write one equation per independent loop.

Kirchhoff's Junction Rule

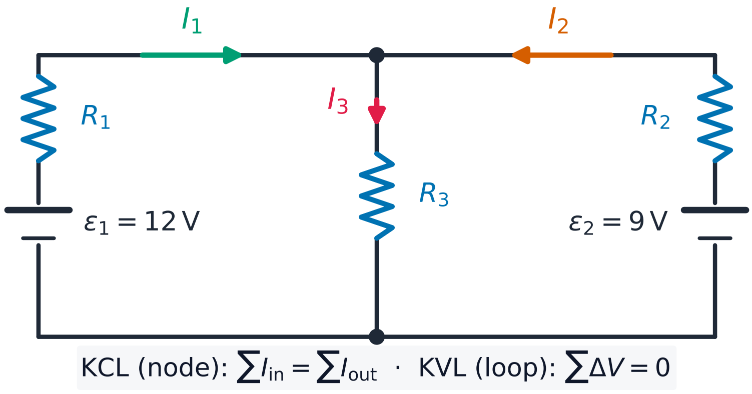
Kirchhoff's junction rule 基尔霍夫节点定则 is conservation of charge at a **junction 节点**:

$$\sum I_{\text{in}} = \sum I_{\text{out}}.$$



Current divides at a junction: what flows in equals what flows out

Together the two rules solve any multi-loop circuit: assign a current to each branch, write junction equations, then loop equations, and solve. A negative answer just means that current flows opposite to your assumed direction.



Two loop equations and one junction equation solve this two-battery circuit

Worked example. In the circuit above, $\varepsilon_1 = 12$ V with $R_1 = 1.0\ \Omega$ on the left, $\varepsilon_2 = 9.0$ V with $R_2 = 1.0\ \Omega$ on the right, and a shared middle resistor $R_3 = 2.0\ \Omega$ carrying $I_3 = I_1 + I_2$ (junction rule). The two loop equations are

$$12 = I_1 + 2(I_1 + I_2) = 3I_1 + 2I_2, \qquad 9 = I_2 + 2(I_1 + I_2) = 2I_1 + 3I_2.$$

Solving: $I_1 = 3.6$ A, $I_2 = 0.60$ A, so $I_3 = 4.2$ A through the middle. Check with the second loop: $2(3.6) + 3(0.60) = 9.0$.

Resistor-Capacitor (RC) Circuits

Capacitor networks reduce like resistors but with the rules swapped: parallel capacitances add ($C_{\text{eq}} = C_1 + C_2$), series add as reciprocals –and capacitors in series must carry the *same charge* on each plate, by conservation of charge. Use the **equivalent capacitance** 等效电容 to analyse the network, then expand back.

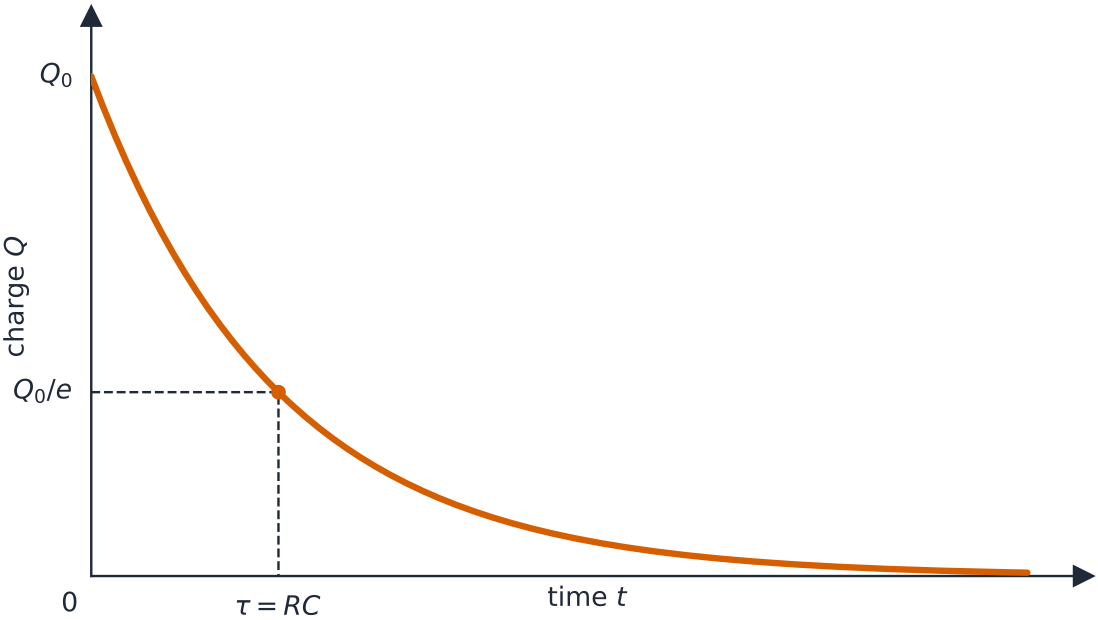
In an **RC circuit** RC 电路, Kirchhoff's loop rule gives the differential equation

$$\varepsilon = R\frac{dq}{dt} + \frac{q}{C},$$

whose solutions are exponentials with **time constant** 时间常数 $\tau = RC$:

