Electric Potential

AP Physics C: Electricity and Magnetism

Electric Potential Energy

When you push two like charges together, you do **work 功** against the electric force. That work is stored by the pair as **electric potential energy 电势能**:

$$U_E = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = k\frac{q_1 q_2}{r}.$$

U_E is defined as the work an external force must do to bring the charges from infinitely far away to a distance r apart. The signs carry the physics:

- **Like charges:** $U_E > 0$. An outside push was needed to bring them together. If released, they fly apart, and the stored energy becomes **kinetic energy 动能**.
- **Opposite charges:** $U_E < 0$. The pair is bound. You must supply $|U_E|$ of work to pull them apart to infinity.

The electric force is a **conservative force 保守力**: the work it does depends only on the start and end points, not the path, and equals $-\Delta U_E$.

For a **system 系统** of several charges, add the potential energy of **every distinct pair 每一对**:

$$U_{\text{total}} = k\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right).$$

Worked example. Three +2.0 μC charges sit at the corners of an equilateral triangle with sides 0.30 m. Each pair stores $U = \frac{kq^2}{r} = \frac{(9.0 \times 10^9)(2.0 \times 10^{-6})^2}{0.30} = 0.12$ J. There are three pairs, so $U_{\text{total}} = 3 \times 0.12\ \text{J} = 0.36$ J. This is the work needed to assemble the triangle from infinity –and the kinetic energy the charges would share if all three were released.

Electric Potential

Electric potential 电势 is electric potential energy **per unit charge 每单位电荷** at a point in space. It is measured in volts (1 V = 1 J/C):

$$V = \frac{U_E}{q}, \qquad V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \text{ (point charge)}.$$

Potential is a **scalar 标量**: it has a sign but no direction. This makes V far easier to work with than the field $\vec{E}$. For several **point charges 点电荷**, use scalar **superposition 叠加**–add the potentials with their signs:

$$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}.$$

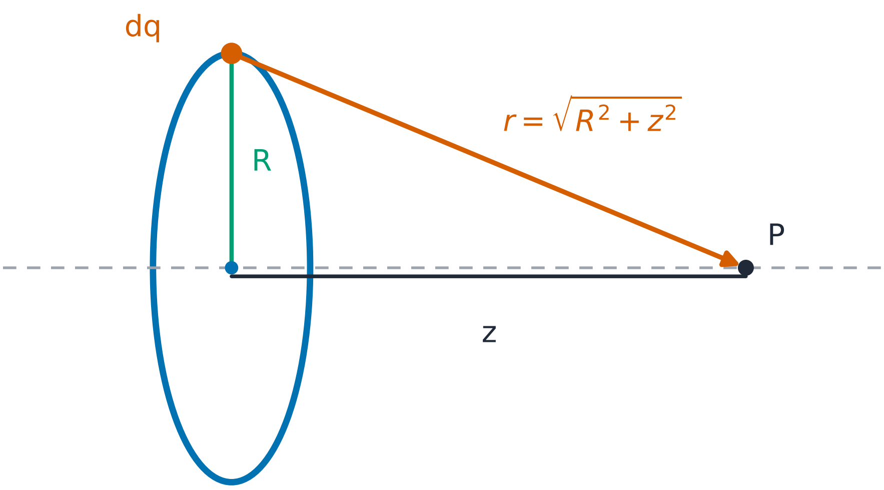
Worked example. The potential 0.10 m from a +3.0 nC point charge is $V = \frac{kq}{r} = \frac{9.0 \times 10^9(3.0 \times 10^{-9})}{0.10} = 270$ V. Now add a −3.0 nC charge 0.30 m from the same point: it contributes $\frac{9.0 \times 10^9(-3.0 \times 10^{-9})}{0.30} = -90$ V, so the total is $270 - 90 = 180$ V. Just a signed sum –no vector components to resolve.

Continuous charge distributions

For a **continuous charge distribution 连续电荷分布**, cut it into small pieces dq, treat each piece as a point charge, and **integrate 积分**:

$$V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r}.$$

This is a scalar integral, so it is usually much easier than the field integral. AP expects you to handle four shapes with calculus: a thin ring (at a point on its axis), an arc (at its centre), a finite line or wire (collinear, or on its perpendicular bisector), and an infinitely long wire or cylinder (at a distance from its axis).



A ring of charge: every element dq is the same distance from an axis point

Worked example (ring of charge). A thin ring of radius R carries total charge Q. For a point P on the axis a distance z from the centre, every element dq sits at the same distance $r = \sqrt{R^2 + z^2}$ from P. The distance is constant, so it comes out of the integral:

$$V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{\sqrt{R^2 + z^2}} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{\sqrt{R^2 + z^2}}.$$

On an FRQ, *say* that r is the same for every element –that statement is the marked step. The same idea gives $V = kQ/R$ at the centre of any arc, whatever its angle.

