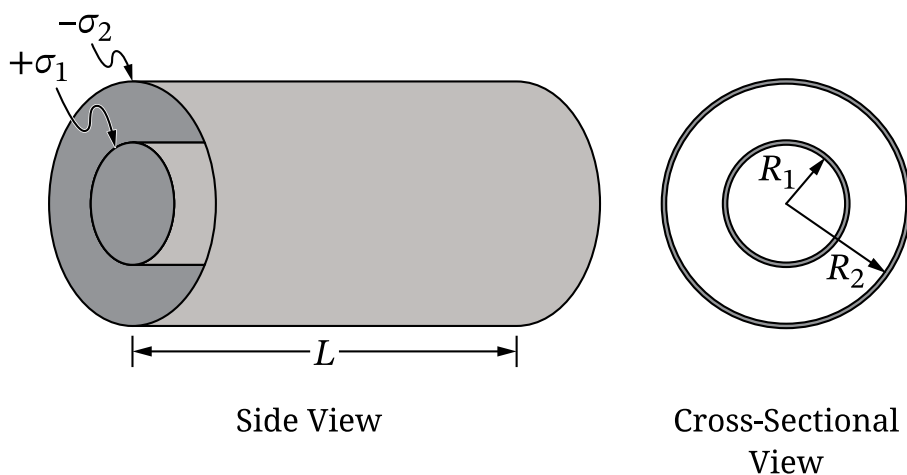


Question 1: Version J

1. An isolated, air-filled, charged capacitor consists of two conducting, coaxial, cylindrical shells that each have length L . The inner shell has radius R_1 and the outer shell has radius R_2 , as shown in Figure 1, where $R_1 < R_2 \ll L$. The surface charge densities (amounts of charge per unit area) of the inner and outer shells are $+\sigma_1$ and $-\sigma_2$, respectively. The absolute values of the total charges on the shells are equal.



Note: Figures not drawn to scale.

Figure 1

A.

- i. Using Gauss's law, **derive** an expression for the magnitude E of the electric field as a function of the radial distance r from the center of the capacitor for the region $R_1 < r < R_2$. Express your answer in terms of R_1 , σ_1 , r , and physical constants, as appropriate.
- ii. **Derive** an expression for the absolute value $|\Delta V|$ of the potential difference between the outer and inner shells in terms of R_1 , R_2 , σ_1 , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

- iii. On the axes shown in Figure 2, **sketch** a graph of E as a function of r from $r = 0$ to a position that is outside the outer shell.

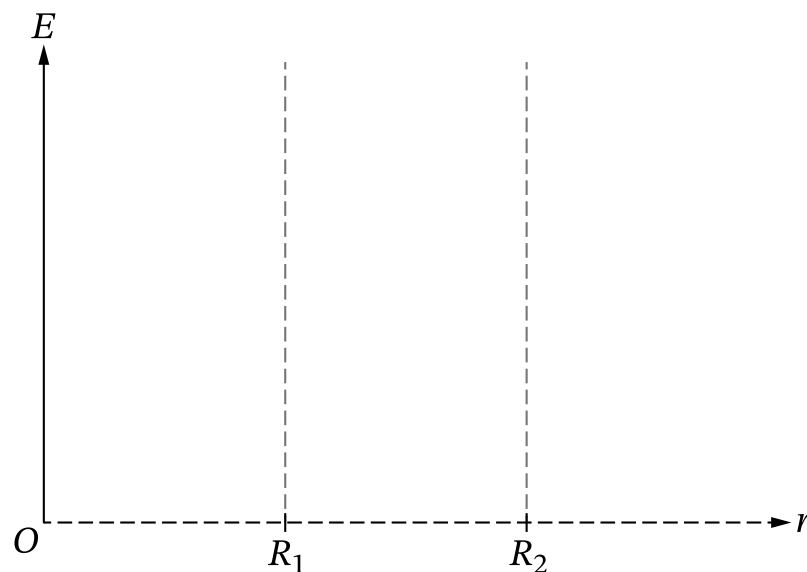
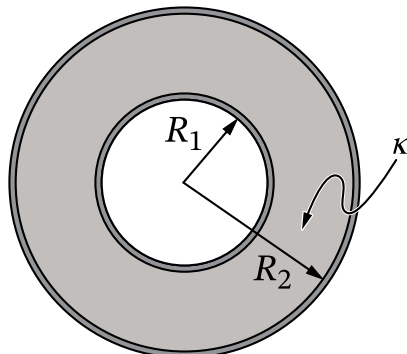


Figure 2

- B. A material of dielectric constant κ is inserted into the isolated, charged capacitor such that the material fills the region $R_1 < r < R_2$, as shown in Figure 3.



Cross-Sectional View

Figure 3

Derive an expression for the capacitance C of the capacitor with the material inserted in terms of L , R_1 , R_2 , κ , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Begin your response to **QUESTION 1** on this page.**PHYSICS C: ELECTRICITY AND MAGNETISM****SECTION II****Time—45 minutes****3 Questions**