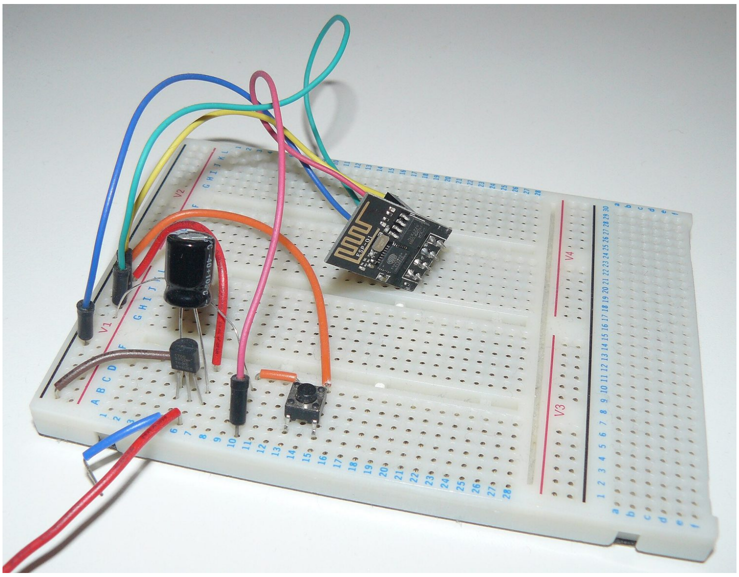
$$q(t) = Q\left(1 - e^{-t/RC}\right)\ \text{(charging)}, \qquad q(t) = Q\,e^{-t/RC}\ \text{(discharging)}, \qquad i(t) = \frac{\varepsilon}{R}e^{-t/RC}.$$

The current is largest at the first instant and decays –it never "waits" for the capacitor. Learn the two limits: at $t = 0$ an uncharged capacitor acts like a plain wire (maximum current); after a long time it is fully charged, no current flows in its branch, and it acts like a break. In any **steady state** 稳态, cover the capacitor branch with your finger, solve the resistor circuit, then read the capacitor's voltage from the element it sits across.



The charge on a capacitor decays exponentially as it discharges

Worked example. With $R = 5.0$ kΩ, $C = 200$ μF, and a 10 V battery: $\tau = RC = 1.0$ s; initial current $\frac{\varepsilon}{R} = 2.0$ mA; after one time constant the charge is $q = CV(1 - e^{-1}) \approx 1.3 \times 10^{-3}$ C, about 63% of full charge, and the current has fallen to 37% of its initial value.



A real circuit on a breadboard: a resistor and capacitor together set how fast the voltage rises and falls

Exam tips

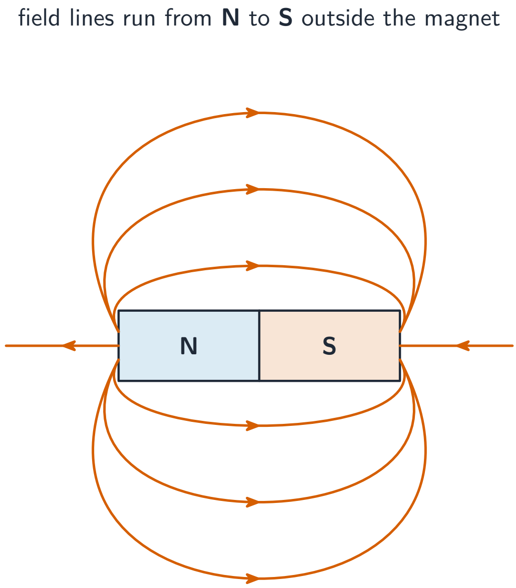
- Relate current to charge flow $I = \frac{dQ}{dt}$ and use $J = \sigma E$, $R = \frac{\rho L}{A}$ for resistance.
- Apply **Kirchhoff's laws** (junction: charge; loop: energy) with consistent sign conventions.
- Analyse **RC circuits** with calculus: charging/discharging give exponentials with time constant $\tau = RC$.
- Combine resistors (series add, parallel reciprocal) and track power $P = IV = I^2R$.
- At $t = 0$ a capacitor acts like a wire; after a long time ($t \to \infty$) it acts like an open branch.

Magnetic Fields and Electromagnetism

AP Physics C: Electricity and Magnetism

Magnetic Fields

A **magnetic field** 磁场 $\vec{B}$ is a **vector field** 矢量场 that surrounds magnets, moving charges, and currents; it determines the magnetic force on any moving charge placed in it. **Magnetic field lines** 磁感线 must form closed loops: they leave the north pole, return to the south pole, and continue through the magnet. Denser lines mean a stronger field.



