Electromagnetic Induction

AP Physics C: Electricity and Magnetism

Magnetic Flux

Magnetic flux 磁通量 measures how much magnetic field passes through a surface. For a uniform field through a flat loop it is the dot product $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$; in general it is the **surface integral** 面积分

$$\Phi_B = \int \vec{B} \cdot d\vec{A}.$$

The **area vector** 面积矢量 is perpendicular to the surface (outward from a closed one), and the sign of the flux comes from the dot product. Flux changes if B changes, the area changes, or the loop turns –keep all three routes in mind, because each one is an exam question.

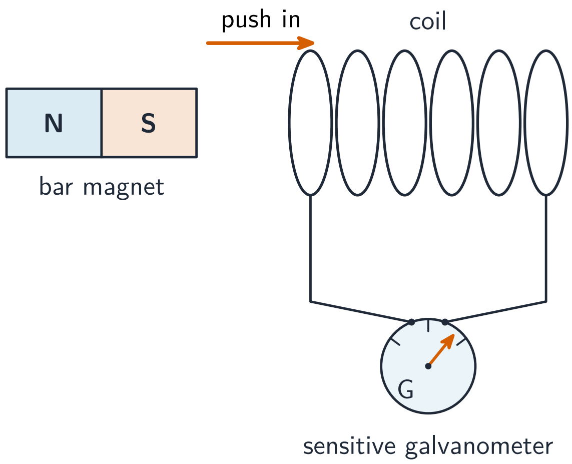
Electromagnetic Induction

A **changing** flux induces an **emf** 电动势–**Faraday's law** 法拉第定律:

$$\varepsilon = -\frac{d\Phi_B}{dt}.$$

With constant area, $\varepsilon = -A\,\frac{dB_\perp}{dt}$; with constant field, $\varepsilon = -B\,\frac{dA_\perp}{dt}$. A coil of N turns multiplies the single-loop emf: $|\varepsilon_{\text{sol}}| = N\left|\frac{d\Phi_B}{dt}\right|$.

Lenz's law 楞次定律 is the minus sign: the **induced current** 感应电流 flows so that its own magnetic field *opposes the change* in flux that created it. Push a magnet toward a loop and the loop pushes back; pull it away and the loop pulls it in. Use the **right-hand rule** 右手定则 to turn "oppose the change" into a current direction.



Moving a magnet into a coil induces an e.m.f. that drives a current

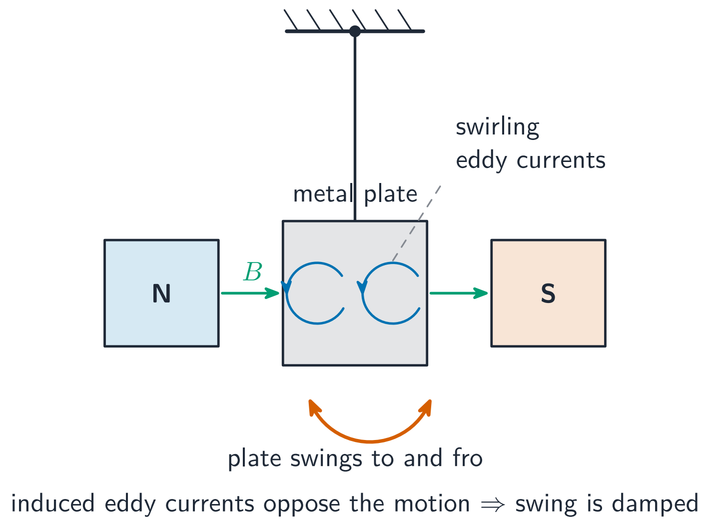
A rod of length L sliding at speed v across a field is the special case worth memorising – the **motional emf 动生电动势** $\varepsilon = BLv$.

Worked example. A 0.20 m rod slides at 3.0 m/s across a 0.50 T field: $\varepsilon = BLv = 0.50(0.20)(3.0) = 0.30$ V. Equivalently, if a single loop's flux drops from 0.020 Wb to 0.008 Wb in 0.030 s, the average emf is $\varepsilon = \frac{0.012}{0.030} = 0.40$ V.

Faraday's law is also the third of **Maxwell's equations** 麦克斯韦方程组, in a deeper form: a changing magnetic flux creates a circulating *electric field*, $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ – that field is what pushes the charges around the loop. Together, Maxwell's equations predict **electromagnetic waves** 电磁波 travelling at $c = 1/\sqrt{\varepsilon_0 \mu_0}$ (you should know this connection, but AP will not ask you to derive it).

