

Question 1: Mathematical Routines (MR)

10 points

- A (i) For a multistep derivation that includes the equation $\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$ **Point A1**

Scoring Note: Vector notation is not required for this point to be earned.

For a correct substitution of the area of an appropriate Gaussian surface with nonzero flux for the region $R_1 < r < R_2$ (e.g., $2\pi r\ell$) **Point A2**

For a correct expression for the enclosed charge (e.g., $\sigma_1(2\pi R_1\ell)$) **Point A3**

Example Response

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

$$E(2\pi r\ell) = \frac{\sigma_1(2\pi R_1\ell)}{\epsilon_0}$$

$$E = \frac{\sigma_1 R_1}{\epsilon_0 r}$$

- (ii) For substituting the expression for E from part A (i) into $\Delta V = -\int_a^b \vec{E} \cdot d\vec{r}$ **Point A4**

Scoring Notes:

- Vector notation is not required for this point to be earned.
- The sign of ΔV is not considered for this point to be earned.

For an attempt to solve the integral for $|\Delta V|$ that includes correct limits **Point A5**

$$\text{(e.g., } |\Delta V| = \frac{\sigma_1 R_1}{\epsilon_0} \int_{R_1}^{R_2} \frac{1}{r} dr \text{)}$$

Scoring Note: This point may be earned regardless of the order of the limits of integration; for example, $\frac{\sigma_1 R_1}{\epsilon_0} \int_{R_2}^{R_1} \frac{1}{r} dr$.

Example Response

$$|\Delta V| = \left| -\int_a^b \vec{E} \cdot d\vec{r} \right|$$

$$|\Delta V| = \int_{R_1}^{R_2} \frac{\sigma_1 R_1}{r \epsilon_0} dr = \frac{\sigma_1 R_1}{\epsilon_0} \int_{R_1}^{R_2} \frac{1}{r} dr$$

$$|\Delta V| = \frac{\sigma_1 R_1}{\epsilon_0} \ln(r) \Big|_{R_1}^{R_2} = \frac{\sigma_1 R_1}{\epsilon_0} \ln\left(\frac{R_2}{R_1}\right)$$

- (iii) For sketching a graph that is zero for both $0 < r < R_1$ and $r > R_2$ **Point A6**

For sketching a graph that is decreasing and concave up for $R_1 < r < R_2$ **Point A7**

Scoring Note: The curve does not have to intersect the vertical dashed lines for this point to be earned.

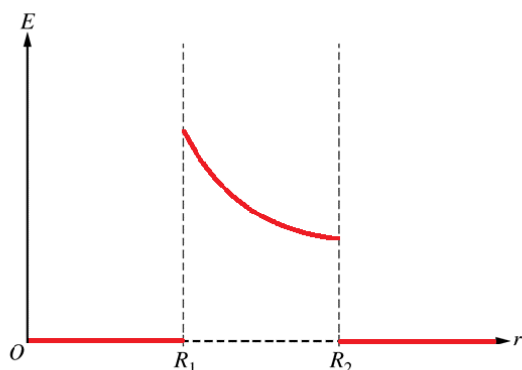
Example Response

Figure 2

B

For a multistep derivation that includes the equation $C = \frac{Q}{\Delta V}$

Point B1

For indicating that C with the dielectric material inserted is C without the dielectric material inserted multiplied by κ (e.g., $C = \kappa \frac{Q}{\Delta V}$)

Point B2

For substitutions of both the total charge Q and the potential difference ΔV that are consistent with part A

Point B3**Example Response**

Without the dielectric material

$$C = \frac{Q}{\Delta V}$$

$$C = \frac{\sigma_1 (2\pi L R_1)}{\left(\frac{\sigma_1 R_1}{\epsilon_0} \ln \left(\frac{R_2}{R_1} \right) \right)}$$

$$C = \frac{2\pi L \epsilon_0}{\ln \left(\frac{R_2}{R_1} \right)}$$

With the dielectric material

$$C = \kappa \frac{Q}{\Delta V}$$

$$C = \frac{2\pi L \kappa \epsilon_0}{\ln \left(\frac{R_2}{R_1} \right)}$$

Question 1: Free-Response Question**15 points**

- (a) For using a correct equation for electric flux **1 point**

Example Response

$$\Phi_E = \frac{Q}{\epsilon_0}$$

For the correct numerical answer **1 point**

Scoring Note: Correct units are not required to earn this point.**Example Response**

$$\Phi_E = 113 \frac{\text{N} \cdot \text{m}^2}{\text{C}}$$

Example Solution

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$\Phi_E = \frac{Q}{\epsilon_0}$$

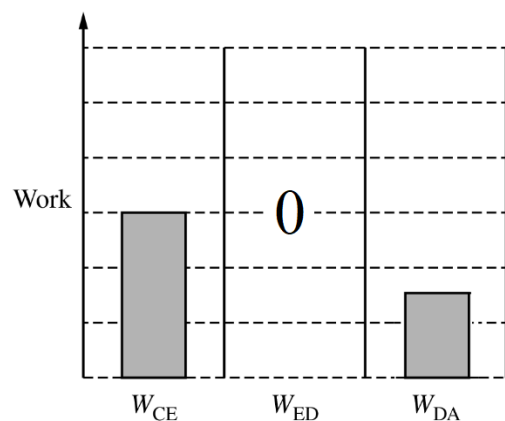
$$\Phi_E = \frac{1.0 \times 10^{-9} \text{ C}}{8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}}$$

$$\Phi_E = 113 \frac{\text{N} \cdot \text{m}^2}{\text{C}}$$

Total for part (a) 2 points

- (b)(i) For indicating that $W_{\text{ED}} = 0$ **1 point**

For drawing a bar representing W_{DA} that has a height of approximately one and a half units **1 point**

Example Response

(b)(ii) For using an equation that relates the electric field to potential difference **1 point**

Scoring Note: This point can be earned if the response begins with a correct relationship between electric field and potential difference in which numerical values are already substituted.

Example Responses

$$E_y = -\frac{dV}{dy} \quad \text{OR} \quad |E_y| = \left| -\frac{dV}{dy} \right| \quad \text{OR} \quad |E_y| = \left| -\frac{\Delta V}{\Delta y} \right| \quad \text{OR} \quad \Delta V = -\int \vec{E} \cdot d\vec{r}$$

For correct substitutions of values of electric potential and the distance between equipotential lines that can be used to calculate the approximate magnitude of the electric field at Position B **1 point**

Example Response

$$|E_y| = \left| -\frac{-20.0 \text{ V} - (-10.0 \text{ V})}{1.4 \text{ m}} \right|$$

Example Solution

$$E_y = -\frac{dV}{dy}$$

$$|E_y| = \left| -\frac{dV}{dy} \right|$$

$$|E_y| = \left| -\frac{\Delta V}{\Delta y} \right|$$

$$|E_y| = \left| -\frac{-20.0 \text{ V} - (-10.0 \text{ V})}{1.4 \text{ m}} \right|$$

$$|E_y| = 7.1 \frac{\text{V}}{\text{m}}$$

Total for part (b) 4 points

(c) For selecting only $-y$ with an attempt at a relevant justification **1 point**

For indicating that the direction of the electric field vector is perpendicular to a line that is tangent to the equipotential line at Position C **1 point**

For indicating one of the following: **1 point**

- The test charge moves from a higher electric potential to a lower electric potential.
- The test charge and the rod have charges of the opposite sign.
- The test charge and the sphere have charges of the same sign.

Example Response

$-y$. The direction of an electric field vector is perpendicular to an equipotential line. Because the test charge has a positive charge, the test charge would move from a position of higher electric potential to a position of lower electric potential when an electric force is exerted on the test charge. Therefore, at Position C, the electric force is downward because that is the direction that is perpendicular to the equipotential line and in the direction of decreasing electric potential.

Total for part (c) 3 points

- (d)(i)** For using an appropriate equation for determining the electric potential from a line of uniform charge **1 point**

Example Responses

$$V = \frac{1}{4\pi\epsilon_0} \sum \frac{q_i}{r_i} \quad \text{OR} \quad V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} \quad \text{OR} \quad \Delta V = -\int \vec{E} \cdot d\vec{r}$$

- For a correct determination of r , the distance between Point P and a point on the line of uniform charge **1 point**

Example Responses

$$V_P = k \sum \frac{Q}{y_P - y} \quad \text{OR} \quad V_P = k \int \left(\frac{1}{y_P - y} \right) dq$$

- For a correct integral with λdy substituted for dq **1 point**

Example Response

$$V_P = -k\lambda \int \left(\frac{1}{y_P - y} \right) dy$$

- For the correct limits of integration **1 point**

Example Response

$$V_P = -k\lambda \int_0^{2L} \left(\frac{1}{y_P - y} \right) dy$$

Example Solutions

$$V = \frac{1}{4\pi\epsilon_0} \sum \frac{q_i}{r_i}$$

$$V_P = k \int \left(\frac{1}{y_P - y} \right) dq$$

$$dq = -\lambda dy$$

$$V_P = -k\lambda \int \left(\frac{1}{y_P - y} \right) dy$$

$$V_P = -k\lambda \int_0^{2L} \left(\frac{1}{y_P - y} \right) dy$$

$$V_P = k\lambda \ln(y_P - y) \Big|_0^{2L} = -k\lambda \ln(y_P - y) \Big|_{2L}^0$$

$$V_P = -k\lambda \ln \left(\frac{y_P}{y_P - 2L} \right)$$

OR

$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

$$E(y) = \int \frac{k}{r^2} dq$$

$$E(y) = \int_0^{2L} \frac{k\lambda}{(y - y')^2} dy' = k\lambda \left(\frac{1}{y - 2L} - \frac{1}{y} \right)$$

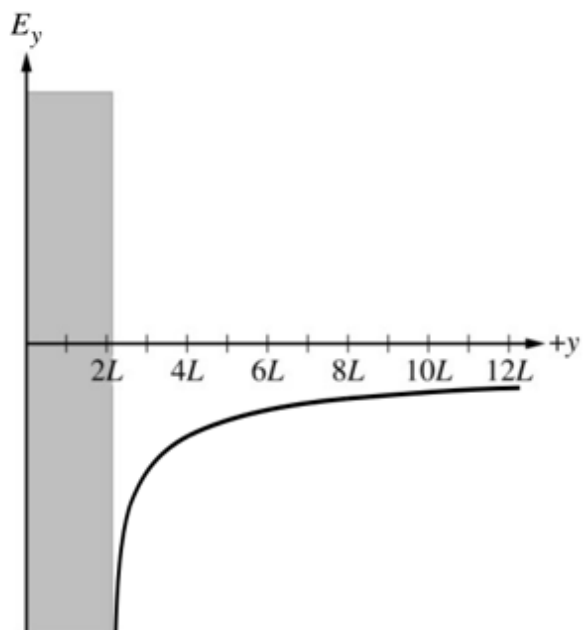
$$V_P = -\int_{\infty}^{y_P} E(y) dy = -k\lambda \int_{\infty}^{y_P} \left(\frac{1}{y - 2L} - \frac{1}{y} \right) dy$$

$$V_P = k\lambda \ln \left(\frac{y_P}{y_P - 2L} \right)$$

(d)(ii) For sketching a curve or line that continually approaches the horizontal axis as position increases **1 point**

For sketching a concave down curve that is always negative **1 point**

Example Response



Total for part (d) 6 points

Total for question 1 15 points

Question 1: Free-Response Question**15 points**

- (a) For correctly determining the charge on the outer surface of the shell **1 point**

Example Response

$$q_{net} = q_{inner} + q_{outer}$$

$$q_{outer} = q_{net} - q_{inner}$$

$$q_{outer} = +3Q - (+Q)$$

$$q_{outer} = +2Q$$

Scoring Note: A correct response may earn a point even if no work is shown.

Total for part (a) 1 points

- (b) For using Gauss's law by substituting for either the area or the enclosed charge **1 point**

Example Response

$$\frac{q_{enc}}{\epsilon_0} = \oint E \cdot dA = E(4\pi r^2)$$

- For using a correct expression for q_{enc} as a function of r **1 point**

Example Response

$$q_{enc} = \rho V = \left(\frac{-Q}{\frac{4}{3}\pi R^3} \right) \left(\frac{4}{3}\pi r^3 \right) = -Q \frac{r^3}{R^3}$$

Scoring Note: The response may earn this point regardless of the sign of q_{enc} .

- For using the correct area **1 point**

Example Response

$$E = q_{enc} \frac{1}{\epsilon_0 (4\pi r^2)} = \left(-Q \frac{r^3}{R^3} \right) \frac{1}{\epsilon_0 (4\pi r^2)}$$

$$E = -Q \frac{r}{4\pi \epsilon_0 R^3}$$

Scoring Note: The response may earn this point regardless of the sign of E .