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

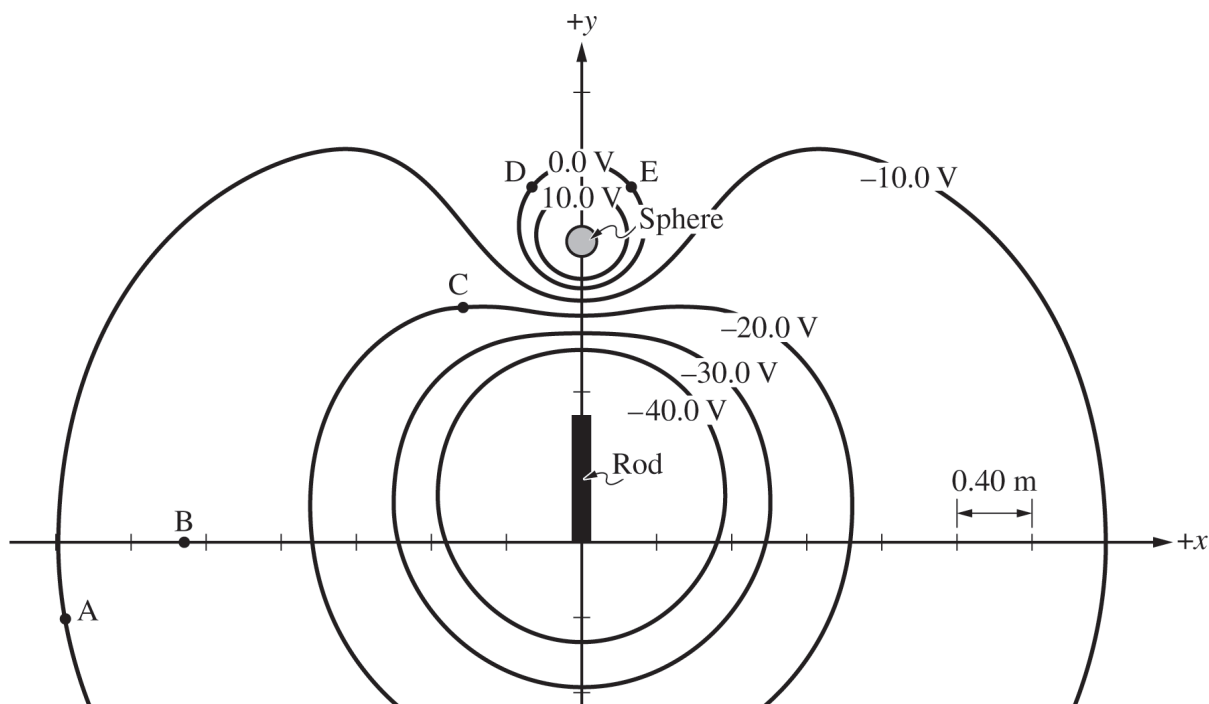


Figure 1

1. A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0 \text{ nC}$. The rod and sphere are held at rest on the y -axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x -axis and on the y -axis are 0.40 m apart.

- (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the 0.0 V equipotential line.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of $C \rightarrow E \rightarrow D \rightarrow A$. The test charge is held momentarily at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE} . If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a “0” in the corresponding columns, as appropriate.

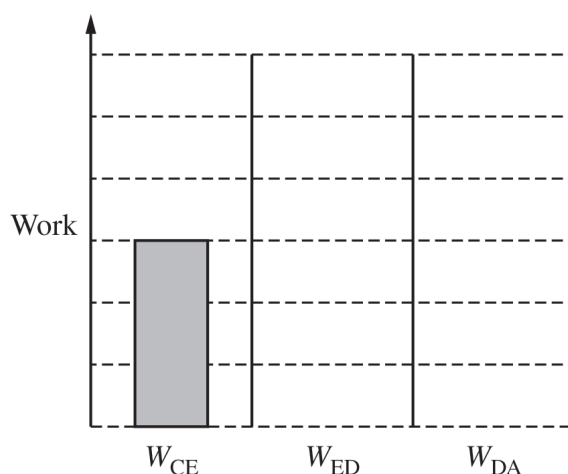


Figure 2

ii. **Calculate** the approximate magnitude of the x -component of the electric field at Position B.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position C. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

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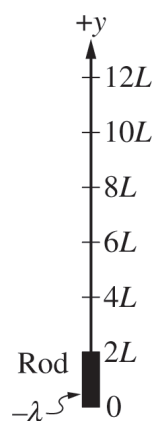
Continue your response to **QUESTION 1** on this page.

Figure 3

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the y -axis in the orientation shown in Figure 3. Position P (not shown) is located on the y -axis a distance y_P from the origin, where $y_P > 2L$.

(d) The electric potential V_P at y_P is $V_P = -k\lambda \ln\left(\frac{y_P}{y_P - 2L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

- ii. On Figure 4, **sketch** a graph of the y -component E_y of the electric field resulting from the rod as a function of y in the region $2L < y < 12L$.

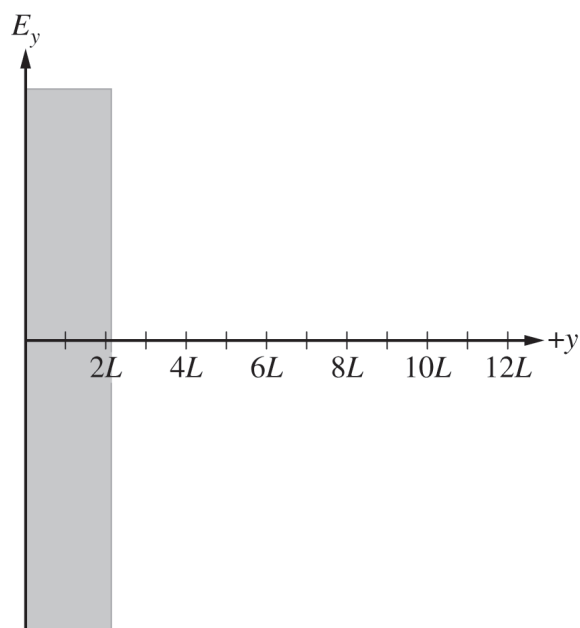
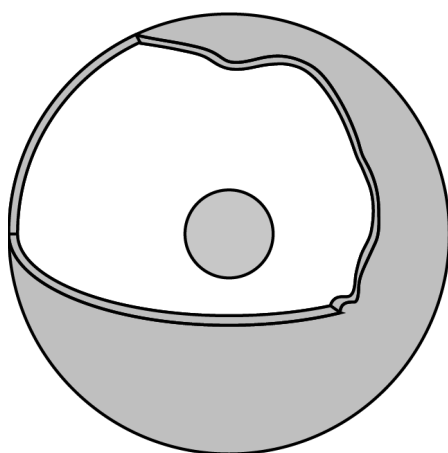


Figure 4

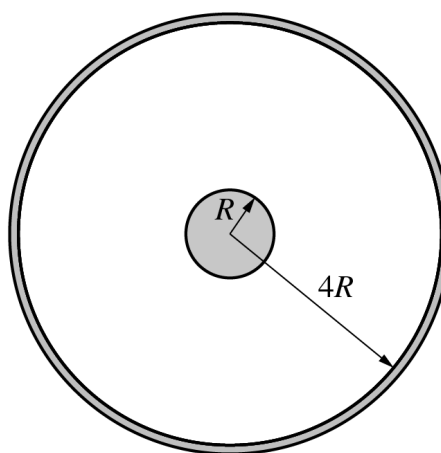
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PHYSICS C: ELECTRICITY AND MAGNETISM**SECTION II****Time—45 minutes****3 Questions**

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Cutout View



Cross-Section View

Note: Figures not drawn to scale.