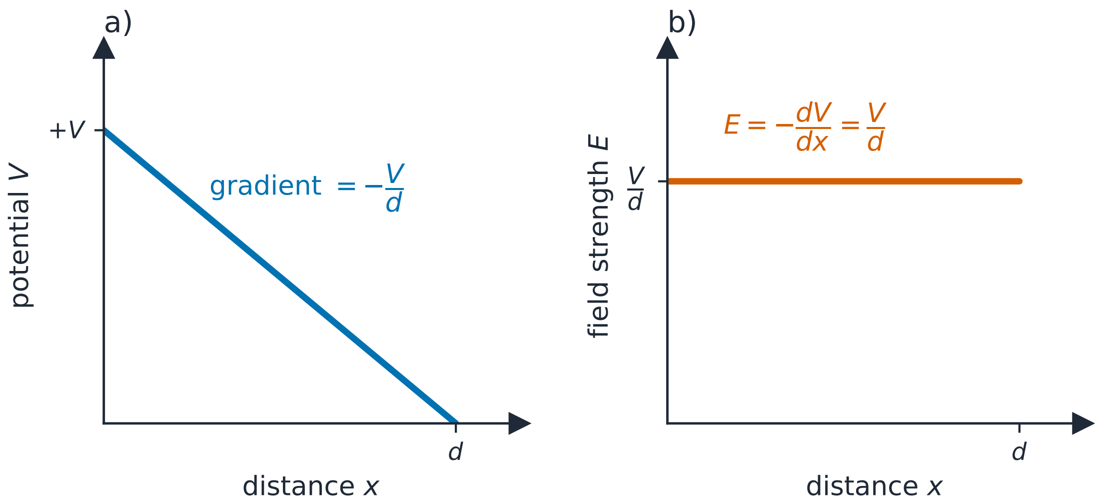
From potential to field

Potential and field contain the same information, linked by a derivative in each direction and by a path integral:

$$E_x = -\frac{dV}{dx}, \qquad \Delta V = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{r}.$$

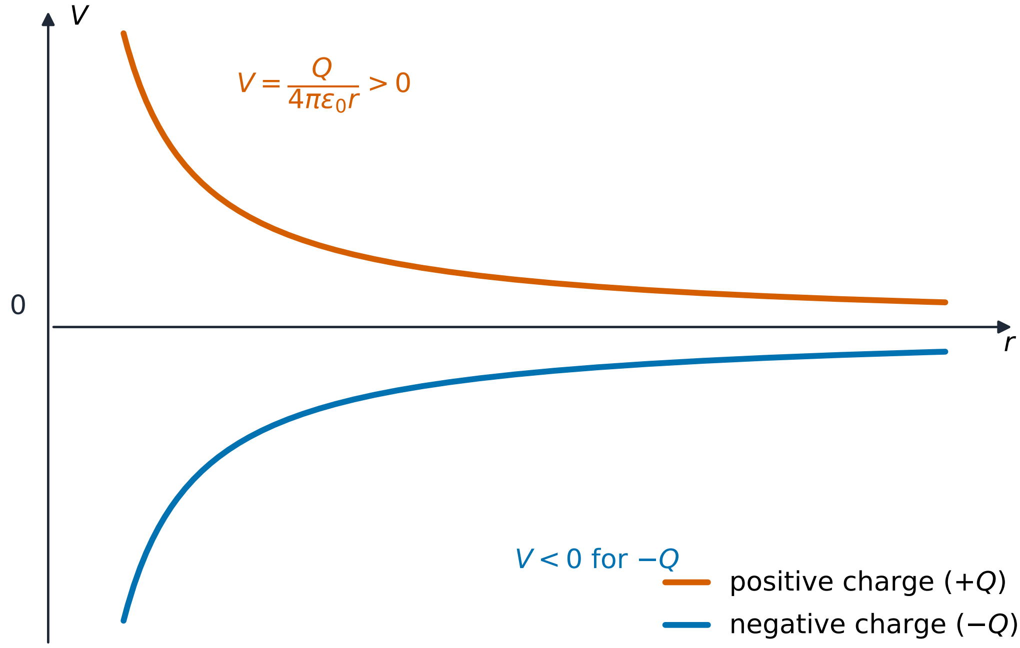
The field is the negative **gradient** 梯度 of the potential: $\vec{E}$ points from high potential to low potential, "downhill". Where V changes quickly, the field is strong.

Worked example. Along the x-axis, $V(x) = 3x^2 - 2x$ (volts, with x in metres). Then $E_x = -\dfrac{dV}{dx} = 2 - 6x$ V/m. At $x = 0.50$ m, $E_x = -1.0$ V/m: the field there points in the $-x$ direction.



