

Begin your response to **QUESTION 1** on this page.**PHYSICS C: ELECTRICITY AND MAGNETISM****SECTION II****Time—45 minutes****3 Questions**

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

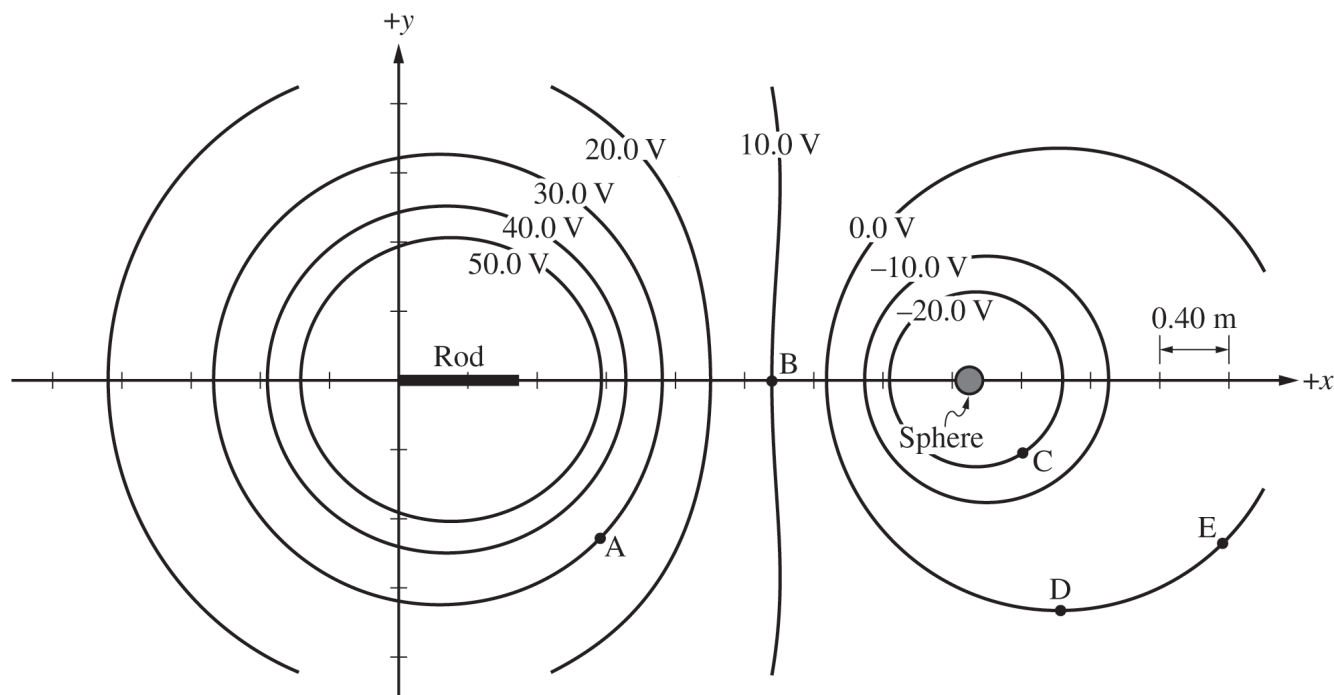


Figure 1

1. A nonconducting rod of uniform positive linear charge density is near a sphere with charge -2.0 nC . The rod and sphere are held at rest on the x -axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x -axis and the y -axis are 0.40 m apart.
- (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the -20.0 V equipotential line.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of $C \rightarrow E \rightarrow D \rightarrow A$. The test charge is momentarily held at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE} . If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a “0” in the corresponding columns, as appropriate.