Field lines run from N to S outside a bar magnet

Closed loops are the content of Gauss's law for magnetism, the second of **Maxwell's equations** 麦克斯韦方程组:

$$\oint \vec{B} \cdot d\vec{A} = 0.$$

The net magnetic flux through *any* closed surface is zero –as many field lines leave as enter. That is exactly the statement that isolated **magnetic monopoles** 磁单极子 do not exist. Every source of magnetism is a **magnetic dipole** 磁偶极子 (a north–south pair), made by circulating charge –in materials, the motion of electrons. Break a bar magnet in half and you get two smaller dipoles, never a lone pole. Like poles repel, opposite poles attract, and a free dipole –a **compass** 指南针–rotates to line up with the local field. Earth's own field is approximately a dipole, which is why a compass works. Dipole fields weaken with distance.

How a material responds to an external field depends on how its internal dipoles behave:

- **Ferromagnetic** 铁磁性 (iron, nickel, cobalt): an external field aligns whole **magnetic domains** 磁畴, and the alignment survives after the field is removed –a permanent magnet.
- **Paramagnetic** 顺磁性 (aluminum, titanium): dipoles align weakly with the field but relax when it is removed.
- **Diamagnetic** 抗磁性 (all materials): the electronic structure produces a usually weak alignment *opposite* to the field.

The strength of a material's response is its **magnetic permeability** 磁导率. Free space has the constant value μ_0 (the vacuum permeability); the permeability of matter differs from μ_0 and is not even constant –it varies with temperature, orientation, and field strength.

An aurora: charged particles from the Sun are steered by Earth's magnetic field toward the poles

Magnetism and Moving Charges

A moving charge does two things: it *creates* a magnetic field, and it *feels* a force in an external one. The field it creates at a point is **perpendicular 垂直** to both its velocity and the position vector from charge to point (right-hand rule again), and is largest where those two are perpendicular –zero directly ahead of or behind the motion.

The force on a charge moving in a field $\vec{B}$ is the cross product

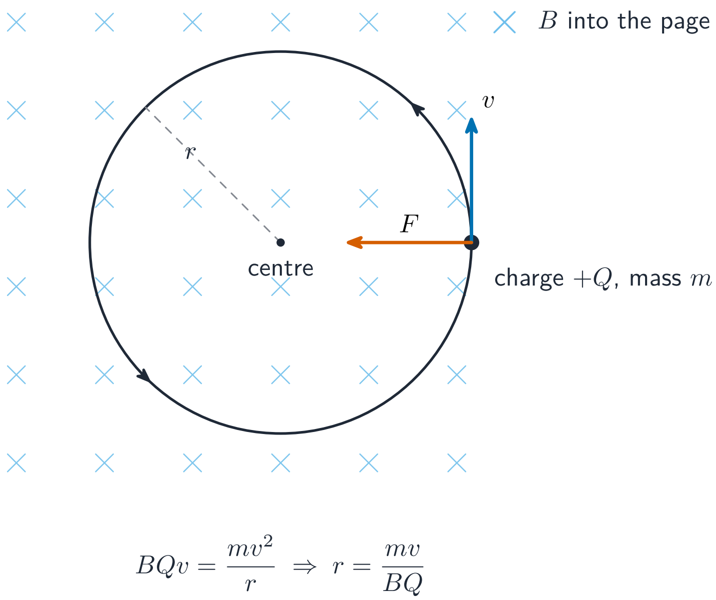
$$\vec{F}_B = q\,\vec{v} \times \vec{B}, \qquad F = qvB\sin\theta,$$

perpendicular to both $\vec{v}$ and $\vec{B}$ –point the fingers of your right hand along $\vec{v}$, curl toward $\vec{B}$, and the thumb gives the force on a *positive* charge (reverse it for negative). Because $\vec{F}_B \perp \vec{v}$, the magnetic force does no work on the charge: it changes direction, never speed.

A charge moving perpendicular to a uniform field therefore travels in a circle, the magnetic force supplying the **centripetal force 向心力**:

$$qvB = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{qB}, \qquad T = \frac{2\pi m}{qB}.$$

Notice the period T does not depend on the speed –faster particles ride larger circles in the same time.



A charged particle moving across a magnetic field follows a circular path

Worked example. A proton ($q = 1.6\times10^{-19}$ C, $m = 1.67\times10^{-27}$ kg) enters a 0.50 T field at 2.0×10^5 m/s, perpendicular to it. The magnetic force $F = qvB = 1.6\times10^{-14}$ N bends it into a circle of radius $r = \dfrac{mv}{qB} = \dfrac{1.67 \times 10^{-27}(2.0 \times 10^5)}{1.6 \times 10^{-19}(0.50)} = 4.2 \times 10^{-3}$ m.

