Conductors and Capacitors

AP Physics C: Electricity and Magnetism

Electrostatics with Conductors

In an ideal **conductor** 导体, electrons move freely. Put extra charge on one and the charges repel each other until, almost instantly, they settle into **electrostatic equilibrium** 静电平衡. In that state the conductor has four properties you must be able to state and use:

- The electric field **inside** the conductor is zero. If it were not, free electrons would still be moving.
- All excess charge sits **on the surface**. (Negative net charge = extra electrons on the surface; positive = a shortage of electrons there.)
- The field just outside is **perpendicular** 垂直 to the surface, with magnitude $E = \sigma/\varepsilon_0$. Any parallel component would push charge sideways along the surface.
- The whole conductor is one **equipotential surface** 等势面: every point, inside and on the surface, is at the same potential.

Charge density is largest where the surface curves sharply –at points and edges –so the outside field is strongest there. In an external field a conductor **polarizes**: charge shifts on its surface so that the interior stays field-free and the body stays an equipotential. Surrounding a region with a closed conducting shell keeps outside fields out entirely – **electrostatic shielding** 静电屏蔽, the idea behind the **Faraday cage** 法拉第笼.

Redistribution of Charge Between Conductors

When two conductors touch (or are wired together), charge flows between them until both surfaces reach the **same potential** –that is the stopping condition, not ”equal charges”. A larger sphere holds more charge at the same potential ($V = kQ/R$), so it takes the larger share.

Worked example. A small sphere of radius R carrying $+6.0\ \mu$C touches a distant sphere of radius $2R$, then they separate. Equal potentials require $\frac{kq_1}{R} = \frac{kq_2}{2R}$, so $q_2 = 2q_1$. With $q_1 + q_2 = 6.0\ \mu$C: $q_1 = 2.0\ \mu$C and $q_2 = 4.0\ \mu$C.

Ground 接地 is an idealized reference at zero potential that can absorb or supply any amount of charge. Grounding a conductor while an external charge is nearby leaves the conductor with a net **induced charge** 感应电荷: the external field pushes charge of one sign to ground, and cutting the ground wire before removing the external charge traps the rest.

Capacitors

A **capacitor** 电容器 stores charge on two conductors separated by a gap: $+Q$ on one plate, $-Q$ on the other. Its **capacitance** 电容 relates the stored charge to the potential difference between the plates:

$$C = \frac{Q}{\Delta V}.$$

Capacitance depends only on geometry and the material in the gap –never on Q or ΔV themselves.

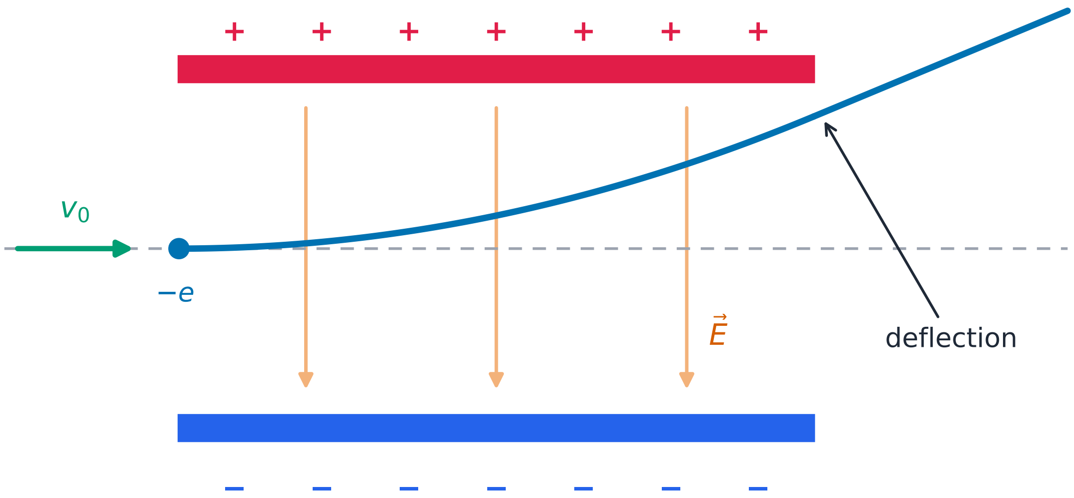
Worked derivation (parallel plates). For a **parallel-plate capacitor 平行板电容器** with plate area A and small gap d: Gauss's law plus **superposition 叠加** gives a **uniform 均匀** field between the plates, $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$ (each plate alone contributes $\sigma/2\varepsilon_0$; between the plates the two add, outside they cancel). A uniform field means $\Delta V = Ed = \frac{Qd}{\varepsilon_0 A}$, so

$$C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}.$$

This $E \to \Delta V \to C$ chain is a standard FRQ derivation –learn it as three steps, and quote each one. The same method handles the other two shapes AP expects: concentric spheres ($C = 4\pi\varepsilon_0 \frac{ab}{b-a}$) and a coaxial cylinder of length L, where $E = \frac{\lambda}{2\pi\varepsilon_0 r}$ gives $\Delta V = \frac{\lambda}{2\pi\varepsilon_0} \ln\frac{b}{a}$ and so $C = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}$.