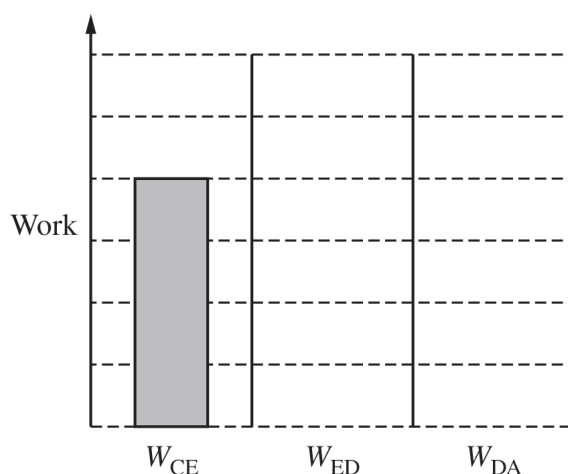


Figure 2

ii. **Calculate** the approximate magnitude of the x -component of the electric field at Position B.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position D. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

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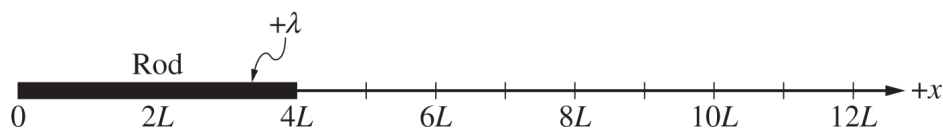
Continue your response to **QUESTION 1** on this page.

Figure 3

The sphere and the test charge are removed. The rod has length $4L$ and uniform positive linear charge density $+\lambda$. The rod is held at rest on the x -axis in the orientation shown in Figure 3. Position P (not shown) is located on the x -axis a distance x_P from the origin, where $x_P > 4L$.

(d) The electric potential V_P at x_P is $V_P = k\lambda \ln\left(\frac{x_P}{x_P - 4L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

- ii. On Figure 4, **sketch** a graph of the x -component E_x of the electric field from the rod as a function of x in the region $4L < x < 12L$.