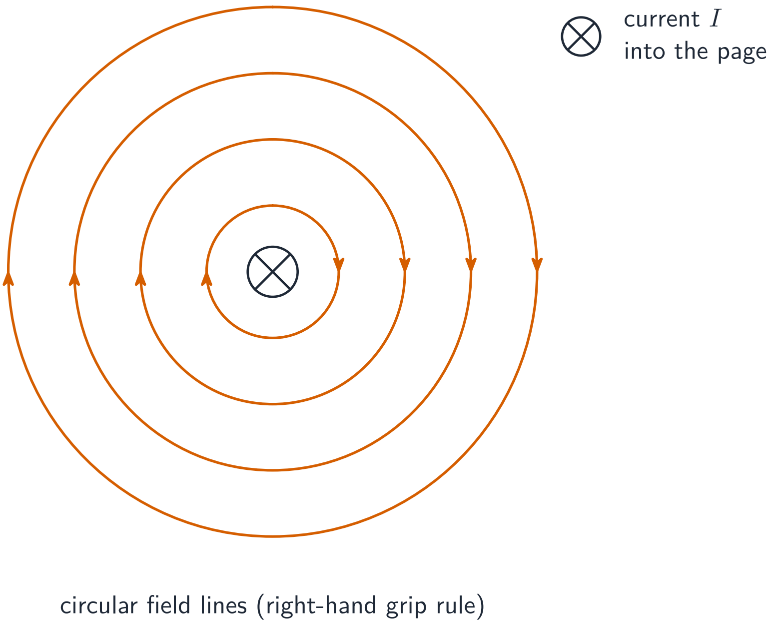
In a region with both an electric and a magnetic field, the two forces act independently and add as vectors. Balancing them makes a **velocity selector** 速度选择器: with $\vec{E}$ and $\vec{B}$ crossed, only charges with $qE = qvB$, i.e. $v = E/B$, pass straight through. The **Hall effect** 霍尔效应 is the same physics inside a conductor: a field with a component perpendicular to the current pushes the moving carriers sideways, charge builds up on one face, and a measurable potential difference appears across the conductor –its sign reveals whether the carriers are positive or negative.

Magnetic Fields of Current-Carrying Wires

A current is a stream of moving charges, so it creates a magnetic field. The **Biot–Savart law** 毕奥-萨伐尔定律 adds up the field of each current element:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}.$$

Around any straight segment the field vectors are **tangent** 相切 to **concentric circles** 同心圆 centred on the wire –no component toward, away from, or along the wire. Curl your right hand around the wire with the thumb along the current: your fingers give the field direction.



Concentric circular field lines surround a straight current-carrying wire

The Biot–Savart integrals AP expects: a long straight wire ($B = \dfrac{\mu_0 I}{2\pi r}$, or a point on the perpendicular bisector of a finite wire), and the centre of a circular loop,

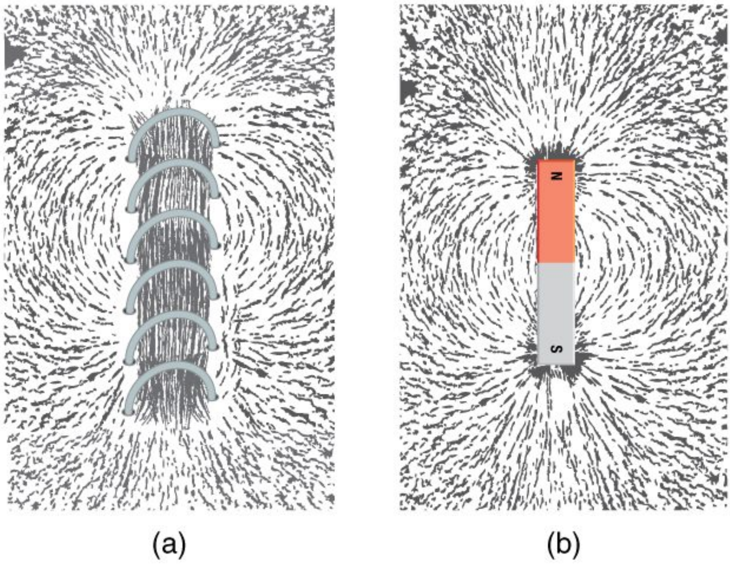
$$B_{\text{centre of loop}} = \frac{\mu_0 I}{2R},$$

where every element $d\vec{l}$ is perpendicular to $\hat{r}$ and equally distant R –say that in an FRQ derivation. An arc that is a fraction of a full circle contributes that same fraction of $\mu_0 I/2R$ at its centre.

A field also pushes on a current-carrying wire, element by element:

$$\vec{F}_B = \int I\, d\vec{l} \times \vec{B} \qquad (\vec{F} = I\vec{L} \times \vec{B} \text{ for a straight wire in a uniform field}).$$

Worked example. Two long parallel wires a distance $d = 0.10$ m apart each carry 5.0 A in the same direction. Wire 1's field at wire 2 is $B = \dfrac{\mu_0 I}{2\pi d} = 1.0 \times 10^{-5}$ T, so wire 2 feels $\dfrac{F}{L} = IB = 5.0 \times 10^{-5}$ N/m, pulled *toward* wire 1. Same-direction currents attract; opposite currents repel.