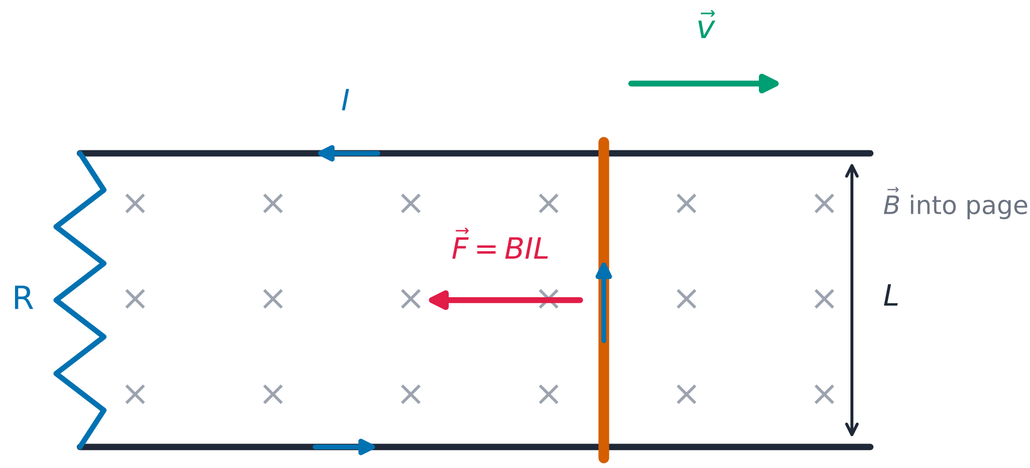
Induced Currents and Magnetic Forces

Once an induced current flows, the external field exerts forces on it ($\vec{F} = \int I \, d\vec{l} \times \vec{B}$) – and by Lenz's law those forces always *resist the motion* that causes the induction. Only the segments of the loop actually inside the field feel a force, which can make a loop accelerate, rotate, or brake. This is the origin of **eddy currents** 涡电流 braking, and you can apply Newton's second law to a moving loop or rod like any other mechanics problem.



Induced eddy currents oppose motion, quickly damping a metal plate swinging in a field

The classic setup is a rod sliding on conducting rails connected by a resistor:



A rod sliding on rails: the induced current feels a force opposing the motion

Worked example (the full chain). Rails $L = 0.20$ m apart with resistance $R = 0.60\ \Omega$ sit in a 0.50 T field into the page. The rod is pushed at a constant 3.0 m/s. Then: $\varepsilon = BLv = 0.30$ V; $I = \varepsilon/R = 0.50$ A; the field pushes back on the rod with $F = BIL = 0.50(0.50)(0.20) = 0.050$ N. The pushing force does work at $P = Fv = 0.15$ W –exactly the $P = I^2R = 0.15$ W dissipated in the resistor. Mechanical work becomes electrical energy: that is a **generator** 发电机, and energy is conserved. Released with no push, the rod slows exponentially: $ma = -\dfrac{B^2L^2}{R}v$.

Exam skill. FRQs walk this exact chain: flux → emf → current → force → Newton's second law. Write each link separately and check the direction with Lenz's law at the end.

Inductance

Inductance 电感 is a conductor's tendency to oppose a change in its own current: changing current changes its own flux, which self-induces an emf. From Faraday's law,

$$\varepsilon = -L\frac{dI}{dt}.$$

An **inductor** 电感器 is a circuit element built to have large inductance –usually a **solenoid** 螺线管, where geometry gives

$$L_{\text{sol}} = \frac{\mu_{\text{core}} N^2 A}{\ell},$$

with N total turns, area A, length ℓ, and the **magnetic permeability** 磁导率 of the core. (Straight wires are modelled as having zero inductance.) An inductor carrying current I stores energy in its magnetic field:

$$U_L = \tfrac{1}{2}LI^2,$$

which can later be dissipated in a resistor or moved into a capacitor –conservation of energy applies as usual.