A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.

(a) Determine the charge on the outer surface of the shell.

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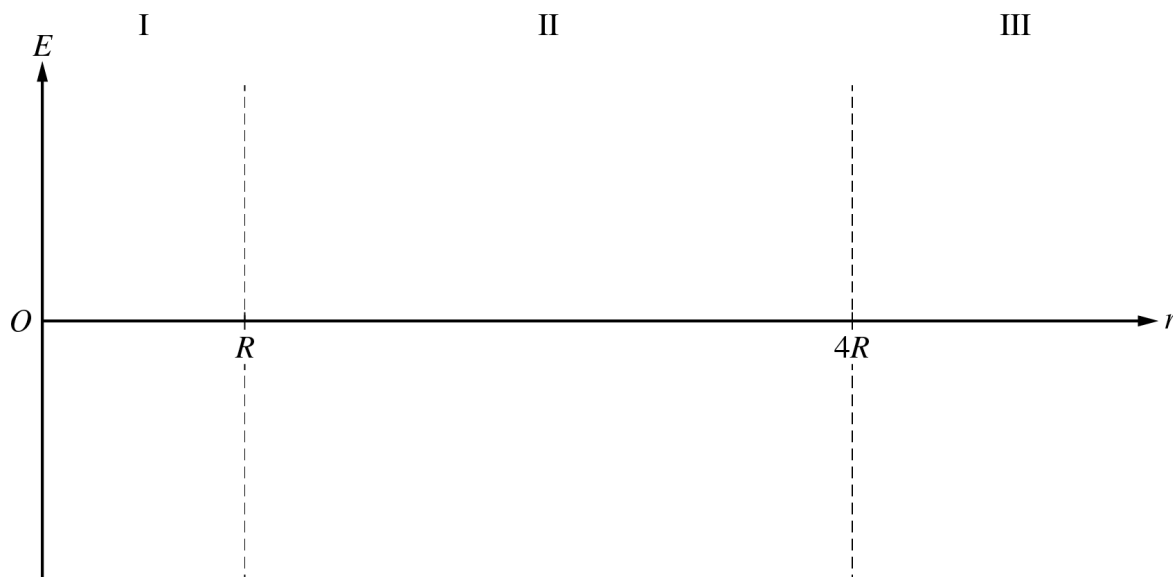
Continue your response to **QUESTION 1** on this page.

- (b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the sphere for $r < R$. Express your answer in terms of Q , R , r , and physical constants, as appropriate.
- (c) The magnitude of the electric field at $r = R$ is 8 N/C . Calculate the value of the electric field at $r = 2R$.
- (d) Derive an expression for the absolute value of the potential difference between the outer surface of the sphere and the inner surface of the shell. Express your answer in terms of Q , R , and physical constants, as appropriate.

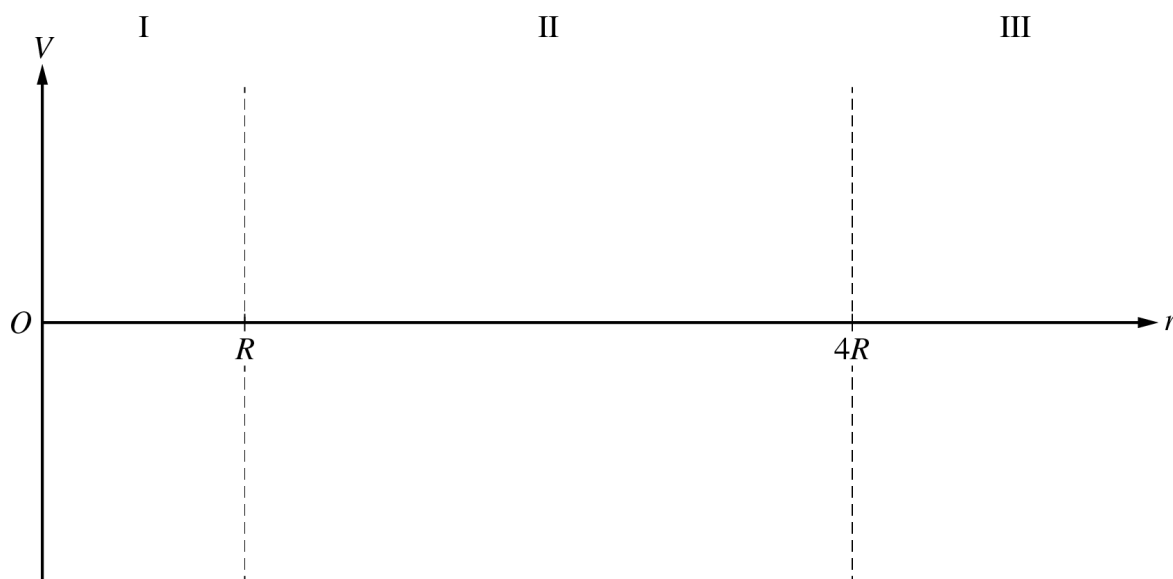
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(e)

i. On the following axes that include regions I, II, and III, sketch a graph of the electric field E as a function of the distance r from the center of the sphere.