A current-carrying coil (solenoid) makes a magnetic field shaped just like a bar magnet's

Ampère's Law

For symmetric current distributions, **Ampère's law** 安培定律 finds the field far faster than Biot–Savart:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}.$$

Choose an **Amperian loop** 安培环路 that matches the symmetry, so that B is constant along the loop (and parallel to it) and comes out of the integral. AP applies it to long straight wires, long solenoids, and conductors carrying a **current density** 电流密度 (slabs and cylinders); combinations are handled by **superposition** 叠加.

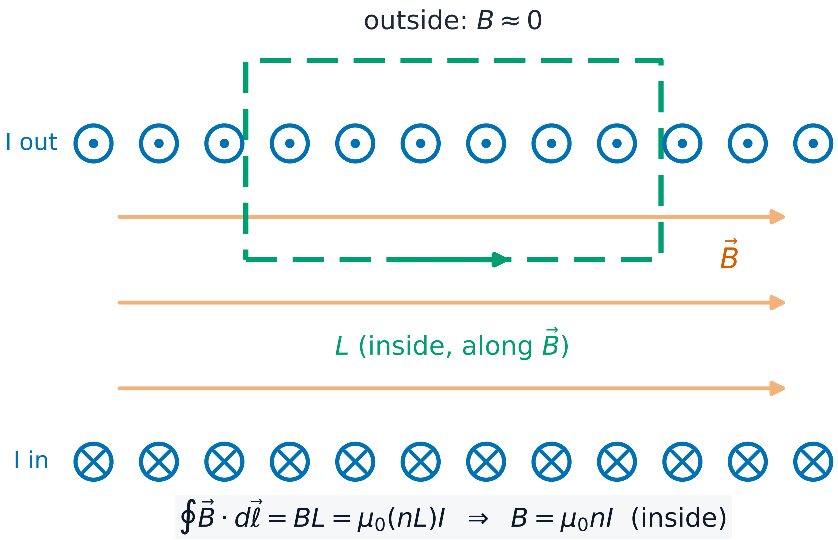
Worked example. Find the field 0.10 m from a long wire carrying 5.0 A. A circular loop of radius r shares the field's symmetry, so $B(2\pi r) = \mu_0 I$ and

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7})(5.0)}{2\pi(0.10)} = 1.0 \times 10^{-5} \text{ T}.$$

A long **solenoid** 螺线管 (assumed: uniform field inside, negligible field outside) is the other classic. Take a rectangular loop with one side of length L inside, parallel to the axis: only that side contributes to the integral, so $BL = \mu_0(nL)I$ and

$$B_{\text{sol}} = \mu_0 n I,$$

with n the turns per metre.



A rectangular Amperian loop derives the uniform field inside a solenoid

Worked example (cylinder). A solid cylindrical conductor of radius R carries current I spread uniformly over its cross-section. Inside ($r < R$), a circular loop encloses $I_{\text{enc}} = I\dfrac{r^2}{R^2}$, so $B = \dfrac{\mu_0 I r}{2\pi R^2}$ –the field grows linearly to the surface, then falls off as $1/r$ outside.

Exam skill. Every Ampère's-law answer earns its marks in the setup: name the loop, state *why* B is constant and parallel to it (symmetry), and count I_{enc} carefully before you solve. Maxwell's fourth equation adds one more idea you should know qualitatively: a *changing electric field* also creates a magnetic field –but AP will not ask you to compute with that term.

Exam tips

- The magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ is **perpendicular** to velocity —use the right-hand rule and note it does **no work**.
- A charge in a uniform field moves in a circle with $r = \frac{mv}{qB}$.
- Find the field of currents with **Biot–Savart** $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}$ or **Ampère's law** $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ when there is symmetry.
- Force on a wire is $\vec{F} = I\vec{L} \times \vec{B}$; keep the cross-product direction straight.
- Distinguish the force **on** a charge from the field the current **creates**.

Electromagnetic Induction

AP Physics C: Electricity and Magnetism

Magnetic Flux

Magnetic flux 磁通量 measures how much magnetic field passes through a surface. For a uniform field through a flat loop it is the dot product $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$; in general it is the **surface integral** 面积分

$$\Phi_B = \int \vec{B} \cdot d\vec{A}.$$

The **area vector** 面积矢量 is perpendicular to the surface (outward from a closed one), and the sign of the flux comes from the dot product. Flux changes if B changes, the area changes, or the loop turns –keep all three routes in mind, because each one is an exam question.

