

# Electromagnetic Induction

AP Physics C: Electricity and Magnetism

## Magnetic Flux

**Magnetic flux** 磁通量 measures how much magnetic field passes through a surface. For a uniform field through a flat loop it is the dot product  $\Phi_B = \vec{B} \cdot \vec{A} = BA \cos \theta$ ; in general it is the **surface integral** 面积分

$$\Phi_B = \int \vec{B} \cdot d\vec{A}.$$

The **area vector** 面积矢量 is perpendicular to the surface (outward from a closed one), and the sign of the flux comes from the dot product. Flux changes if  $B$  changes, the area changes, or the loop turns – keep all three routes in mind, because each one is an exam question.

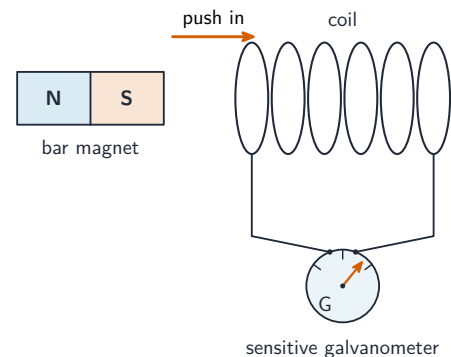
## Electromagnetic Induction

A **changing** flux induces an **emf** 电动势 – **Faraday’s law** 法拉第定律:

$$\varepsilon = -\frac{d\Phi_B}{dt}.$$

With constant area,  $\varepsilon = -A \frac{dB_{\perp}}{dt}$ ; with constant field,  $\varepsilon = -B \frac{dA_{\perp}}{dt}$ . A coil of  $N$  turns multiplies the single-loop emf:  $|\varepsilon_{\text{sol}}| = N \left| \frac{d\Phi_B}{dt} \right|$ .

**Lenz’s law** 楞次定律 is the minus sign: the **induced current** 感应电流 flows so that its own magnetic field *opposes the change* in flux that created it. Push a magnet toward a loop and the loop pushes back; pull it away and the loop pulls it in. Use the **right-hand rule** 右手定则 to turn “oppose the change” into a current direction.

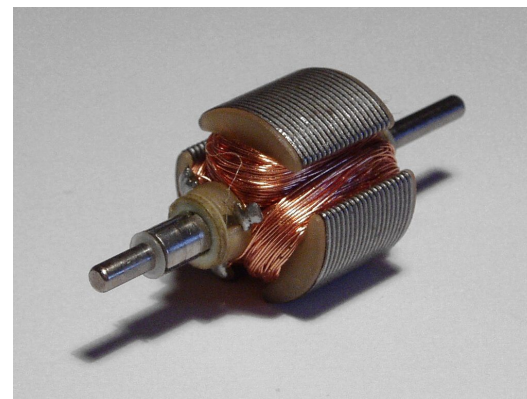


*Moving a magnet into a coil induces an e.m.f. that drives a current*

A rod of length  $L$  sliding at speed  $v$  across a field is the special case worth memorising – the **motional emf** 动生电动势  $\varepsilon = BLv$ .

**Worked example.** A 0.20 m rod slides at 3.0 m/s across a 0.50 T field:  $\varepsilon = BLv = 0.50(0.20)(3.0) = 0.30$  V. Equivalently, if a single loop’s flux drops from 0.020 Wb to 0.008 Wb in 0.030 s, the average emf is  $\varepsilon = \frac{0.012}{0.030} = 0.40$  V.

Faraday’s law is also the third of **Maxwell’s equations** 麦克斯韦方程组, in a deeper form: a changing magnetic flux creates a circulating *electric field*,  $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$  – that field is what pushes the charges around the loop. Together, Maxwell’s equations predict **electromagnetic waves** 电磁波 travelling at  $c = 1/\sqrt{\varepsilon_0\mu_0}$  (you should know this connection, but AP will not ask you to derive it).