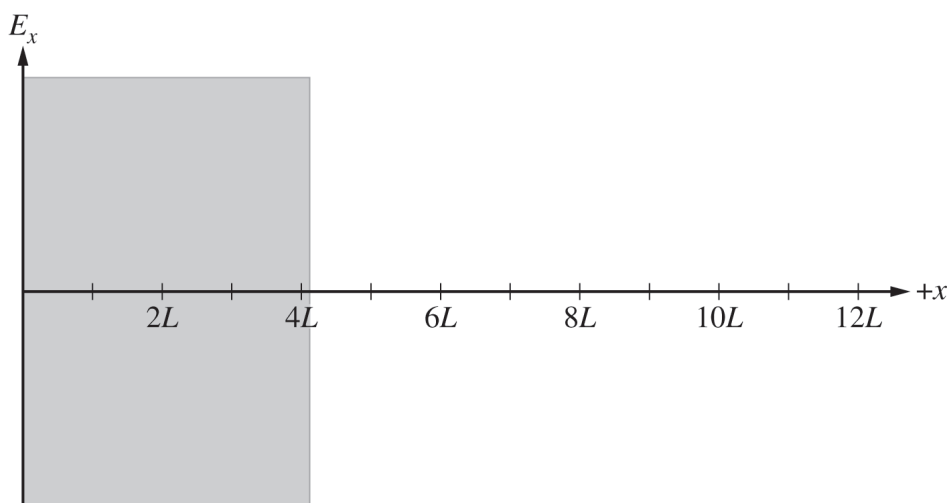


Figure 4

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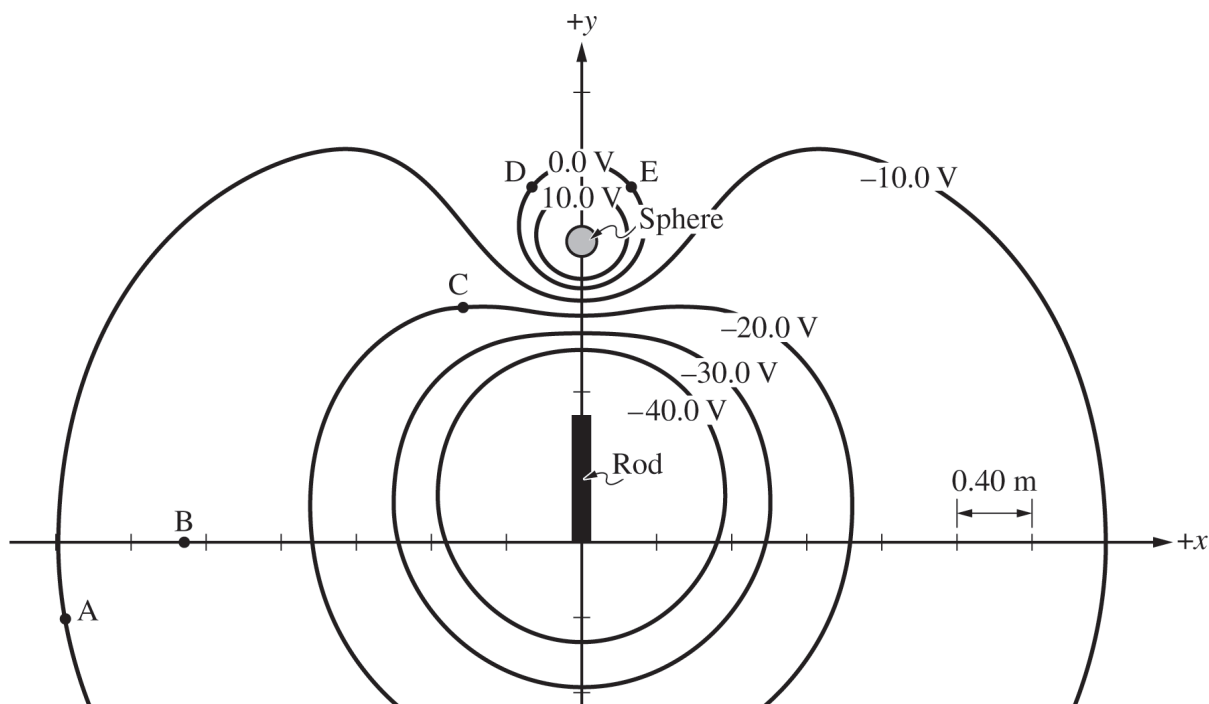


Figure 1

1. A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0 \text{ nC}$. The rod and sphere are held at rest on the y -axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x -axis and on the y -axis are 0.40 m apart.

(a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the 0.0 V equipotential line.

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A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of $C \rightarrow E \rightarrow D \rightarrow A$. The test charge is held momentarily at rest at each position.

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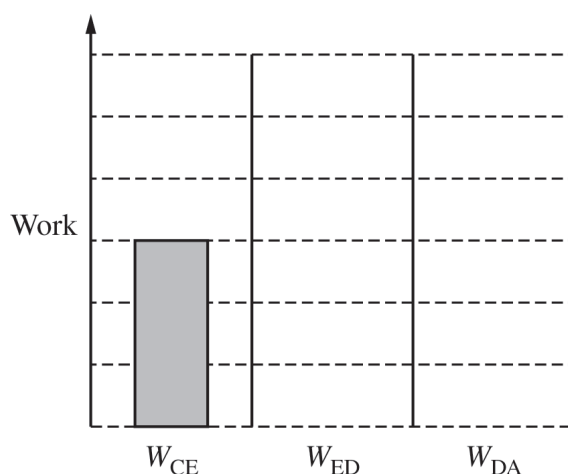


Figure 2

ii. **Calculate** the approximate magnitude of the x -component of the electric field at Position B.

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The positive test charge is placed at Position C. The test charge is then released from rest.

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_____ $+x$ _____ $+y$ _____ Directly away from the sphere

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Without using equations, **justify** your answer using physics principles.

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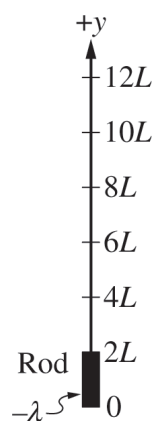
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Figure 3

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the y -axis in the orientation shown in Figure 3. Position P (not shown) is located on the y -axis a distance y_P from the origin, where $y_P > 2L$.

(d) The electric potential V_P at y_P is $V_P = -k\lambda \ln\left(\frac{y_P}{y_P - 2L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

GO ON TO THE NEXT PAGE.

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- ii. On Figure 4, **sketch** a graph of the y -component E_y of the electric field resulting from the rod as a function of y in the region $2L < y < 12L$.