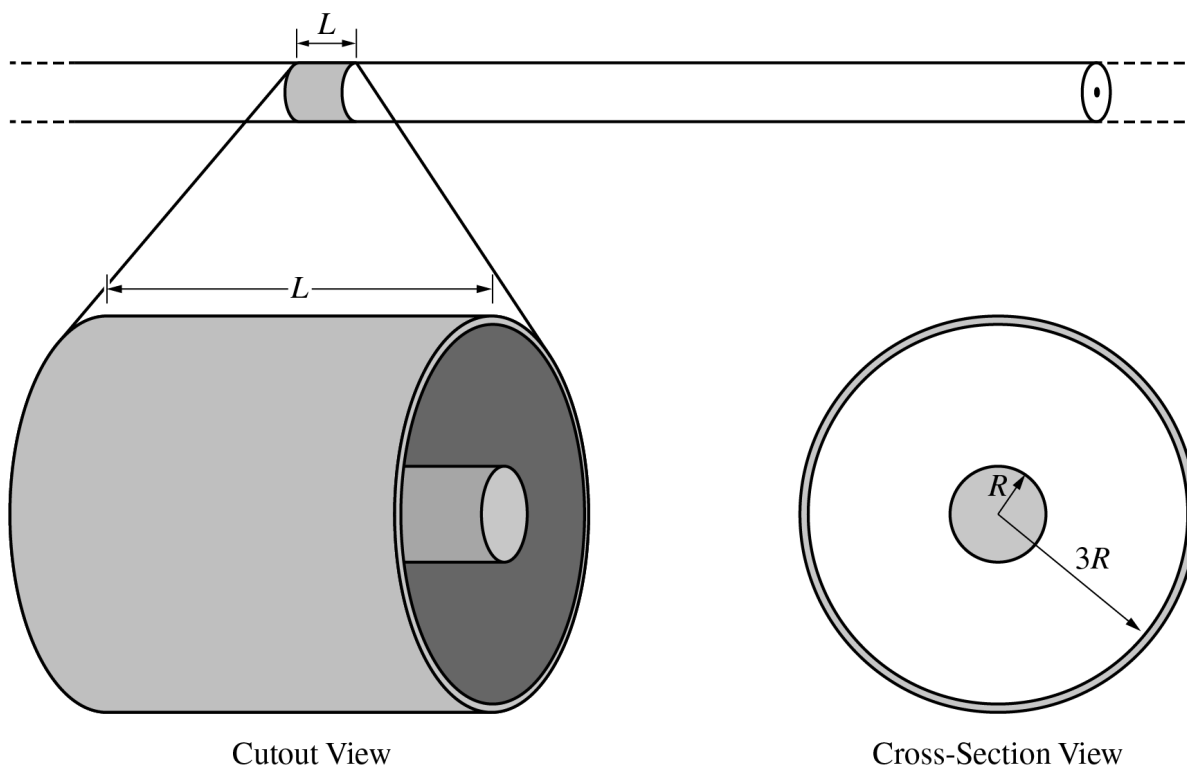
ii. On the following axes that include regions I, II, and III, sketch a graph of the electric potential V as a function of the distance r from the center of the sphere.



GO ON TO THE NEXT PAGE.

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Note: Figures not drawn to scale.

A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

- (a) Determine the charge on the outer surface of the cylindrical shell within length L .
- (b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the inner cylinder for $r < R$. Express your answers in terms of Q , R , r , L , and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(c) The magnitude of the electric field at $r = R$ is 12 N/C . Calculate the value of the electric field at $r = 2R$.

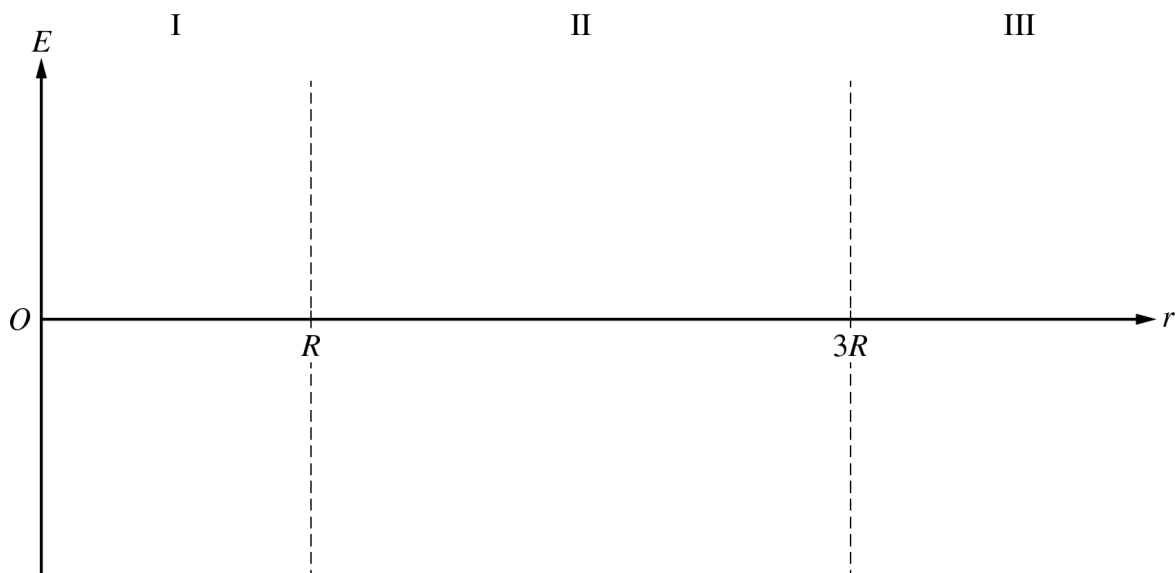
(d) Derive an expression for the absolute value of the potential difference between the surface of the nonconducting cylinder and the inner surface of the cylindrical shell.