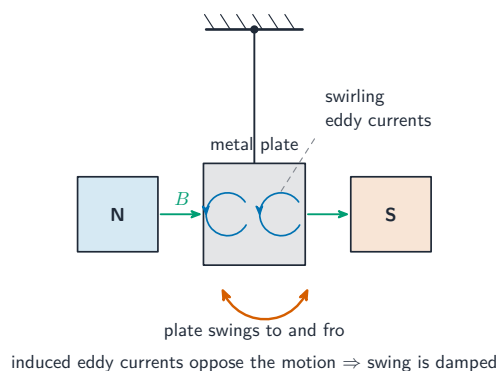
*Coils of wire spinning in a magnetic field: moving them past the field induces a current, the basis of a generator*



A substation transformer: changing magnetic flux in coils induces the voltages that power the grid

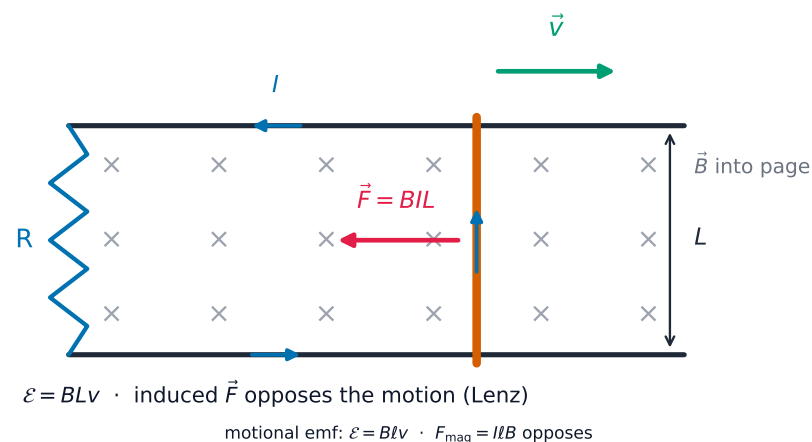
## Induced Currents and Magnetic Forces

Once an induced current flows, the external field exerts forces on it ( $\vec{F} = \int I d\vec{l} \times \vec{B}$ ) – and by Lenz’s law those forces always *resist the motion* that causes the induction. Only the segments of the loop actually inside the field feel a force, which can make a loop accelerate, rotate, or brake. This is the origin of **eddy currents** 涡电流 braking, and you can apply Newton’s second law to a moving loop or rod like any other mechanics problem.



Induced eddy currents oppose motion, quickly damping a metal plate swinging in a field

The classic setup is a rod sliding on conducting rails connected by a resistor:



A rod sliding on rails: the induced current feels a force opposing the motion

**Worked example (the full chain).** Rails  $L = 0.20$  m apart with resistance  $R = 0.60 \Omega$  sit in a  $0.50$  T field into the page. The rod is pushed at a constant  $3.0$  m/s. Then:  $\varepsilon = BLv = 0.30$  V;  $I = \varepsilon/R = 0.50$  A; the field pushes back on the rod with  $F = BIL = 0.50(0.50)(0.20) = 0.050$  N. The pushing force does work at  $P = Fv = 0.15$  W – exactly the  $P = I^2R = 0.15$  W dissipated in the resistor. Mechanical work becomes electrical energy: that is a **generator** 发电机, and energy is conserved.

Released with no push, the rod slows exponentially:  $ma = -\frac{B^2L^2}{R}v$ .

**Exam skill.** FRQs walk this exact chain: flux  $\rightarrow$  emf  $\rightarrow$  current  $\rightarrow$  force  $\rightarrow$  Newton’s second law. Write each link separately and check the direction with Lenz’s law at the end.

## Inductance

**Inductance** 电感 is a conductor’s tendency to oppose a change in its own current: changing current changes its own flux, which self-induces an emf. From Faraday’s law,

$$\varepsilon = -L \frac{dI}{dt}.$$

An **inductor** 电感器 is a circuit element built to have large inductance – usually a **solenoid** 螺线管, where geometry gives

$$L_{\text{sol}} = \frac{\mu_{\text{core}} N^2 A}{\ell},$$

with  $N$  total turns, area  $A$ , length  $\ell$ , and the **magnetic permeability** 磁导率 of the core. (Straight wires are modelled as having zero inductance.) An inductor carrying current  $I$  stores energy in its magnetic field:

$$U_L = \frac{1}{2} L I^2,$$

which can later be dissipated in a resistor or moved into a capacitor – conservation of energy applies as usual.