Total for part (b) 3 points

-
- (c) For recognizing the electric field varies as an inverse square of the separation distance from the center of the nonconducting sphere **1 point**

Example Response

$$E \propto \frac{1}{r^2}$$

For the correct magnitude, including units, of the electric field at $2R$ **1 point**

Example Response

$$E_{new} = \frac{E_{old}}{4} = \frac{8 \text{ N/C}}{4}$$

$$E_{new} = 2 \text{ N/C}$$

Total for part (c) 2 points

-
- (d) For using the equation relating potential difference to the electric field with an attempt at either integration limits or evaluating the integral **1 point**

Example Response

$$V_f - V_i = - \int_{s_i}^{s_f} E dr$$

$$\Delta V = - \int_{r=R}^{r=4R} E dr$$

For substituting the correct expression for the electric field or an expression for E that is consistent with the explicit functional dependence of $E(r)$ from part (c) **1 point**

Example Response

$$\Delta V = - \int_{r=R}^{r=4R} - \frac{Q}{4\pi\epsilon_0 r^2} dr$$

$$\Delta V = \frac{Q}{4\pi\epsilon_0} \int_{r=R}^{r=4R} \frac{1}{r^2} dr$$

For correctly integrating the electric field expression that was substituted in the previous point with correct limits of integration **1 point**

Example Response

$$\Delta V = \frac{Q}{4\pi\epsilon_0} \left[-\frac{1}{r} \right]_{r=R}^{r=4R} = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r} \right]_{r=4R}^{r=R} = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{R} - \frac{1}{4R} \right) = \frac{Q}{4\pi\epsilon_0} \left(\frac{4-1}{4R} \right)$$

$$\Delta V = \frac{3Q}{16\pi\epsilon_0 R}$$

Alternate Solution

For using a difference in potentials treating the inner sphere as a point charge by symmetry **1 Point**

$$\Delta V = \frac{Q}{4\pi\epsilon_0 r_f} - \frac{Q}{4\pi\epsilon_0 r_i}$$

For using the correct charge on both of the terms; Q not Q and $3Q$ **1 Point**

For using the correct values for r **1 Point**

$$\Delta V = \frac{Q}{4\pi\epsilon_0 R} - \frac{Q}{4\pi\epsilon_0 4R}$$

$$\Delta V = \frac{3Q}{16\pi\epsilon_0 R}$$

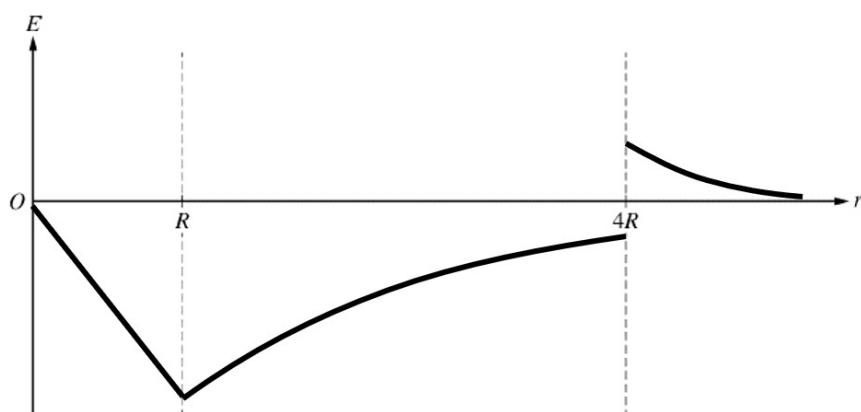
Total for part (d) 3 points

(e)(i) For indicating that E is negative and the magnitude of E increases linearly from 0 to R **1 point**

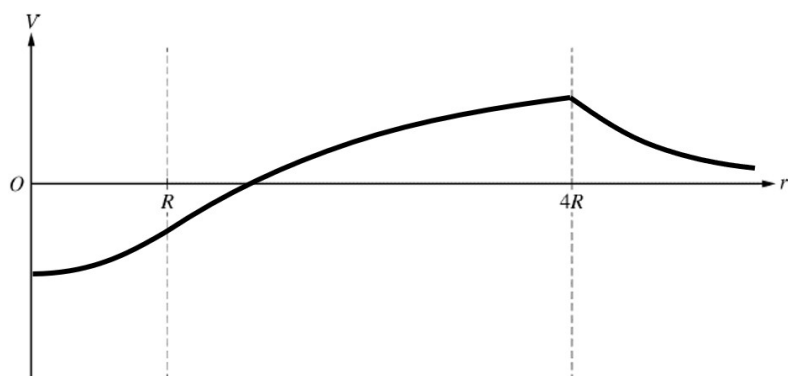
For a curve that is continuous at R and asymptotically approaches zero in the region $R < r < 4R$ **1 point**

For a positive, concave-up, and decreasing curve for $r > 4R$ **1 point**

Example Response



(e)(ii)	For a potential curve that is always increasing from 0 to $4R$	1 point
	For a potential curve that is decreasing for $r > 4R$	1 point
	For a continuous graph across all three regions	1 point

Example Response

Scoring Note: The intercept of the curve on the vertical axis is irrelevant. The intercept of the curve on the horizontal axis is irrelevant. The curve can, hence, cross the horizontal axis at any location or even be entirely on the negative side of the horizontal axis as long as the other criteria are met.

Total for part (e) 6 points

Total for question 1 15 points

Question 1: Free-Response Question**15 points**

- (a) For correctly determining the charge on the outer surface of the shell **1 point**

Example Response

$$q_{net} = q_{inner} + q_{outer}$$

$$q_{outer} = q_{net} - q_{inner}$$

$$q_{outer} = +4Q - (-Q)$$

$$q_{outer} = +5Q$$

Scoring Note: A correct response may earn a point even if no work is shown.

Total for part (a) 1 point

- (b) For using Gauss's law by substituting for either the area or the enclosed charge **1 point**

Example Response

$$\frac{q_{enc}}{\epsilon_0} = \oint E \cdot dA = E(2\pi rL)$$

For using a correct expression for q_{enc} as a function of r **1 point**

Example Response

$$q_{enc} = \rho V = \left(\frac{Q}{\pi R^2 L} \right) (\pi r^2 L) = Q \frac{r^2}{R^2}$$

For using the correct area **1 point**

Example Response

$$E = q_{enc} \frac{1}{\epsilon_0 (2\pi rL)} = \left(Q \frac{r^2}{R^2} \right) \frac{1}{\epsilon_0 (2\pi rL)}$$

$$E = Q \frac{r}{2\pi \epsilon_0 R^2 L}$$

Total for part (b) 3 points

- (c) For recognizing that the E -field varies as the reciprocal of the separation distance from the center of the nonconducting cylinder **1 point**

Example Response

$$E \propto \frac{1}{r}$$

For the correct value, including units, of electric field at $2R$ **1 point**

Example Response

$$E_{new} = \frac{E_{old}}{2} = \frac{12 \text{ N/C}}{2}$$

$$E_{new} = 6 \text{ N/C}$$

Total for part (c) 2 points

- (d) For using the equation relating potential difference to the electric field with an attempt at either integration limits or evaluating the integral **1 point**

Example Response

$$V_f - V_i = - \int_{s_i}^{s_f} E dr$$

$$\Delta V = - \int_{r=R}^{r=3R} E dr$$

For substituting the correct expression for the electric field or an expression consistent with the explicit functional dependence of $E(r)$ from part (c) **1 point**

Example Response

$$\Delta V = - \int_{r=R}^{r=3R} \left(\frac{Q}{2\pi\epsilon_0 r L} \right) dr = - \frac{Q}{2\pi\epsilon_0 L} \int_{r=R}^{r=3R} \frac{1}{r} dr$$

For correctly integrating the electric field expression that was substituted in the previous point with the correct limits of integration **1 point**

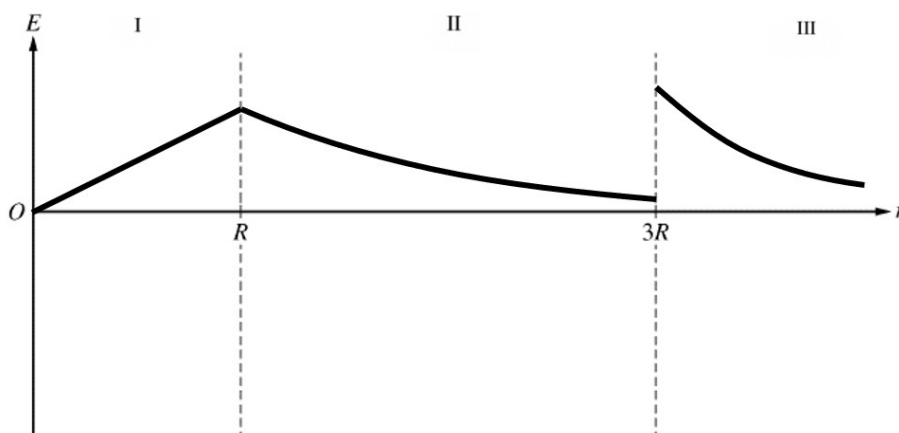
Example Response

$$\Delta V = - \frac{Q}{2\pi\epsilon_0 L} [\ln(r)]_{r=R}^{r=3R} = - \frac{Q}{2\pi\epsilon_0 L} [\ln(3R) - \ln(R)]$$

$$\Delta V = - \frac{Q}{2\pi\epsilon_0 L} \ln(3)$$

Total for part (d) 3 points

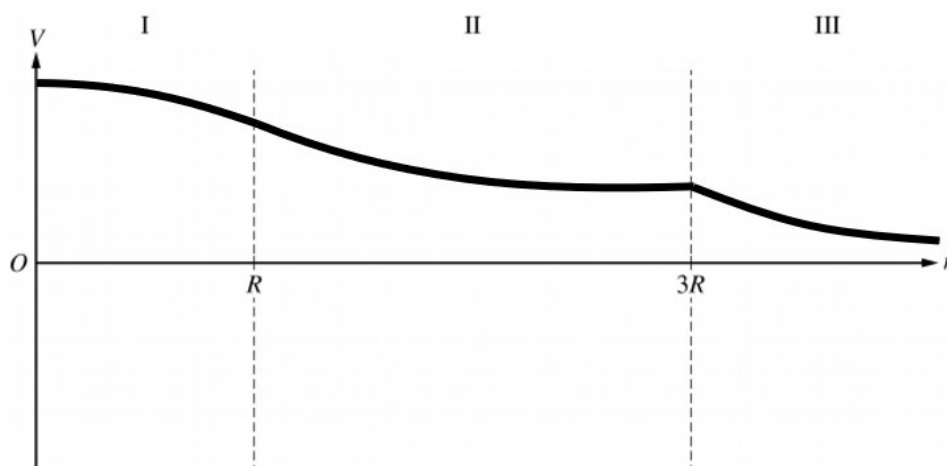
(e)(i)	For indicating E is positive and the magnitude increases linearly from 0 to R	1 point
	For a positive, decreasing concave up graph for $r > R$	1 point
	For a discontinuous increase in magnitude at $r = 3R$	1 point

Example Response

(e)(ii)	For a graph that is concave down and always decreasing as r increases within Region I	1 point
	For a graph that is concave up and always decreasing as r increases within each of the Regions II and III	1 point
	For a continuous graph across all three regions	1 point

Scoring Notes:

- The intercept of the curve on the vertical axis is irrelevant. The intercept of the curve on the horizontal axis is irrelevant. The curve can, hence, cross the horizontal axis at any location or even be entirely on the negative side of the horizontal axis as long as the other criteria are met.
- The response can earn a maximum of 2 points if all three segments are reflected across the horizontal axis.

Example Response**Total for part (e) 6 points****Total for question 1 15 points**

2019

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AP[®] Physics C: Electricity and Magnetism

Scoring Guidelines Set 1

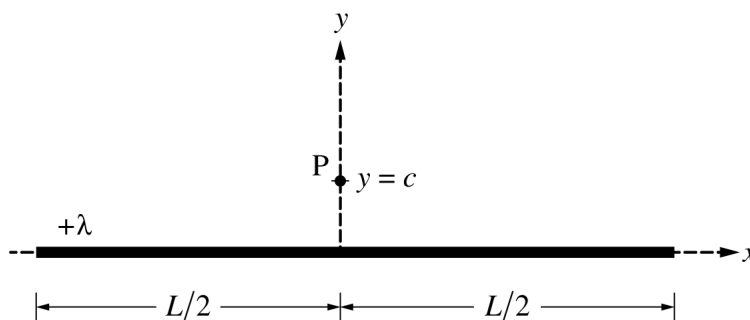
2019 SCORING GUIDELINES**General Notes About 2019 AP Physics Scoring Guidelines**

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.
2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at <https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf>.
3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.
5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.
6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

2019 SCORING GUIDELINES

Question 1

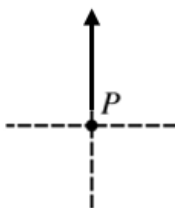
15 points

Note: Figure not drawn to scale.

A very long, thin, nonconducting cylinder of length L is centered on the y -axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y -axis at $y = c$, where $L \gg c$.

- (a)
- LO CNV-3.B.a, SP 7.A
1 point

On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the long cylinder. The arrow should start on and point away from the dot.

For drawing an arrow at point P that points upward

1 point

- LO CNV-2.F, SP 7.C
1 point

Describe the shape and location of a Gaussian surface that can be used to determine the electric field at point P due to the long cylinder.

For describing a Gaussian surface that could be used to determine the electric field at point P

1 point

Example: Drawing a cylinder that is coaxial with the thin cylinder and whose surface contains point P can be used to determine the electric field at point P .

Note: Credit is earned if the student draws the correct surface on the figure.