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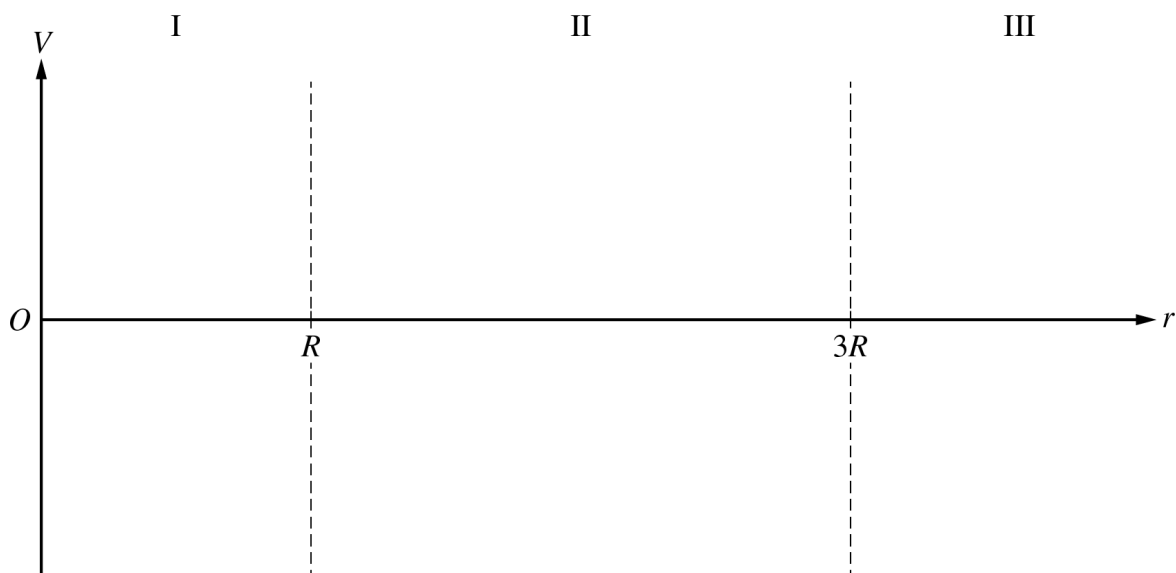
Continue your response to **QUESTION 1** on this page.

(e)

- i. On the following axes that include regions I, II, and III, sketch the graph of the electric field E as a function of the distance r from the axis of the inner cylinder.

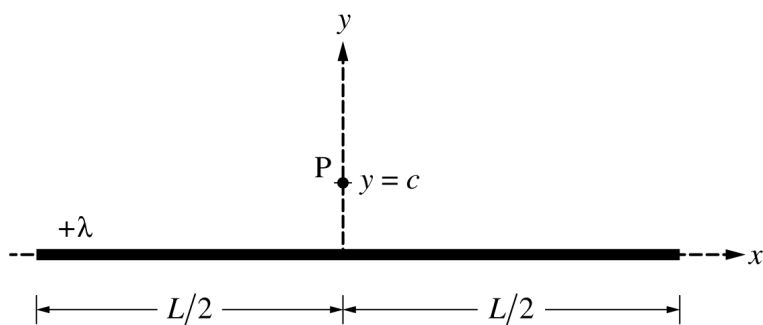


- ii. On the following axes that include regions I, II, and III, sketch the graph of the electric potential V as a function of the distance r from the axis of the inner cylinder.

**GO ON TO THE NEXT PAGE.**

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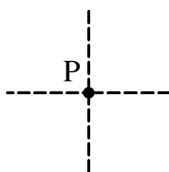


Note: Figure not drawn to scale.

1. A very long, thin, nonconducting cylinder of length L is centered on the y -axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y -axis at $y = c$, where $L \gg c$.

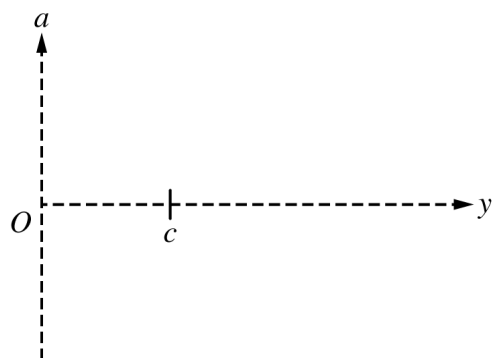
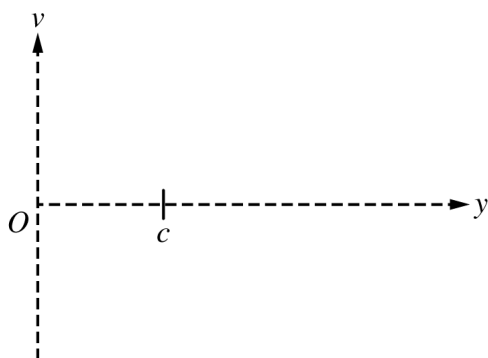
(a)

- i. On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the long cylinder. The arrow should start on and point away from the dot.

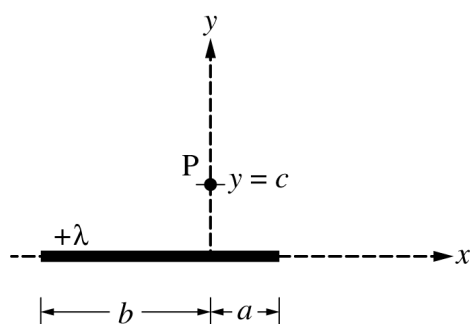


- ii. Describe the shape and location of a Gaussian surface that can be used to determine the electric field at point P due to the long cylinder.
- iii. Use your Gaussian surface to derive an expression for the magnitude of the electric field at point P. Express your answer in terms of λ , c , L , and physical constants, as appropriate.

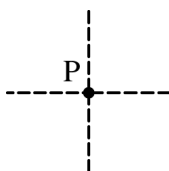
- (b) A proton is released from rest at point P. On the axes below, sketch the velocity v as a function of position y and the acceleration a as a function of position y for the proton.



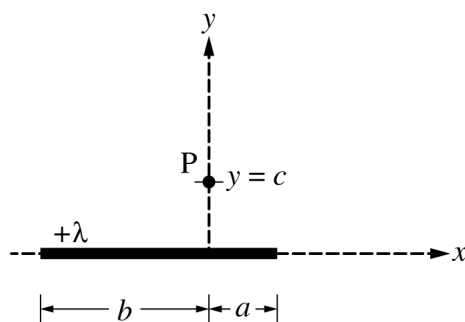
The original cylinder is now replaced with a much shorter thin, nonconducting cylinder with the same uniform linear charge density $+\lambda$, as shown in the figure below. The length of the cylinder to the right of the y -axis is a , and the length of the cylinder to the left of the y -axis is b , where $a < b$.



