Electric Circuits

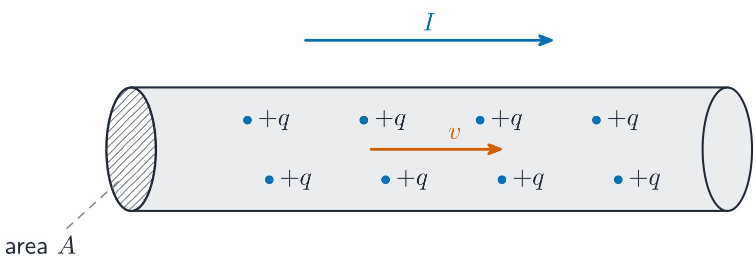
AP Physics C: Electricity and Magnetism

Electric Current

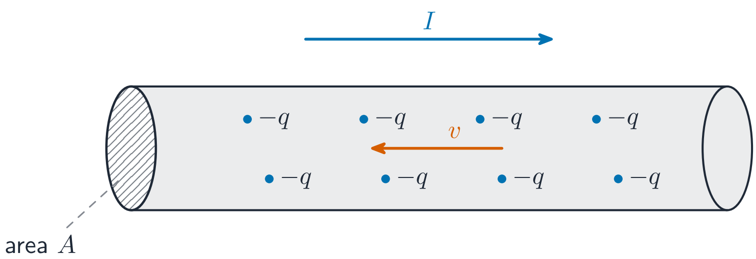
Electric current 电流 is the rate at which charge passes a cross-section of wire, $I = \frac{dq}{dt}$, measured in **amperes** 安培. Conventional current points the way *positive* charge would move. Microscopically, a current is a slow drift of many carriers:

$$I = nqv_dA,$$

with n the number of carriers per volume, q the charge each carries, v_d the **drift velocity** 漂移速度, and A the **cross-sectional area** 横截面积.



(a) positive carriers drift with I



(b) electrons drift against I

Charge carriers drift slowly through a conductor to make a current

Worked example. A copper wire with $A = 1.0 \times 10^{-6}$ m^2 and $n = 8.5 \times 10^{28}$ m^{-3} carries 1.7 A. Then $v_d = \frac{I}{nqA} = \frac{1.7}{(8.5 \times 10^{28})(1.6 \times 10^{-19})(1.0 \times 10^{-6})} \approx 1.3 \times 10^{-4}$ m/s –the carriers drift slower than a snail, even though the signal travels near light speed.

Current density 电流密度 is charge flow per unit area, $\vec{J} = nq\vec{v}_d$, linked to the field driving it by $\vec{E} = \rho\vec{J}$. In general $I = \int \vec{J} \cdot d\vec{A}$; if $J(r)$ varies across the wire, integrate it over the cross-section to get the total current. One care point: current has a direction along its wire, but it is a **scalar** 标量–currents do not add as vectors, and there are no "components of current".

Electric Circuits

A circuit is a set of closed loops built from wires, batteries, resistors, lightbulbs, capacitors, inductors, switches, and meters; charge can flow only around a *closed* path. One element can belong to several loops at once –that is what makes multi-loop problems interesting. Every analysis starts by reading the **circuit diagram** 电路图: trace each loop and identify which elements share the same current (**series** 串联) and which share the same potential difference (**parallel** 并联).

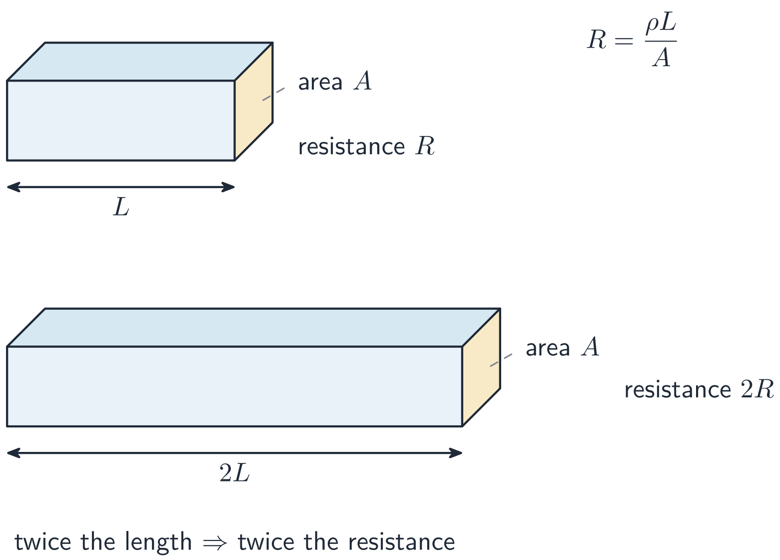
Resistance, Resistivity, and Ohm's Law

Resistance 电阻 measures how strongly an object opposes charge flow. It grows with the material's **resistivity** 电阻率 and the conductor's length, and shrinks with its area:

