

Question 1: Free-Response Question

15 points

- (a) For using a correct equation for electric flux 1 point

Example Response

$$\Phi_E = \frac{Q}{\epsilon_0}$$

For the correct numerical answer 1 point

Scoring Note: This point can be earned if a negative sign is included in the final answer or if units are missing and/or the units are incorrect.

Example Response

$$|\Phi_E| = 226 \frac{\text{N} \cdot \text{m}^2}{\text{C}}$$

Example Solution

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$\Phi_E = \frac{Q}{\epsilon_0}$$

$$\Phi_E = \frac{-2.0 \times 10^{-9} \text{ C}}{8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}}$$

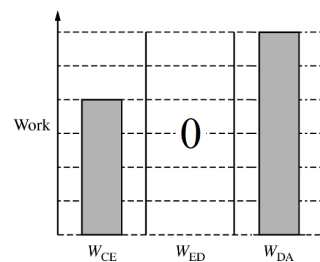
$$|\Phi_E| = 226 \frac{\text{N} \cdot \text{m}^2}{\text{C}}$$

Total for part (a) 2 points

- (b)(i) For indicating that
- $W_{ED} = 0$
- 1 point

For drawing a bar representing W_{DA} that has a height of six units 1 point

Example Response



- (b)(ii) For using an equation that relates the electric field to potential difference 1 point

Scoring Note: This point can be earned if the response begins with a correct relationship between electric field and potential difference in which numerical values are already substituted.

Example Responses

$$E_x = -\frac{dV}{dx} \quad \text{OR} \quad |E_x| = \left| -\frac{dV}{dx} \right| \quad \text{OR} \quad |E_x| = \left| -\frac{\Delta V}{\Delta x} \right| \quad \text{OR} \quad \Delta V = -\int \vec{E} \cdot d\vec{r}$$

For correct substitutions of values of electric potential and the distance between equipotential lines that can be used to calculate the approximate magnitude of the electric field at Position B 1 point

Example Response

$$|E_x| = \left| -\frac{20.0 \text{ V} - 0.0 \text{ V}}{0.65 \text{ m}} \right|$$

Example Solution

$$E_x = -\frac{dV}{dx}$$

$$|E_x| = \left| -\frac{dV}{dx} \right|$$

$$|E_x| = \left| -\frac{\Delta V}{\Delta x} \right|$$

$$|E_x| = \left| -\frac{20.0 \text{ V} - 0.0 \text{ V}}{0.65 \text{ m}} \right|$$

$$|E_x| = 31 \frac{\text{V}}{\text{m}}$$

Total for part (b) 4 points

(c)	For selecting only $+y$ with an attempt at a relevant justification	1 point
	For indicating that the direction of the electric field vector is perpendicular to a line that is tangent to the equipotential line at Position D	1 point
	For indicating one of the following:	1 point
	- The test charge moves from a higher electric potential to a lower electric potential. - The test charge and the sphere have charges of opposite sign. - The test charge moves in the direction of the electric field, which is directed upward.	
Example Response		
$+y$. The direction of an electric field vector is perpendicular to an equipotential line. Because the test charge has a positive charge, the test charge would move from a position of higher electric potential to a position of lower electric potential when an electric force is exerted on the test charge. Therefore, at Position D, the electric force is upward because that is the direction that is perpendicular to the equipotential line and in the direction of decreasing electric potential.		
Total for part (c)		3 points
(d)(i)	For using an appropriate equation for determining the electric potential from a line of uniform charge	1 point
Example Responses		
$V = \frac{1}{4\pi\epsilon_0} \sum \frac{q_i}{r_i} \quad \text{OR} \quad V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} \quad \text{OR} \quad \Delta V = -\int \vec{E} \cdot d\vec{r}$		
	For a correct determination of r , the distance between Point P and a point on the line of uniform charge	1 point
Example Responses		
$V_P = k \sum \frac{Q}{x_P - x} \quad \text{OR} \quad V_P = k \int \left(\frac{1}{x_P - x} \right) dq$		
	For a correct integral with λdx substituted for dq	1 point
Example Response		
$V_P = k\lambda \int \left(\frac{1}{x_P - x} \right) dx$		
	For the correct limits of integration	1 point
Example Response		
$V_P = k\lambda \int_0^{4L} \left(\frac{1}{x_P - x} \right) dx$		

Example Solutions

$$V = \frac{1}{4\pi\epsilon_0} \sum \frac{q_i}{r_i}$$

$$V_P = k \int \left(\frac{1}{x_P - x} \right) dq$$

$$dq = \lambda dx$$

$$V_P = k\lambda \int \left(\frac{1}{x_P - x} \right) dx$$

$$V_P = k\lambda \int_0^{4L} \left(\frac{1}{x_P - x} \right) dx$$

$$V_P = -k\lambda \ln(x_P - x) \Big|_0^{4L} = k\lambda \ln(x_P - x) \Big|_{4L}^0$$

$$V_P = k\lambda \ln \left(\frac{x_P}{x_P - 4L} \right)$$

OR

$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

$$E(x) = \int \frac{k}{r^2} dq$$

$$E(x) = \int_0^{4L} \frac{k\lambda}{(x - x')^2} dx' = k\lambda \left(\frac{1}{x - 4L} - \frac{1}{x} \right)$$