2019 SCORING GUIDELINES**Question 1 (continued)**

(a) continued

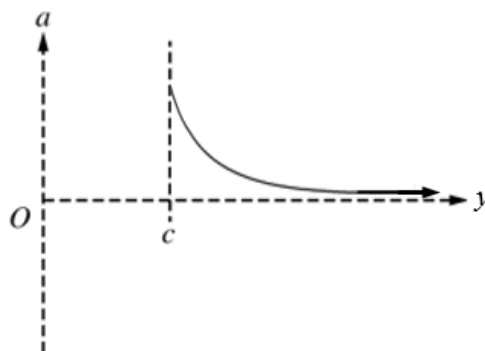
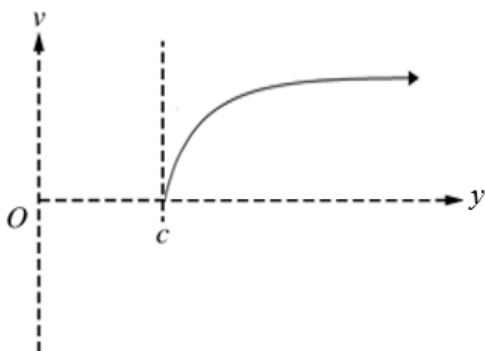
- iii. LO CNV-2.C, SP 5.A, 5.E
3 points

Use your Gaussian surface to derive an expression for the magnitude of the electric field at point P.
Express your answer in terms of λ , c , L , and physical constants, as appropriate.

For using Gauss's law to determine the electric field at point P		1 point
$\frac{q_{enc}}{\epsilon_0} = \int E \cdot dA \therefore \frac{Q}{\epsilon_0} = EA$		
For correctly substituting for the charge into the equation above		1 point
For correctly substituting for the area or into the equation above		1 point
$\frac{\lambda L}{\epsilon_0} = E(2\pi cL) \therefore E = \frac{\lambda}{2\pi\epsilon_0 c} = \frac{2k\lambda}{c}$		

- (b) LO ACT-1.D, SP 3.C
2 points

A proton is released from rest at point P. On the axes below, sketch the velocity v as a function of position y and the acceleration a as a function of position y for the proton.

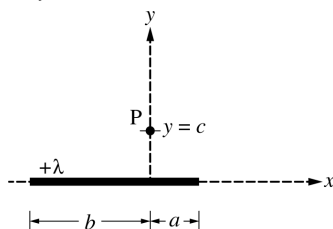


For a concave down graph for v as a function of position x that does not start at the origin		1 point
For a concave up graph for a as a function of position x that has an asymptote at the horizontal axis		1 point

2019 SCORING GUIDELINES

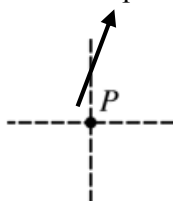
Question 1 (continued)

The original cylinder is now replaced with a much shorter thin, nonconducting cylinder with the same uniform linear charge density $+\lambda$, as shown in the figure below. The length of the cylinder to the right of the y -axis is a , and the length of the cylinder to the left of the y -axis is b , where $a < b$.



- (c) LO CNV-3.B.a, SP 7.A
2 points

On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the shorter cylinder. The arrow should start on and point away from the dot.



For drawing an arrow at point P that points to the right	1 point
For drawing an arrow at point P that points up and to the right	1 point

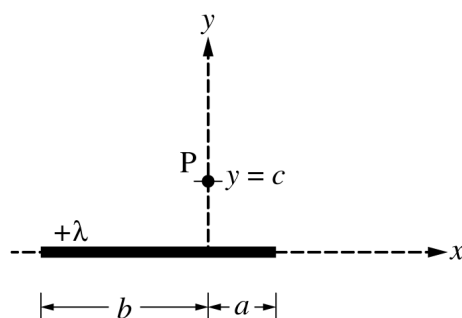
- (d) i. and ii. LO CNV-2.F, SP 7.A
1 point

Is there a single Gaussian surface that can be used with Gauss's law to derive an expression for the electric field at point P ?

____ Yes ____ No

If your answer to part (d)(i) is yes, explain how you can use Gauss's law to derive an expression for the field at point P . If your answer to part (d)(i) is no, explain why Gauss's law cannot be applied to derive an expression for the electric field in this case.

For selecting "No" with a valid explanation	1 point
Claim: Select "No."	
Evidence: The length of the cylinder is not much greater than the distance from the cylinder to point P and the charge distribution is asymmetric.	
Reasoning: Therefore, cannot use the approximation of the constant magnitude of electric field over a cylindrical surface.	

2019 SCORING GUIDELINES**Question 1 (continued)**

Note: This figure is shown again for reference.

A student in class argues that using the integral shown below might be a useful approach for determining the electric field at point P.

$$E = \int \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} dq$$

The student uses this approach and writes the following two integrals for the magnitude of the horizontal and vertical components of the electric field at point P.

$$\text{Horizontal component: } |E_x| = \frac{\lambda}{4\pi\epsilon_0} \int_{-b}^a \frac{x}{(c^2 + x^2)^{3/2}} dx$$

$$\text{Vertical component: } |E_y| = \frac{\lambda}{4\pi\epsilon_0} \int_{-b}^a \frac{y}{(c^2 + x^2)} dy$$

- (e)
- i. LO CNV-3.A, SP 7.A
1 point

One of the two expressions above is not correct. Which expression is not correct?

_____ Horizontal component _____ Vertical component

For correctly selecting “Vertical component”	1 point
--	---------

2019 SCORING GUIDELINES**Question 1 (continued)**

(e) continued

- ii. LO CNV-3.A, SP 7.D
4 points

Identify two mistakes in the incorrect expression, and explain how to correct the mistakes.

For indicating the integral is not along the length of the cylinder	1 point
For an appropriate correction	1 point
Claim: Change dy to dx . Evidence: The integral is not along the cylinder. Reasoning: This change will make the integral valid.	
For indicating the power on the denominator term for the vertical component is incorrect	1 point
For an appropriate correction	1 point
Claim: The power on the term in the denominator should be $3/2$. Evidence: The units of the integrand are not valid. Reasoning: This change will make the integrand valid.	

Learning Objectives

ACT-1.D: Determine the motion of a charged object of specified charge and mass under the influence of an electrostatic force.

CNV-2.C: State and use Gauss's law in integral form to derive unknown electric fields for planar, spherical, or cylindrically symmetrical charge distributions.

CNV-2.F: Describe the general features of an unknown charge distribution, given other features of the system.

CNV-3.A: Derive expressions for the electric field of specified charge distributions using integration and the principle of superposition. Examples of such charge distributions include a uniformly charged wire, a thin ring of charge (along the axis of the ring), and a semicircular or part of a semicircular arc.

CNV-3.B.a: Identify and qualitatively describe situations in which the direction and magnitude of the electric field can be deduced from symmetry considerations and understanding the general behavior of certain charge distributions.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.

5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.

5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

7.A: Make a scientific claim.

7.C: Support a claim with evidence from physical representations.

7.D: Provide reasoning to justify a claim using physical principles or laws.

2019 SCORING GUIDELINES

Question 2

15 points

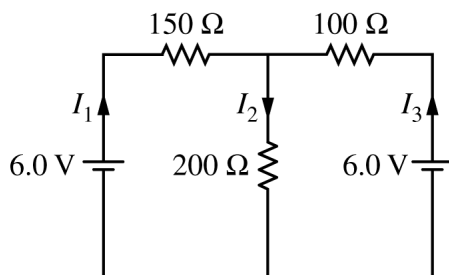


Figure 1

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1 , I_2 , and I_3 in each branch of the circuit are indicated.

- (a)
- LO CNV-6.F.b, SP 6.A
3 points

Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

For an equation representing the sum of the currents at one of the junctions		1 point
$I_1 - I_2 + I_3 = 0$		
For an equation representing the sum of the potential differences around one of the loops		1 point
For an equation representing the sum of the potential differences around a loop different from the above loop		1 point
$6 - 150I_1 - 200I_2 = 0$		
$6 - 100I_3 - 200I_2 = 0$		
$6 - 150I_1 + 100I_3 - 6 = 0$		
<u>Note:</u> Full credit is earned for two correct loop equations using loop currents.		

- LO CNV-6.F.b, SP 6.C
2 points

Calculate the current in the $200\ \Omega$ resistor.

For combining the equations from part (a)(i)		1 point
$I_1 - I_2 + I_3 = 0$ $-I_1 - 1.33I_2 = -.04 \therefore -4.33I_2 = -.10$ $-I_3 - 2I_2 = -.06$		
<u>Note:</u> Credit is earned if students indicate they used a calculator function to solve the system of equations.		
For a correct answer with correct units		1 point
$-4.33I_2 = -.10 \therefore I_2 = 0.023\text{ A}$		

2019 SCORING GUIDELINES**Question 2 (continued)**

(a) continued

iii. LO CNV-5.A.a, SP 6.C

1 point

Calculate the power dissipated by the $200\ \Omega$ resistor.

For using a correct equation to calculate the power in the $200\ \Omega$ resistor		1 point
$P = I^2 R = (0.023\ \text{A})^2 (200\ \Omega) \therefore P = 0.107\ \text{W}$		

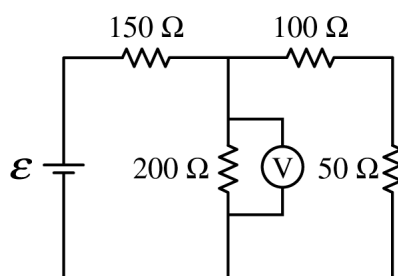


Figure 2

The two $6.0\ \text{V}$ batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is $4.4\ \text{V}$.

(b) LO CNV-6.C.a, SP 6.B, 6.C

2 points

Calculate the current through the $50\ \Omega$ resistor.

For correctly calculating the equivalent resistance of the branch with the $50\ \Omega$ resistor		1 point
$R = 100\ \Omega + 50\ \Omega = 150\ \Omega$		
For using the correct potential difference in Ohm's law to calculate the current in the $50\ \Omega$ resistor		1 point
$I = \frac{V}{R} = \frac{(4.4\ \text{V})}{(100\ \Omega + 50\ \Omega)} \therefore I = 0.029\ \text{A}$		

2019 SCORING GUIDELINES**Question 2 (continued)**

- (c) LO CNV-6.C.a, SP 6.A, 6.C
3 points

Calculate the voltage $\mathcal{E}$ of the battery.

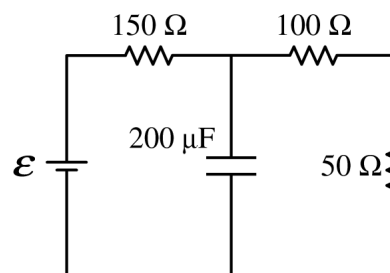
For using a correct equation to determine the current through the $150\ \Omega$ resistor	1 point
For correctly substituting the current from part (b) as I_3	1 point
$I_1 = I_2 + I_3 = \frac{(4.4\ \text{V})}{(200\ \Omega)} + 0.029\ \text{A} = 0.051\ \text{A}$	
For using a correct equation to determine the emf of the battery	1 point
$\mathcal{E} = I_1 R_1 + V_2 = (0.051\ \text{A})(150\ \Omega) + 4.4\ \text{V} = 12.1\ \text{V}$	
<i>Alternate Third Point</i>	
<i>For calculating the equivalent resistance of the circuit and substituting this resistance into a correct equation to determine the emf of the battery</i>	<i>1 point</i>
$R_T = 150\ \Omega + \frac{1}{\frac{1}{200\ \Omega} + \frac{1}{150\ \Omega}} = 236\ \Omega$	
$\mathcal{E} = I_1 R_T = (0.051\ \text{A})(236\ \Omega) = 12.0\ \text{V}$	
<i>Alternate Solution</i>	<i>Alternate Points</i>
<i>For using a correct equation to determine the equivalent resistance of the parallel resistors</i>	<i>1 point</i>
$\frac{1}{R_P} = \frac{1}{200\ \Omega} + \frac{1}{150\ \Omega} \therefore R_P = 86\ \Omega$	
<i>For correctly substituting the given potential difference and the calculated equivalent resistance to determine the total current of the circuit</i>	<i>1 point</i>
$I_T = \frac{(4.4\ \text{V})}{(86\ \Omega)} = 0.051\ \text{A}$	
<i>For calculating the equivalent resistance of the circuit and substituting into a correct equation to determine the emf of the battery</i>	<i>1 point</i>
$R_T = 150\ \Omega + 86\ \Omega = 236\ \Omega$	
$\mathcal{E} = I_1 R_T = (0.051\ \text{A})(236\ \Omega) = 12.0\ \text{V}$	

2019 SCORING GUIDELINES

Question 2 (continued)

- (d)
i. LO CNV-7.B.a, SP 6.A, 6.C
2 points

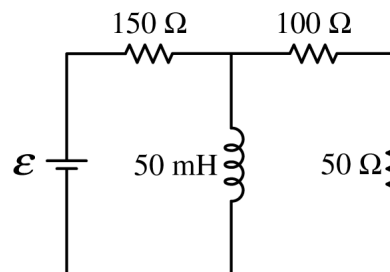
The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu\text{F}$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.



For substituting the voltage consistent with part (c) into Ohm's law		1 point
For correctly calculating the equivalent resistance of the circuit		1 point
$I = \frac{\mathcal{E}}{R_{tot}} = \frac{(12.1\ \text{V})}{(150\ \Omega + 100\ \Omega + 50\ \Omega)} = 40.3\ \text{mA}$		

- ii. LO CNV-10.C.a, SP 7.A, 7.C
2 points

The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal $50\ \text{mH}$ inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b)?