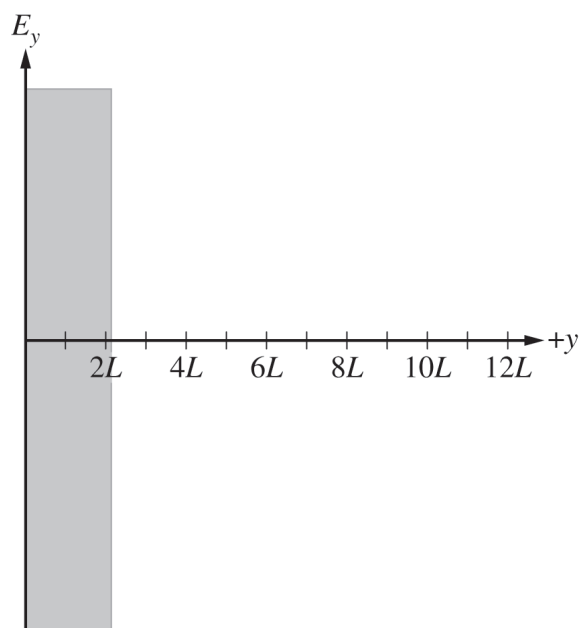
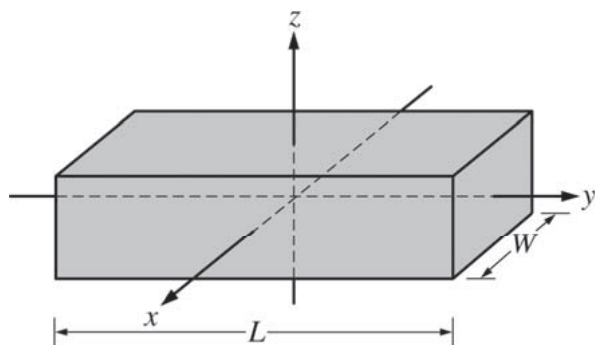


Figure 4

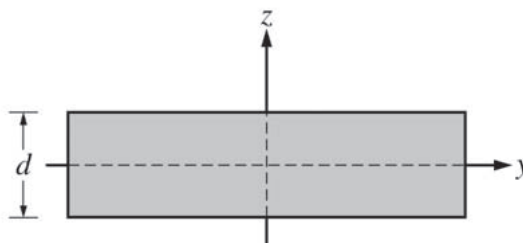
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Perspective View

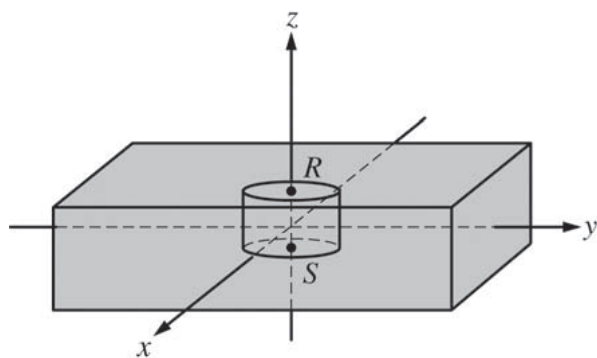


Side View

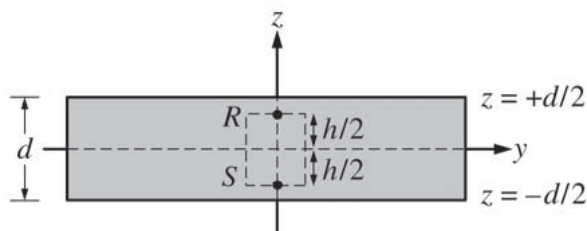
Note: Figures not drawn to scale.

1. A very large nonconducting slab with a uniform positive volume charge density ρ_0 is fixed with the origin of the xyz -axes at its center, as shown in the figure above. The thickness of the slab is d , the length is L , and the width is W , where $L \gg d$ and $W \gg d$. The large faces of the slab are parallel to the xy -plane.

Consider a Gaussian cylinder with a cross-sectional area A and height h that is positioned with its axis along the z -axis, as shown in the figure below.



Perspective View



Side View

Note: Figures not drawn to scale.