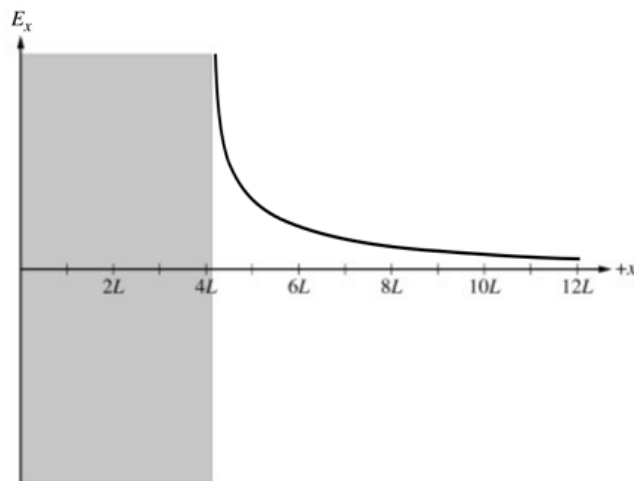
$$V_P = -\int_{\infty}^{x_P} E(x) dx = -k\lambda \int_{\infty}^{x_P} \left(\frac{1}{x - 4L} - \frac{1}{x} \right) dx$$

$$V_P = k\lambda \ln \left(\frac{x_P}{x_P - 4L} \right)$$

- (d)(ii) For sketching a curve or line that continually approaches the horizontal axis as position increases **1 point**

For sketching a concave up curve that is always positive **1 point**

Example Response



Total for part (d) 6 points

Total for question 1 15 points

Question 1: Free-Response Question

15 points

- (a) For using a correct equation for electric flux **1 point**

Example Response

$$\Phi_E = \frac{Q}{\epsilon_0}$$

For the correct numerical answer **1 point**

Scoring Note: Correct units are not required to earn this point.

Example Response

$$\Phi_E = 113 \frac{\text{N} \cdot \text{m}^2}{\text{C}}$$

Example Solution

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$\Phi_E = \frac{Q}{\epsilon_0}$$

$$\Phi_E = \frac{1.0 \times 10^{-9} \text{ C}}{8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}}$$

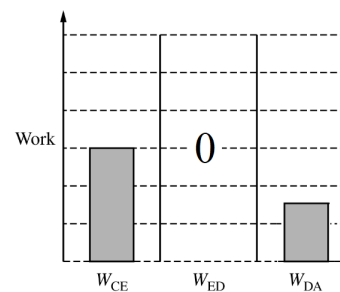
$$\Phi_E = 113 \frac{\text{N} \cdot \text{m}^2}{\text{C}}$$

Total for part (a) 2 points

- (b)(i) For indicating that $W_{\text{ED}} = 0$ **1 point**

For drawing a bar representing W_{DA} that has a height of approximately one and a half units **1 point**

Example Response



- (b)(ii) For using an equation that relates the electric field to potential difference **1 point**

Scoring Note: This point can be earned if the response begins with a correct relationship between electric field and potential difference in which numerical values are already substituted.

Example Responses

$$E_y = -\frac{dV}{dy} \quad \text{OR} \quad |E_y| = \left| -\frac{dV}{dy} \right| \quad \text{OR} \quad |E_y| = \left| -\frac{\Delta V}{\Delta y} \right| \quad \text{OR} \quad \Delta V = -\int \vec{E} \cdot d\vec{r}$$

For correct substitutions of values of electric potential and the distance between equipotential lines that can be used to calculate the approximate magnitude of the electric field at Position B **1 point**

Example Response

$$|E_y| = \left| -\frac{-20.0 \text{ V} - (-10.0 \text{ V})}{1.4 \text{ m}} \right|$$

Example Solution

$$E_y = -\frac{dV}{dy}$$

$$|E_y| = \left| -\frac{dV}{dy} \right|$$

$$|E_y| = \left| -\frac{\Delta V}{\Delta y} \right|$$

$$|E_y| = \left| -\frac{-20.0 \text{ V} - (-10.0 \text{ V})}{1.4 \text{ m}} \right|$$

$$|E_y| = 7.1 \frac{\text{V}}{\text{m}}$$

Total for part (b) 4 points

- (c) For selecting only $-y$ with an attempt at a relevant justification **1 point**

For indicating that the direction of the electric field vector is perpendicular to a line that is tangent to the equipotential line at Position C **1 point**

For indicating one of the following: **1 point**

- The test charge moves from a higher electric potential to a lower electric potential.
- The test charge and the rod have charges of the opposite sign.
- The test charge and the sphere have charges of the same sign.

Example Response

$-y$. The direction of an electric field vector is perpendicular to an equipotential line. Because the test charge has a positive charge, the test charge would move from a position of higher electric potential to a position of lower electric potential when an electric force is exerted on the test charge. Therefore, at Position C, the electric force is downward because that is the direction that is perpendicular to the equipotential line and in the direction of decreasing electric potential.

