Electric Charges, Fields, and Gauss's Law

AP Physics C: Electricity and Magnetism

Electric Charge and Electric Force

Electric charge 电荷 comes in positive and negative; like charges repel, opposites attract. Charge is **quantized** 量子化–every charge is a whole-number multiple of the **elementary charge** 基本电荷 $e = 1.6 \times 10^{-19}$ C –and obeys **conservation of charge** 电荷守恒: charge is never created or destroyed, only moved. The force between two **point charges** 点电荷 is **Coulomb's law** 库仑定律:

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\,\hat{r},$$

an **inverse-square** 平方反比 force along the line joining them, with $\frac{1}{4\pi\varepsilon_0} = k = 9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$. For several charges (AP uses four or fewer, unless symmetry helps), add the force *vectors* on the charge you care about –**superposition** 叠加.

Worked example. Two $+2.0\ \mu$C charges sit 0.30 m apart: $F = \frac{kq_1q_2}{r^2} = \frac{9.0 \times 10^9 (2.0 \times 10^{-6})^2}{(0.30)^2} = 0.40$ N, repulsive.

Worked example (vectors). Charges $+3.0\ \mu$C and $-3.0\ \mu$C sit at two corners of a right angle, each 0.30 m from a $+1.0\ \mu$C charge at the corner. Each exerts $F = \frac{(9.0 \times 10^9)(3.0 \times 10^{-6})(1.0 \times 10^{-6})}{0.09} = 0.30$ N –one a push, one a pull, at right angles to each other. The net force is $F_{\text{net}} = \sqrt{0.30^2 + 0.30^2} = 0.42$ N, pointing between them. Never add magnitudes blindly: components first.

Electric Charge and the Process of Charging

A **conductor** 导体 lets charge move freely; an **insulator** 绝缘体 holds it in place. Objects gain net charge three ways:

- **Charging by friction** 摩擦起电: rubbing transfers electrons from one surface to the other.
- **Charging by conduction** 传导起电: touching a charged object shares charge of the *same* sign.
- **Charging by induction** 感应起电: a nearby charge pushes the conductor's charge apart; **ground** 接地 the far side, remove the ground wire *first*, and the conductor is left with the *opposite* sign –all without contact.

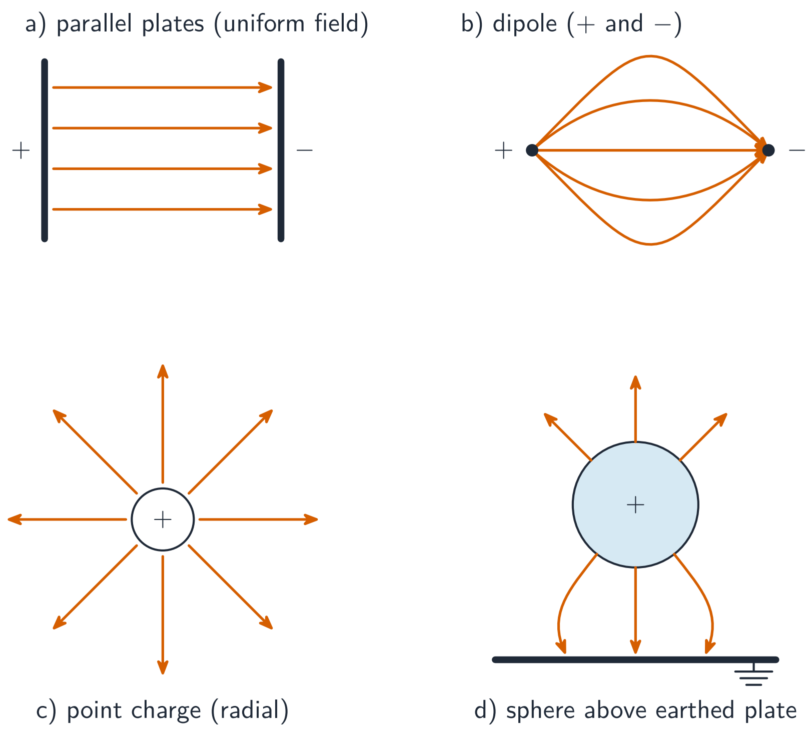
An insulator cannot pass charge, but it can develop **polarization** 极化: its molecules stretch or turn so one face is slightly positive and the other slightly negative. That is why a charged comb picks up neutral paper scraps. An **electroscope** 验电器 shows net charge by its leaves repelling; on any isolated conductor the excess charge sits entirely on the outer surface.

