Electric Potential

AP Physics C: Electricity and Magnetism

Electric Potential Energy

When you push two like charges together, you do **work 功** against the electric force. That work is stored by the pair as **electric potential energy 电势能**:

$$U_E = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = k\frac{q_1 q_2}{r}.$$

U_E is defined as the work an external force must do to bring the charges from infinitely far away to a distance r apart. The signs carry the physics:

- **Like charges:** $U_E > 0$. An outside push was needed to bring them together. If released, they fly apart, and the stored energy becomes **kinetic energy 动能**.
- **Opposite charges:** $U_E < 0$. The pair is bound. You must supply $|U_E|$ of work to pull them apart to infinity.

The electric force is a **conservative force 保守力**: the work it does depends only on the start and end points, not the path, and equals $-\Delta U_E$.

For a **system 系统** of several charges, add the potential energy of every distinct pair:

$$U_{\text{total}} = k\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right).$$

Worked example. Three $+2.0\ \mu$C charges sit at the corners of an equilateral triangle with sides 0.30 m. Each pair stores $U = \frac{kq^2}{r} = \frac{(9.0\times10^9)(2.0\times10^{-6})^2}{0.30} = 0.12$ J. There are three pairs, so $U_{\text{total}} = 3 \times 0.12$ J $= 0.36$ J. This is the work needed to assemble the triangle from infinity – and the kinetic energy the charges would share if all three were released.

Electric Potential

Electric potential 电势 is electric potential energy per unit charge at a point in space. It is measured in volts (1 V = 1 J/C):

$$V = \frac{U_E}{q}, \qquad V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \text{ (point charge).}$$

Potential is a **scalar 标量**: it has a sign but no direction. This makes V far easier to work with than the field $\vec{E}$. For several **point charges 点电荷**, use scalar **superposition 叠加** – add the potentials with their signs:

$$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}.$$

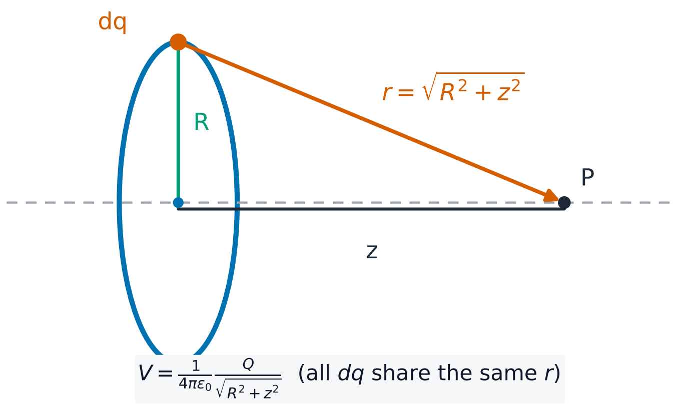
Worked example. The potential 0.10 m from a +3.0 nC point charge is $V = \frac{kq}{r} = \frac{9.0\times10^9(3.0\times10^{-9})}{0.10} = 270$ V. Now add a -3.0 nC charge 0.30 m from the same point: it contributes $\frac{9.0\times10^9(-3.0\times10^{-9})}{0.30} = -90$ V, so the total is $270 - 90 = 180$ V. Just a signed sum – no vector components to resolve.

Continuous charge distributions

For a **continuous charge distribution 连续电荷分布**, cut it into small pieces dq, treat each piece as a point charge, and **integrate 积分**:

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}.$$

This is a scalar integral, so it is usually much easier than the field integral. AP expects you to handle four shapes with calculus: a thin ring (at a point on its axis), an arc (at its centre), a finite line or wire (collinear, or on its perpendicular bisector), and an infinitely long wire or cylinder (at a distance from its axis).



A ring of charge: every element dq is the same distance from an axis point

Worked example (ring of charge). A thin ring of radius R carries total charge Q. For a point P on the axis a distance z from the centre, every element dq sits at the same distance $r = \sqrt{R^2 + z^2}$ from P. The distance is constant, so it comes out of the integral:

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{\sqrt{R^2+z^2}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+z^2}}.$$

On an FRQ, *say* that r is the same for every element – that statement is the marked step. The same idea gives $V = kQ/R$ at the centre of any arc, whatever its angle.

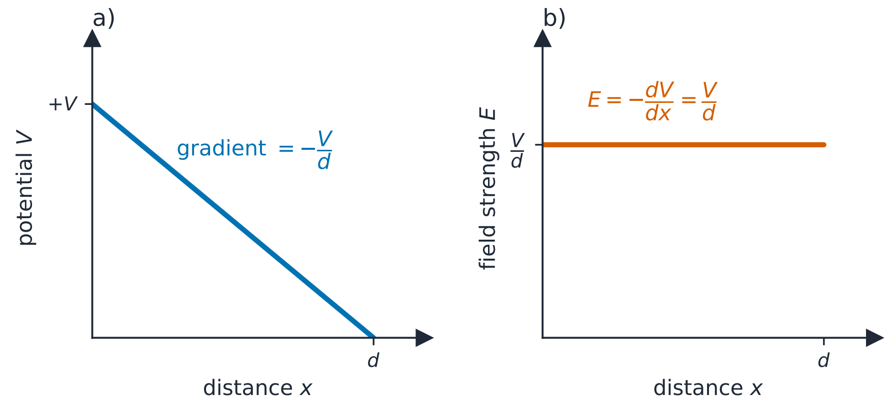
From potential to field

Potential and field contain the same information, linked by a derivative in each direction and by a path integral:

$$E_x = -\frac{dV}{dx}, \qquad \Delta V = V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{r}.$$