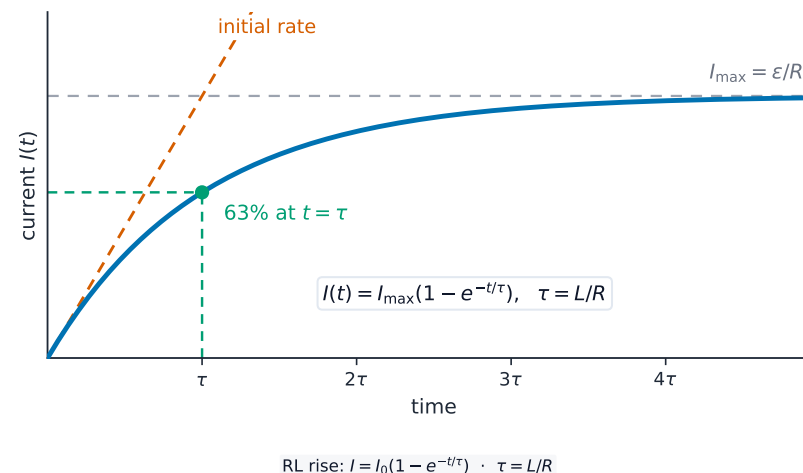
## Circuits with Resistors and Inductors

In an **RL circuit** RL 电路, Kirchhoff's loop rule for a battery  $\varepsilon$ , resistor  $R$ , and inductor  $L$  in series gives a **differential equation** 微分方程:

$$\varepsilon = IR + L \frac{dI}{dt} \Rightarrow I(t) = \frac{\varepsilon}{R} (1 - e^{-t/\tau}), \quad \tau = \frac{L}{R}.$$

The **time constant** 时间常数  $\tau$  sets the pace: after one  $\tau$  a rising current reaches about 63% of its final value (a decaying one falls to 37%); it is also how long the change *would* take at the initial rate. Learn the two limits – they answer most conceptual questions:

- **Just after the switch closes** ( $t = 0$ ): current cannot jump, so the inductor momentarily blocks it, self-inducing an emf equal and opposite to the applied potential difference.
- **Long after** ( $t \gg \tau$ ): the current is steady,  $dI/dt = 0$ , and the inductor behaves as a plain wire – the exact opposite of a capacitor.



The current in an RL circuit rises exponentially, reaching 63% at one time constant

**Worked example.**  $\varepsilon = 12$  V,  $R = 6.0$   $\Omega$ ,  $L = 3.0$  H: final current  $\varepsilon/R = 2.0$  A,  $\tau = L/R = 0.50$  s. At  $t = 0.50$  s the current is  $2.0(1 - e^{-1}) \approx 1.3$  A. At  $t = 0$  the inductor's potential difference is the full 12 V; as  $t \rightarrow \infty$  it falls to zero. Current, inductor voltage, and stored energy are all exponential in time.

## Circuits with Capacitors and Inductors

An **LC circuit** LC 电路 has no resistance, so nothing dissipates energy: it **oscillates** 振荡, sloshing energy between the capacitor's electric field and the inductor's magnetic field. The loop rule gives

$$\frac{d^2 q}{dt^2} = -\frac{1}{LC} q,$$

the same equation as a mass on a spring – **simple harmonic motion** 简谐运动 with charge playing the role of displacement and

$$\omega = \frac{1}{\sqrt{LC}}.$$

The total energy  $\frac{q^2}{2C} + \frac{1}{2} L I^2$  stays constant: all in the capacitor at maximum charge, all in the inductor at maximum current.

**Worked example.**  $L = 2.0 \text{ H}$ ,  $C = 8.0 \text{ }\mu\text{F}$ :  $\omega = \frac{1}{\sqrt{2.0(8.0 \times 10^{-6})}} = 250 \text{ rad/s}$ , period  $T = 2\pi/\omega \approx 0.025 \text{ s}$ . If the capacitor starts charged to  $12 \text{ V}$ , then  $U = \frac{1}{2}CV^2 = 5.8 \times 10^{-4} \text{ J}$ , and the maximum current follows from  $\frac{1}{2}LI_{\text{max}}^2 = U$ :  $I_{\text{max}} = \sqrt{2U/L} = 0.024 \text{ A}$  – **conservation of energy** 能量守恒, no calculus needed.

## Exam tips

- Compute **flux**  $\Phi_B = \int \vec{B} \cdot d\vec{A}$  and get the induced EMF from **Faraday’s law**  $\varepsilon = -\frac{d\Phi_B}{dt}$ .
- Use **Lenz’s law** (the minus sign) to fix the direction: the induced current opposes the change in flux.
- Flux changes three ways — changing  $B$ , changing area, or changing angle — identify which and differentiate.
- For a rod of length  $L$  moving at speed  $v$ , the motional EMF is  $BLv$ .
- An inductor stores energy  $\frac{1}{2}LI^2$  and resists **changes** in current (RL time constant  $\tau = L/R$ ).