Electric Fields

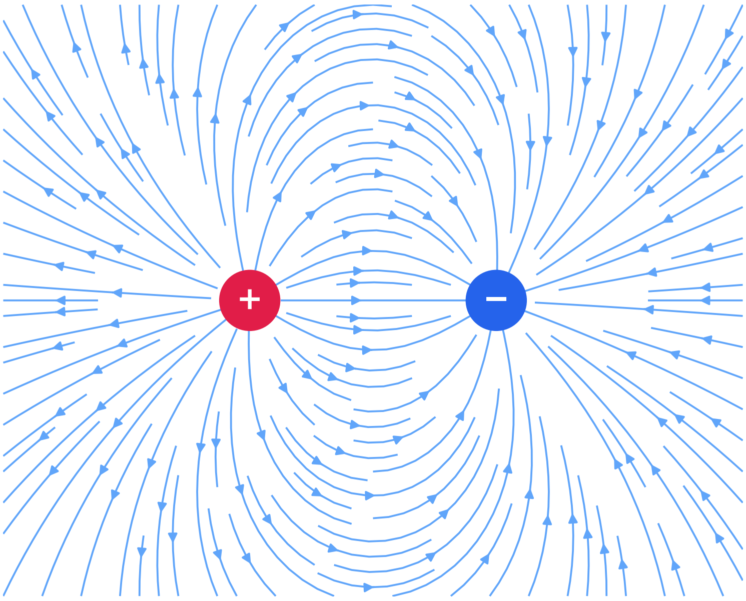
The **electric field** 电场 at a point is the force per unit charge that a small positive **test charge** 检验电荷 would feel there:

$$\vec{E} = \frac{\vec{F}}{q}, \qquad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,\hat{r} \text{ (point charge).}$$

A charge placed in a field feels $\vec{F} = q\vec{E}$ –positive charges along the field, negative charges against it. Fields from several sources add as vectors (superposition again). **Field lines** 电场线 make the field visible: they point away from positive charge and toward negative, their density shows the strength, and they never cross.

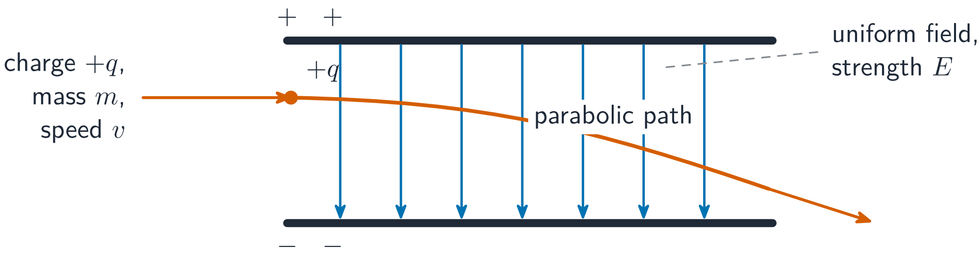


Electric field-line patterns for parallel plates, a dipole, and a point charge



Electric field lines of a dipole point from the positive to the negative charge

In a **uniform** 均匀 field a charge feels a constant force, so it accelerates uniformly – launched sideways, it follows a parabola, just like a projectile in gravity.



A charge crossing a uniform field follows a parabolic path

Electric Fields of Charge Distributions

For a **continuous charge distribution** 连续电荷分布, cut the object into pieces dq and integrate their point-charge fields:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\,\hat{r}.$$

Write dq using the right density: $\lambda\,dl$ with the **linear charge density** 线电荷密度 (a line or ring), $\sigma\,dA$ with the **surface charge density** 面电荷密度, or $\rho\,dV$ with the **volume charge density** 体电荷密度. Then use **symmetry** 对称性 to cancel components *before* integrating –that sentence is usually a scored step. AP's calculus cases: a finite line (collinear point or perpendicular bisector), an infinite wire or cylinder, a ring on its axis, and an arc at its centre.

Worked example (ring, on axis). A ring of radius R carries charge Q. At a point z along the axis, each element is a distance $\sqrt{R^2+z^2}$ away, and by symmetry the sideways components cancel, leaving only the axial part ($\cos\alpha = z/\sqrt{R^2+z^2}$):

$$E_z = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{R^2+z^2}\cdot\frac{z}{\sqrt{R^2+z^2}} = \frac{1}{4\pi\varepsilon_0}\frac{Qz}{(R^2+z^2)^{3/2}}.$$

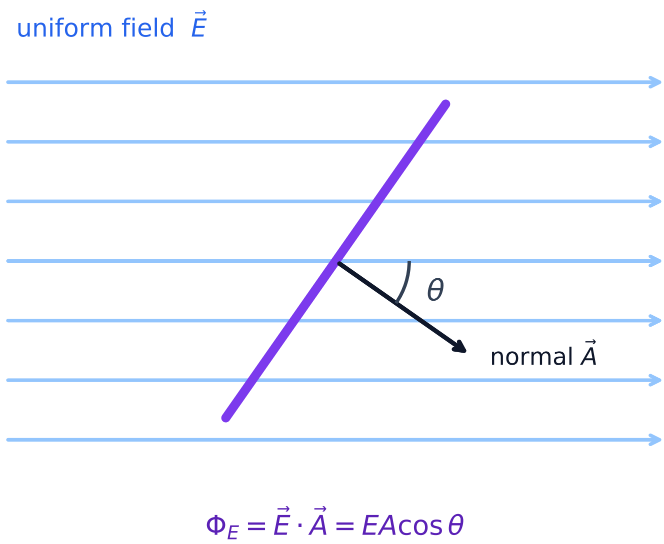
Check the limits: $E=0$ at the centre ($z=0$), and far away ($z \gg R$) it becomes kQ/z^2 –a point charge, as it must.