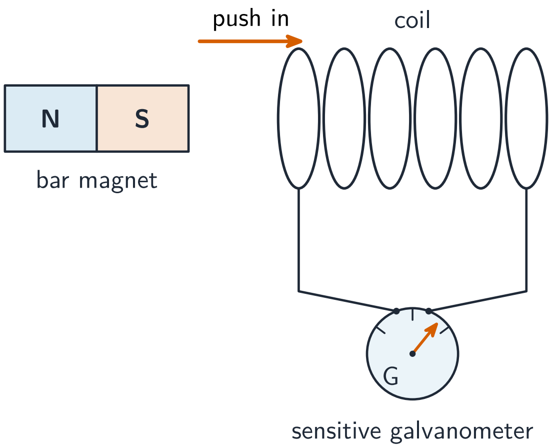
Electromagnetic Induction

A **changing** flux induces an **emf** 电动势–**Faraday's law** 法拉第定律:

$$\varepsilon = -\frac{d\Phi_B}{dt}.$$

With constant area, $\varepsilon = -A\dfrac{dB_\perp}{dt}$; with constant field, $\varepsilon = -B\dfrac{dA_\perp}{dt}$. A coil of N turns multiplies the single-loop emf: $|\varepsilon_{\text{sol}}| = N\left|\dfrac{d\Phi_B}{dt}\right|$.

Lenz's law 楞次定律 is the minus sign: the **induced current** 感应电流 flows so that its own magnetic field *opposes the change* in flux that created it. Push a magnet toward a loop and the loop pushes back; pull it away and the loop pulls it in. Use the **right-hand rule** 右手定则 to turn "oppose the change" into a current direction.

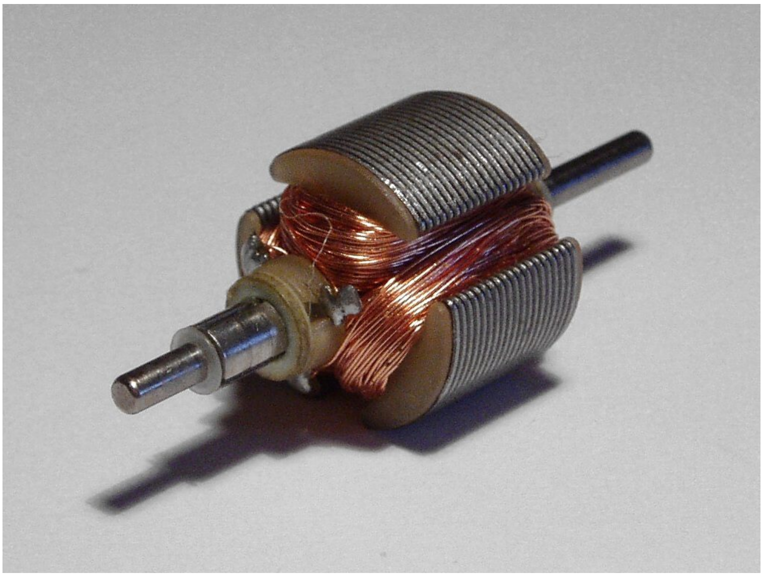


Moving a magnet into a coil induces an e.m.f. that drives a current

A rod of length L sliding at speed v across a field is the special case worth memorising –the **motional emf** 动生电动势 $\varepsilon = BLv$.

Worked example. A 0.20 m rod slides at 3.0 m/s across a 0.50 T field: $\varepsilon = BLv = 0.50(0.20)(3.0) = 0.30$ V. Equivalently, if a single loop's flux drops from 0.020 Wb to 0.008 Wb in 0.030 s, the average emf is $\varepsilon = \dfrac{0.012}{0.030} = 0.40$ V.

Faraday's law is also the third of **Maxwell's equations** 麦克斯韦方程组, in a deeper form: a changing magnetic flux creates a circulating *electric field*, $\oint \vec{E}\cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$ – that field is what pushes the charges around the loop. Together, Maxwell's equations predict **electromagnetic waves** 电磁波 travelling at $c = 1/\sqrt{\varepsilon_0\mu_0}$ (you should know this connection, but AP will not ask you to derive it).

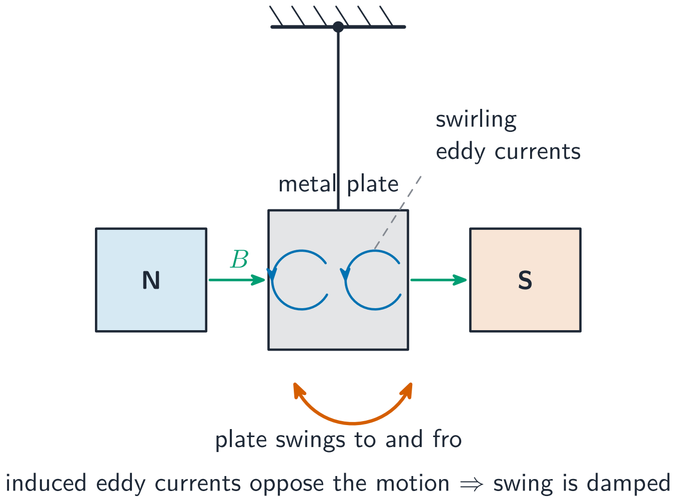


Coils of wire spinning in a magnetic field: moving them past the field induces a current, the basis of a generator