Total for part (c) 3 points

- (d)(i) For using an appropriate equation for determining the electric potential from a line of uniform charge **1 point**

Example Responses

$$V = \frac{1}{4\pi\epsilon_0} \sum \frac{q_i}{r_i} \quad \text{OR} \quad V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} \quad \text{OR} \quad \Delta V = -\int \vec{E} \cdot d\vec{r}$$

For a correct determination of r , the distance between Point P and a point on the line of uniform charge **1 point**

Example Responses

$$V_P = k \sum \frac{Q}{y_P - y} \quad \text{OR} \quad V_P = k \int \left(\frac{1}{y_P - y} \right) dq$$

For a correct integral with λdy substituted for dq **1 point**

Example Response

$$V_P = -k\lambda \int \left(\frac{1}{y_P - y} \right) dy$$

For the correct limits of integration **1 point**

Example Response

$$V_P = -k\lambda \int_0^{2L} \left(\frac{1}{y_P - y} \right) dy$$

Example Solutions

$$V = \frac{1}{4\pi\epsilon_0} \sum \frac{q_i}{r_i}$$

$$V_P = k \int \left(\frac{1}{y_P - y} \right) dq$$

$$dq = -\lambda dy$$

$$V_P = -k\lambda \int \left(\frac{1}{y_P - y} \right) dy$$

$$V_P = -k\lambda \int_0^{2L} \left(\frac{1}{y_P - y} \right) dy$$

$$V_P = k\lambda \ln(y_P - y) \Big|_0^{2L} = -k\lambda \ln(y_P - y) \Big|_{2L}^0$$

$$V_P = -k\lambda \ln \left(\frac{y_P}{y_P - 2L} \right)$$

OR

$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

$$E(y) = \int \frac{k}{r^2} dq$$

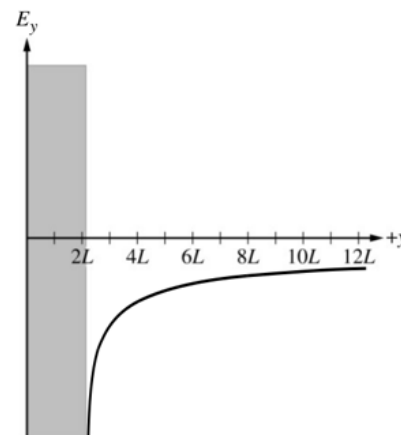
$$E(y) = \int_0^{2L} \frac{k\lambda}{(y - y')^2} dy' = k\lambda \left(\frac{1}{y - 2L} - \frac{1}{y} \right)$$

$$V_P = -\int_{\infty}^{y_P} E(y) dy = -k\lambda \int_{\infty}^{y_P} \left(\frac{1}{y - 2L} - \frac{1}{y} \right) dy$$

$$V_P = k\lambda \ln \left(\frac{y_P}{y_P - 2L} \right)$$

(d)(ii) For sketching a curve or line that continually approaches the horizontal axis as position increases **1 point**

For sketching a concave down curve that is always negative **1 point**

Example Response

Total for part (d) 6 points

Total for question 1 15 points

2019

AP[®] CollegeBoard

AP[®] Physics C: Electricity and Magnetism

Scoring Guidelines Set 2

2019 SCORING GUIDELINES

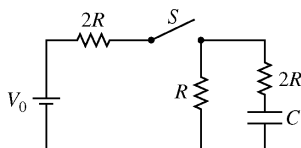
General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.
2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at <https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf>.
3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as “derive” and “calculate” on the exams, and what is expected for each, see “The Free-Response Sections — Student Presentation” in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or “Terms Defined” in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.
5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.
6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

2019 SCORING GUIDELINES

Question 1

15 points



The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0 , a capacitor of capacitance C , and a switch S . The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0 , R , C , and fundamental constants, as appropriate.

- (a) LO CNV-7.B.a, SP 5.A, 5.E
2 points

Derive an expression for the steady-state current supplied by the battery.

For using Ohm's law		1 point
$I = \frac{V}{R_{\text{eff}}} = \frac{V_0}{(2R + R)}$		
For correct substitution leading to correct answer		1 point
$I = \frac{V_0}{3R}$		

- (b) LO CNV-7.B.b, SP 5.A, 5.E
2 points

Derive an expression for the charge on the capacitor.

For using the equation relating stored charge to capacitance		1 point
$q = CV = CV_C = CV_R$		
For determining V_R and substituting into the above equation		1 point
$V_R = IR = \left(\frac{V_0}{3R}\right)R = \frac{1}{3}V_0$		
$q = CV_R = C\left(\frac{1}{3}V_0\right) = \frac{1}{3}CV_0$		

2019 SCORING GUIDELINES

Question 1 (continued)

- (c) LO CNV-7.B.b, SP 5.A, 5.E
2 points

Derive an expression for the energy stored in the capacitor.

For any correct equation for energy stored in a capacitor		1 point
$U = \frac{q^2}{2C}$		
Substitute charge and/or voltage from part (b)		1 point
$U = \frac{(CV_0/3)^2}{2C} = \frac{1}{18}CV_0^2$		

Now the switch is opened at time $t = 0$.

- (d) LO CNV-7.D.a, SP 5.A, 5.E
2 points

Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time t after the switch is opened.

For any correct voltage loop equation		1 point
$V_C - V_R - V_{2R} = 0 \therefore V_C = V_R + V_{2R}$		
$\frac{q(t)}{C} = I(R + 2R)$		
<u>Note:</u> Any correct loop equation for when the switch is open earns the point		
For substituting $-dq/dt$ or dq/dt for the current, consistent with loop equation		1 point
$\frac{q(t)}{C} = -3R \frac{dq}{dt}$		

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Question 1 (continued)

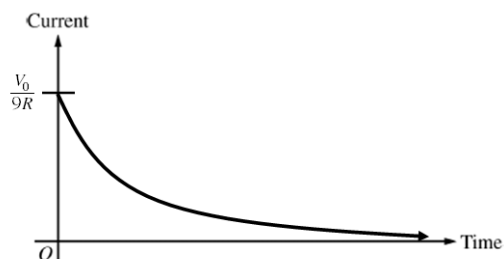
- (e)
i. LO CNV-7.B.a, SP 5.A, 5.E
2 points

Calculate the current in resistor R immediately after the switch is opened.

