

Electric Charges, Fields, and Gauss's Law

AP Physics C: Electricity and Magnetism

Electric Charge and Electric Force

Electric charge 电荷 comes in positive and negative; like charges repel, opposites attract. Charge is **quantized** 量子化 – every charge is a whole-number multiple of the **elementary charge** 基本电荷 $e = 1.6 \times 10^{-19} \text{ C}$ – and obeys **conservation of charge** 电荷守恒: charge is never created or destroyed, only moved. The force between two **point charges** 点电荷 is **Coulomb's law** 库仑定律:

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r},$$

an **inverse-square** 平方反比 force along the line joining them, with $\frac{1}{4\pi\epsilon_0} = k = 9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$. For several charges (AP uses four or fewer, unless symmetry helps), add the force *vectors* on the charge you care about – **superposition** 叠加.

Charge is a **fundamental property** 基本属性 of matter, and a **point charge** is a model that ignores the size of a charged object. The constant ϵ_0 is the **permittivity of free space** 真空介电常数 – a material property of the vacuum. The **permittivity of matter** differs from ϵ_0 , set by how easily the material polarizes, which is why a dielectric between charges weakens the force. Note too that although the electric force is far **stronger** than gravity, gravity dominates at astronomical scales because large bodies are nearly **neutral**.

Worked example. Two $+2.0 \mu\text{C}$ charges sit 0.30 m apart: $F = \frac{kq_1 q_2}{r^2} = \frac{9.0 \times 10^9 (2.0 \times 10^{-6})^2}{(0.30)^2} = 0.40 \text{ N}$, repulsive.

Worked example (vectors). Charges $+3.0 \mu\text{C}$ and $-3.0 \mu\text{C}$ sit at two corners of a right angle, each 0.30 m from a $+1.0 \mu\text{C}$ charge at the corner. Each exerts $F = \frac{(9.0 \times 10^9)(3.0 \times 10^{-6})(1.0 \times 10^{-6})}{0.09} = 0.30 \text{ N}$ – one a push, one a pull, at right angles to each other. The net force is $F_{\text{net}} = \sqrt{0.30^2 + 0.30^2} = 0.42 \text{ N}$, pointing between them. Never add magnitudes blindly: components first.

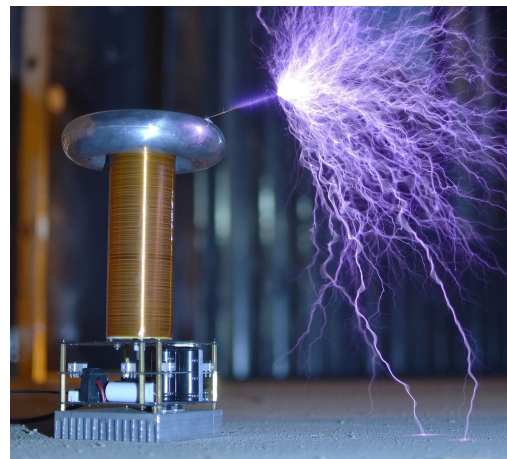
Electric Charge and the Process of Charging

A **conductor** 导体 lets charge move freely; an **insulator** 绝缘体 holds it in place. Objects gain net charge three ways:

- **Charging by friction** 摩擦起电: rubbing transfers electrons from one surface to the other.
- **Charging by conduction** 传导起电: touching a charged object shares charge of the *same* sign.
- **Charging by induction** 感应起电: a nearby charge pushes the conductor's charge apart; **ground** 接地 the far side, remove the ground wire *first*, and the conductor is left with the *opposite* sign – all without contact.

An insulator cannot pass charge, but it can develop **polarization** 极化: its molecules stretch or turn so one face is slightly positive and the other slightly negative. That is why a charged comb picks up neutral paper scraps. An **electroscope** 验电器 shows net charge by its leaves repelling; on any isolated conductor the excess charge sits entirely on the outer surface.

Electric Fields



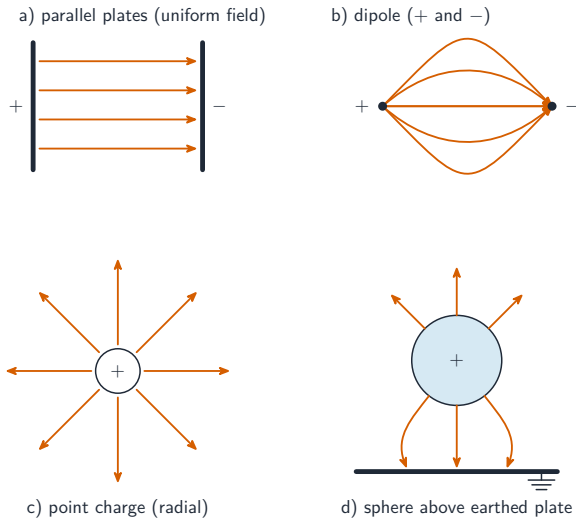
A Tesla coil throws sparks: it raises the voltage so high that the electric field ionizes the surrounding air, and charge leaps across as a visible discharge