____ Greater than ____ Less than ____ Equal to

Justify your answer.

For correctly selecting "Less than" with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because steady state is reached, the inductor will act as a short circuit. So all the current will pass through the inductor and no current will pass through the $50\ \Omega$ resistor.		

2019 SCORING GUIDELINES**Question 2 (continued)****Learning Objectives**

CNV-5.A.a: Derive expressions that relate current, voltage, and resistance to the rate at which heat is produced in a resistor.

CNV-6.C.a: Calculate voltage, current, and power dissipation for any resistor in a circuit containing a network of known resistors with a single battery or energy source.

CNV-6.F.b: Set up simultaneous equations to calculate at least two unknowns (currents or resistance values) in a multi-loop circuit.

CNV-7.B.a: Calculate the potential difference across a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.

CNV-10.C.a: Calculate initial transient currents and final steady-state currents through any part of a series or parallel circuit containing an inductor and one or more resistors.

Science Practices

6.A: Extract quantities from narratives or mathematical relationships to solve problems.

6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.

6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

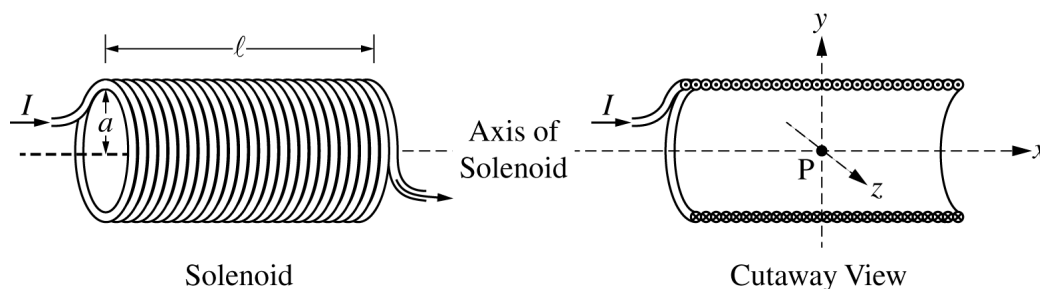
7.A: Make a scientific claim.

7.C: Support a claim with evidence from physical representations.

2019 SCORING GUIDELINES

Question 3

15 points



Note: Figures not drawn to scale.

A solenoid is used to generate a magnetic field. The solenoid has an inner radius a , length ℓ , and N total turns of wire. A power supply, not shown, is connected to the solenoid and generates current I , as shown in the figure on the left above. The x -axis runs along the axis of the solenoid. Point P is in the middle of the solenoid at the origin of the xyz -coordinate system, as shown in the cutaway view on the right above. Assume $\ell \gg a$.

- (a) LO CNV-8.E.a, SP 7.A, 7.C
2 points

Select the correct direction of the magnetic field at point P.

☐ + x -direction ☐ + y -direction ☐ + z -direction
☐ - x -direction ☐ - y -direction ☐ - z -direction

Justify your selection.

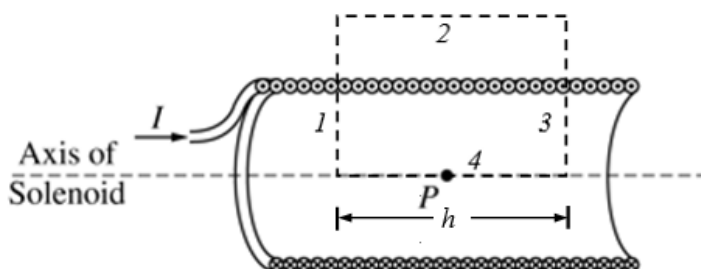
For choosing the “+ x -direction” and providing a justification	1 point
For a correct justification	1 point
Example: Using the right-hand rule for current on the left side of the solenoid, the fingers curl into the loop, so the magnetic field points to the right, or in the + x -direction.	
Example: Using the right-hand rule for solenoids, when the fingers curl around the solenoid in the direction of the current, the thumb points to the right, therefore the magnetic field is to the right, or in the + x -direction.	

2019 SCORING GUIDELINES

Question 3 (continued)

- (b)
- i. LO CNV-8.C.c, SP 3.D
1 point

On the cutaway view below, clearly draw an Amperian loop that can be used to determine the magnetic field at point P at the center of the solenoid.



For drawing a rectangle with one side along the central axis of the solenoid and another side outside the solenoid and whose edges do not extend beyond the solenoid	1 point
--	---------

- ii. LO CNV-8.C.c, SP 5.A, 5.E
2 points

Use Ampere's law to derive an expression for the magnetic field strength at point P. Express your answer in terms of I , ℓ , N , a , and physical constants, as appropriate.

For using Ampere's law to calculate the magnetic field along the axis of the solenoid	1 point
$\int \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I_{enc} \therefore \left(\int \mathbf{B} \cdot d\boldsymbol{\ell} \right)_1 + \left(\int \mathbf{B} \cdot d\boldsymbol{\ell} \right)_2 + \left(\int \mathbf{B} \cdot d\boldsymbol{\ell} \right)_3 + \left(\int \mathbf{B} \cdot d\boldsymbol{\ell} \right)_4 = \mu_0 I_{enc}$	
$\left(\int \mathbf{B} \cdot d\boldsymbol{\ell} \right)_1 + 0 + \left(- \left(\int \mathbf{B} \cdot d\boldsymbol{\ell} \right)_1 \right) + Bh = \mu_0 \frac{N}{\ell} hI$	
$Bh = \mu_0 \frac{N}{\ell} hI$	
For a correct answer	1 point
$B = \frac{\mu_0 NI}{\ell}$	

Some physics students conduct an experiment to determine the resistance R_S of a solenoid with radius $a = 0.015$ m, total turns $N = 100$, and total length $\ell = 0.40$ m. The students connect the solenoid to a variable power supply. A magnetic field sensor is used to measure the magnetic field strength along the central axis at the center of the solenoid. The plot of the magnetic field strength B as a function of the emf $\mathcal{E}$ of the power supply is shown below.

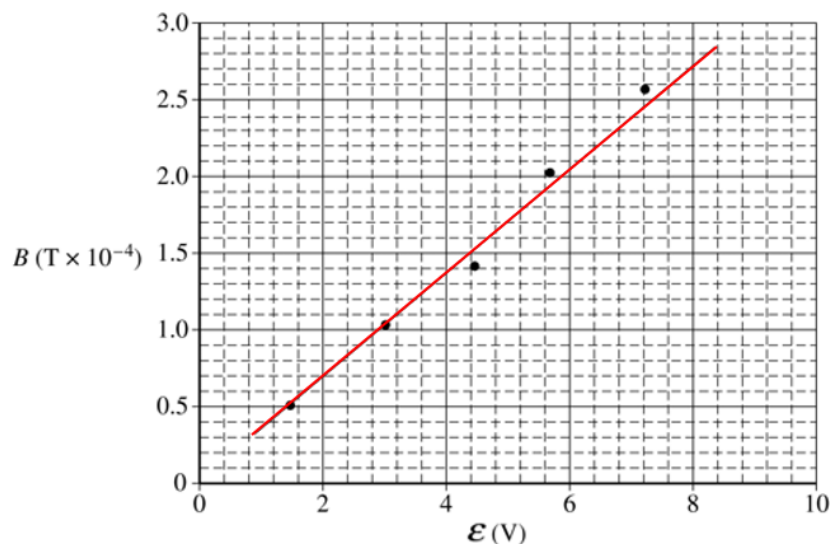
2019 SCORING GUIDELINES

Question 3 (continued)

(c) LO CNV-8.C.c, SP 4.C

i. 1 point

On the graph above, draw a best-fit line for the data.



For drawing a best-fit line with at least one point above and one point below the line

1 point

ii. LO CNV-8.C.c, SP 6.B, 6.C

2 points

Use the straight line to determine the resistance R_S of the solenoid used in the experiment.

For calculating the slope using the best-fit line and not the data points unless they fall on the best-fit line

1 point

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{(2.5 - 0.9)(\times 10^{-4} \text{ T})}{(6.4 - 2.0) \text{ V}} = 0.36 \times 10^{-4} = 3.6 \times 10^{-5} \text{ T/V}$$

For correctly giving the expression that relates the slope to the resistance of the solenoid

1 point

$$B = \frac{\mu_0 N I}{\ell} = \frac{\mu_0 N \mathcal{E}}{\ell R} \therefore \text{slope} = \frac{\mu_0 N}{\ell R_S} \therefore R_S = \frac{\mu_0 N}{\ell \times \text{slope}}$$

$$R_S = \frac{(4\pi \times 10^{-7} \text{ (T}\cdot\text{m)/A})(100 \text{ turns})}{(0.40 \text{ m})(3.6 \times 10^{-5} \text{ T/V})} = 8.7 \Omega$$

One of the students notes that the horizontal component of the magnetic field of Earth is $2.5 \times 10^{-5} \text{ T}$.

2019 SCORING GUIDELINES**Question 3 (continued)**

- (d)
- i. LO CNV-8.E.a, SP 2.D
1 point

Is there evidence from the graph that the horizontal orientation of the solenoid affects the measured values for B ?

____ Yes ____ No

Justify your answer.

If the line on the graph does not go through the origin, select “Yes”		
For a correct justification		1 point
Example: The horizontal component of Earth’s magnetic field will add or subtract from the magnetic field of the solenoid depending on the orientation of the solenoid.		
<i>Alternate Solution</i>	<i>Alternate Points</i>	
If the line on the graph does pass through the origin, select “No”		
For a correct justification		1 point
Example: Based on the graph, the line passes through the origin, so the magnetic field is zero when the emf of the power supply is zero, therefore Earth’s magnetic field is not affecting the values of B .		

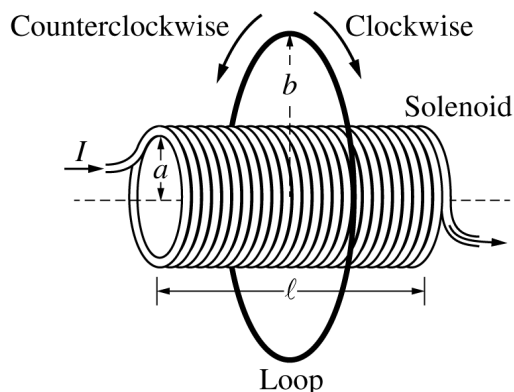
- ii. LO CNV-8.E.a, SP 2.E
1 point

Would the horizontal orientation of the solenoid affect the calculated value for R_S ?

____ Yes ____ No

Justify your answer.

Select “No”		
For a correct justification		1 point
Example: The horizontal component of Earth’s magnetic field will not affect the change in magnetic field as the emf is changed. Therefore, the value for the resistance of the solenoid will not change.		

2019 SCORING GUIDELINES**Question 3 (continued)**

A thin conducting loop of radius b and resistance R_L is placed concentric with the solenoid, as shown above. The current in the solenoid is decreased from I to zero over time Δt .

(e)

- i. LO FIE-6.A.b, SP 7.A, 7.C
2 points

Is the direction of the induced current in the loop clockwise or counterclockwise during the time period that the current in the solenoid is decreasing?

_____ Clockwise _____ Counterclockwise

Justify your answer.

Select "Clockwise"		
For a justification indicating that the magnetic field inside the solenoid, and therefore the loop, will decrease		1 point
For a justification using Lenz's law to relate the change in magnetic field to the direction of the current		1 point
Example: As the current in the solenoid decreases, the magnetic field inside the solenoid decreases. As the solenoid's magnetic field decreases, the induced current in the loop will create a magnetic field to oppose this change. Because the solenoid's magnetic field is toward the right and decreasing, the magnetic field due to the current in the loop must be toward the right. Therefore, the current in the loop must be clockwise.		

2019 SCORING GUIDELINES

Question 3 (continued)

(e) continued

- ii. LO FIE-6.A.d, SP 5.A, 5.E
3 points

Derive an equation for the average induced current i_{IND} in the loop during the time period that the current in the solenoid is decreasing. Express your answer in terms of I , ℓ , N , a , b , R_L , R_S , Δt , and physical constants, as appropriate.

For using Faraday's law to calculate the emf in the loop	1 point
$\mathcal{E} = \left \frac{d\Phi}{dt} \right = \frac{d(BA)}{dt} = A \frac{\Delta B}{\Delta t}$	
For using Ohm's law to calculate the current in the loop	1 point
$I_{\text{IND}} = \frac{V}{R} = \frac{A\Delta B/\Delta t}{R_L} = \frac{A\Delta B}{R_L\Delta t}$	
For using the correct radius for the area in the equation above	1 point
$I_{\text{IND}} = \frac{\pi a^2 \mu_0 N \Delta I / \ell}{R_L \Delta t} = \frac{\pi a^2 \mu_0 N I}{R_L \ell \Delta t}$	

Learning Objectives

CNV-8.C.c: Derive the expression for the magnetic field of an ideal solenoid (length dimension is much larger than the radius of the solenoid) using Ampère's law.

CNV-8.E.a: Describe the direction of a magnetic field at a point in space due to various combinations of conductors, wires, cylindrical conductors, or loops.

FIE-6.A.b: Describe the direction of an induced current in a conductive loop that is placed in a changing magnetic field.

FIE-6.A.d: Calculate the magnitude and direction of induced EMF and induced current in a conductive loop (or conductive bar) when the magnitude of either the field or area of loop is changing at a constant rate.