Worked example. Plates of area $A = 0.020\ \text{m}^2$ and gap $d = 1.0$ mm: $C = \frac{8.85 \times 10^{-12}(0.020)}{1.0 \times 10^{-3}} = 1.8 \times 10^{-10}$ F. Charged to 100 V it holds $Q = CV = 1.8 \times 10^{-8}$ C.

Because the field between the plates is uniform, a charged particle there feels a constant force, so it moves with constant acceleration –exactly like **projectile motion 抛体运动** in gravity: constant speed across, uniform acceleration towards a plate.

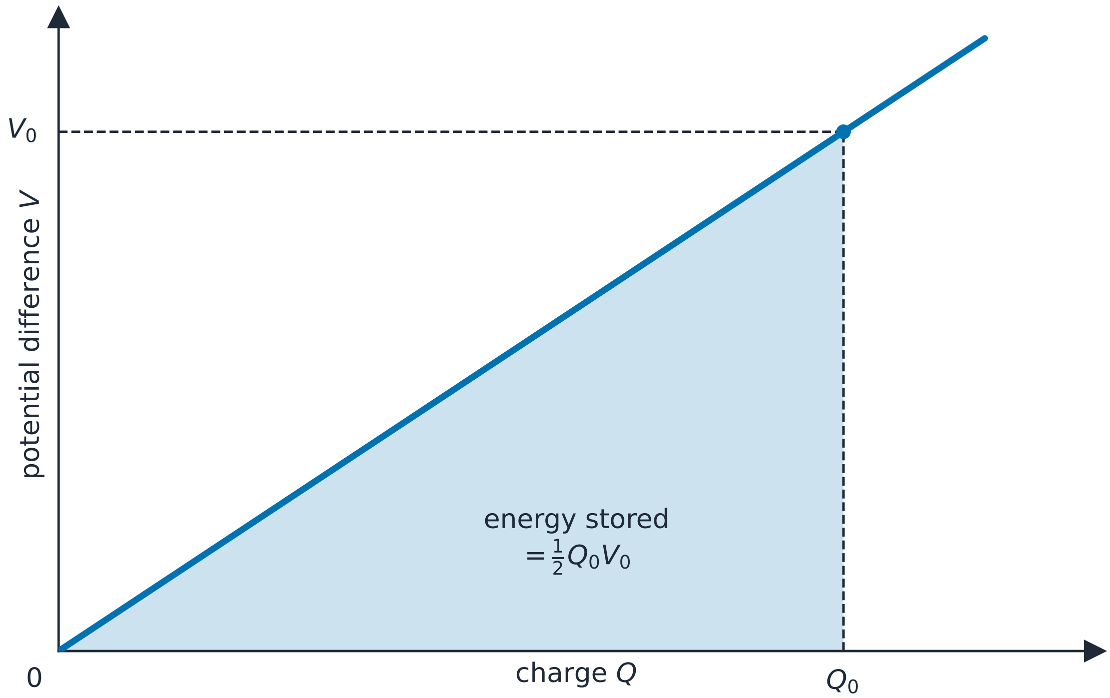


A charged particle between the plates follows a parabola, like a projectile

Worked example. An electron enters midway between plates at $v_0 = 2.0 \times 10^7$ m/s, parallel to them. The field is $E = 1.0 \times 10^3$ N/C and the plates are 4.0 cm long. Acceleration: $a = \dfrac{eE}{m} = \dfrac{(1.6 \times 10^{-19})(1.0 \times 10^3)}{9.11 \times 10^{-31}} = 1.8 \times 10^{14}$ m/s^2. Time between the plates: $t = \dfrac{0.040}{2.0 \times 10^7} = 2.0 \times 10^{-9}$ s. Deflection: $y = \frac{1}{2}at^2 = \frac{1}{2}(1.8 \times 10^{14})(2.0 \times 10^{-9})^2 \approx 3.5 \times 10^{-4}$ m –about 0.35 mm towards the positive plate.

Storing charge takes **work** 功: an external force must move each bit of charge against the field of the charge already there. The total work ends up as stored potential energy,

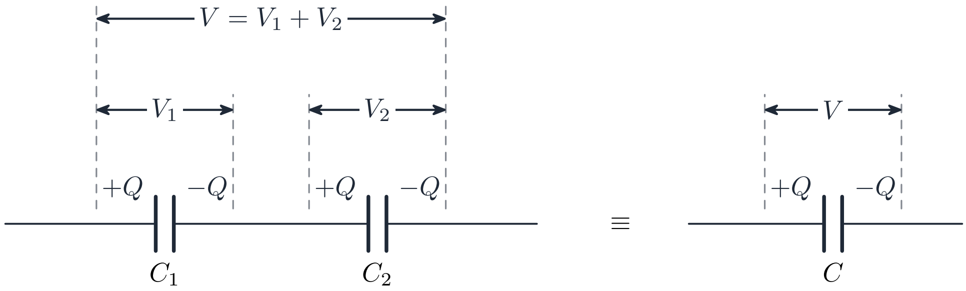
$$U_C = \tfrac{1}{2}\, Q\, \Delta V = \tfrac{1}{2} C (\Delta V)^2 = \frac{Q^2}{2C}.$$



The energy stored in a capacitor is the area under its charge-voltage line

The factor $\frac{1}{2}$ is the area of the triangle under the Q–ΔV line: the first charge moved across costs almost nothing, the last costs the full ΔV.