$$R = \frac{\rho \ell}{A}.$$

Ohm's law 欧姆定律 relates the current through an element to the potential difference across it:

$$I = \frac{\Delta V}{R}.$$



A longer conductor has more resistance; a wider one has less

Worked example. Stretch a wire to double its length: the volume is fixed, so the area halves, and $R = \rho\ell/A$ becomes $\rho(2\ell)/(A/2) = 4R$ –four times the resistance. An element is **ohmic** 欧姆性 if R stays constant (a straight line through the origin on an I–ΔV graph); a lightbulb filament, which heats up, is not.

Electric Power

A charge q falling through a potential difference ΔV gives up energy $q\Delta V$, so the rate of energy transfer in an element is

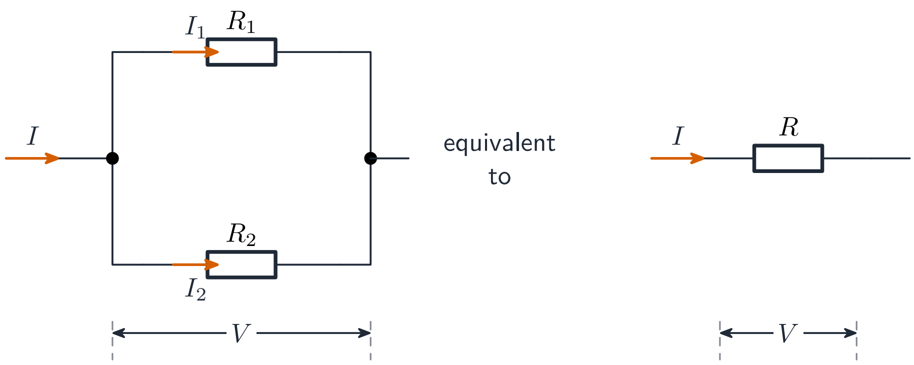
$$P = I\,\Delta V = I^2 R = \frac{(\Delta V)^2}{R}.$$

In a resistor all of it becomes heat. Use the form whose quantities you actually know – and use power to rank lightbulb brightness: brighter = more power, not necessarily more resistance. In series, the *larger* resistance is brighter ($P = I^2R$, same I); in parallel, the *smaller* one is ($P = \Delta V^2/R$, same ΔV).

Compound Direct Current Circuits

Reduce resistor networks to an **equivalent resistance** 等效电阻: **series** resistances add ($R_{eq} = R_1 + R_2 + \cdots$), while **parallel** resistances add as reciprocals ($\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$ –always less than the smallest branch). Collapse the network step by step to find the battery current, then expand back out to find each element's current and voltage.

Worked example. A 12 V battery drives a 4.0 Ω and a 2.0 Ω resistor in series: $R_{eq} =$ 6.0 Ω, I = 2.0 A, the voltages split 8.0 V and 4.0 V, and the 4.0 Ω resistor dissipates $P = I^2R = 16$ W.



Resistors in parallel combine to a smaller equivalent resistance

Real batteries are not ideal. Model a battery as an **ideal battery** 理想电池 of emf ε in series with its own **internal resistance** 内阻 r. When current flows, some emf is used up inside, so the **terminal voltage** 端电压–what a voltmeter across the battery actually reads –drops:

$$\Delta V_{\text{terminal}} = \varepsilon - Ir.$$

Worked example. A battery with ε = 12 V and r = 0.50 Ω supplies 2.0 A: the terminals sit at $\Delta V = 12 - 2.0(0.50) = 11$ V. With no current, a voltmeter reads the full 12 V.