Science Practices

2.D: Make observations or collect data from representations of laboratory setups or results.

2.E: Identify or describe potential sources of experimental error.

3.D: Create appropriate diagrams to represent physical situations.

4.C: Linearize data and/or determine a best-fit line or curve.

5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.

5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.

6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

7.A: Make a scientific claim.

7.C: Support a claim with evidence from physical representations.

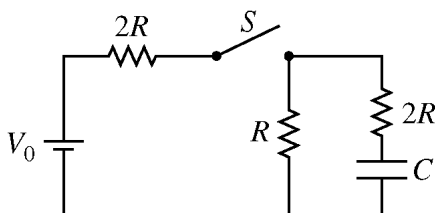
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AP[®] Physics C: Electricity and Magnetism

Scoring Guidelines Set 2

2019 SCORING GUIDELINES**General Notes About 2019 AP Physics Scoring Guidelines**

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.
2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at <https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf>.
3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as “derive” and “calculate” on the exams, and what is expected for each, see “The Free-Response Sections — Student Presentation” in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or “Terms Defined” in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.
5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.
6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

2019 SCORING GUIDELINES**Question 1****15 points**

The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0 , a capacitor of capacitance C , and a switch S . The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0 , R , C , and fundamental constants, as appropriate.

- (a) LO CNV-7.B.a, SP 5.A, 5.E
2 points

Derive an expression for the steady-state current supplied by the battery.

For using Ohm's law		1 point
$I = \frac{V}{R_{\text{eff}}} = \frac{V_0}{(2R + R)}$		
For correct substitution leading to correct answer		1 point
$I = \frac{V_0}{3R}$		

- (b) LO CNV-7.B.b, SP 5.A, 5.E
2 points

Derive an expression for the charge on the capacitor.

For using the equation relating stored charge to capacitance		1 point
$q = CV = CV_C = CV_R$		
For determining V_R and substituting into the above equation		1 point
$V_R = IR = \left(\frac{V_0}{3R}\right)R = \frac{1}{3}V_0$		
$q = CV_R = C\left(\frac{1}{3}V_0\right) = \frac{1}{3}CV_0$		

2019 SCORING GUIDELINES**Question 1 (continued)**

- (c) LO CNV-7.B.b, SP 5.A, 5.E
2 points

Derive an expression for the energy stored in the capacitor.

For any correct equation for energy stored in a capacitor		1 point
$U = \frac{q^2}{2C}$		
Substitute charge and/or voltage from part (b)		1 point
$U = \frac{(CV_0/3)^2}{2C} = \frac{1}{18}CV_0^2$		

Now the switch is opened at time $t = 0$.

- (d) LO CNV-7.D.a, SP 5.A, 5.E
2 points

Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time t after the switch is opened.

For any correct voltage loop equation		1 point
$V_C - V_R - V_{2R} = 0 \therefore V_C = V_R + V_{2R}$		
$\frac{q(t)}{C} = I(R + 2R)$		
<u>Note:</u> Any correct loop equation for when the switch is open earns the point		
For substituting $-dq/dt$ or dq/dt for the current, consistent with loop equation		1 point
$\frac{q(t)}{C} = -3R \frac{dq}{dt}$		

2019 SCORING GUIDELINES**Question 1 (continued)**

(e)

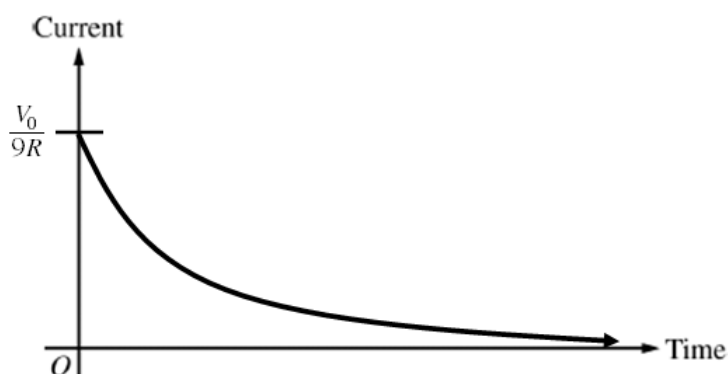
- i. LO CNV-7.B.a, SP 5.A, 5.E
2 points

Calculate the current in resistor R immediately after the switch is opened.

For using a voltage consistent with part (b) in $I = \frac{V}{R}$		1 point
For substituting the correct resistance ($3R$) into Ohm's law		1 point
$I = \frac{V_0/3}{3R} = \frac{V_0}{9R}$		

- ii. LO CNV-7.E.b, SP 3.C
3 points

On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



For a curve that is concave up throughout graph		1 point
For having the horizontal axis as an asymptote		1 point
For labeling the maximum current consistent with part (e)(i)		1 point

2019 SCORING GUIDELINES**Question 1 (continued)**

- (f) LO CNV-7.G.a, SP 7.A, 7.C
2 points

Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c)?

___ Greater than ___ Less than ___ Equal to

Justify your answer.

For selecting “Equal to”	1 point
For a correct justification invoking conservation of energy	1 point
Example: After the switch is opened, the capacitor will discharge all of its stored energy and charge. Assuming no energy is lost in the wire, then the only parts of the circuit that will dissipate this energy are the two resistors in series with the capacitor.	
<i>Alternate Solution</i>	<i>Alternate Points</i>
For selecting “Less than”	1 point
For a correct justification invoking conservation of energy	1 point
Example: After the switch is opened, the capacitor will discharge all of its stored energy and charge. Additionally there is energy lost due to resistance in the wire and/or energy loss in the capacitor.	

2019 SCORING GUIDELINES**Question 1 (continued)****Learning Objectives**

CNV-7.B.a: Calculate the potential difference across a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.

CNV-7.B.b: Calculate the stored charge on a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.

CNV-7.D.a: Derive expressions using calculus to describe the time dependence of the stored charge or potential difference across the capacitor, or the current or potential difference across the resistor in an RC circuit when charging or discharging a capacitor.

CNV-7.E.b: Describe the behavior of the voltage or current behavior over time for a circuit that contains resistors and capacitors in a multi-loop arrangement.

CNV-7.G.a: Describe the energy transfer in charging or discharging a capacitor in an RC circuit.

Science Practices

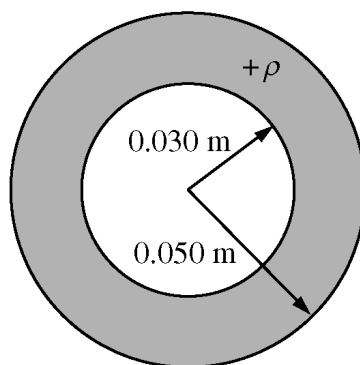
3.C: Sketch a graph that shows a functional relationship between two quantities.

5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.

5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

7.A: Make a scientific claim.

7.C: Support a claim with evidence from physical representations.

2019 SCORING GUIDELINES**Question 2****15 points**

A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ , as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m: } \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m: } \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

$$r > 0.050 \text{ m: } \rho = 0$$

- (a) LO CNV-2.D.a, SP 6.B, 6.C
3 points

Calculate the total charge of the sphere.

For indicating the need to integrate the expression for charge density to determine the total charge on the sphere		1 point
$Q = \int \rho dV$		
For proper substitutions into the integration		1 point
$Q = \int \frac{(1.6 \times 10^{-6})}{r} (4\pi r^2) dr$		
For using the proper limits of integration		1 point
$Q = 4\pi (1.6 \times 10^{-6}) \int_{r=0.03}^{r=0.05} r dr = 4\pi (1.6 \times 10^{-6}) \left[\frac{r^2}{2} \right]_{r=0.03}^{r=0.05}$		
$Q = (2\pi)(1.6 \times 10^{-6})(0.05^2 - 0.03^2) = 1.61 \times 10^{-8} \text{ C}$		

2019 SCORING GUIDELINES

Question 2 (continued)

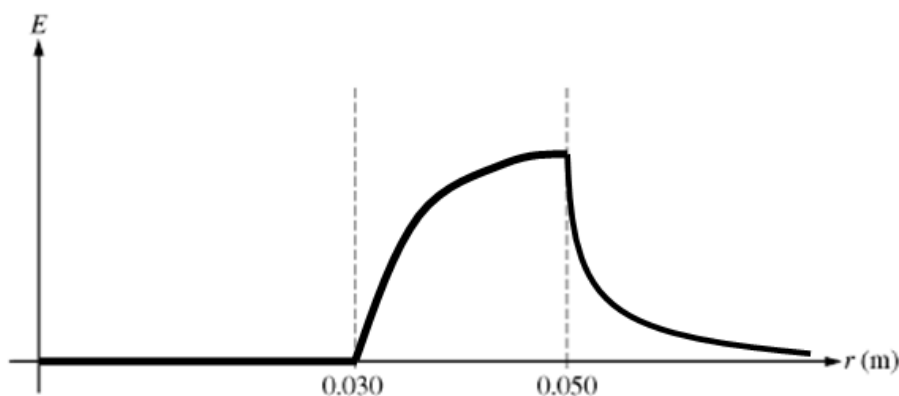
- (b) LO CNV-2.D.a, SP 6.C
3 points

Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.

For correctly evaluating the surface integral in Gauss's law		1 point
$\oint E \cdot dA = E(4\pi r^2)$		
For correctly substituting the answer from part (a) and correct radius into above equation		1 point
$\frac{Q_{enc}}{\epsilon_0} = E(4\pi r^2) \therefore E = \frac{Q_{enc}}{4\pi\epsilon_0 r^2} = \frac{(1.61 \times 10^{-8} \text{ C})}{(4\pi)(8.85 \times 10^{-12})(0.05)^2}$		
For an answer consistent with part (a) with correct units		1 point
$E = 5.79 \times 10^4 \text{ N/C}$		

- (c) LO CNV-2.C, SP 3.C
3 points

On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.



For clearly showing a graph with a value of $E = 0$ for $r < 0.030 \text{ m}$		1 point
For a continuous graph that starts at zero, is concave down, and increases in value from $r = 0.030$ to $r = 0.050$		1 point
For a continuous graph that decreases asymptotically toward the horizontal axis for $r > 0.050 \text{ m}$		1 point

2019 SCORING GUIDELINES

Question 2 (continued)

- (d) LO CNV-1.G.a, SP 6.B, 6.C
2 points

Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

For substituting the total charge from part (a) into a correct expression for electric potential	1 point
For substituting $r = 0.05$ m into a correct expression for electric potential	1 point
$V_R = \frac{Q_{\text{tot}}}{4\pi\epsilon_0 r} = \frac{(9 \times 10^9)(1.61 \times 10^{-8} \text{ C})}{(0.05 \text{ m})} = 2900 \text{ V}$	
<i>Alternate Solution</i>	<i>Alternate Points</i>
For substituting the total charge from part (a) into an integration for electric potential	1 point
$\Delta V_R = V_R - V_\infty = V_R = -\int E dr = -\int \frac{Q_{\text{enc}}}{4\pi\epsilon_0 r^2} dr = -\int \frac{(9 \times 10^9)(1.61 \times 10^{-8} \text{ C})}{r^2} dr$	
For integrating with correct limits of integration	1 point
$V_R = -\int_{r=\infty}^{r=0.05 \text{ m}} \frac{145}{r^2} dr = -145 \left[-\frac{1}{r} \right]_{r=\infty}^{r=0.05 \text{ m}} = 145 \left(\frac{1}{(0.05 \text{ m})} - \frac{1}{\infty} \right) = 2900 \text{ V}$	

A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.

- (e)
i. LO ACT-1.D, SP 6.B, 6.C
2 points

Calculate the magnitude of the initial acceleration of the proton.

For using a correct expression of Newton's second law in terms of the electric field	1 point
$F = ma \therefore qE = ma \therefore a = \frac{qE}{m}$	
For correctly substituting into equation above	1 point
$a = \frac{qE}{m} = \frac{(1.6 \times 10^{-19} \text{ C})(5.79 \times 10^4 \text{ N/C})}{1.67 \times 10^{-27} \text{ kg}} = 5.55 \times 10^{12} \text{ m/s}^2$	

2019 SCORING GUIDELINES**Question 2 (continued)**

(e) continued

ii. LO CNV-1.E, SP 6.B, 6.C

2 points

Calculate the speed of the proton after a long time.