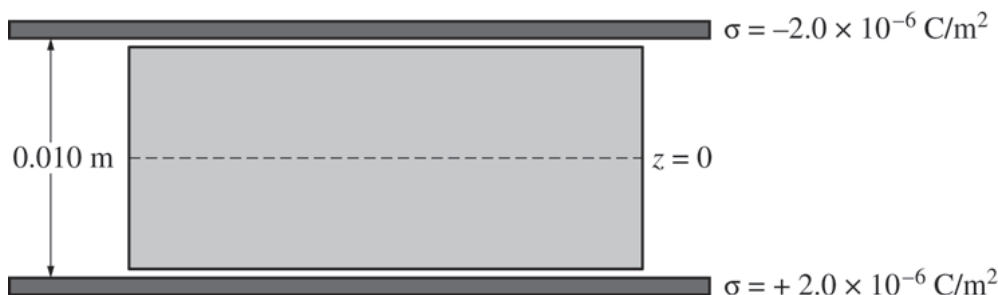
- (a) Draw a single vector on each of the dots below representing the direction of the electric field at the given points. If the electric field at either point is zero, write “ $E = 0$ ” next to the point.

i.

ii.

• R • S

- b) Use Gauss’s law to derive expressions for the following. Express your answers in terms of ρ_0 , A , d , h , z , and physical constants, as appropriate.
- Derive an expression for the total flux Φ through the Gaussian surface shown.
 - Derive an expression for the magnitude of the electric field as a function of z for any position inside the slab, and show that it is equal to $E = \frac{\rho_0 z}{\epsilon_0}$.



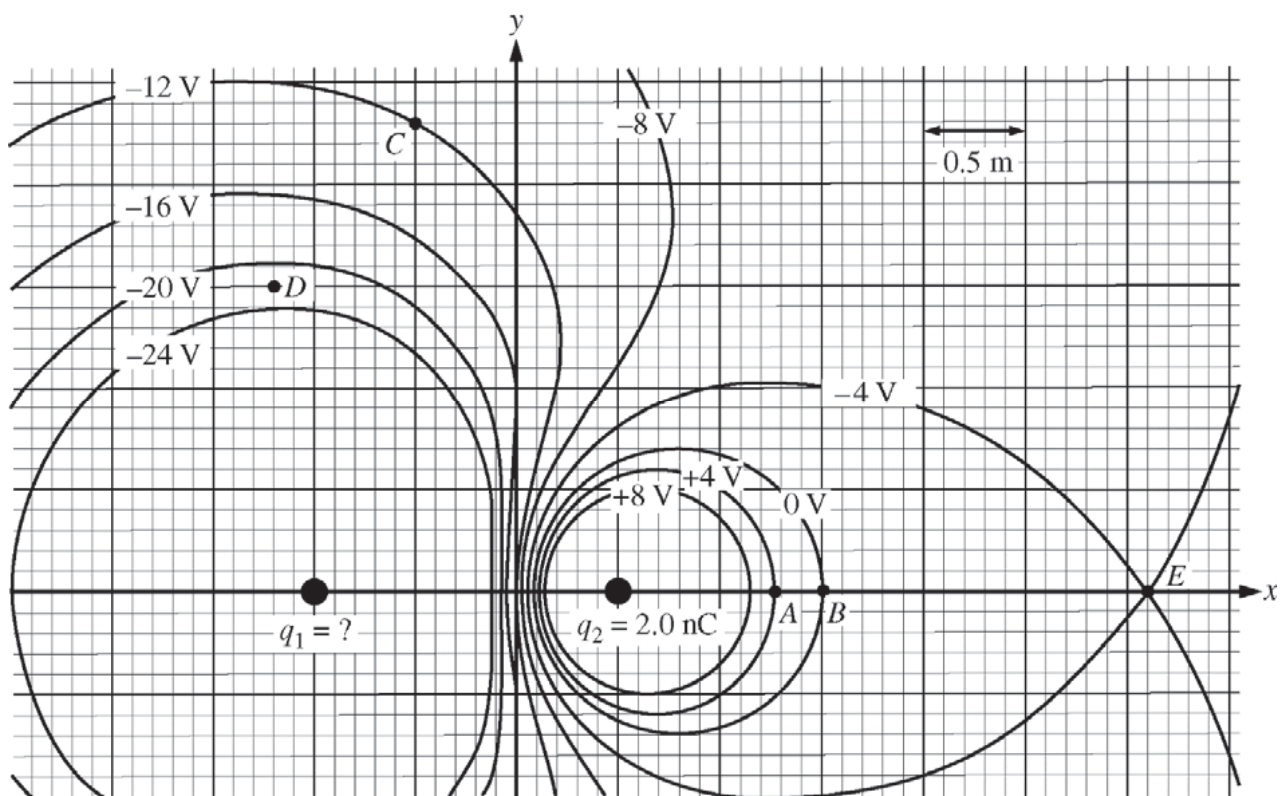
Note: Figure not drawn to scale.