Capacitors combine oppositely to resistors: in **parallel** 并联 the capacitances add ($C_{\text{eq}} = C_1 + C_2$, same ΔV, charges add), while in **series** 串联 the reciprocals add ($\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$, same Q, voltages add).



Capacitors in series carry the same charge, and their p.d.s add

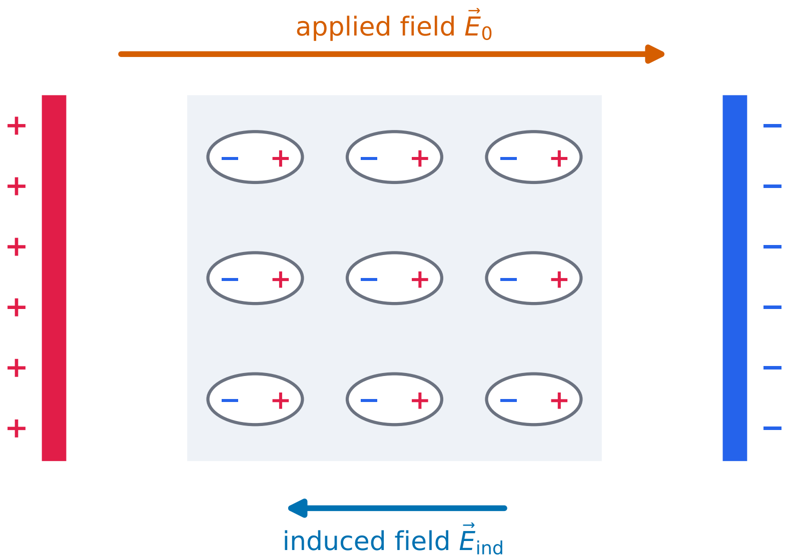
Worked example. A 2 μF and a 4 μF capacitor in series: $\frac{1}{C} = \frac{1}{2} + \frac{1}{4}$, so $C = \frac{4}{3}$ μF. The same pair in parallel: 6 μF.

Dielectrics

A **dielectric** 电介质 is an insulating material. Its charges cannot travel, but in an external field each molecule stretches or turns slightly –the material becomes **polarized** 极化. The lined-up molecules create their own small field *opposite* to the applied one, so the net field inside the material drops:

$$E = \frac{E_0}{\kappa}, \qquad \kappa = \frac{\varepsilon}{\varepsilon_0} \quad (\kappa > 1),$$

where κ is the **dielectric constant** 介电常数, the ratio of the material's permittivity to the **permittivity of free space** 真空介电常数.



A polarized dielectric creates an internal field opposing the applied field

Filling a capacitor with dielectric multiplies its capacitance:

$$C = \kappa\, C_0 \qquad \left(\text{parallel plates: } C = \frac{\kappa\varepsilon_0 A}{d}\right).$$

What happens next depends on what stays fixed –a favourite exam trap:

	battery stays connected (ΔV fixed)	**battery removed first (Q fixed)**
charge Q	rises to κQ_0	unchanged
voltage ΔV	unchanged	drops to $\Delta V_0/\kappa$
field E	unchanged	drops to E_0/κ
energy U	rises to κU_0	drops to U_0/κ

Worked example. A 100 pF capacitor is charged to 12 V, disconnected, then filled with a $\kappa = 3$ dielectric. Q is trapped, so ΔV falls to 4 V and the stored energy falls to one-third –the missing energy went into pulling the dielectric in. Reconnected to the 12 V battery instead, the capacitor would hold three times the charge and three times the energy.

Exam skill. Always begin dielectric questions by writing down which quantity is held fixed (Q or ΔV), then let $C = \kappa C_0$ drive everything else through $Q = C\Delta V$ and $U = \frac{1}{2}C(\Delta V)^2$.

Exam tips

- In electrostatic equilibrium a conductor has $\vec{E} = 0$ **inside** and all excess charge on the **surface**.
- Apply **Gauss's law** $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$ with a symmetry-matched Gaussian surface (sphere, cylinder, pillbox).
- The whole conductor is one **equipotential**, and the surface field is perpendicular to it.
- For a capacitor use $C = \frac{Q}{V}$, energy $U = \frac{1}{2}CV^2$, and how a **dielectric** raises C.
- Pick the Gaussian surface so $\vec{E}$ is constant and parallel (or zero) on each part.