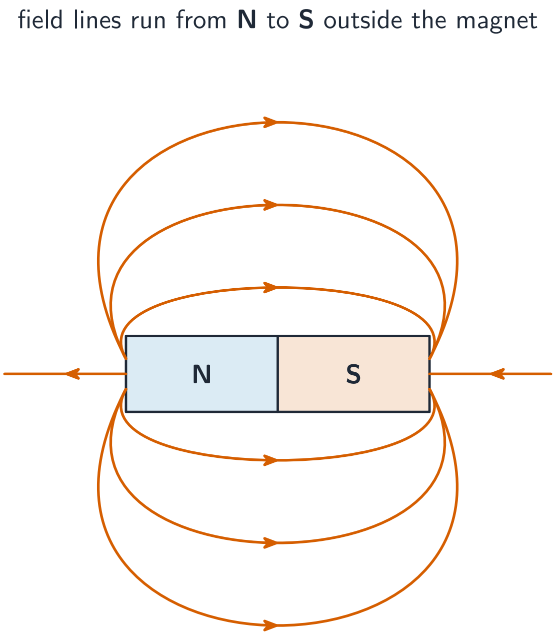
Magnetic Fields and Electromagnetism

AP Physics C: Electricity and Magnetism

Magnetic Fields

A **magnetic field** 磁场 $\vec{B}$ is a **vector field** 矢量场 that surrounds magnets, moving charges, and currents; it determines the magnetic force on any moving charge placed in it. **Magnetic field lines** 磁感线 must form closed loops: they leave the north pole, return to the south pole, and continue through the magnet. Denser lines mean a stronger field.



Field lines run from N to S outside a bar magnet

Closed loops are the content of Gauss's law for magnetism, the second of **Maxwell's equations** 麦克斯韦方程组:

$$\oint \vec{B} \cdot d\vec{A} = 0.$$

The net magnetic flux through *any* closed surface is zero –as many field lines leave as enter. That is exactly the statement that isolated **magnetic monopoles** 磁单极子 do not exist. Every source of magnetism is a **magnetic dipole** 磁偶极子 (a north–south pair), made by circulating charge –in materials, the motion of electrons. Break a bar magnet in half and you get two smaller dipoles, never a lone pole. Like poles repel, opposite poles attract, and a free dipole –a **compass** 指南针–rotates to line up with the local field. Earth's own field is approximately a dipole, which is why a compass works. Dipole fields weaken with distance.

How a material responds to an external field depends on how its internal dipoles behave:

- **Ferromagnetic** 铁磁性 (iron, nickel, cobalt): an external field aligns whole **magnetic domains** 磁畴, and the alignment survives after the field is removed –a permanent magnet.
- **Paramagnetic** 顺磁性 (aluminum, titanium): dipoles align weakly with the field but relax when it is removed.
- **Diamagnetic** 抗磁性 (all materials): the electronic structure produces a usually weak alignment *opposite* to the field.

The strength of a material's response is its **magnetic permeability** 磁导率. Free space has the constant value μ_0 (the vacuum permeability); the permeability of matter differs from μ_0 and is not even constant –it varies with temperature, orientation, and field strength.

Magnetism and Moving Charges

A moving charge does two things: it *creates* a magnetic field, and it *feels* a force in an external one. The field it creates at a point is **perpendicular** 垂直 to both its velocity and the position vector from charge to point (right-hand rule again), and is largest where those two are perpendicular –zero directly ahead of or behind the motion.

The force on a charge moving in a field $\vec{B}$ is the cross product

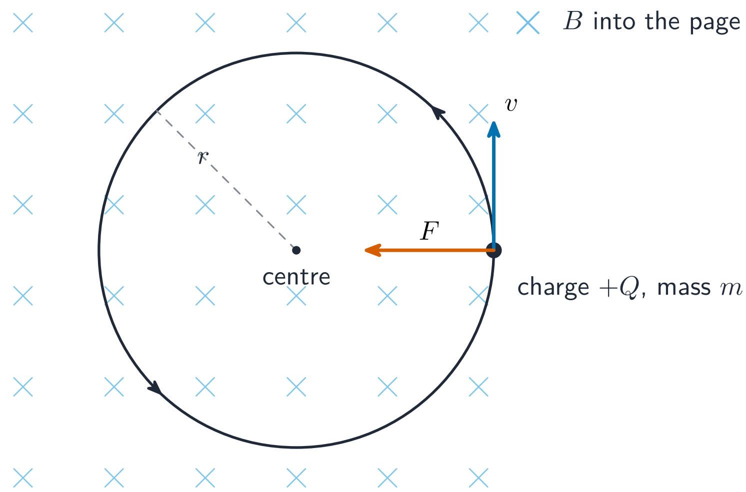
$$\vec{F}_B = q\,\vec{v} \times \vec{B}, \qquad F = qvB\sin\theta,$$

perpendicular to both $\vec{v}$ and $\vec{B}$ –point the fingers of your right hand along $\vec{v}$, curl toward $\vec{B}$, and the thumb gives the force on a *positive* charge (reverse it for negative). Because $\vec{F}_B \perp \vec{v}$, the magnetic force does **no work** 不做功 on the charge: it changes direction, never speed.