The charged slab is now placed between two large metal plates separated by a distance of 0.010 m, which is approximately the thickness of the slab, but the slab does not contact either metal plate. The metal plates are charged, resulting in the surface charge densities $\sigma = \pm 2.0 \times 10^{-6} \text{ C/m}^2$, as shown in the figure above. Assume the charge distribution inside the slab remains unchanged by the presence of the charged plates and that the slab’s volume charge density is $\rho_0 = 1.00 \times 10^{-3} \text{ C/m}^3$.

- (c)
- The magnitude of the electric field inside the slab is zero on the z -axis at position z_0 . Which of the following correctly indicates the value for z_0 ?
 _____ $z_0 > 0$ _____ $z_0 = 0$ _____ $z_0 < 0$
 Justify your answer.
 - Calculate the value z_0 .
- (d) Calculate the magnitude of the electric potential difference from the center of the slab to the top of the slab.

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E&M.1.

Two point charges, q_1 and q_2 , are fixed in place on the x -axis at positions $x_1 = -1.00$ m and $x_2 = +0.50$ m, respectively. Charge q_2 has a value of $+2.0$ nC. Values of electric potential are illustrated by the given equipotentials in the diagram shown above, which is drawn to scale.

- Calculate the value of q_1 .
- At point C on the diagram, draw a vector representing the direction of the electric field at that point.
- Calculate the approximate magnitude of the electric field strength at point D on the diagram.
- The equipotential labeled 0 V is the cross section of a nearly spherical surface. Calculate the electric flux for this surface.

(e) A proton is placed at point A and then released from rest.

- i. Calculate the work done by the electric field on the proton as it moves from point A to point E.
- ii. Calculate the speed of the proton when it reaches point E.

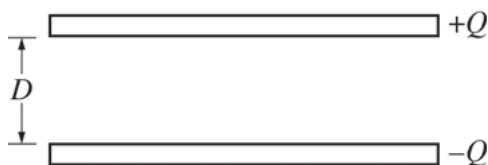
(f) An electron is released from rest at point B. Which of the following indicates the direction of the initial acceleration, if any, of the electron?

- | | |
|---|--|
| ☐ Up | ☐ Down |
| ☐ Left | ☐ Right |
| ☐ Into the page | ☐ Out of the page |
| ☐ The direction is undefined since the acceleration is zero. | |

Justify your answer.

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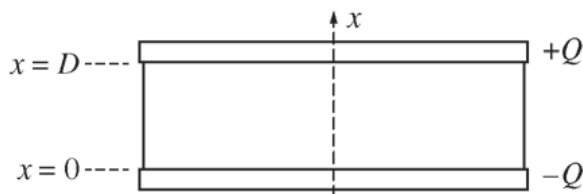


E&M.1.

A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D . The plates of the capacitor are each given a charge of magnitude Q , as shown in the figure above. Ignore edge effects.

(a)

- i. On the figure above, draw an arrow to indicate the direction of the electric field between the plates.
- ii. On the figure above, draw an appropriate Gaussian surface that will be used to derive an expression for the magnitude of the electric field E between the plates.
- iii. Using Gauss's law and the Gaussian surface from part (a)-ii, derive an expression for the magnitude of the electric field E between the plates. Express your answer in terms of A , D , Q , and physical constants, as appropriate.



The space between the plates is now filled with a dielectric material that is engineered so that its dielectric constant varies with the distance from the bottom plate to the top plate, defined by the x -axis indicated in the diagram above. As a result, the electric field between the plates is given by $\vec{E} = -\frac{Q}{\epsilon_0 \kappa_0 e^{-x/D} A} \hat{i}$, where κ_0 is a

positive constant. Express all algebraic answers to the remaining parts in terms of A , D , Q , κ_0 , x , and physical constants, as appropriate.