- (c) On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the shorter cylinder. The arrow should start on and point away from the dot.



- (d)
- Is there a single Gaussian surface that can be used with Gauss's law to derive an expression for the electric field at point P?
 ____ Yes ____ No
 - If your answer to part (d)(i) is yes, explain how you can use Gauss's law to derive an expression for the field at point P. If your answer to part (d)(i) is no, explain why Gauss's law cannot be applied to derive an expression for the electric field in this case.



Note: This figure is shown again for reference.

A student in class argues that using the integral shown below might be a useful approach for determining the electric field at point P.

$$E = \int \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} dq$$

The student uses this approach and writes the following two integrals for the magnitude of the horizontal and vertical components of the electric field at point P.

Horizontal component: $|E_x| = \frac{\lambda}{4\pi\epsilon_0} \int_{-b}^a \frac{x}{(c^2 + x^2)^{3/2}} dx$

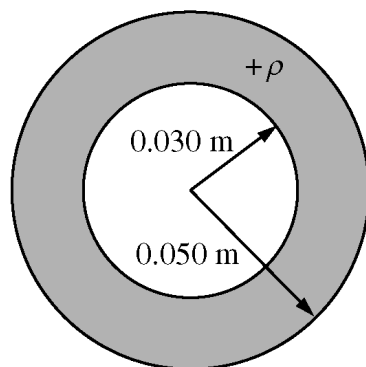
Vertical component: $|E_y| = \frac{\lambda}{4\pi\epsilon_0} \int_{-b}^a \frac{y}{(c^2 + x^2)} dy$

(e)

- i. One of the two expressions above is not correct. Which expression is not correct?

_____ Horizontal component _____ Vertical component

- ii. Identify two mistakes in the incorrect expression, and explain how to correct the mistakes.



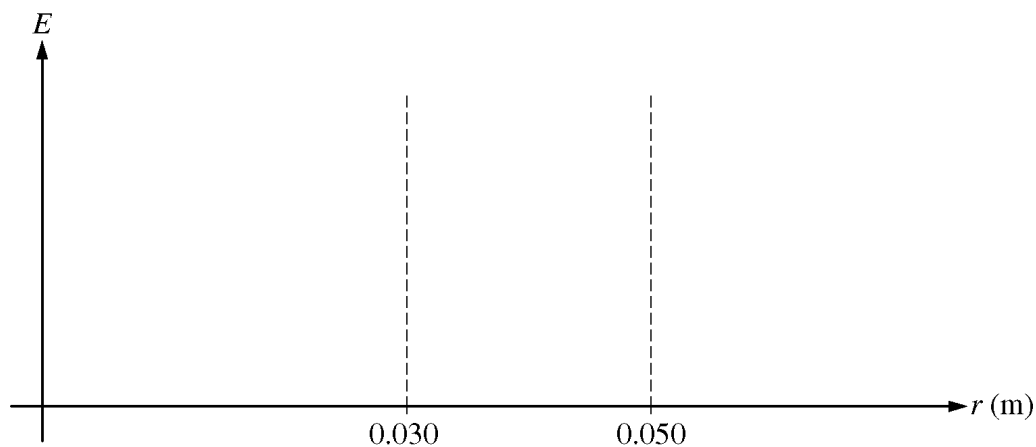
2. A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ , as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m: } \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m: } \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

$$r > 0.050 \text{ m: } \rho = 0$$

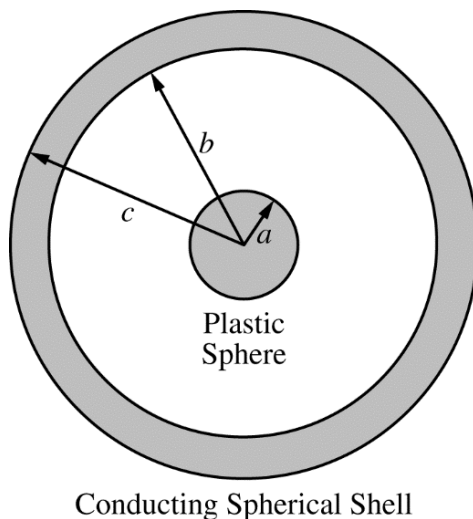
- (a) Calculate the total charge of the sphere.
- (b) Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.
- (c) On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.



- (d) Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.
- (e) A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.
 - i. Calculate the magnitude of the initial acceleration of the proton.
 - ii. Calculate the speed of the proton after a long time.

PHYSICS C: ELECTRICITY AND MAGNETISM**SECTION II****Time—45 minutes****3 Questions**

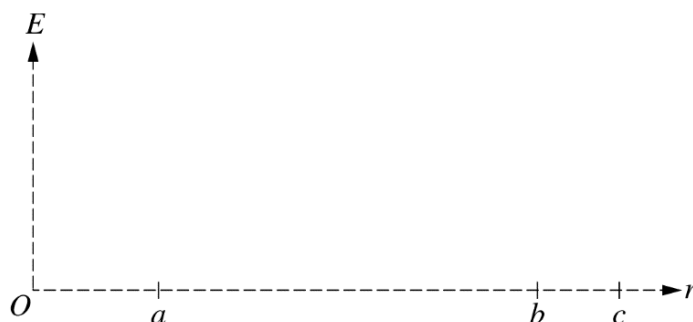
Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β , r , a , and physical constants, as appropriate.
 - (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.
 - i. $r < a$
 - ii. $a < r < b$
 - (b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.
 - i. $r < a$
 - ii. $a < r < b$
 - (c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.
 - i. Determine the charge on the inner surface of the conducting shell.
Justify your answer.
 - ii. Determine the charge on the outer surface of the conducting shell.