The **electric field** 电场 at a point is the force per unit charge that a small positive **test charge** 检验电荷 would feel there:

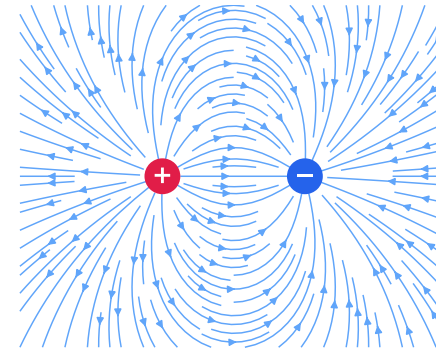
$$\vec{E} = \frac{\vec{F}}{q}, \quad \vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \text{ (point charge).}$$

A charge placed in a field feels $\vec{F} = q\vec{E}$ – positive charges along the field, negative charges against it. Fields from several sources add as vectors (superposition again).

Field lines 电场线 make the field visible: they point away from positive charge and toward negative, their density shows the strength, and they never cross.



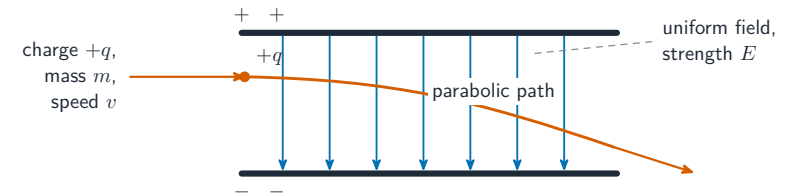
Electric field-line patterns for parallel plates, a dipole, and a point charge



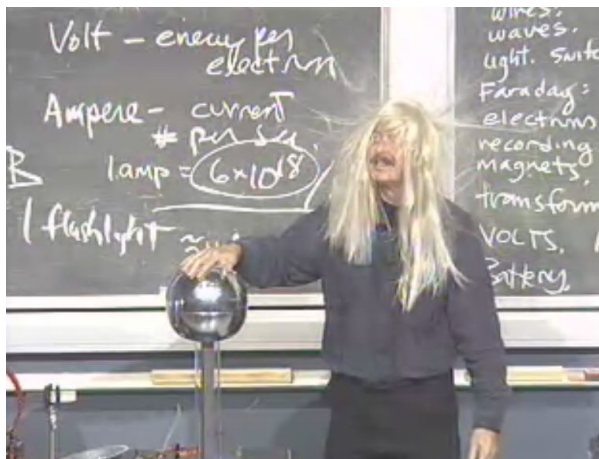
$$\vec{p} = q\vec{d} \cdot \text{lines leave + enter -} \cdot \text{denser} \Leftrightarrow \text{stronger } |\vec{E}|$$

Electric field lines of a dipole point from the positive to the negative charge

In a **uniform** 均匀 field a charge feels a constant force, so it accelerates uniformly – launched sideways, it follows a parabola, just like a projectile in gravity.



A charge crossing a uniform field follows a parabolic path



Charge spreads over the dome and onto the hair; like charges repel, so the strands push apart

Electric Fields of Charge Distributions

For a **continuous charge distribution** 连续电荷分布, cut the object into pieces dq and integrate their point-charge fields:

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}.$$

Write dq using the right density: λdl with the **linear charge density** 线电荷密度 (a line or ring), σdA with the **surface charge density** 面电荷密度, or ρdV with the **volume charge density** 体电荷密度. Then use **symmetry** 对称性 to cancel components *before* integrating – that sentence is usually a scored step. AP's calculus cases: a finite line (collinear point or perpendicular bisector), an infinite wire or cylinder, a ring on its axis, and an arc at its centre.

Worked example (ring, on axis). A ring of radius R carries charge Q . At a point z along the axis, each element is a distance $\sqrt{R^2 + z^2}$ away, and by symmetry the sideways components cancel, leaving only the axial part ($\cos \alpha = z/\sqrt{R^2 + z^2}$):

$$E_z = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{R^2 + z^2} \cdot \frac{z}{\sqrt{R^2 + z^2}} = \frac{1}{4\pi\epsilon_0} \frac{Qz}{(R^2 + z^2)^{3/2}}.$$