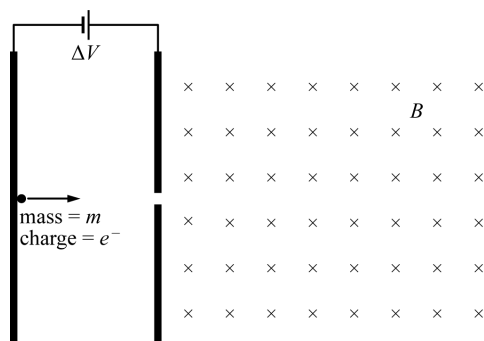
For a correct expression of kinetic energy in terms of the electric potential difference		1 point
$-q\Delta V = \frac{1}{2}mv^2$		
For correctly substituting into equation above		1 point
$v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{-2(1.6 \times 10^{-19} \text{ C})(0 - 2900 \text{ V})}{(1.67 \times 10^{-27} \text{ kg})}} = 7.45 \times 10^5 \text{ m/s}$		

Learning Objectives**ACT-1.D:** Determine the motion of a charged object of specified charge and mass under the influence of an electrostatic force.**CNV-1.E:** Calculate the work done or changes in kinetic energy (or changes in speed) of a charged particle when it is moved through some known potential difference.**CNV-1.G.a:** Use the general relationship between electric field and electric potential to calculate the relationships between the magnitude of electric field or the potential difference as a function of position.**CNV-2.C:** State and use Gauss's law in integral form to derive unknown electric fields for planar, spherical, or cylindrically symmetrical charge distributions.**CNV-2.D.a:** Using appropriate mathematics (which may involve calculus), calculate the total charge contained in lines, surfaces, or volumes when given a linear-charge density, a surface-charge density, or a volume-charge density of the charge configuration.**Science Practices****3.C:** Sketch a graph that shows a functional relationship between two quantities.**6.B:** Apply an appropriate law, definition, or mathematical relationship to solve a problem.**6.C:** Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

2019 SCORING GUIDELINES

Question 3

15 points



Two plates are set up with a potential difference V between them. A small sphere of mass m and charge $-e$ is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude B directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of V , m , e , B , and fundamental constants, as appropriate.

(a)

- i. LO CHG-1.A.a, SP 7.A
1 point

What is the initial direction of the force on the sphere as it enters the magnetic field?

☐ Into the page

☐ Out of the page

☐ Toward the top of the page

☐ Toward the bottom of the page

For selecting "Toward the bottom of the page"	1 point
---	---------

- ii. LO CHG-1.B, SP 7.A
1 point

Describe the path taken by the sphere after it enters the magnetic field.

For describing a circular path for the sphere consistent with the selection from part (a)(i)	1 point
Example: The sphere will move in a circular path toward the bottom of the page.	

2019 SCORING GUIDELINES**Question 3 (continued)**

- (b) LO CNV-1.E, SP 5.A, 5.E
2 points

Derive an expression for the speed of the sphere as it passes through the small opening.

For using a valid equation relating potential difference to kinetic energy of the sphere		1 point
$\Delta K = -\Delta U \therefore \frac{1}{2}mv^2 = -q\Delta V$		
$\frac{1}{2}mv^2 = eV$		
For the correct answer with supporting work		1 point
$v = \sqrt{\frac{2eV}{m}}$		

- (c) LO CHG-1.C, SP 5.A, 5.E
3 points

Derive an expression for the radius of the path taken by the sphere as it moves through the magnetic field.

For an expression relating the magnetic force to the centripetal force		1 point
$\frac{mv^2}{r} = Bqv$		
For substituting the charge of the sphere into the above equation and solving for r		1 point
$\frac{mv}{r} = Be \therefore r = \frac{mv}{Be}$		
For substituting the answer from part (b)		1 point
$r = \frac{m}{Be} \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2mV}{eB^2}}$		

An experiment is performed in which a beam of electrons is accelerated across the space between the plates and passes through the small opening. After passing through the opening, the electrons travel in a semicircular path and strike the right-hand plate. The potential difference between the plates is varied in regular increments, as shown in the table below. For each potential difference, the magnetic field is varied in order to cause the beam to strike the right-hand plate at a distance of 0.020 m from the opening.

Potential difference (V)	60	70	100	110	120	140
Magnetic field ($T \times 10^{-3}$)	2.62	2.78	3.39	3.54	3.78	3.99

2019 SCORING GUIDELINES**Question 3 (continued)**

- (d) LO CHG-1.C, SP 4.C
1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the mass-to-charge ratio of an electron.

Vertical axis: _____

Horizontal axis: _____

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

$$r = \sqrt{\frac{2m_e V}{eB^2}} \therefore B^2 = \frac{2m_e}{er^2} V$$

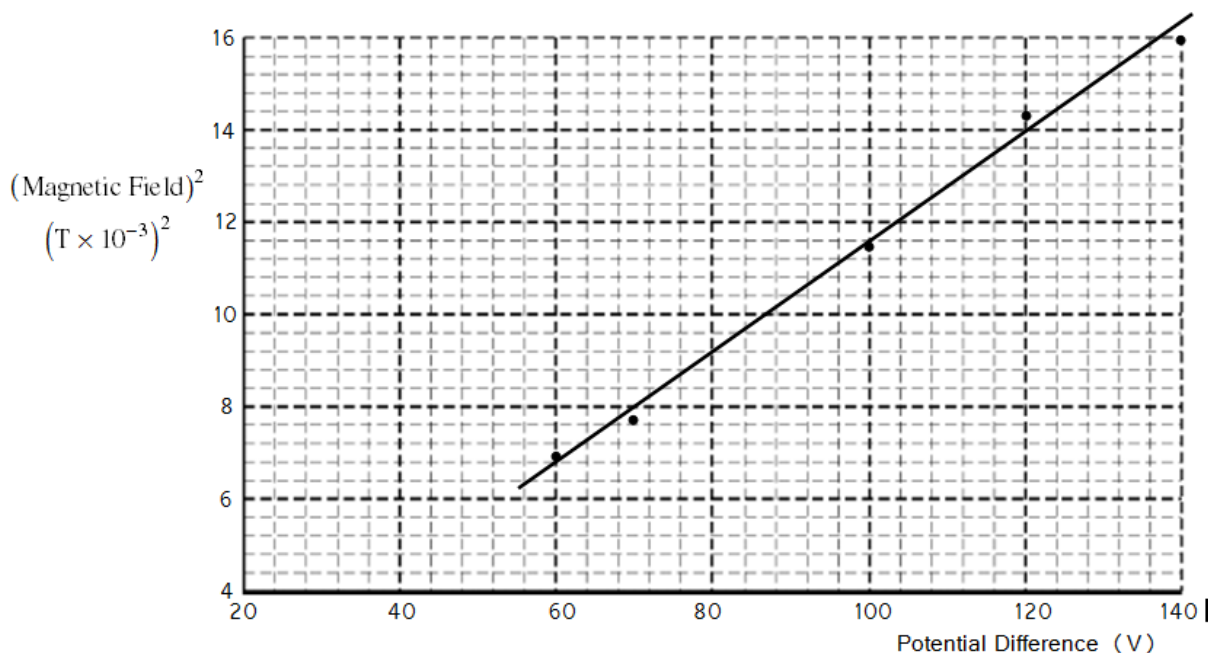
Potential difference (V)	60	70	100	110	120	140
Magnetic field ($\text{T} \times 10^{-3}$)	2.62	2.78	3.39	3.54	3.78	3.99
(Magnetic field) ² ($\text{T} \times 10^{-3}$) ²	6.86	7.73	11.5	12.5	14.3	15.9

For indicating axis choices with appropriate quantities to produce a straight line		1 point
Note: There are several possible choices.		

2019 SCORING GUIDELINES**Question 3 (continued)**

- (e) LO CHG-1.C, SP 3.A, 4.C
4 points

On the graph below, plot the relationship determined in part (d). Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



For correctly labeling both axes with quantities and units	1 point
For correctly scaling and labeling the axes so that the points use at least half the grid	1 point
For correctly plotting the data	1 point
For drawing a straight line that represents the data	1 point

- (f) LO CHG-1.C, SP 4.D, 5.E
3 points

Using the straight line from part (e), determine the mass-to-charge ratio of an electron.

For correctly calculating the slope from the best-fit straight line and not from the data points unless the points fall on the best-fit line	1 point
$\text{slope} = \frac{14 - 9.2}{120 - 80} = 0.12 \frac{(\text{T} \times 10^{-3})^2}{\text{V}} = 1.2 \times 10^{-7} \text{ T}^2/\text{V}$	
For a correct expression relating the slope to the mass to charge ratio of an electron	1 point
$B^2 = \frac{2m_e}{er^2} V \therefore \text{slope} = \frac{2m_e}{er^2} \therefore \frac{m_e}{e} = \frac{r^2 \times \text{slope}}{2}$	
For substituting correct values into the above equation	1 point
$\frac{m_e}{e} = \frac{(0.01 \text{ m})^2 \times (1.2 \times 10^{-7} \text{ T}^2/\text{V})}{2} = 6.0 \times 10^{-12} \text{ kg/C}$	

2019 SCORING GUIDELINES**Question 3 (continued)****Learning Objectives**

CHG-1.A.a: Calculate the magnitude and direction of the magnetic force of interaction between a moving charged particle of specified charge and velocity moving in a region of a uniform magnetic field.

CHG-1.B: Describe the path of different moving charged particles (i.e., of different type of charge or mass) in a uniform magnetic field.

CHG-1.C: Derive an expression for the radius of a circular path for a charged particle of specified characteristics moving in a specified magnetic field.

CNV-1.E: Calculate the work done or changes in kinetic energy (or changes in speed) of a charged particle when it is moved through some known potential difference.

Science Practices

3.A: Select and plot appropriate data.

4.C: Linearize data and/or determine a best-fit line or curve.

4.D: Select relevant features of a graph to describe a physical situation or solve problems.

5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.

5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

7.A: Make a scientific claim.

2018**AP[®]** **CollegeBoard**

AP Physics C: Electricity and Magnetism

Scoring Guidelines

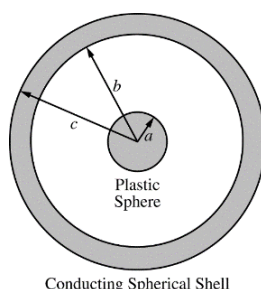
2018 SCORING GUIDELINES**General Notes About 2018 AP Physics Scoring Guidelines**

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.
2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at <https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf>.
3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as “derive” and “calculate” on the exams, and what is expected for each, see “The Free-Response Sections — Student Presentation” in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or “Terms Defined” in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.
5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.
6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

2018 SCORING GUIDELINES

Question 1

15 points total

Distribution
of points

A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β , r , a , and physical constants, as appropriate.

- (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

- i. 3 points

$$r < a$$

For using the integral to determine the charge enclosed	1 point
$q_{enc} = \int \rho dV = \int \beta r dV$	
For correctly substituting for dV into the integration to determine the charge enclosed	1 point
$q_{enc} = \int (4\pi r'^2)(\beta r') dr' = 4\beta\pi \int_{r'=0}^{r'=r} r'^3 dr' = 4\beta\pi \left[\frac{r'^4}{4} \right]_{r'=0}^{r'=r}$	
For a correct answer	1 point
$q_{enc} = \beta\pi r^4$	

- ii. 1 points

$$a < r < b$$

For an answer consistent with (a)(i)	1 point
$q_{enc} = \beta\pi a^4$	

2018 SCORING GUIDELINES

Question 1 (continued)

**Distribution
of points**

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. 2 points

$$r < a$$

For correctly substituting the area of a Gaussian sphere into Gauss's law		1 point
$\frac{q_{enc}}{\epsilon_0} = \oint \vec{E} \cdot d\vec{A}$		
$\frac{q(r)}{\epsilon_0} = EA_{sphere} = E(4\pi r^2)$		
For correctly substituting the charge from part (a)(i) into the equation above		1 point
$\frac{\beta\pi r^4}{\epsilon_0} = E(4\pi r^2)$		
$E = \frac{\beta r^2}{4\epsilon_0}$		

ii. 1 point

$$a < r < b$$

For correctly substituting the charge from part (a)(ii) into Gauss's law to calculate the electric field		1 point
$\frac{\beta\pi a^4}{\epsilon_0} = E(4\pi r^2)$		
$E = \frac{\beta a^4}{4\epsilon_0 r^2}$		

2018 SCORING GUIDELINES

Question 1 (continued)

**Distribution
of points**

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. 2 points

Determine the charge on the inner surface of the conducting shell.
Justify your answer.

For an answer consistent with part (a)(ii) (must have opposite sign)	1 point
$q_{\text{inner}} = -\beta\pi a^4$	
For a correct justification	1 point
$E_{\text{shell}} = 0 = \frac{q_{\text{enc}}}{\epsilon_0} \therefore q_{\text{enc}} = 0 = q_{\text{sphere}} + q_{\text{inner}} \therefore q_{\text{inner}} = -q_{\text{sphere}}$	

ii. 1 point

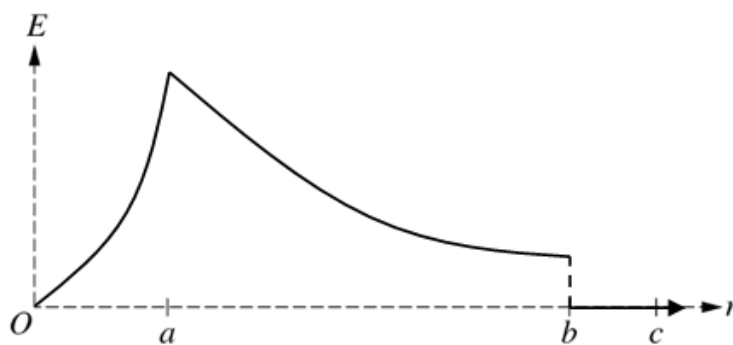
Determine the charge on the outer surface of the conducting shell.

A correct answer	1 point
$q_{\text{outer}} = 0$	

(d)

i. 3 points

On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



For concave up graph that begins at the origin and increases to $r = a$	1 point
For a concave up graph that is continuous with the first section and decreases from $r = a$ to nonzero value at $r = b$	1 point
For a graph with a value of $E = 0$ for $r > b$	1 point
Note: Due to scale, discontinuity at point b is not required.	