(d)

- i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



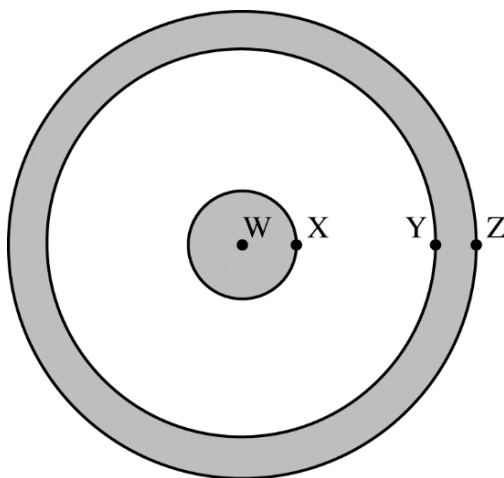
- ii. The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

___ W

___ X

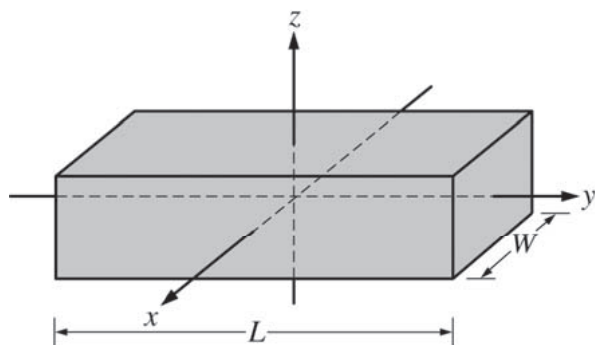
___ Y

___ Z

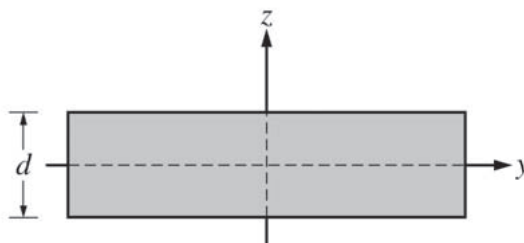


PHYSICS C: ELECTRICITY AND MAGNETISM**SECTION II****Time—45 minutes****3 Questions**

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Perspective View

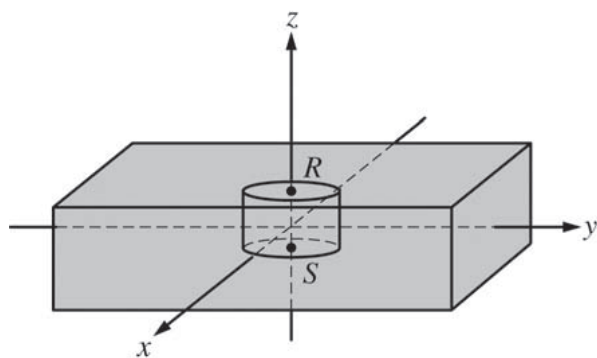


Side View

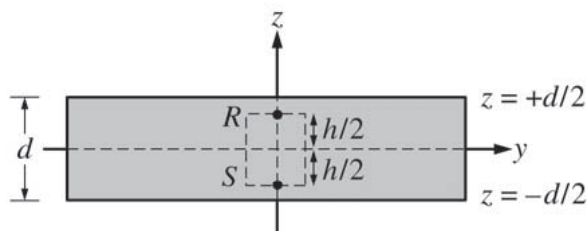
Note: Figures not drawn to scale.

1. A very large nonconducting slab with a uniform positive volume charge density ρ_0 is fixed with the origin of the xyz -axes at its center, as shown in the figure above. The thickness of the slab is d , the length is L , and the width is W , where $L \gg d$ and $W \gg d$. The large faces of the slab are parallel to the xy -plane.

Consider a Gaussian cylinder with a cross-sectional area A and height h that is positioned with its axis along the z -axis, as shown in the figure below.



Perspective View



Side View

Note: Figures not drawn to scale.

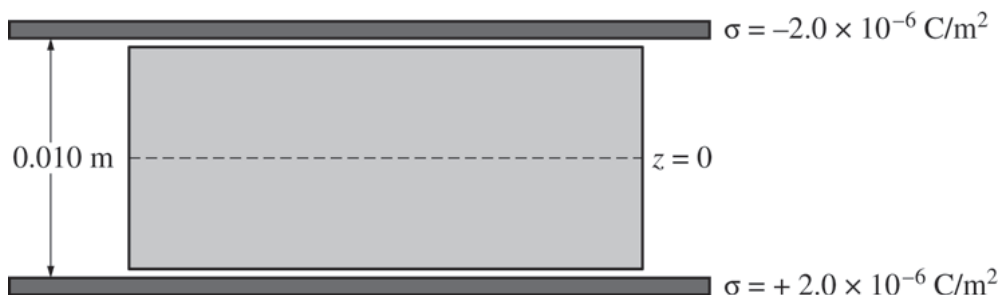
- (a) Draw a single vector on each of the dots below representing the direction of the electric field at the given points. If the electric field at either point is zero, write “ $E = 0$ ” next to the point.

i.

ii.

• R • S

- b) Use Gauss’s law to derive expressions for the following. Express your answers in terms of ρ_0 , A , d , h , z , and physical constants, as appropriate.
- Derive an expression for the total flux Φ through the Gaussian surface shown.
 - Derive an expression for the magnitude of the electric field as a function of z for any position inside the slab, and show that it is equal to $E = \frac{\rho_0 z}{\epsilon_0}$.



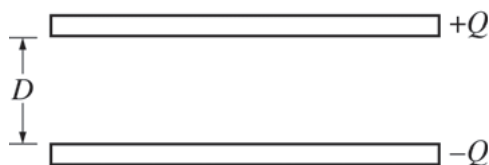
Note: Figure not drawn to scale.

The charged slab is now placed between two large metal plates separated by a distance of 0.010 m, which is approximately the thickness of the slab, but the slab does not contact either metal plate. The metal plates are charged, resulting in the surface charge densities $\sigma = \pm 2.0 \times 10^{-6} \text{ C/m}^2$, as shown in the figure above. Assume the charge distribution inside the slab remains unchanged by the presence of the charged plates and that the slab’s volume charge density is $\rho_0 = 1.00 \times 10^{-3} \text{ C/m}^3$.