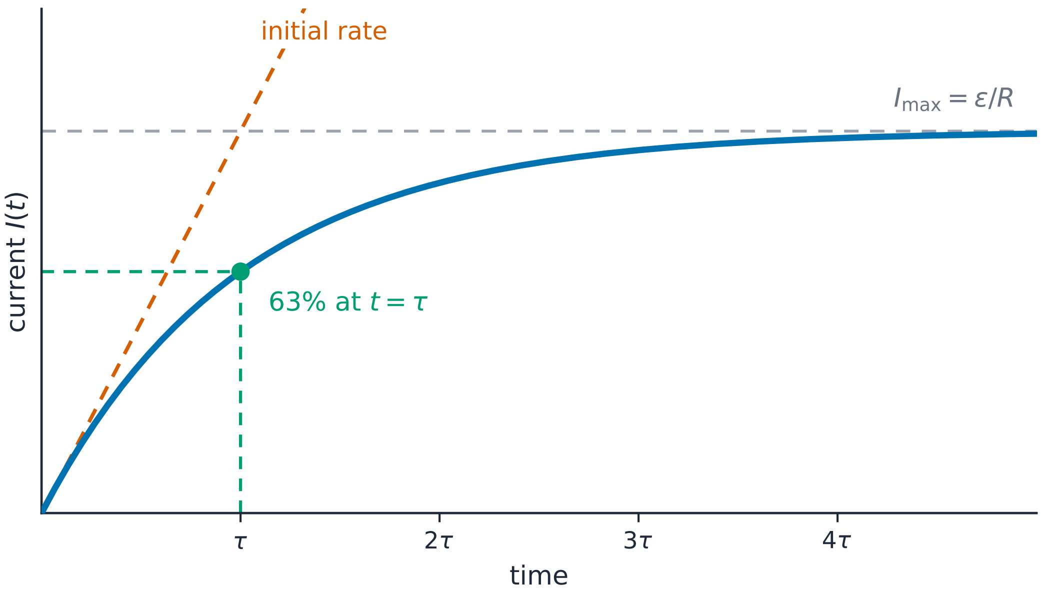
Circuits with Resistors and Inductors

In an **RL circuit** RL 电路, Kirchhoff's loop rule for a battery ε, resistor R, and inductor L in series gives a **differential equation** 微分方程:

$$\varepsilon = IR + L\frac{dI}{dt} \quad \Rightarrow \quad I(t) = \frac{\varepsilon}{R}\big(1 - e^{-t/\tau}\big), \qquad \tau = \frac{L}{R}.$$

The **time constant** 时间常数 τ sets the pace: after one τ a rising current reaches about 63% of its final value (a decaying one falls to 37%); it is also how long the change *would* take at the initial rate. Learn the two limits –they answer most conceptual questions:

- **Just after the switch closes** ($t = 0$): current cannot jump, so the inductor momentarily blocks it, self-inducing an emf equal and opposite to the applied potential difference.
- **Long after** ($t \gg \tau$): the current is steady, $dI/dt = 0$, and the inductor behaves as a plain wire –the exact opposite of a capacitor.



The current in an RL circuit rises exponentially, reaching 63% at one time constant

Worked example. $\varepsilon = 12$ V, $R = 6.0\ \Omega$, $L = 3.0$ H: final current $\varepsilon/R = 2.0$ A, $\tau = L/R = 0.50$ s. At $t = 0.50$ s the current is $2.0(1 - e^{-1}) \approx 1.3$ A. At $t = 0$ the inductor's potential difference is the full 12 V; as $t \to \infty$ it falls to zero. Current, inductor voltage, and stored energy are all exponential in time.

Circuits with Capacitors and Inductors

An **LC circuit** LC 电路 has no resistance, so nothing dissipates energy: it **oscillates** 振荡, sloshing energy between the capacitor's electric field and the inductor's magnetic field. The loop rule gives

$$\frac{d^2q}{dt^2} = -\frac{1}{LC}\,q,$$

the same equation as a mass on a spring –**simple harmonic motion** 简谐运动 with charge playing the role of displacement and

$$\omega = \frac{1}{\sqrt{LC}}.$$

The total energy $\dfrac{q^2}{2C} + \frac{1}{2}LI^2$ stays constant: all in the capacitor at maximum charge, all in the inductor at maximum current.

Worked example. $L = 2.0$ H, $C = 8.0\ \mu$F: $\omega = \dfrac{1}{\sqrt{2.0(8.0 \times 10^{-6})}} = 250$ rad/s, period $T = 2\pi/\omega \approx 0.025$ s. If the capacitor starts charged to 12 V, then $U = \frac{1}{2}CV^2 = 5.8 \times 10^{-4}$ J, and the maximum current follows from $\frac{1}{2}LI_{\text{max}}^2 = U$: $I_{\text{max}} = \sqrt{2U/L} = 0.024$ A –**conservation of energy** 能量守恒, no calculus needed.

Exam tips

- Compute **flux** $\Phi_B = \int \vec{B} \cdot d\vec{A}$ and get the induced EMF from **Faraday's law** $\varepsilon = -\frac{d\Phi_B}{dt}$.
- Use **Lenz's law** (the minus sign) to fix the direction: the induced current opposes the change in flux.
- Flux changes three ways —changing B, changing area, or changing angle —identify which and differentiate.
- For a rod of length L moving at speed v, the motional EMF is BLv.
- An inductor stores energy $\frac{1}{2}LI^2$ and resists **changes** in current (RL time constant $\tau = L/R$).