A charge moving perpendicular to a uniform field therefore travels in a circle, the magnetic force supplying the **centripetal force** 向心力:

$$qvB = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{qB}, \qquad T = \frac{2\pi m}{qB}.$$

Notice the period T does not depend on the speed –faster particles ride larger circles in the same time.



$$BQv = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{BQ}$$

A charged particle moving across a magnetic field follows a circular path

Worked example. A proton ($q = 1.6 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) enters a 0.50 T field at 2.0×10^5 m/s, perpendicular to it. The magnetic force $F = qvB = 1.6 \times 10^{-14}$ N bends it into a circle of radius $r = \dfrac{mv}{qB} = \dfrac{1.67 \times 10^{-27}(2.0 \times 10^5)}{1.6 \times 10^{-19}(0.50)} = 4.2 \times 10^{-3}$ m.

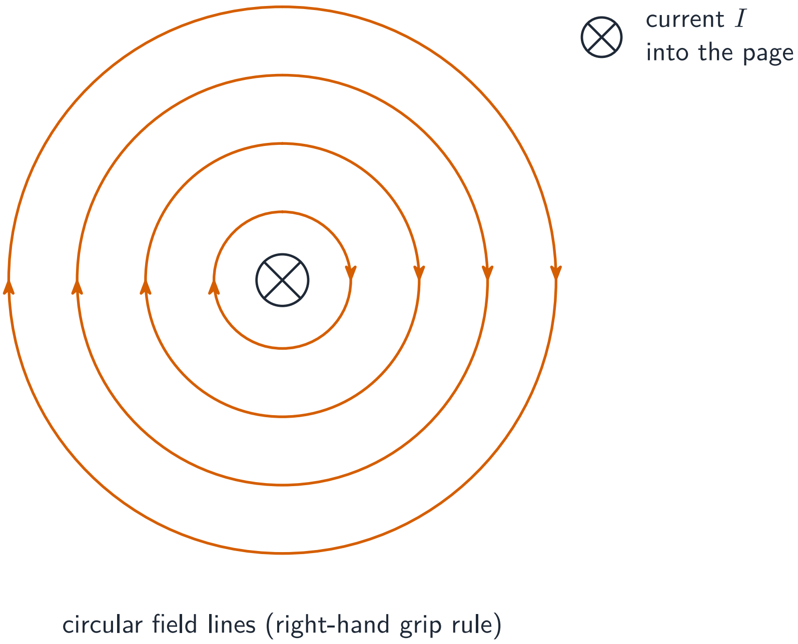
In a region with both an electric and a magnetic field, the two forces act independently and add as vectors. Balancing them makes a **velocity selector** 速度选择器: with $\vec{E}$ and $\vec{B}$ crossed, only charges with $qE = qvB$, i.e. $v = E/B$, pass straight through. The **Hall effect** 霍尔效应 is the same physics inside a conductor: a field with a component perpendicular to the current pushes the moving carriers sideways, charge builds up on one face, and a measurable potential difference appears across the conductor –its sign reveals whether the carriers are positive or negative.

Magnetic Fields of Current-Carrying Wires

A current is a stream of moving charges, so it creates a magnetic field. The **Biot–Savart law** 毕奥-萨伐尔定律 adds up the field of each current element:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}.$$

Around any straight segment the field vectors are **tangent** 相切 to **concentric circles** 同心圆 centred on the wire –no component toward, away from, or along the wire. Curl your right hand around the wire with the thumb along the current: your fingers give the field direction.



Concentric circular field lines surround a straight current-carrying wire

The Biot–Savart integrals AP expects: a long straight wire ($B = \dfrac{\mu_0 I}{2\pi r}$, or a point on the perpendicular bisector of a finite wire), and the centre of a circular loop,

$$B_{\text{centre of loop}} = \frac{\mu_0 I}{2R},$$

where every element $d\vec{l}$ is perpendicular to $\hat{r}$ and equally distant R –say that in an FRQ derivation. An arc that is a fraction of a full circle contributes that same fraction of $\mu_0 I/2R$ at its centre.