- (c)
- The magnitude of the electric field inside the slab is zero on the z -axis at position z_0 . Which of the following correctly indicates the value for z_0 ?
 _____ $z_0 > 0$ _____ $z_0 = 0$ _____ $z_0 < 0$
 Justify your answer.
 - Calculate the value z_0 .
- (d) Calculate the magnitude of the electric potential difference from the center of the slab to the top of the slab.

PHYSICS C: ELECTRICITY AND MAGNETISM**SECTION II****Time—45 minutes****3 Questions**

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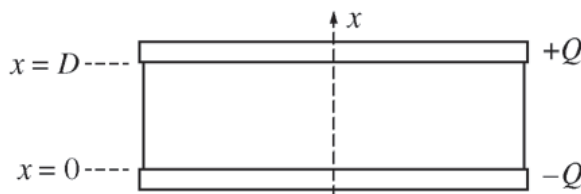


E&M.1.

A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D . The plates of the capacitor are each given a charge of magnitude Q , as shown in the figure above. Ignore edge effects.

(a)

- i. On the figure above, draw an arrow to indicate the direction of the electric field between the plates.
- ii. On the figure above, draw an appropriate Gaussian surface that will be used to derive an expression for the magnitude of the electric field E between the plates.
- iii. Using Gauss's law and the Gaussian surface from part (a)-ii, derive an expression for the magnitude of the electric field E between the plates. Express your answer in terms of A , D , Q , and physical constants, as appropriate.



The space between the plates is now filled with a dielectric material that is engineered so that its dielectric constant varies with the distance from the bottom plate to the top plate, defined by the x -axis indicated in the diagram above. As a result, the electric field between the plates is given by $\vec{E} = -\frac{Q}{\epsilon_0 \kappa_0 e^{-x/D} A} \hat{i}$, where κ_0 is a

positive constant. Express all algebraic answers to the remaining parts in terms of A , D , Q , κ_0 , x , and physical constants, as appropriate.

(b) Determine an expression for the dielectric constant κ as a function of x .

(c)

- i. Write, but do NOT solve, an equation that could be used to determine the potential difference V between the plates of the capacitor.
- ii. Using the equation from part (c)-i, derive an expression for the potential difference $V_D - V_0$, where V_D is the potential of the top plate and V_0 is the potential of the bottom plate.

(d) Determine the capacitance of the capacitor.

- (e) The energy stored in the capacitor that has a varying dielectric is U_V . A second capacitor that has a constant dielectric of value κ_0 is also given a charge Q . The energy stored in the second capacitor is U_C . How do the values of U_V and U_C compare?

_____ $U_V < U_C$ _____ $U_V > U_C$ _____ $U_V = U_C$

Justify your answer.

E&M.3.

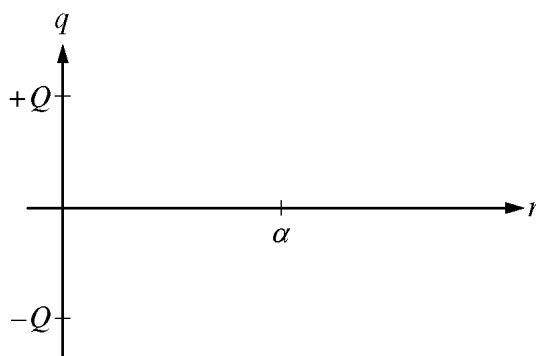
A scientist describes an electrically neutral atom with a model that consists of a nucleus that is a point particle with positive charge $+Q$ at the center of the atom and an electron volume charge density of the form

$$\rho(r) = \begin{cases} -\frac{\beta}{r^2} e^{-r/\alpha} & r < \alpha \\ 0 & r > \alpha \end{cases}$$

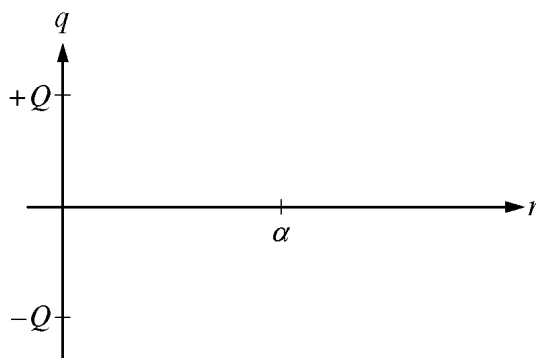
where α and β are positive constants and r is the distance from the center of the atom.

- (a) On the axes below, let r stand for the radius of a Gaussian sphere. Sketch the graph for each of the following charges enclosed by the Gaussian sphere as a function of r . Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

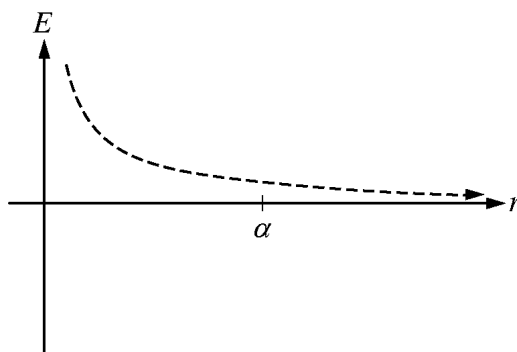
- i. The nuclear charge only



- ii. The electron charge only



- (b) The dashed curve on the graph below represents the electric field as a function of distance r due to the positive nucleus of the atom without any electrons. The nucleus is modeled as a point particle of charge $+Q$. On the same graph, sketch the electric field as a function of distance r for the neutral atom as defined by the scientist's model, which includes the nucleus and the negative electrons surrounding it.



- (c) Use Gauss's law to derive an expression for the electric field strength due to the neutral atom for the following positions in terms of Q , α , β , r , and fundamental constants, as appropriate.
- $r > \alpha$
 - $r < \alpha$
- (d) Based on the model proposed by the scientist, what is the physical meaning of the constant α ?

STOP
END OF EXAM