2018 SCORING GUIDELINES

Question 1 (continued)

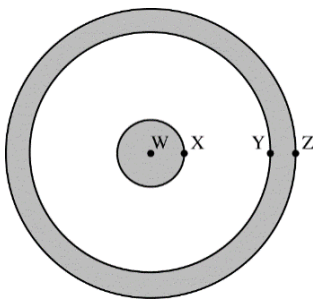
**Distribution
of points**

(d) (continued)

ii. 2 points

The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

__1__ W __2__ X __3__ Y __3__ Z

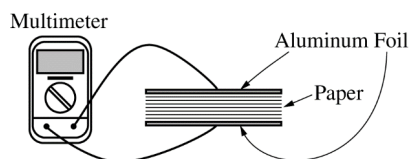


For any ranking that has Y and Z at the same potential	1 point
For ranking the electric potentials: $W > X > Y \text{ \& } Z$ (because $Y = Z$ is the first point)	1 point

2018 SCORING GUIDELINES

Question 2

15 points total

Distribution
of points

An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060 \text{ m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)		
500	0.045	6.5×10^{-11}		
400	0.036	7.4×10^{-11}		
300	0.027	8.9×10^{-11}		
200	0.018	11.9×10^{-11}		
100	0.010	21.0×10^{-11}		

(a) 1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: _____

Horizontal axis: _____

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

For indicating variables that will create a straight line whose slope can be used to determine the dielectric constant of the paper		1 point
Example: Vertical axis: C Horizontal axis: $\frac{1}{d}$		
Note: Student earns full credit if the axes are reversed or if they use another acceptable combination.		

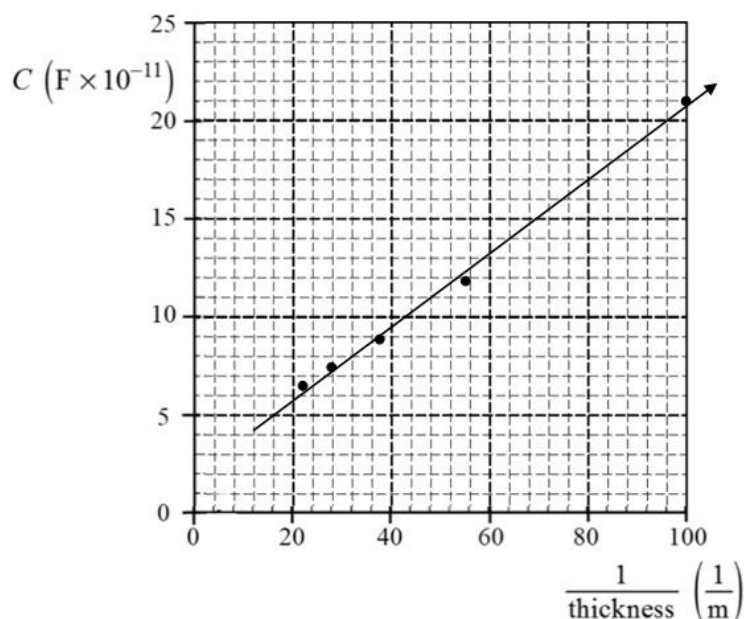
2018 SCORING GUIDELINES

Question 2 (continued)

Distribution
of points

(b) 4 points

Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



For a correct scale that uses more than half the grid	1 point
For correctly labeling the axes including units	1 point
For correctly plotting the data	1 point
For drawing a straight line consistent with the plotted data	1 point

(c) 3 points

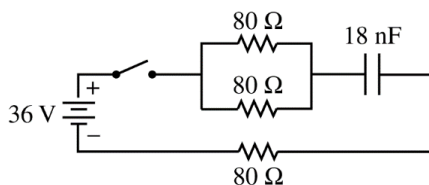
Using the straight line, calculate a dielectric constant for the paper.

For correctly calculating the slope from the best-fit line and not the data points unless the points fall on the best-fit line	1 point
$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{(17 - 8)(\text{F} \times 10^{-11})}{(80 - 32)(1/\text{m})} = 0.19 \text{ F} \cdot \text{m} \quad (\text{Linear regression} = 0.187 \text{ F} \cdot \text{m})$	
For correctly relating the slope to the dielectric constant	1 point
$\text{slope} = \kappa \epsilon_0 A$	
$\kappa = \frac{\text{slope}}{\epsilon_0 A} = \frac{(0.19 \text{ F} \cdot \text{m})}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.060 \text{ m}^2)}$	
For a correct answer	1 point
$\kappa = 3.58 \quad (\text{Linear regression} = 3.52)$	

2018 SCORING GUIDELINES

Question 2 (continued)

Distribution
of points



The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical 80 Ω resistors, and an open switch, as shown in the figure above.

(d) 3 points

Calculate the current in the battery immediately after the switch is closed.

For calculating the equivalent resistance for the parallel resistors	1 point
$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{80\ \Omega} + \frac{1}{80\ \Omega} = \frac{2}{80\ \Omega} = \frac{1}{40\ \Omega}$	
$R_p = 40\ \Omega$	
For using Ohm's law with the potential difference across the capacitor equal to zero	1 point
$I = \frac{V}{R} = \frac{V}{R_p + R_3} = \frac{(36\ \text{V})}{R_p + R_3}$	
For substitution of values for resistance including the value for combined resistance above	1 point
$I = \frac{V}{R_p + R_3} = \frac{(36\ \text{V})}{(40\ \Omega + 80\ \Omega)} = 0.30\ \text{A}$	

(e) 2 points

Determine the time constant for this circuit.

For using the equation for the time constant with the equivalent resistance from above	1 point
$\tau = R_{eq}C = (120\ \Omega)(18\ \text{nF})$	
For an answer with units consistent with part (d)	1 point
$t = 2.16\ \mu\text{s}$	

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Question 2 (continued)

**Distribution
of points**

- (f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. 1 point

Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

____ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

Select “Larger than measured”		
For an appropriate explanation		1 point
<i>Example: The battery is not ideal and has internal resistance, so the actual resistance for the circuit is larger than the measured resistance.</i>		

ii. 1 points

Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

____ Larger than measured ____ Smaller than measured

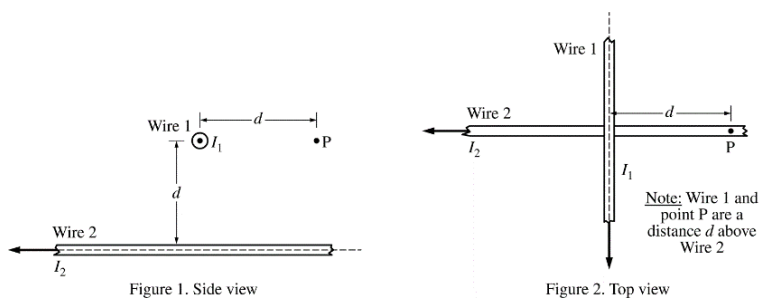
Explain experimentally what could account for this.

Select “Larger than measured”		
For an appropriate explanation		1 point
<i>Example: Some of the sheets of paper may be thinner than expected, so the actual capacitance for the circuit is larger than the measured capacitance.</i>		

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Question 3

15 points total

Distribution
of points

The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I , d , and physical constants, as appropriate.

(a) 2 points

Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

For attempting to use Ampere's law to calculate the magnetic field at point P	1 point
$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$	
$B(2\pi d) = \mu_0 I$	
For a correct answer	1 point
$B = \frac{\mu_0 I}{2\pi d}$	

(b) 2 points

Derive an expression for the magnitude of the net magnetic field at point P.

For indicating $B_2 = 2B_1$	1 point
$B_1 = \frac{\mu_0 I}{2\pi d}$ & $B_2 = \frac{\mu_0 (2I)}{2\pi d} = \frac{\mu_0 I}{\pi d}$	
For an indication that the magnitude of the magnetic field is the vector sum of B_1 and B_2	1 point
$B_{\text{net}} = \sqrt{B_1^2 + B_2^2} = \sqrt{B^2 + B^2} = \sqrt{\left(\frac{\mu_0 I}{2\pi d}\right)^2 + \left(\frac{\mu_0 (2I)}{2\pi d}\right)^2} = \frac{\sqrt{5}\mu_0 I}{2\pi d}$	

2018 SCORING GUIDELINES

Question 3 (continued)

Distribution
of points

(c) 2 points

Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

For correctly relating the angle to the individual magnetic fields	1 point
$\theta = \tan^{-1}\left(\frac{B_1}{B_2}\right)$	
For correctly substituting B_1 and B_2 into the equation	1 point
$\theta = \tan^{-1}\left(\frac{B_1}{B_2}\right) = \tan^{-1}\left(\frac{\left(\frac{\mu_0 I}{2\pi d}\right)}{\left(\frac{\mu_0 (2I)}{2\pi d}\right)}\right) = \tan^{-1}\left(\frac{1}{2}\right) = 26.6^\circ$	

(d) 2 points

Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2? Assume gravitational effects are negligible.

____ Wire 1 will not move.

____ Wire 1 will move upward as viewed in Figure 1.

____ Wire 1 will move downward as viewed in Figure 1.

____ Wire 1 will rotate clockwise as viewed in Figure 2.

____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

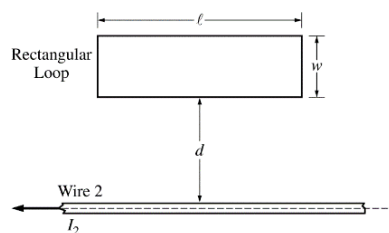
Justify your answer.

For stating there is no translational motion since the magnetic forces on the wire cancel	1 point
For stating there is a net torque which causes rotation	1 point
<i>Example: In Figure 2, the top portion of wire 1 will be in a magnetic field into the page from wire 2 and, thus, will experience a force to the right. The bottom portion of wire 1 will be in a magnetic field out of the page from wire 2 and, thus, will experience a force to the left. So the net force will be zero, but there will be a net clockwise torque, so the wire will rotate clockwise.</i>	

2018 SCORING GUIDELINES

Question 3 (continued)

**Distribution
of points**



Wire 1 is now replaced by a conducting rectangular loop of length ℓ , width w , and resistance R . The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1} .

(e) 1 point

Of the following, select the integration that will give an expression for the flux Φ as a function of time t .

_____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

_____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

___X___ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

_____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

For selecting $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$	1 point
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2018 SCORING GUIDELINES

Question 3 (continued)

**Distribution
of points**

(f) 3 points

Given that the flux through the rectangular loop as a function of time t is given by the equation

$\Phi = \frac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\frac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0 , d , r , R , w , k , ℓ , and physical constants, as appropriate.

For attempting to take the time derivative of the magnetic flux to calculate the emf	1 point
$\mathcal{E} = \left -\frac{d\Phi}{dt} \right = \frac{\mu_0 I_0 \ell}{\pi} \ln\left(\frac{d + w}{d}\right) \left \frac{d}{dt} [(1 - kt)] \right $	
$\mathcal{E} = \frac{\mu_0 I_0 k \ell}{\pi} \ln\left(\frac{d + w}{d}\right)$	
For dividing the emf by the resistance to calculate the current	1 point
$I = \frac{\mathcal{E}}{R} = \frac{\mu_0 I_0 k \ell}{\pi R} \ln\left(\frac{d + w}{d}\right)$	
For a correct answer	1 point
$I = \frac{\mu_0 I_0 k \ell}{\pi R} \ln\left(\frac{d + w}{d}\right)$	

(g) 3 points

What is the direction of the current, if any, induced in the loop as seen in Figure 3?

___ Clockwise ___ Counterclockwise

___ Undefined because there is no current induced in the loop

Justify your answer.

For selecting “Clockwise” with an attempt at a relevant justification	1 point
For indicating that the flux inside the loop will decrease	1 point
For using Lenz’s law to relate the decrease in the flux to the clockwise current	1 point
<i>Example: Because the current in the wire is decreasing, the flux in the loop will decrease. According to Lenz’s law, the induced current should create a magnetic field to oppose this decrease. Thus the induced magnetic field must be into the page, and the current in the loop must be clockwise.</i>	

2017**AP[®]** **CollegeBoard**

AP Physics C: Electricity and Magnetism

Scoring Guidelines

AP[®] PHYSICS
2017 SCORING GUIDELINES

General Notes About 2017 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.
2. The requirements that have been established for the paragraph length response in Physics 1 and Physics 2 can be found on AP Central at <https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf>.
3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth one point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections—Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based and AP Physics 2: Algebra-Based Course and Exam Description*.
5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.
6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

2017 SCORING GUIDELINES

15 points total	Question	Distribution of ints
(a) i. 1 point		1 point
ii. 1 point		1 point
(b) i. 2 points	For using Gauss's law to calculate the electric flux	1 point
	$\Phi = \frac{Q}{\epsilon_0} = \frac{\rho V}{\epsilon_0}$	1 point
	For a correct answer	
	$\Phi = \frac{\rho_0 Ah}{\epsilon_0}$	
ii. 3 points	For correctly applying Gauss's law to the slab	1 point
	$\frac{Q}{\epsilon_0} = \oint \vec{E} \cdot d\vec{A} = EA_{top} + EA_{bottom}$	1 point
	For substituting the flux from part (b)(i) into equation above and for including the contributions of the top and bottom parts of the Gaussian surface	
	$\frac{\rho_0 Ah}{\epsilon_0} = 2EA$	
	For correctly relating h to $2z$	1 point
	$\frac{\rho_0 A(2z)}{\epsilon_0} = 2EA$	
	$E = \frac{\rho_0 z}{\epsilon_0}$	

2017 SCORING GUIDELINES

Question 1 (continued)

Distribution
of points

(c)

i. 1 point

Selecting “ $z_0 < 0$ ”

For a correct justification

1 point

Example: The electric field due to the plates alone is directed toward the top of the page.