For using a voltage consistent with part (b) in $I = \frac{V}{R}$		1 point
For substituting the correct resistance ($3R$) into Ohm's law		1 point
$I = \frac{V_0/3}{3R} = \frac{V_0}{9R}$		

- ii. LO CNV-7.E.b, SP 3.C
3 points

On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



For a curve that is concave up throughout graph		1 point
For having the horizontal axis as an asymptote		1 point
For labeling the maximum current consistent with part (e)(i)		1 point

2019 SCORING GUIDELINES

Question 1 (continued)

- (f) LO CNV-7.G.a, SP 7.A, 7.C
2 points

Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c)?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

For selecting "Equal to"		1 point
For a correct justification invoking conservation of energy		1 point
Example: After the switch is opened, the capacitor will discharge all of its stored energy and charge. Assuming no energy is lost in the wire, then the only parts of the circuit that will dissipate this energy are the two resistors in series with the capacitor.		
<i>Alternate Solution</i>		<i>Alternate Points</i>
For selecting "Less than"		1 point
For a correct justification invoking conservation of energy		1 point
Example: After the switch is opened, the capacitor will discharge all of its stored energy and charge. Additionally there is energy lost due to resistance in the wire and/or energy loss in the capacitor.		

2019 SCORING GUIDELINES

Question 1 (continued)

Learning Objectives

CNV-7.B.a: Calculate the potential difference across a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.

CNV-7.B.b: Calculate the stored charge on a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.

CNV-7.D.a: Derive expressions using calculus to describe the time dependence of the stored charge or potential difference across the capacitor, or the current or potential difference across the resistor in an RC circuit when charging or discharging a capacitor.

CNV-7.E.b: Describe the behavior of the voltage or current behavior over time for a circuit that contains resistors and capacitors in a multi-loop arrangement.

CNV-7.G.a: Describe the energy transfer in charging or discharging a capacitor in an RC circuit.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.

5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.

5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

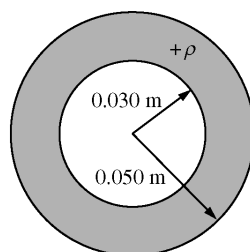
7.A: Make a scientific claim.

7.C: Support a claim with evidence from physical representations.

2019 SCORING GUIDELINES

Question 2

15 points



A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ , as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m: } \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m: } \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

$$r > 0.050 \text{ m: } \rho = 0$$

- (a) LO CNV-2.D.a, SP 6.B, 6.C
3 points

Calculate the total charge of the sphere.

For indicating the need to integrate the expression for charge density to determine the total charge on the sphere	1 point
$Q = \int \rho dV$	
For proper substitutions into the integration	1 point
$Q = \int \frac{(1.6 \times 10^{-6})}{r} (4\pi r^2) dr$	
For using the proper limits of integration	1 point
$Q = 4\pi (1.6 \times 10^{-6}) \int_{r=0.03}^{r=0.05} r dr = 4\pi (1.6 \times 10^{-6}) \left[\frac{r^2}{2} \right]_{r=0.03}^{r=0.05}$	
$Q = (2\pi) (1.6 \times 10^{-6}) (0.05^2 - 0.03^2) = 1.61 \times 10^{-8} \text{ C}$	

2019 SCORING GUIDELINES

Question 2 (continued)

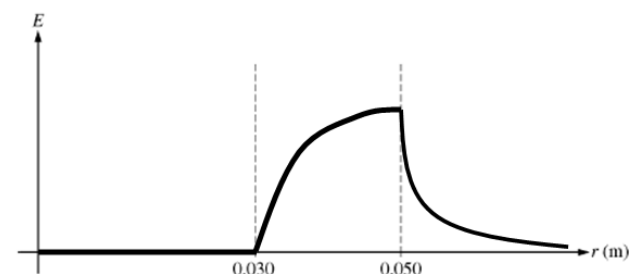
- (b) LO CNV-2.D.a, SP 6.C
3 points

Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.

For correctly evaluating the surface integral in Gauss's law	1 point
$\oint E \cdot dA = E(4\pi r^2)$	
For correctly substituting the answer from part (a) and correct radius into above equation	1 point
$\frac{Q_{enc}}{\epsilon_0} = E(4\pi r^2) \therefore E = \frac{Q_{enc}}{4\pi\epsilon_0 r^2} = \frac{(1.61 \times 10^{-8} \text{ C})}{(4\pi)(8.85 \times 10^{-12})(0.05)^2}$	
For an answer consistent with part (a) with correct units	1 point
$E = 5.79 \times 10^4 \text{ N/C}$	

- (c) LO CNV-2.C, SP 3.C
3 points

On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.