Electric Flux

Electric flux 电通量 measures how much field passes through a surface:

$$\Phi_E = \int \vec{E}\cdot d\vec{A}.$$

For a uniform field through a flat area, $\Phi_E = EA\cos\theta$, where the **area vector** 面积矢量 is perpendicular to the surface. Flux is positive where field lines *exit* a closed surface and negative where they *enter* –only the perpendicular component of $\vec{E}$ counts.



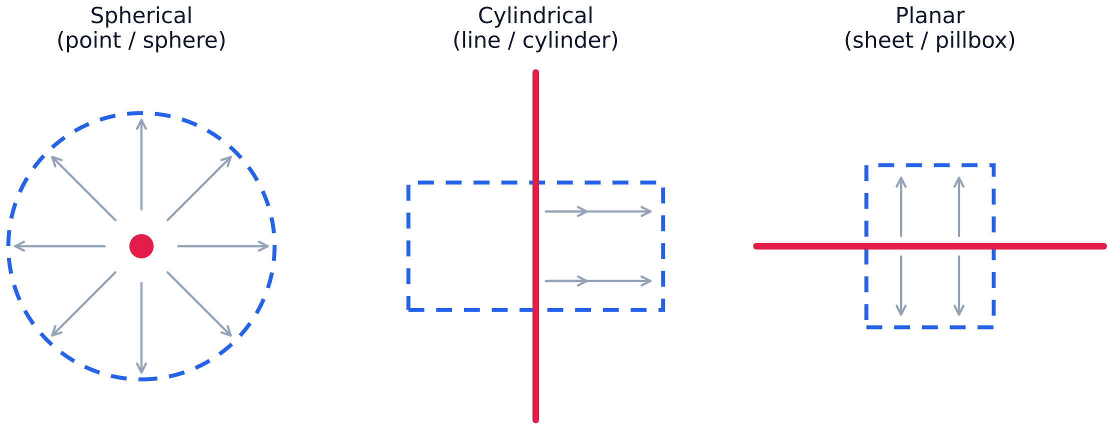
Electric flux through a surface depends on the angle between the field and the surface normal

Gauss's Law

Gauss's law 高斯定律–the first of Maxwell's equations –relates the flux through any closed surface to the charge inside it:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\varepsilon_0}.$$

It is true for *any* closed surface, but it *solves* for E only when symmetry lets you choose a **Gaussian surface** 高斯面 on which E is constant and perpendicular (or parallel, contributing zero). The three symmetries AP tests: **spherical symmetry** 球对称 (concentric sphere), **cylindrical symmetry** 柱对称 (coaxial cylinder), and **planar symmetry** 平面对称 (a straddling "pillbox").



A Gaussian surface is chosen to match the symmetry of the charge

Worked example (sphere). Outside a sphere of total charge Q, a spherical Gaussian surface of radius r gives $E(4\pi r^2) = Q/\varepsilon_0$, so $E = \dfrac{kQ}{r^2}$ –identical to a point charge at the centre. *Inside* a uniformly charged solid sphere of radius R, the surface encloses only $Q_{\text{enc}} = Q\,r^3/R^3$, so $E = \dfrac{kQr}{R^3}$: zero at the centre, growing linearly to the surface.

Worked example (wire). For an infinite wire with charge per length λ, take a coaxial cylinder of radius r and length L. The ends contribute nothing ($\vec{E} \perp d\vec{A}$), so $E(2\pi rL) = \dfrac{\lambda L}{\varepsilon_0}$ and $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$. The same pillbox method gives an infinite sheet's field, $E = \dfrac{\sigma}{2\varepsilon_0}$, uniform on both sides.

Exam skill. A full-credit Gauss's-law answer has four parts: name the surface, state the symmetry argument (why E is constant and perpendicular on it), count Q_{enc}, then solve. Skipping the symmetry sentence loses the reasoning point –and remember Gauss's law also explains why $E = 0$ inside any conductor in equilibrium.

Exam tips

- Use **Coulomb's law** $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$ and superpose forces as **vectors** (components, not magnitudes).
- For a continuous charge, integrate $d\vec{E}$ over $dq = \lambda\,dl,\ \sigma\,dA,\ \rho\,dV$; use symmetry to cancel components.

- The field points **away from** positive and **toward** negative charge; draw field lines correctly.
- Distinguish the force on a charge ($\vec{F} = q\vec{E}$) from the field the charge creates.
- Keep the constant $k = \frac{1}{4\pi\varepsilon_0}$ straight and check units.