A field also pushes on a current-carrying wire, element by element:

$$\vec{F}_B = \int I\, d\vec{l} \times \vec{B} \qquad (\vec{F} = I\vec{L} \times \vec{B} \text{ for a straight wire in a uniform field}).$$

Worked example. Two long parallel wires a distance $d = 0.10$ m apart each carry 5.0 A in the same direction. Wire 1's field at wire 2 is $B = \dfrac{\mu_0 I}{2\pi d} = 1.0 \times 10^{-5}$ T, so wire 2 feels $\dfrac{F}{L} = IB = 5.0 \times 10^{-5}$ N/m, pulled *toward* wire 1. Same-direction currents attract; opposite currents repel.

Ampère's Law

For symmetric current distributions, **Ampère's law** 安培定律 finds the field far faster than Biot–Savart:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}.$$

Choose an **Amperian loop** 安培环路 that matches the symmetry, so that B is constant along the loop (and parallel to it) and comes out of the integral. AP applies it to long straight wires, long solenoids, and conductors carrying a **current density** 电流密度 (slabs and cylinders); combinations are handled by **superposition** 叠加.

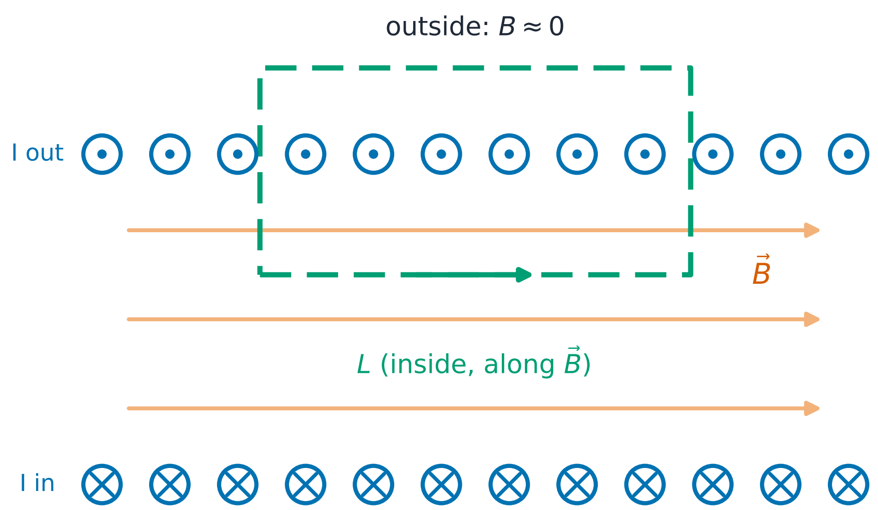
Worked example. Find the field 0.10 m from a long wire carrying 5.0 A. A circular loop of radius r shares the field's symmetry, so $B(2\pi r) = \mu_0 I$ and

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7})(5.0)}{2\pi(0.10)} = 1.0 \times 10^{-5} \text{ T}.$$

A long **solenoid** 螺线管 (assumed: uniform field inside, negligible field outside) is the other classic. Take a rectangular loop with one side of length L inside, parallel to the axis: only that side contributes to the integral, so $BL = \mu_0(nL)I$ and

$$B_{\text{sol}} = \mu_0 n I,$$

with n the turns per metre.



A rectangular Amperian loop derives the uniform field inside a solenoid

Worked example (cylinder). A solid cylindrical conductor of radius R carries current I spread uniformly over its cross-section. Inside ($r < R$), a circular loop encloses $I_{\text{enc}} = I\dfrac{r^2}{R^2}$, so $B = \dfrac{\mu_0 I r}{2\pi R^2}$ –the field grows linearly to the surface, then falls off as $1/r$ outside.

Exam skill. Every Ampère's-law answer earns its marks in the setup: name the loop, state *why* B is constant and parallel to it (symmetry), and count I_{enc} carefully before you solve. Maxwell's fourth equation adds one more idea you should know qualitatively: a *changing electric field* also creates a magnetic field –but AP will not ask you to compute with that term.

Exam tips

- The magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ is **perpendicular** to velocity —use the right-hand rule and note it does **no work**.
- A charge in a uniform field moves in a circle with $r = \frac{mv}{qB}$.
- Find the field of currents with **Biot–Savart** $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$ or **Ampère's law** $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$ when there is symmetry.
- Force on a wire is $\vec{F} = I\vec{L} \times \vec{B}$; keep the cross-product direction straight.
- Distinguish the force **on** a charge from the field the current **creates**.