For clearly showing a graph with a value of $E = 0$ for $r < 0.030 \text{ m}$	1 point
For a continuous graph that starts at zero, is concave down, and increases in value from $r = 0.030$ to $r = 0.050$	1 point
For a continuous graph that decreases asymptotically toward the horizontal axis for $r > 0.050 \text{ m}$	1 point

2019 SCORING GUIDELINES

Question 2 (continued)

- (d) LO CNV-1.G.a, SP 6.B, 6.C
2 points

Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

For substituting the total charge from part (a) into a correct expression for electric potential	1 point
For substituting $r = 0.05$ m into a correct expression for electric potential	1 point
$V_R = \frac{Q_{\text{tot}}}{4\pi\epsilon_0 r} = \frac{(9 \times 10^9)(1.61 \times 10^{-8} \text{ C})}{(0.05 \text{ m})} = 2900 \text{ V}$	
<i>Alternate Solution</i>	<i>Alternate Points</i>
For substituting the total charge from part (a) into an integration for electric potential	1 point
$\Delta V_R = V_R - V_\infty = V_R = -\int E dr = -\int \frac{Q_{\text{enc}}}{4\pi\epsilon_0 r^2} dr = -\int \frac{(9 \times 10^9)(1.61 \times 10^{-8} \text{ C})}{r^2} dr$	
For integrating with correct limits of integration	1 point
$V_R = -\int_{r=\infty}^{r=0.05 \text{ m}} \frac{145}{r^2} dr = -145 \left[-\frac{1}{r} \right]_{r=\infty}^{r=0.05 \text{ m}} = 145 \left(\frac{1}{(0.05 \text{ m})} - \frac{1}{\infty} \right) = 2900 \text{ V}$	

A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.

- (e) i. LO ACT-1.D, SP 6.B, 6.C
2 points

Calculate the magnitude of the initial acceleration of the proton.

For using a correct expression of Newton's second law in terms of the electric field	1 point
$F = ma \therefore qE = ma \therefore a = \frac{qE}{m}$	
For correctly substituting into equation above	1 point
$a = \frac{qE}{m} = \frac{(1.6 \times 10^{-19} \text{ C})(5.79 \times 10^4 \text{ N/C})}{1.67 \times 10^{-27} \text{ kg}} = 5.55 \times 10^{12} \text{ m/s}^2$	

2019 SCORING GUIDELINES

Question 2 (continued)

- (e) continued
ii. LO CNV-1.E, SP 6.B, 6.C
2 points

Calculate the speed of the proton after a long time.

For a correct expression of kinetic energy in terms of the electric potential difference	1 point
$-q\Delta V = \frac{1}{2}mv^2$	
For correctly substituting into equation above	1 point
$v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{-2(1.6 \times 10^{-19} \text{ C})(0 - 2900 \text{ V})}{(1.67 \times 10^{-27} \text{ kg})}} = 7.45 \times 10^5 \text{ m/s}$	

Learning Objectives

ACT-1.D: Determine the motion of a charged object of specified charge and mass under the influence of an electrostatic force.

CNV-1.E: Calculate the work done or changes in kinetic energy (or changes in speed) of a charged particle when it is moved through some known potential difference.

CNV-1.G.a: Use the general relationship between electric field and electric potential to calculate the relationships between the magnitude of electric field or the potential difference as a function of position.

CNV-2.C: State and use Gauss's law in integral form to derive unknown electric fields for planar, spherical, or cylindrically symmetrical charge distributions.

CNV-2.D.a: Using appropriate mathematics (which may involve calculus), calculate the total charge contained in lines, surfaces, or volumes when given a linear-charge density, a surface-charge density, or a volume-charge density of the charge configuration.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.

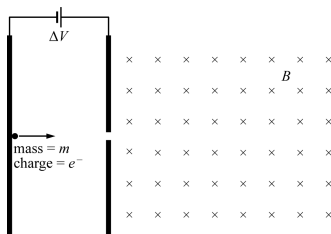
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.

6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

2019 SCORING GUIDELINES

Question 3

15 points



Two plates are set up with a potential difference V between them. A small sphere of mass m and charge $-e$ is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude B directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of V , m , e , B , and fundamental constants, as appropriate.

- (a)
i. LO CHG-1.A.a, SP 7.A
1 point

What is the initial direction of the force on the sphere as it enters the magnetic field?

☐ Into the page ☐ Out of the page
☐ Toward the top of the page ☐ Toward the bottom of the page

For selecting "Toward the bottom of the page"	1 point
---	---------

- ii. LO CHG-1.B, SP 7.A
1 point

Describe the path taken by the sphere after it enters the magnetic field.

For describing a circular path for the sphere consistent with the selection from part (a)(i)	1 point
Example: The sphere will move in a circular path toward the bottom of the page.	

2019 SCORING GUIDELINES

Question 3 (continued)

- (b) LO CNV-1.E, SP 5.A, 5.E
2 points

Derive an expression for the speed of the sphere as it passes through the small opening.

For using a valid equation relating potential difference to kinetic energy of the sphere	1 point
$\Delta K = -\Delta U \therefore \frac{1}{2}mv^2 = -q\Delta V$	
$\frac{1}{2}mv^2 = eV$	
For the correct answer with supporting work	1 point
$v = \sqrt{\frac{2eV}{m}}$	

- (c) LO CHG-1.C, SP 5.A, 5.E
3 points

Derive an expression for the radius of the path taken by the sphere as it moves through the magnetic field.