Meters: an **ammeter** 电流表 goes *in series* at the point whose current you want (ideal ammeter: zero resistance); a **voltmeter** 电压表 goes *in parallel* across the element (ideal voltmeter: infinite resistance). Non-ideal meters disturb the circuit they measure –a real ammeter adds series resistance, a real voltmeter steals current.

Kirchhoff's Loop Rule

Charges moving through potential differences exchange energy ($\Delta U_E = q\Delta V$), and energy must balance around any closed path. That is **Kirchhoff's loop rule** 基尔霍夫回路定则:

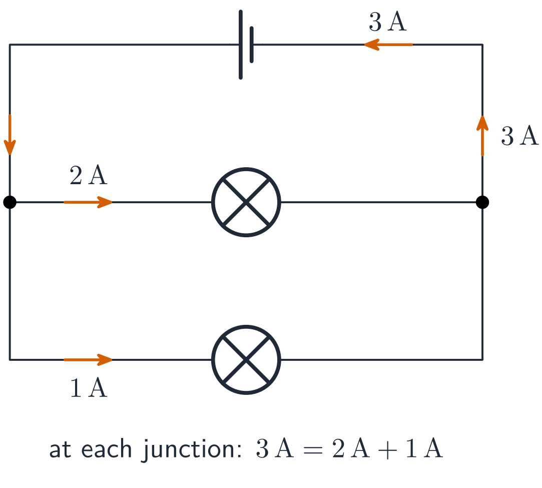
$$\sum \Delta V = 0 \text{ around any closed loop.}$$

Sign discipline wins these problems: crossing a battery from $-$ to $+$ is $+\varepsilon$; crossing a resistor *with* the assumed current is $-IR$ (against it, $+IR$). Write one equation per independent loop.

Kirchhoff's Junction Rule

Kirchhoff's junction rule 基尔霍夫节点定则 is conservation of charge at a **junction** 节点:

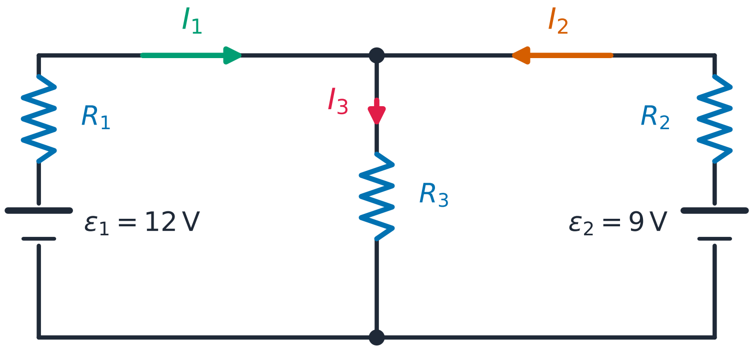
$$\sum I_{\text{in}} = \sum I_{\text{out}}.$$



at each junction: $3\,\text{A} = 2\,\text{A} + 1\,\text{A}$

Current divides at a junction: what flows in equals what flows out

Together the two rules solve any multi-loop circuit: assign a current to each branch, write junction equations, then loop equations, and solve. A negative answer just means that current flows opposite to your assumed direction.



Two loop equations and one junction equation solve this two-battery circuit

Worked example. In the circuit above, $\varepsilon_1 = 12$ V with $R_1 = 1.0\ \Omega$ on the left, $\varepsilon_2 = 9.0$ V with $R_2 = 1.0\ \Omega$ on the right, and a shared middle resistor $R_3 = 2.0\ \Omega$ carrying $I_3 = I_1 + I_2$ (junction rule). The two loop equations are

$$12 = I_1 + 2(I_1 + I_2) = 3I_1 + 2I_2, \qquad 9 = I_2 + 2(I_1 + I_2) = 2I_1 + 3I_2.$$

Solving: $I_1 = 3.6$ A, $I_2 = 0.60$ A, so $I_3 = 4.2$ A through the middle. Check with the second loop: $2(3.6) + 3(0.60) = 9.0$.

Resistor-Capacitor (RC) Circuits

Capacitor networks reduce like resistors but with the rules swapped: parallel capacitances add ($C_{\text{eq}} = C_1 + C_2$), series add as reciprocals –and capacitors in series must carry the *same charge* on each plate, by conservation of charge. Use the **equivalent capacitance** 等效电容 to analyse the network, then expand back.