Therefore, the net electric field can only be zero where the electric field of the slab is directed toward the bottom of the page. So the net electric field can only be zero when $z_0 < 0$.

ii. 3 points

For setting the net electric field between the plates equal to zero and relating it to the sum of the electric fields of the plates and the slab

1 point

$$E_{total} = 0 = E_{plate} + E_{slab}$$

For using the correct formula for the electric field between the plates regardless of sign

1 point

$$\frac{\sigma}{\epsilon_0} = -\frac{\rho_0 z_0}{\epsilon_0}$$

$$z_0 = -\frac{\sigma}{\rho_0}$$

Substitute into equation above

$$z_0 = -\frac{(2.0 \times 10^{-6} \text{ C/m}^2)}{(1.0 \times 10^{-3} \text{ C/m}^3)}$$

For an answer that is consistent with the formula provided by the student above (must include units)

1 point

$$z_0 = -2.0 \times 10^{-3} \text{ m}$$

2017 SCORING GUIDELINES

Question 1 (continued)

Distribution
of points

(d) 4 points

For using the sum of the electric potential of the slab and the plates as the total potential difference between the center and top of the slab 1 point

$$\Delta V = \Delta V_{slab} + \Delta V_{plate}$$

$$\Delta V = \int E_{total} dr = \int (E_{slab} + E_{plate}) dr$$

$$\Delta V = \int E_{slab} dr + \int E_{plate} dr$$

For integrating with proper limits or constants of integration 1 point

$$\Delta V = \int_{z=0}^{z=0.005} \frac{\rho_0 z}{\epsilon_0} dz + \int_{z=0}^{z=0.005} \frac{\sigma}{\epsilon_0} dz$$

For correctly integrating each term regardless of whether they contain the correct expressions 1 point

$$\Delta V = \frac{\rho_0}{\epsilon_0} \left[\frac{z^2}{2} \right]_{z=0}^{z=0.005} + \frac{\sigma}{\epsilon_0} [z]_{z=0}^{z=0.005} = \frac{\rho_0}{2\epsilon_0} (0.005^2 - 0) + \frac{\sigma}{\epsilon_0} (0.005 - 0)$$

$$\Delta V = \frac{\rho_0 z^2}{2\epsilon_0} + \frac{\sigma z}{\epsilon_0}$$

$$\Delta V = \frac{(1.0 \times 10^{-3} \text{ C/m}^3)(5.0 \times 10^{-3} \text{ m})^2}{(2)(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)} + \frac{(2.0 \times 10^{-6} \text{ C/m}^2)(5.0 \times 10^{-3} \text{ m})}{(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)}$$

For a correct answer 1 point

$$\Delta V = 2540 \text{ V}$$

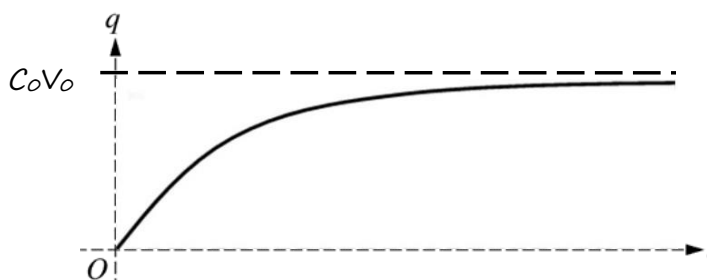
2017 SCORING GUIDELINES

Question

15 points total

Distribution
of points

(a) 3 points



For a concave down curve starting at the origin

1 point

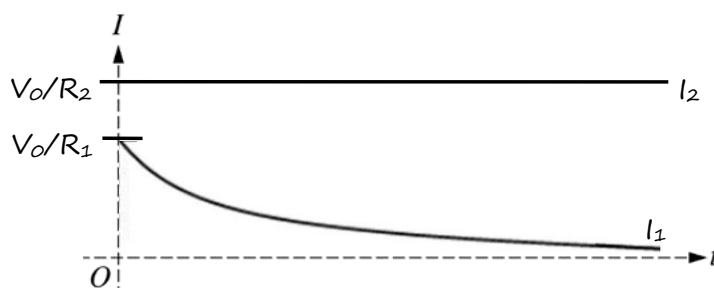
For a horizontal asymptote

1 point

For properly labeling the horizontal asymptote

1 point

(b) 4 points

For showing that I_2 is horizontal with the correct vertical intercept

1 point

For showing that I_1 is concave up and asymptotic to $I = 0$

1 point

For correctly labeling the vertical intercept of the I_1 graph

1 point

For drawing the I_2 graph always above the I_1 graph

1 point

(c) 2 points

Select " $\Delta t_C < \Delta t_D$ "

For indicating that the equivalent resistance during discharging is greater than during charging

1 point

For a statement relating the greater resistance to the greater time constant, or to a smaller current

1 point

Example: Because the resistance as the capacitor discharges is greater than when it charges, the time constant is larger for discharging. Therefore, the time to charge to 50% of its maximum charge is less than the time to discharge to 50% of its maximum charge.

2017 SCORING GUIDELINES

Question 2 (continued)

Distribution
of points

(d)

i. 2 points

For using a correct statement of Ohm's law or loop rule with the switch open

1 point

$$V = IR$$

$$I = \frac{V_C}{R_{tot}} = \frac{V_0}{R_1 + R_2}$$

Substitute correct values into equation above

$$I = \frac{(1.5 \text{ V})}{(100 \, \Omega + 150 \, \Omega)}$$

For a correct answer

1 point

 $I = 6.0 \text{ mA}$ with units, or 0.006 without units

ii. 1 point

For a correct justification

1 point

Example: Once the capacitor begins to discharge, the charge stored in the capacitor will decrease and the potential difference across the capacitor decreases. Therefore, the current through resistor R_2 decreases.

(e)

i. 1 point

For correctly substituting into an equation for the energy stored in a capacitor, including units for numerical values

1 point

$$U = \frac{1}{2} CV^2 = \frac{1}{2} (80 \, \mu\text{F}) (1.5 \text{ V})^2 = 90 \, \mu\text{J}$$

ii. 2 points

For setting up a valid equation or argument to calculate the energy dissipated by resistor R_1

1 point

$$E_{dis} = Pt = I^2 R t \therefore E_{dis} \sim R \therefore E_{dis,1} = E_{dis,tot} \frac{R_1}{(R_1 + R_2)}$$

For correctly substituting into the equation above

1 point

$$E_{dis,1} = (90 \, \mu\text{J}) \frac{(150 \, \Omega)}{(150 \, \Omega + 100 \, \Omega)} = 54 \, \mu\text{J}$$

Alternate Solution on next page.

2017 SCORING GUIDELINES

Question 2 (continued)

Distribution
of points

(e)(ii) continued

*Alternate Solution**Alternate
Points**Derive an equation for the current through resistor R_1 as a function of time*

$$I(t) = I_{\text{MAX}} e^{-t/\tau} = \frac{V_{\text{MAX}}}{(R_1 + R_2)} e^{-t/(R_1 + R_2)C} = \frac{(1.5 \text{ V})}{(250 \, \Omega)} e^{-t/(250 \, \Omega)(80 \, \mu\text{F})}$$

$$I(t) = (6.0 \text{ mA}) e^{-t/(0.02)}$$

For using a correct formula with integral calculus to calculate the energy dissipated by resistor R_1

1 point

$$E = \int P dt = \int I^2 R dt = \int I(t)^2 R_1 dt = \int \left((6.0 \text{ mA}) e^{-t/(0.02)} \right)^2 (150 \, \Omega) dt$$

For integrating the above equation with correct limits or constants of integration

1 point

$$E = (0.0054) \int_{t=0}^{t=\infty} \left(e^{-t/(0.02)} \right)^2 dt = (0.0054) \int_{t=0}^{t=\infty} e^{-100t} dt = (0.0054) \left[\frac{e^{-100t}}{(-100)} \right]_{t=0}^{t=\infty}$$

$$E = (-5.4 \times 10^{-5}) (e^{-\infty} - e^0) = 54 \, \mu\text{J}$$

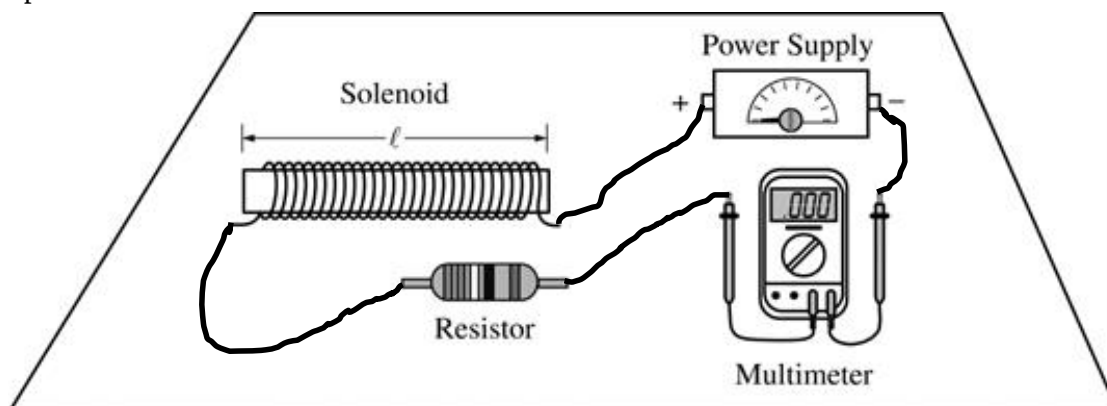
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Question

15 points total

Distribution
of points

- (a)
- i. 2 points



For connections that create a circuit with the power supply, solenoid, and resistor in series with each other

1 point

For connections that create a circuit with the multimeter in series with the inductor/resistor combination

1 point

- ii. 1 point

For selecting the answer consistent with the answer in part (a)(i)

1 point

- iii. 3 points

For selecting point *B*

1 point

For any indication that the magnetic field outside the solenoid is less than inside the solenoid

1 point

For any indication that the closer you are to the center of the solenoid, the better the approximation of an ideal solenoid due to the lack of edge effects

1 point

Example: The equation for the magnetic field for a solenoid is for an ideal solenoid.

This does not work outside the solenoid (this eliminates *C* and *D*). In addition, as you approach the edge of the solenoid, the ideal approximation is not as accurate (this eliminates *A*). Thus, point *B* gives the best approximation of an ideal solenoid.

2017 SCORING GUIDELINES

Question 3 (continued)

Distribution
of points

(b) 2 points

For any indication that Solenoid 1 has more turns per unit length

1 point

For any indication that Solenoid 1 is longer and therefore more like an ideal solenoid

1 point

Examples:

The equation for the magnetic field of a solenoid ($B = \mu_0 nI$) is based on an ideal solenoid in which you can ignore edge effects. Because Solenoid 1 has more turns per unit length, it is a better approximation of an ideal solenoid and thus its slope will give a better value for the permeability of free space.

The slope for Solenoid 1 is closer to the accepted value than the slope for Solenoid 2; therefore, the graph for Solenoid 1 is better.

(c) i. 3 points

For correctly calculating the slope for the solenoid chosen in part (b) using the best-fit line and not the data points

1 point

$$\text{slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(10 - 4)(\text{T} \times 10^{-5})}{(75 - 24)(\text{A/m})} = 1.17 \times 10^{-6} \text{ T} \cdot \text{m/A}$$

For correctly relating the magnetic permeability of free space to the slope

1 point

 $\mu_0 = \text{slope}$

For correct units

1 point

$$\mu_0 = 1.17 \times 10^{-6} \text{ T} \cdot \text{m/A}$$

For Solenoid 2

$$\text{slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(8 - 4)(\text{T} \times 10^{-5})}{(82 - 35)(\text{A/m})} = 8.51 \times 10^{-7} \text{ T} \cdot \text{m/A}$$

ii. 1 point

For using a correct equation for % error including the substitution of the accepted and the measured values

1 point

$$\% \text{ error} = \frac{|\text{acc} - \text{exp}|}{\text{acc}} \times 100\% = \frac{|(1.26 \times 10^{-6}) - (1.18 \times 10^{-6})|}{(1.26 \times 10^{-6})} \times 100\% = 6.35\%$$

For Solenoid 2

$$\% \text{ error} = \frac{|\text{acc} - \text{exp}|}{\text{acc}} \times 100\% = \frac{|(1.26 \times 10^{-6}) - (8.51 \times 10^{-7})|}{(1.26 \times 10^{-6})} \times 100\% = 32.4\%$$

2017 SCORING GUIDELINES**Question 3 (continued)****Distribution
of points**

(d)

i. 1 point

For a correct explanation involving any extraneous B fields

1 point

Example: A component of Earth's magnetic field is in the direction of the inductor's magnetic field.

ii. 2 points

For stating that either the solenoid or the circuit has near zero resistance

1 point

For stating that as a result of near zero resistance the current becomes very high

1 point

Example: The inductor has near zero resistance. Therefore, without the resistor in series with it, the inductor will draw a near infinite current that the multimeter cannot handle.