For an expression relating the magnetic force to the centripetal force	1 point
$\frac{mv^2}{r} = Bqv$	
For substituting the charge of the sphere into the above equation and solving for r	1 point
$\frac{mv}{r} = Be \therefore r = \frac{mv}{Be}$	
For substituting the answer from part (b)	1 point
$r = \frac{m}{Be} \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2mV}{eB^2}}$	

An experiment is performed in which a beam of electrons is accelerated across the space between the plates and passes through the small opening. After passing through the opening, the electrons travel in a semicircular path and strike the right-hand plate. The potential difference between the plates is varied in regular increments, as shown in the table below. For each potential difference, the magnetic field is varied in order to cause the beam to strike the right-hand plate at a distance of 0.020 m from the opening.

Potential difference (V)	60	70	100	110	120	140
Magnetic field ($T \times 10^{-3}$)	2.62	2.78	3.39	3.54	3.78	3.99

2019 SCORING GUIDELINES

Question 3 (continued)

- (d) LO CHG-1.C, SP 4.C
1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the mass-to-charge ratio of an electron.

Vertical axis: _____
Horizontal axis: _____

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

$$r = \sqrt{\frac{2m_e V}{eB^2}} \therefore B^2 = \frac{2m_e}{er^2} V$$

Potential difference (V)	60	70	100	110	120	140
Magnetic field ($T \times 10^{-3}$)	2.62	2.78	3.39	3.54	3.78	3.99
(Magnetic field) ² ($T \times 10^{-3}$) ²	6.86	7.73	11.5	12.5	14.3	15.9

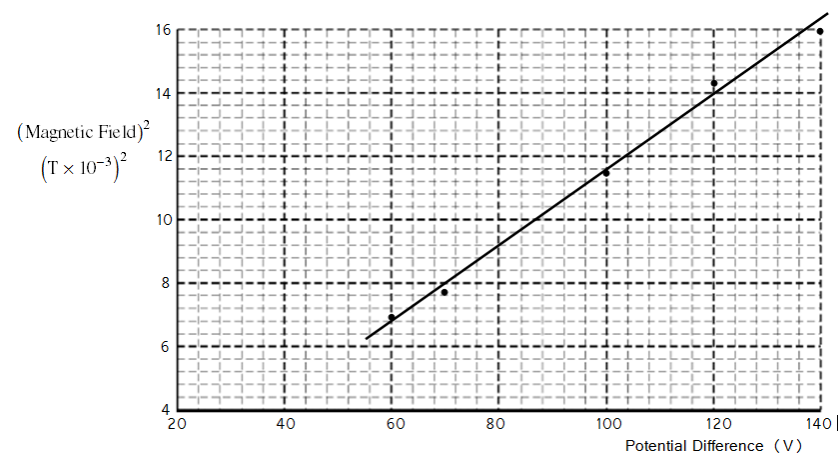
For indicating axis choices with appropriate quantities to produce a straight line		1 point
Note: There are several possible choices.		

2019 SCORING GUIDELINES

Question 3 (continued)

- (e) LO CHG-1.C, SP 3.A, 4.C
4 points

On the graph below, plot the relationship determined in part (d). Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



For correctly labeling both axes with quantities and units	1 point
For correctly scaling and labeling the axes so that the points use at least half the grid	1 point
For correctly plotting the data	1 point
For drawing a straight line that represents the data	1 point

- (f) LO CHG-1.C, SP 4.D, 5.E
3 points

Using the straight line from part (e), determine the mass-to-charge ratio of an electron.

For correctly calculating the slope from the best-fit straight line and not from the data points unless the points fall on the best-fit line	1 point
$\text{slope} = \frac{14 - 9.2}{120 - 80} = 0.12 \frac{(T \times 10^{-3})^2}{V} = 1.2 \times 10^{-7} T^2/V$	
For a correct expression relating the slope to the mass to charge ratio of an electron	1 point
$B^2 = \frac{2m_e}{er^2} V \therefore \text{slope} = \frac{2m_e}{er^2} \therefore \frac{m_e}{e} = \frac{r^2 \times \text{slope}}{2}$	
For substituting correct values into the above equation	1 point
$\frac{m_e}{e} = \frac{(0.01 \text{ m})^2 \times (1.2 \times 10^{-7} T^2/V)}{2} = 6.0 \times 10^{-12} \text{ kg/C}$	

2019 SCORING GUIDELINES**Question 3 (continued)****Learning Objectives**

CHG-1.A.a: Calculate the magnitude and direction of the magnetic force of interaction between a moving charged particle of specified charge and velocity moving in a region of a uniform magnetic field.

CHG-1.B: Describe the path of different moving charged particles (i.e., of different type of charge or mass) in a uniform magnetic field.

CHG-1.C: Derive an expression for the radius of a circular path for a charged particle of specified characteristics moving in a specified magnetic field.

CNV-1.E: Calculate the work done or changes in kinetic energy (or changes in speed) of a charged particle when it is moved through some known potential difference.

Science Practices

3.A: Select and plot appropriate data.

4.C: Linearize data and/or determine a best-fit line or curve.

4.D: Select relevant features of a graph to describe a physical situation or solve problems.

5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.

5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

7.A: Make a scientific claim.