A substation transformer: changing magnetic flux in coils induces the voltages that power the grid

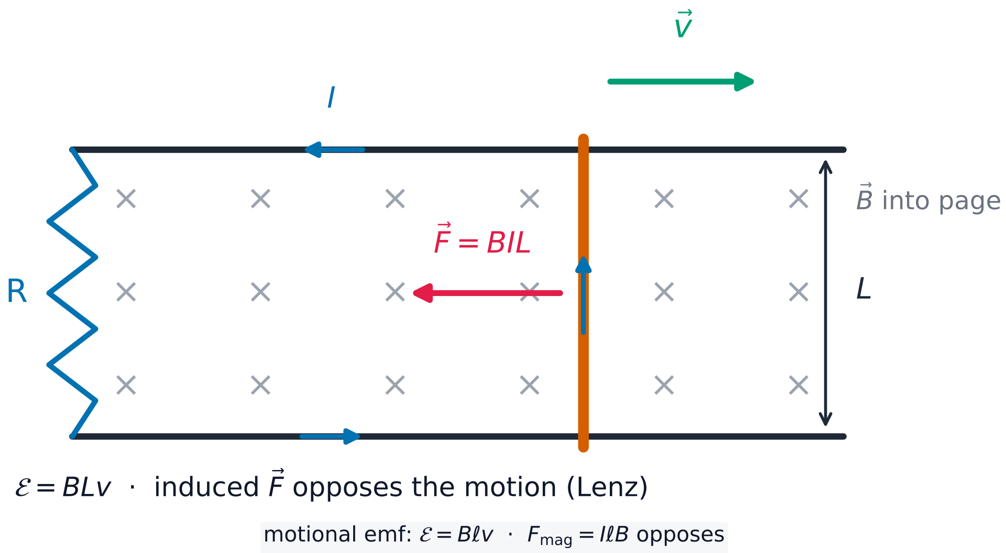
Induced Currents and Magnetic Forces

Once an induced current flows, the external field exerts forces on it ($\vec{F} = \int I\, d\vec{l} \times \vec{B}$) – and by Lenz's law those forces always *resist the motion* that causes the induction. Only the segments of the loop actually inside the field feel a force, which can make a loop accelerate, rotate, or brake. This is the origin of **eddy currents** 涡电流 braking, and you can apply Newton's second law to a moving loop or rod like any other mechanics problem.



Induced eddy currents oppose motion, quickly damping a metal plate swinging in a field

The classic setup is a rod sliding on conducting rails connected by a resistor:



A rod sliding on rails: the induced current feels a force opposing the motion

Worked example (the full chain). Rails $L = 0.20$ m apart with resistance $R = 0.60\ \Omega$ sit in a 0.50 T field into the page. The rod is pushed at a constant 3.0 m/s. Then: $\varepsilon = BLv = 0.30$ V; $I = \varepsilon/R = 0.50$ A; the field pushes back on the rod with $F = BIL = 0.50(0.50)(0.20) = 0.050$ N. The pushing force does work at $P = Fv = 0.15$ W –exactly the $P = I^2R = 0.15$ W dissipated in the resistor. Mechanical work becomes electrical energy: that is a **generator** 发电机, and energy is conserved. Released with no push, the rod slows exponentially: $ma = -\frac{B^2L^2}{R}v$.

Exam skill. FRQs walk this exact chain: flux → emf → current → force → Newton's second law. Write each link separately and check the direction with Lenz's law at the end.

Inductance

Inductance 电感 is a conductor's tendency to oppose a change in its own current: changing current changes its own flux, which self-induces an emf. From Faraday's law,

$$\varepsilon = -L\frac{dI}{dt}.$$

An **inductor** 电感器 is a circuit element built to have large inductance –usually a **solenoid** 螺线管, where geometry gives

$$L_{\text{sol}} = \frac{\mu_{\text{core}}N^2A}{\ell},$$

with N total turns, area A, length ℓ, and the **magnetic permeability** 磁导率 of the core. (Straight wires are modelled as having zero inductance.) An inductor carrying current I stores energy in its magnetic field:

$$U_L = \tfrac{1}{2}LI^2,$$

which can later be dissipated in a resistor or moved into a capacitor –conservation of energy applies as usual.