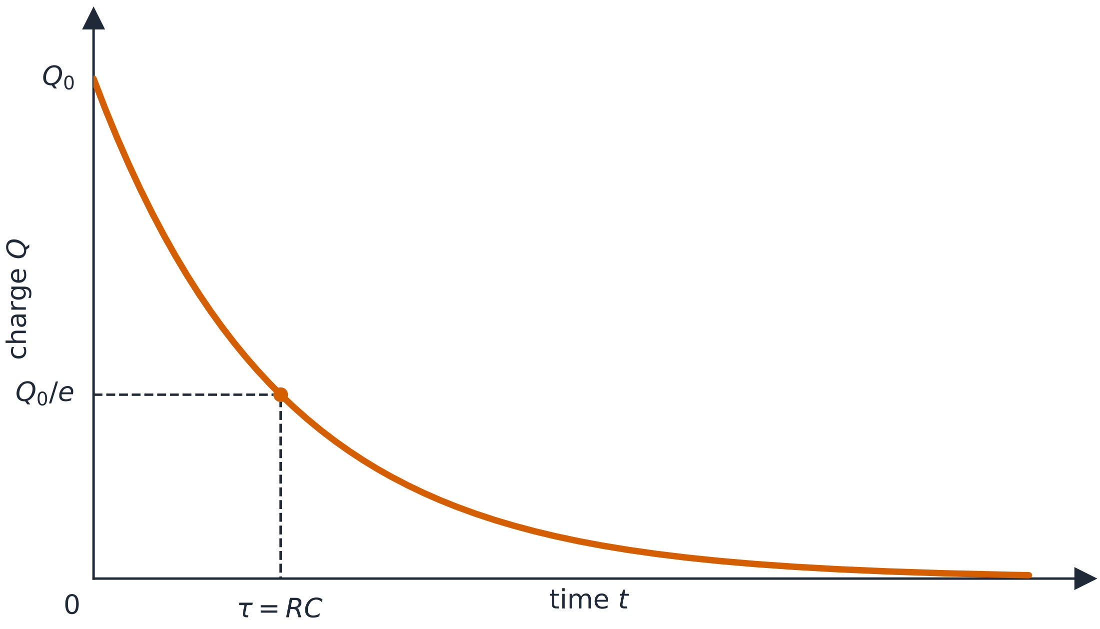
In an **RC circuit** RC 电路, Kirchhoff's loop rule gives the differential equation

$$\varepsilon = R\frac{dq}{dt} + \frac{q}{C},$$

whose solutions are exponentials with **time constant** 时间常数 $\tau = RC$:

$$q(t) = Q\left(1 - e^{-t/RC}\right)\ \text{(charging)}, \qquad q(t) = Q\,e^{-t/RC}\ \text{(discharging)}, \qquad i(t) = \frac{\varepsilon}{R}e^{-t/RC}.$$

The current is largest at the first instant and decays –it never "waits" for the capacitor. Learn the two limits: at $t = 0$ an uncharged capacitor acts like a plain wire (maximum current); after a long time it is fully charged, no current flows in its branch, and it acts like a break. In any **steady state** 稳态, cover the capacitor branch with your finger, solve the resistor circuit, then read the capacitor's voltage from the element it sits across.



The charge on a capacitor decays exponentially as it discharges

Worked example. With $R = 5.0\ \text{k}\Omega$, $C = 200\ \mu\text{F}$, and a 10 V battery: $\tau = RC = 1.0$ s; initial current $\frac{\mathcal{E}}{R} = 2.0$ mA; after one time constant the charge is $q = CV(1 - e^{-1}) \approx 1.3 \times 10^{-3}$ C, about 63% of full charge, and the current has fallen to 37% of its initial value.

Exam tips

- Relate current to charge flow $I = \frac{dQ}{dt}$ and use $J = \sigma E$, $R = \frac{\rho L}{A}$ for resistance.
- Apply **Kirchhoff's laws** (junction: charge; loop: energy) with consistent sign conventions.
- Analyse **RC circuits** with calculus: charging/discharging give exponentials with time constant $\tau = RC$.
- Combine resistors (series add, parallel reciprocal) and track power $P = IV = I^2R$.
- At $t = 0$ a capacitor acts like a wire; after a long time ($t \to \infty$) it acts like an open branch.