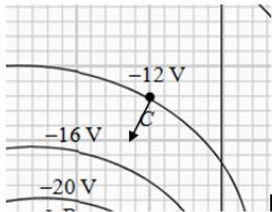
**AP[®] Physics C: Electricity and Magnetism
2016 Scoring Guidelines**

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SCORING GUIDELINES

15 points total	Question	Distribution of points
3 points		
	For indicating that the total potential is the sum of the potential from individual point charges	1 point
	ample using point B: $V_B = 0 = V_1 + V_2$	
	orrectly substituting into the above equation (signs are ignored at this step)	1 point
	$-V_1 = V_2$	
	$-\frac{kq_1}{r_1} = \frac{kq_2}{r_2}$	
	$-\frac{q_1}{(5 \times 0.5\text{ m})} = \frac{(2.0\text{ nC})}{(2 \times 0.5\text{ m})}$	
	For a correct answer with correct sign and units	1 point
	$q_1 = -5.0\text{ nC}$	
2 points		
		
	For drawing a vector perpendicular to the equipotential line for C	1 point
	For drawing a vector in the direction of the -16 V line	1 point
2 points		
	For using the equation relating the electric field to potential difference	1 point
	$E = -\frac{dV}{dx}$	
	$ E \approx \frac{\Delta V}{\Delta x}$	
	For substituting values from the figure	1 point
	$E = \frac{(-20\text{ V} - (-24\text{ V}))}{(2 \times 0.1\text{ m})}$	
	$E = 20\text{ N/C}$	

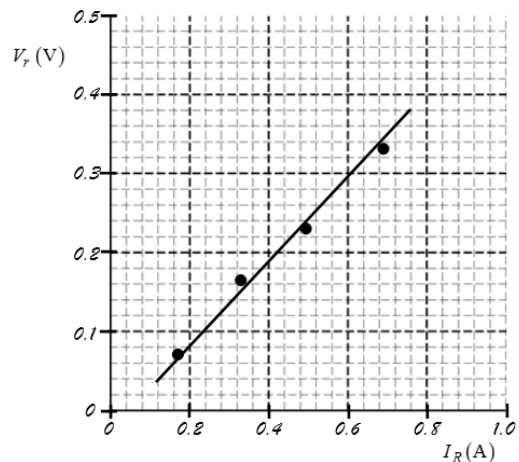
SCORING GUIDELINES

Question 1 (continued)	Distribution of points
2 points	
For using a correct equation for the electric flux	1 point
$\Phi_E = \frac{q_{enc}}{\epsilon_0}$	
$\Phi_E = \frac{(2.0\text{ nC})}{(8.85 \times 10^{-12}\text{ C}^2/(\text{N}\cdot\text{m}^2))}$	
For a correct answer with units	1 point
$\Phi_E = 226\text{ (N}\cdot\text{m}^2)/\text{C}$	
(	
2 points	
sing an equation that relates the work done to the change in potential energy	1 point
$W = -q\Delta V = -(1.6 \times 10^{-19}\text{ C})(-4\text{ V} - (4\text{ V}))$	
For a correct answer including sign and units	1 point
$W = 1.28 \times 10^{-18}\text{ J} = 8.0\text{ eV}$	
2 points	
ndicating that the kinetic energy of the proton at E will be equal to the work done	1 point
$W = \Delta K = \frac{1}{2}mv^2$	
orrectly substituting the answer from part (e) i. into the above equation	1 point
$(1.28 \times 10^{-18}\text{ J}) = \left(\frac{1}{2}\right)(1.67 \times 10^{-27}\text{ kg})v^2$	
$v = 3.92 \times 10^4\text{ m/s}$	
)	
2 points	
orrectly selecting Left	1 point
For a correct justification	1 point
Example: Electrons accelerate in the direction perpendicular to equipotential surfaces and toward higher potential. Therefore, at point B, an electron would accelerate toward the left.	
No points are earned if the wrong answer is selected.	

SCORING GUIDELINES

15 points total	Question	Distribution of points
2 points	For selecting “It is the same through both.” For a correct justification Example: The sample of wire and the resistor are in series with each other and therefore will have the same current. Note: Because the voltmeter is ideal, it will have no effect on the result. No points are earned if the wrong answer is selected.	1 point 1 point
2 points	For selecting “It will depend on the resistance of the sample.” For a correct justification including argument using Ohm’s law, voltage drop across resistors, etc. Example: Because the sample of wire has resistance, it cannot be known whether the wire of the resistor has the greatest resistance and which has the higher potential difference across it. Therefore, it will depend on the resistance of the wire sample. No points are earned if the wrong answer is selected.	1 point 1 point
(1 point	indicating the correct quantities for each axis Horizontal axis: I_R Vertical axis: $V_r = \mathcal{E} - V_R$ Note: Credit is received if the axes are reversed.	1 point

SCORING GUIDELINES

Question 2 (continued)	Distribution of points
3 points	 For a correct scale that uses more than half the grid and for correctly labeling the axes including units For correctly plotting given data For drawing a straight line consistent with the given data 1 point 1 point 1 point
(2 points	For correctly calculating slope using the best-fit straight line and not data points $\text{slope} = \frac{(V_2 - V_1)}{(I_2 - I_1)} = \frac{(0.30 - 0.10) \text{ V}}{(0.60 - 0.22) \text{ A}} = 0.526 \text{ V/A}$ Note: Linear regression gives slope = 0.485 V/A. (Student must indicate that the value comes from a linear regression from the calculator, and the equation of the line must be present.) For correctly relating r to the slope $V_r = I_R r$, therefore the slope equals r $r = 0.526 \Omega$ Note: Linear regression gives $r = 0.485 \Omega$. 1 point

SCORING GUIDELINES

Question 2 (continued)

Distribution
of points

) 1 point

For using the equation relating resistance to resistivity with the correct or consistent substitutions

1 point

$$R = \frac{\rho L}{A} \quad \text{so} \quad \rho = \frac{AR}{L}$$

$$\rho = \frac{(\pi)(1.00 \times 10^{-3} \text{ m})^2 (0.526 \, \Omega)}{(3.00 \text{ m})}$$

$$\rho = 5.51 \times 10^{-7} \, \Omega \cdot \text{m}$$

Note: Linear regression gives $\rho = 5.08 \times 10^{-7} \, \Omega \cdot \text{m}$.