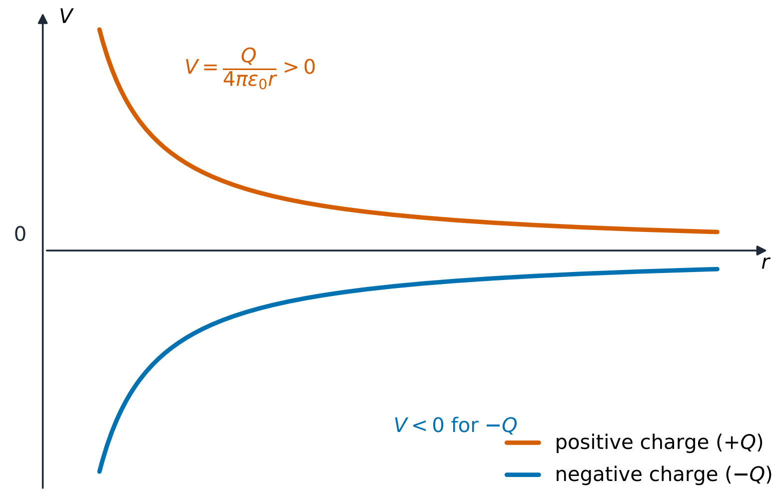
The field is the negative **gradient** 梯度 of the potential: $\vec{E}$ points from high potential to low potential, "downhill". Where V changes quickly, the field is strong.

Worked example. Along the x-axis, $V(x) = 3x^2 - 2x$ (volts, with x in metres). Then $E_x = -\dfrac{dV}{dx} = 2 - 6x$ V/m. At $x = 0.50$ m, $E_x = -1.0$ V/m: the field there points in the $-x$ direction.



In a uniform field the potential falls steadily with distance

The **potential difference** 电势差 between two points is the change in potential energy per unit charge moved between them: $\Delta V = \Delta U_E/q$. A battery makes a potential difference chemically: reactions inside separate positive from negative charge and hold the terminals a fixed ΔV apart.

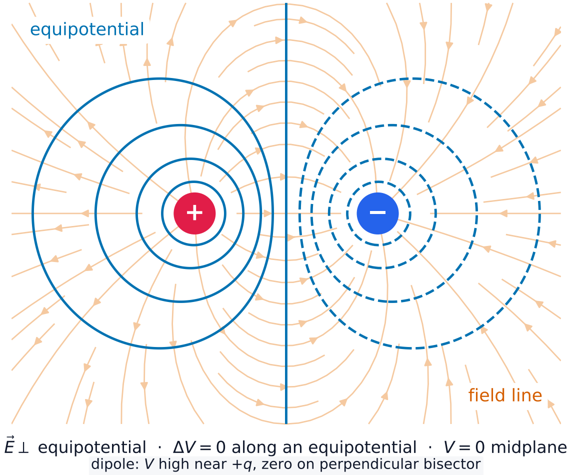


The potential near a point charge varies as $1/r$

Equipotential maps

An **equipotential** 等势面 line (also called an **isoline** 等值线) joins points that share the same potential. Four rules let you read any map:

- Isolines are **perpendicular** 垂直 to field lines 电场线 everywhere they cross.
- $\vec{E}$ points from high V to low V – never along an isoline. Moving a charge along an isoline takes no work.
- Closely spaced isolines mean a strong field: $E \approx -\Delta V/\Delta x$.
- You can sketch the field map from an isoline map, and the isoline map from a field map.



Equipotential lines around a dipole cross the field lines at right angles

Exam skill. Given a map with isolines every 10 V spaced about 2 cm apart, estimate $E \approx \dfrac{10}{0.02} = 500$ V/m, pointing from the higher-value line to the lower one. The work an external agent does moving a charge q slowly from A to B is $W = q(V_B - V_A)$ – the path taken does not matter.

High-voltage power lines: electric potential energy is converted and transmitted as current at high voltage

Conservation of Electric Energy

When a charge q moves between two points that differ in potential by ΔV, the potential energy of the charge–field system changes by

$$\Delta U_E = q\,\Delta V.$$

If only the electric force acts, total energy is conserved, so the kinetic energy changes by the opposite amount:

$$\Delta K = -\Delta U_E = -q\,\Delta V.$$

Watch the two signs together: a positive charge speeds up when it moves to *lower* potential, while a negative charge speeds up when it moves to *higher* potential. Both are just the system trading potential energy for kinetic energy. This is how particle accelerators give charged particles their energy, and it underlies energy analysis in circuits.

Worked example. A proton ($q = 1.6 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) starts at rest and is accelerated through a potential drop of 500 V (it moves to lower potential, so $\Delta V = -500$ V and $\Delta K = -q\,\Delta V = +8.0 \times 10^{-17}$ J). Setting $\Delta K = \frac{1}{2}mv^2$:

$$v = \sqrt{\frac{2(8.0 \times 10^{-17})}{1.67 \times 10^{-27}}} = 3.1 \times 10^5 \text{ m/s}.$$

Exam skill. Choose energy methods, not kinematics, whenever the field is non-uniform or the path curves: only the end-point potentials matter. A typical FRQ chain is $q\,\Delta V \to \Delta K \to v$, with one line of justification: "the electric force is conservative, so energy is conserved."

Exam tips

- Relate field and potential by $V = -\int \vec{E} \cdot d\vec{l}$ and $\vec{E} = -\nabla V$ (in 1-D, $E_x = -\frac{dV}{dx}$).
- Potential is a **scalar** — add contributions with signs, no vector components needed.
- Potential energy of a pair is $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$; use energy conservation for a charge's speed.
- Know that no work is done moving along an **equipotential**, which is perpendicular to $\vec{E}$.
- Choose the zero of potential (usually infinity) and state it.