In a uniform field the potential falls steadily with distance

The **potential difference** 电势差 between two points is the change in potential energy per unit charge moved between them: $\Delta V = \Delta U_E/q$. A battery makes a potential difference chemically: reactions inside separate positive from negative charge and hold the terminals a fixed ΔV apart.

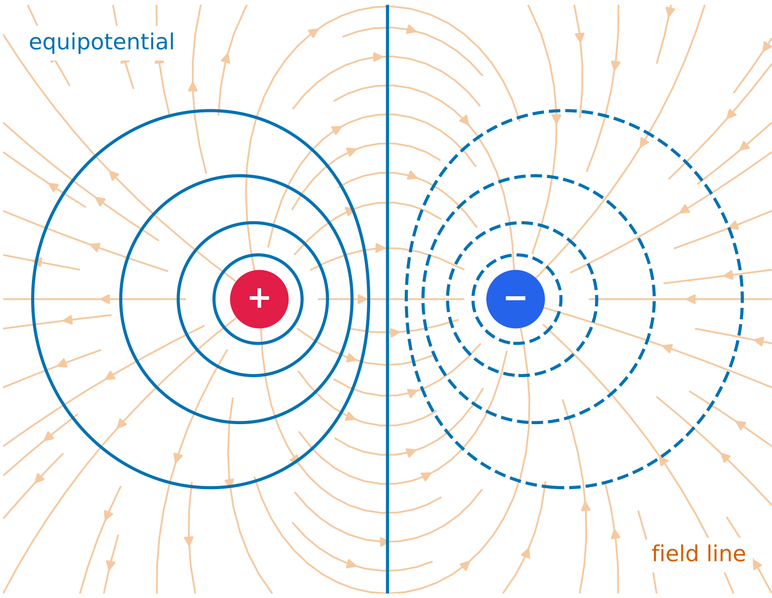


The potential near a point charge varies as 1/r

Equipotential maps

An **equipotential** 等势面 line (also called an **isoline** 等值线) joins points that share the same potential. Four rules let you read any map:

- Isolines are **perpendicular** 垂直 to field lines 电场线 everywhere they cross.
- $\vec{E}$ points from high V to low V –never along an isoline. Moving a charge along an isoline takes **no work** 不做功.
- Closely spaced isolines mean a strong field: $E \approx -\Delta V/\Delta x$.
- You can sketch the field map from an isoline map, and the isoline map from a field map.



Equipotential lines around a dipole cross the field lines at right angles

Exam skill. Given a map with isolines every 10 V spaced about 2 cm apart, estimate $E \approx \frac{10}{0.02} = 500$ V/m, pointing from the higher-value line to the lower one. The work an external agent does moving a charge q slowly from A to B is $W = q(V_B - V_A)$ –the path taken does not matter.

Conservation of Electric Energy

When a charge q moves between two points that differ in potential by ΔV, the potential energy of the charge–field system changes by

$$\Delta U_E = q\,\Delta V.$$

If only the electric force acts, total energy is conserved, so the kinetic energy changes by the opposite amount:

$$\Delta K = -\Delta U_E = -q\,\Delta V.$$

Watch the two signs together: a positive charge speeds up when it moves to *lower* potential, while a negative charge speeds up when it moves to *higher* potential. Both are just the system trading potential energy for kinetic energy. This is how particle accelerators give charged particles their energy, and it underlies energy analysis in circuits.

Worked example. A proton ($q = 1.6 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) starts at rest and is accelerated through a potential drop of 500 V (it moves to lower potential, so $\Delta V = -500$ V and $\Delta K = -q\,\Delta V = +8.0 \times 10^{-17}$ J). Setting $\Delta K = \frac{1}{2}mv^2$:

$$v = \sqrt{\frac{2(8.0 \times 10^{-17})}{1.67 \times 10^{-27}}} = 3.1 \times 10^5 \text{ m/s}.$$

Exam skill. Choose energy methods, not kinematics, whenever the field is non-uniform or the path curves: only the end-point potentials matter. A typical FRQ chain is $q\,\Delta V \rightarrow \Delta K \rightarrow v$, with one line of justification: "the electric force is conservative, so energy is conserved."

Exam tips

- Relate field and potential by $V = -\int \vec{E} \cdot d\vec{l}$ and $\vec{E} = -\nabla V$ (in 1-D, $E_x = -\frac{dV}{dx}$).
- Potential is a **scalar** —add contributions with signs, no vector components needed.
- Potential energy of a pair is $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$; use energy conservation for a charge's speed.
- Know that no work is done moving along an **equipotential**, which is perpendicular to $\vec{E}$.
- Choose the zero of potential (usually infinity) and state it.