2 points

electing “Less than” with an attempt at a justification
For a correct justification

1 point
1 point

Example: The resistance calculated from the graph is the sum of the sample resistance plus the ammeter resistance because it is not ideal. The actual resistance is the calculated resistance minus the ammeter resistance, and therefore less than the calculated resistance.
oints are earned if the wrong answer is selected.

2 points

For selecting “Greater than” with an attempt at a justification
For a correct justification

1 point
1 point

Example: If the voltmeter is not ideal, that would add an additional resistor in parallel. A parallel resistor reduces the total resistance of the circuit. This would lead to an increase in current and a higher reading on the ammeter.
No points are earned if the wrong answer is selected.

SCORING GUIDELINES

Question

Distribution
of points

15 points total

1 point

For selecting “Left”

1 point

2 points

For selecting “Less than”

1 point

For a correct justification

1 point

Example: Because as the bar falls the flux at point C is increasing, the emf generated must create a magnetic field to oppose this change. Therefore, it will create a magnetic field to decrease the flux and thus decrease the magnetic field.

No points are earned if the wrong answer is selected.

2 points

For selecting “Greater than”

1 point

For a correct justification

1 point

Example: The field at point C, which is above the bar, is less than the original magnetic field, and point D is on the other side of the bar. Therefore, the direction of the magnetic field from the bar at point D is the opposite of the direction at point C, so the net magnetic field at D when the bar is falling must be greater than the original magnetic field.

No points are earned if the wrong answer is selected.

(4 points

For correctly applying Newton’s second law to the motion of the bar

1 point

$$F_{\text{net}} = Mg - F_M = Mg - BIL$$

$$I = \mathcal{E}/R$$

For attempting to use Faraday’s law to obtain an expression for the emf in the bar

1 point

$$\mathcal{E} = d\Phi/dt = BL(dx/dt)$$

For correctly using the expression for emf to obtain an expression for the current

1 point

$$I = \frac{BL(dx/dt)}{R} = \frac{BLv}{R}$$

$$Ma = Mg - B \frac{BLv}{R} L$$

$$a = g - \frac{B^2 L^2 v}{MR}$$

riting the acceleration as dv/dt

1 point

$$\frac{dv}{dt} = g - \frac{B^2 L^2 v}{MR}$$

SCORING GUIDELINES

Question 3 (continued)

Distribution
of points

2 points

For setting the net force at terminal velocity equal to zero

1 point

$$0 = Mg - \frac{B^2 L^2 v_T}{R}$$

$$\frac{B^2 L^2 v_T}{R} = Mg$$

in answer consistent with part (c)

1 point

$$v_T = \frac{MgR}{B^2 L^2}$$

(1 point

For a correct or consistent substitution in an appropriate power equation

1 point

$$P = I^2 R \quad \text{or} \quad V = IR$$

$$\frac{BLv_T}{R} = I$$

$$P = \frac{B^2 L^2 v_T^2}{R}$$

Alternate Solution

For a correct or consistent substitution in an appropriate power equation

$$P = I^2 R$$

$$Mg = BIL$$

$$I = \frac{Mg}{BL}$$

$$P = \frac{M^2 g^2 R}{B^2 L^2}$$

Alternate points
1 point

SCORING GUIDELINES

Question 3 (continued)

Distribution
of points

) 3 points

Using the equation from part (c)

$$\frac{dv}{dt} = g - \frac{B^2 L^2 v}{MR}$$

$$\frac{dv}{dt} = \frac{-B^2 L^2}{MR} \left(v - \frac{MRg}{B^2 L^2} \right)$$

For attempting separation of variables

1 point

$$\frac{dv}{v - \frac{MRg}{B^2 L^2}} = \frac{-B^2 L^2}{MR} dt$$

For attempting to integrate with the correct limits or the correct constant of integration

1 point

$$\int_{v'=0}^{v'=v(t)} \frac{1}{\left(v' - \frac{MRg}{B^2 L^2} \right)} dv' = \int_{t'=0}^{t'=t} \frac{-B^2 L^2}{MR} dt'$$

$$\ln \left[v' - \frac{MRg}{B^2 L^2} \right]_{v'=0}^{v'=v(t)} = \frac{-B^2 L^2}{MR} t$$

$$\ln \left(\frac{v(t) - \frac{MRg}{B^2 L^2}}{\frac{MRg}{B^2 L^2}} \right) = \frac{-B^2 L^2}{MR} t$$

For a correct answer

1 point

$$v(t) = \frac{MRg}{B^2 L^2} \left(1 - e^{-\frac{B^2 L^2}{MR} t} \right)$$

ng a trial solution in the differential equation and verifying its correctness is also acceptable.