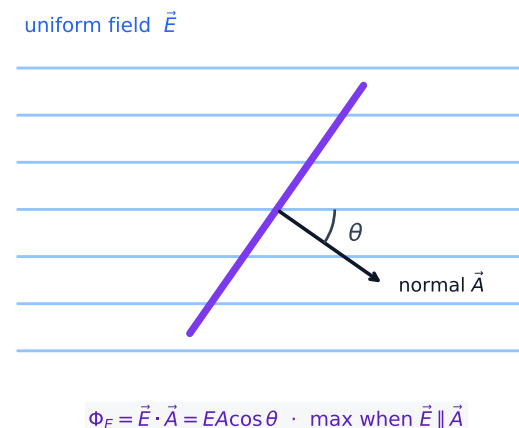
Check the limits: $E = 0$ at the centre ($z = 0$), and far away ($z \gg R$) it becomes kQ/z^2 – a point charge, as it must.

Electric Flux

Electric flux 电通量 measures how much field passes through a surface:

$$\Phi_E = \int \vec{E} \cdot d\vec{A}.$$

For a uniform field through a flat area, $\Phi_E = EA \cos \theta$, where the **area vector** 面积矢量 is perpendicular to the surface. Flux is positive where field lines *exit* a closed surface and negative where they *enter* – only the perpendicular component of $\vec{E}$ counts.



Electric flux through a surface depends on the angle between the field and the surface normal

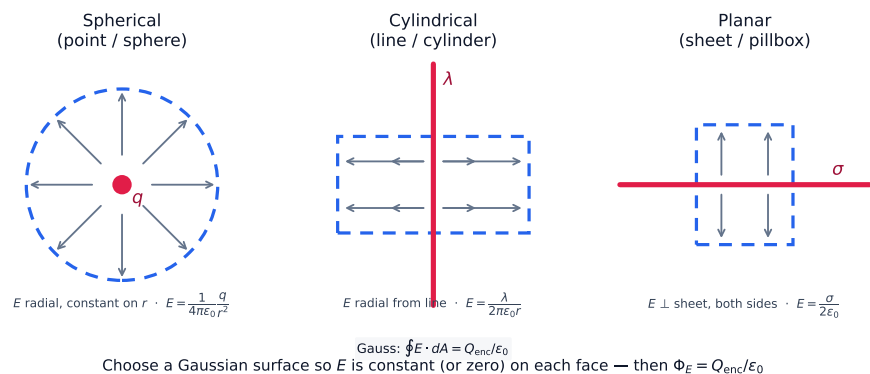
Gauss's Law

Gauss's law 高斯定律 – the first of Maxwell's equations – relates the flux through any closed surface to the charge inside it:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}.$$

It is true for *any* closed surface, but it *solves* for E only when symmetry lets you choose a **Gaussian surface** 高斯面 on which E is constant and perpendicular (or parallel, contributing zero). The three symmetries AP tests: **spherical symmetry**

球对称 (concentric sphere), **cylindrical symmetry** 柱对称 (coaxial cylinder), and **planar symmetry** 平面对称 (a straddling "pillbox").



A Gaussian surface is chosen to match the symmetry of the charge

Worked example (sphere). Outside a sphere of total charge Q , a spherical Gaussian surface of radius r gives $E(4\pi r^2) = Q/\epsilon_0$, so $E = \frac{kQ}{r^2}$ — identical to a point charge at the centre. *Inside* a uniformly charged solid sphere of radius R , the surface encloses only $Q_{\text{enc}} = Q r^3/R^3$, so $E = \frac{kQr}{R^3}$: zero at the centre, growing linearly to the surface.

Worked example (wire). For an infinite wire with charge per length λ , take a coaxial cylinder of radius r and length L . The ends contribute nothing ($\vec{E} \perp d\vec{A}$), so $E(2\pi rL) = \frac{\lambda L}{\epsilon_0}$ and $E = \frac{\lambda}{2\pi\epsilon_0 r}$. The same pillbox method gives an infinite sheet's field, $E = \frac{\sigma}{2\epsilon_0}$, uniform on both sides.

Exam skill. A full-credit Gauss's-law answer has four parts: name the surface, state the symmetry argument (why E is constant and perpendicular on it), count Q_{enc} , then solve. Skipping the symmetry sentence loses the reasoning point — and remember Gauss's law also explains why $E = 0$ inside any conductor in equilibrium.

Exam tips

- Use **Coulomb's law** $F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$ and superpose forces as **vectors** (components, not magnitudes).
- For a continuous charge, integrate $d\vec{E}$ over $dq = \lambda dl$, σdA , ρdV ; use symmetry to cancel components.

- The field points **away from** positive and **toward** negative charge; draw field lines correctly.
- Distinguish the force on a charge ($\vec{F} = q\vec{E}$) from the field the charge creates.
- Keep the constant $k = \frac{1}{4\pi\epsilon_0}$ straight and check units.