(b) Determine an expression for the dielectric constant κ as a function of x .

(c)

- i. Write, but do NOT solve, an equation that could be used to determine the potential difference V between the plates of the capacitor.
- ii. Using the equation from part (c)-i, derive an expression for the potential difference $V_D - V_0$, where V_D is the potential of the top plate and V_0 is the potential of the bottom plate.

(d) Determine the capacitance of the capacitor.

- (e) The energy stored in the capacitor that has a varying dielectric is U_V . A second capacitor that has a constant dielectric of value κ_0 is also given a charge Q . The energy stored in the second capacitor is U_C . How do the values of U_V and U_C compare?

_____ $U_V < U_C$ _____ $U_V > U_C$ _____ $U_V = U_C$

Justify your answer.

E&M.3.

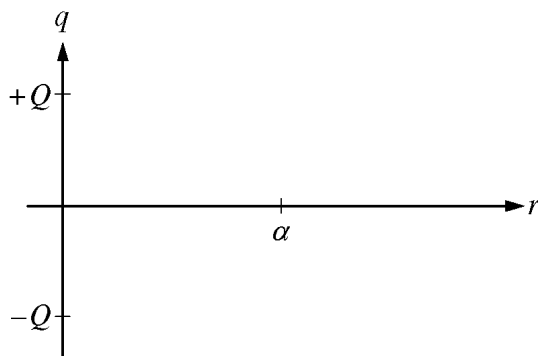
A scientist describes an electrically neutral atom with a model that consists of a nucleus that is a point particle with positive charge $+Q$ at the center of the atom and an electron volume charge density of the form

$$\rho(r) = \begin{cases} -\frac{\beta}{r^2} e^{-r/\alpha} & r < \alpha \\ 0 & r > \alpha \end{cases}$$

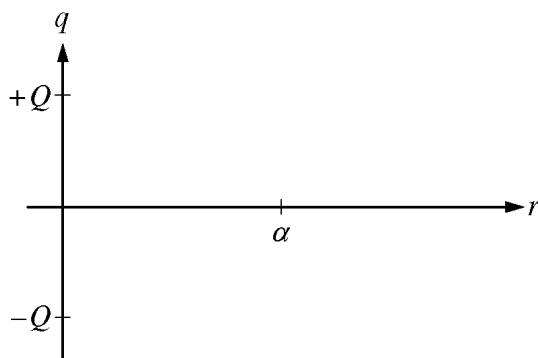
where α and β are positive constants and r is the distance from the center of the atom.

- (a) On the axes below, let r stand for the radius of a Gaussian sphere. Sketch the graph for each of the following charges enclosed by the Gaussian sphere as a function of r . Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

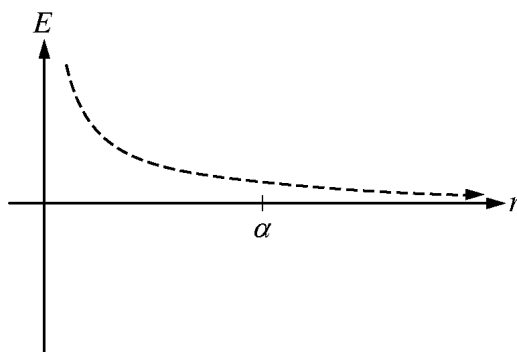
- i. The nuclear charge only



- ii. The electron charge only



- (b) The dashed curve on the graph below represents the electric field as a function of distance r due to the positive nucleus of the atom without any electrons. The nucleus is modeled as a point particle of charge $+Q$. On the same graph, sketch the electric field as a function of distance r for the neutral atom as defined by the scientist's model, which includes the nucleus and the negative electrons surrounding it.



- (c) Use Gauss's law to derive an expression for the electric field strength due to the neutral atom for the following positions in terms of Q , α , β , r , and fundamental constants, as appropriate.
- $r > \alpha$
 - $r < \alpha$
- (d) Based on the model proposed by the scientist, what is the physical meaning of the constant α ?

STOP
END OF EXAM