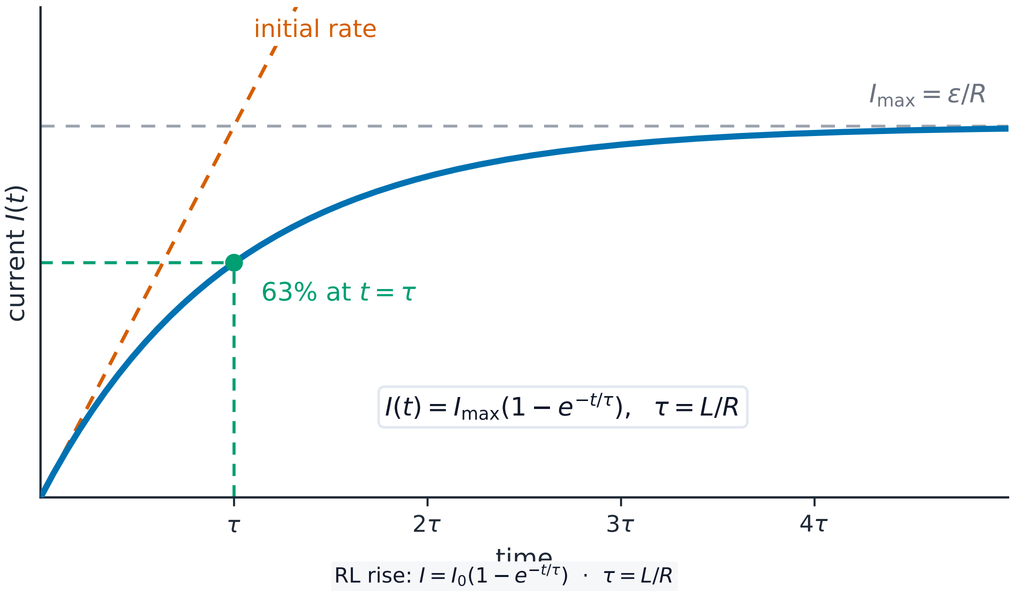
Circuits with Resistors and Inductors

In an **RL circuit** RL 电路, Kirchhoff's loop rule for a battery ε, resistor R, and inductor L in series gives a **differential equation** 微分方程:

$$\varepsilon = IR + L\frac{dI}{dt} \quad\Rightarrow\quad I(t) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right), \qquad \tau = \frac{L}{R}.$$

The **time constant** 时间常数 τ sets the pace: after one τ a rising current reaches about 63% of its final value (a decaying one falls to 37%); it is also how long the change *would* take at the initial rate. Learn the two limits –they answer most conceptual questions:

- **Just after the switch closes** ($t = 0$): current cannot jump, so the inductor momentarily blocks it, self-inducing an emf equal and opposite to the applied potential difference.
- **Long after** ($t \gg \tau$): the current is steady, $dI/dt = 0$, and the inductor behaves as a plain wire –the exact opposite of a capacitor.



The current in an RL circuit rises exponentially, reaching 63% at one time constant

Worked example. $\varepsilon = 12$ V, $R = 6.0\ \Omega$, $L = 3.0$ H: final current $\varepsilon/R = 2.0$ A, $\tau = L/R = 0.50$ s. At $t = 0.50$ s the current is $2.0(1 - e^{-1}) \approx 1.3$ A. At $t = 0$ the inductor's potential difference is the full 12 V; as $t \to \infty$ it falls to zero. Current, inductor voltage, and stored energy are all exponential in time.

Circuits with Capacitors and Inductors

An **LC circuit** LC 电路 has no resistance, so nothing dissipates energy: it **oscillates** 振荡, sloshing energy between the capacitor's electric field and the inductor's magnetic field. The loop rule gives

$$\frac{d^2q}{dt^2} = -\frac{1}{LC}\,q,$$

the same equation as a mass on a spring –**simple harmonic motion** 简谐运动 with charge playing the role of displacement and

$$\omega = \frac{1}{\sqrt{LC}}.$$

The total energy $\dfrac{q^2}{2C} + \frac{1}{2}LI^2$ stays constant: all in the capacitor at maximum charge, all in the inductor at maximum current.

Worked example. $L = 2.0$ H, $C = 8.0$ μF: $\omega = \dfrac{1}{\sqrt{2.0(8.0 \times 10^{-6})}} = 250$ rad/s, period $T = 2\pi/\omega \approx 0.025$ s. If the capacitor starts charged to 12 V, then $U = \frac{1}{2}CV^2 = 5.8 \times 10^{-4}$ J, and the maximum current follows from $\frac{1}{2}LI_{\text{max}}^2 = U$: $I_{\text{max}} = \sqrt{2U/L} = 0.024$ A –**conservation of energy 能量守恒**, no calculus needed.

Exam tips

- Compute **flux** $\Phi_B = \int \vec{B} \cdot d\vec{A}$ and get the induced EMF from **Faraday's law** $\varepsilon = -\frac{d\Phi_B}{dt}$.
- Use **Lenz's law** (the minus sign) to fix the direction: the induced current opposes the change in flux.
- Flux changes three ways —changing B, changing area, or changing angle —identify which and differentiate.
- For a rod of length L moving at speed v, the motional EMF is BLv.
- An inductor stores energy $\frac{1}{2}LI^2$ and resists **changes** in current (RL time constant $\tau = L/R$).