Question 1: Free-Response Question — 15 points

(a) For using a correct equation for electric flux — **1 point**

Example Response

$$\Phi_E = \frac{Q}{\varepsilon_0}$$

For the correct numerical answer — **1 point**

Scoring Note: Correct units are not required to earn this point.

Example Response

$$\Phi_E = 113 \frac{\mathrm{N \cdot m^2}}{\mathrm{C}}$$

Example Solution

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$\Phi_E = \frac{Q}{\varepsilon_0}$$

$$\Phi_E = \frac{1.0 \times 10^{-9}\ \mathrm{C}}{8.85 \times 10^{-12} \frac{\mathrm{C^2}}{\mathrm{N \cdot m^2}}}$$

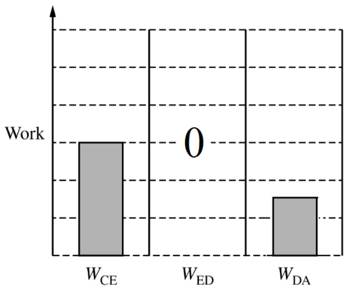
$$\Phi_E = 113 \frac{\mathrm{N \cdot m^2}}{\mathrm{C}}$$

Total for part (a) — 2 points

(b)(i) For indicating that $W_{\mathrm{ED}} = 0$ — **1 point**

For drawing a bar representing W_{DA} that has a height of approximately one and a half units — **1 point**

Example Response



(b)(ii) For using an equation that relates the electric field to potential difference — **1 point**

Scoring Note: This point can be earned if the response begins with a correct relationship between electric field and potential difference in which numerical values are already substituted.

Example Responses

$$E_y = -\frac{dV}{dy} \quad \textbf{OR} \quad \left|E_y\right| = \left|-\frac{dV}{dy}\right| \quad \textbf{OR} \quad \left|E_y\right| = \left|-\frac{\Delta V}{\Delta y}\right| \quad \textbf{OR} \quad \Delta V = -\int \vec{E} \cdot d\vec{r}$$

For correct substitutions of values of electric potential and the distance between equipotential lines that can be used to calculate the approximate magnitude of the electric field at Position B — **1 point**

Example Response

$$\left|E_y\right| = \left|-\frac{-20.0\ \mathrm{V} - (-10.0\ \mathrm{V})}{1.4\ \mathrm{m}}\right|$$

Example Solution

$$E_y = -\frac{dV}{dy}$$

$$\left|E_y\right| = \left|-\frac{dV}{dy}\right|$$

$$\left|E_y\right| = \left|-\frac{\Delta V}{\Delta y}\right|$$

$$\left|E_y\right| = \left|-\frac{-20.0\ \mathrm{V} - (-10.0\ \mathrm{V})}{1.4\ \mathrm{m}}\right|$$

$$\left|E_y\right| = 7.1 \frac{\mathrm{V}}{\mathrm{m}}$$

Total for part (b) — 4 points

(c) For selecting only $-y$ with an attempt at a relevant justification — **1 point**

For indicating that the direction of the electric field vector is perpendicular to a line that is tangent to the equipotential line at Position C — **1 point**

For indicating one of the following: — **1 point**

- The test charge moves from a higher electric potential to a lower electric potential.
- The test charge and the rod have charges of the opposite sign.
- The test charge and the sphere have charges of the same sign.

Example Response

$-y$. The direction of an electric field vector is perpendicular to an equipotential line. Because the test charge has a positive charge, the test charge would move from a position of higher electric potential to a position of lower electric potential when an electric force is exerted on the test charge. Therefore, at Position C, the electric force is downward because that is the direction that is perpendicular to the equipotential line and in the direction of decreasing electric potential.

Total for part (c) — 3 points

(d)(i)	For using an appropriate equation for determining the electric potential from a line of uniform charge	**1 point**

Example Responses

$$V = \frac{1}{4\pi\varepsilon_0}\sum\frac{q_i}{r_i} \quad \textbf{OR} \quad V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} \quad \textbf{OR} \quad \Delta V = -\int\vec{E}\cdot d\vec{r}$$

For a correct determination of r, the distance between Point P and a point on the line of uniform charge	**1 point**

Example Responses

$$V_P = k\sum\frac{Q}{y_P - y} \quad \textbf{OR} \quad V_P = k\int\left(\frac{1}{y_P - y}\right)dq$$

For a correct integral with $\lambda\, dy$ substituted for dq	**1 point**

Example Response

$$V_P = -k\lambda\int\left(\frac{1}{y_P - y}\right)dy$$

For the correct limits of integration	**1 point**

Example Response

$$V_P = -k\lambda\int_0^{2L}\left(\frac{1}{y_P - y}\right)dy$$

AP Physics C: **Electricity and Magnetism: 2024 Set 2**

Example Solutions

$$V = \frac{1}{4\pi\varepsilon_0}\sum\frac{q_i}{r_i}$$

$$V_P = k\int\left(\frac{1}{y_P - y}\right)dq$$

$$dq = -\lambda\, dy$$

$$V_P = -k\lambda\int\left(\frac{1}{y_P - y}\right)dy$$

$$V_P = -k\lambda\int_0^{2L}\left(\frac{1}{y_P - y}\right)dy$$

$$V_P = k\lambda\ln(y_P - y)\Big|_0^{2L} = -k\lambda\ln(y_P - y)\Big|_{2L}^{0}$$

$$V_P = -k\lambda\ln\left(\frac{y_P}{y_P - 2L}\right)$$

OR

$$\Delta V = -\int\vec{E}\cdot d\vec{r}$$

$$E(y) = \int\frac{k}{r^2}dq$$

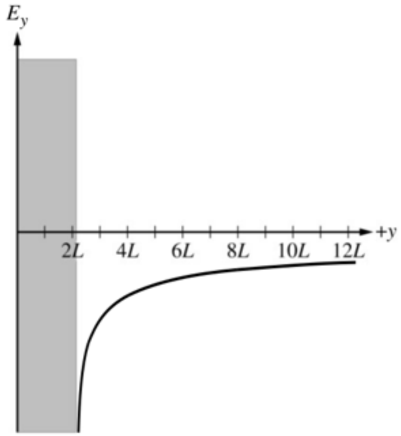
$$E(y) = \int_0^{2L}\frac{k\lambda}{(y - y')^2}dy' = k\lambda\left(\frac{1}{y - 2L} - \frac{1}{y}\right)$$

$$V_P = -\int_\infty^{y_P}E(y)\,dy = -k\lambda\int_\infty^{y_P}\left(\frac{1}{y - 2L} - \frac{1}{y}\right)dy$$

$$V_P = k\lambda\ln\left(\frac{y_P}{y_P - 2L}\right)$$

(d)(ii)	For sketching a curve or line that continually approaches the horizontal axis as position increases	**1 point**
	For sketching a concave down curve that is always negative	**1 point**

Example Response



Total for part (d) **6 points**

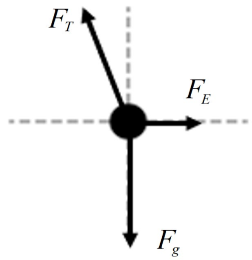
Total for question 1 **15 points**

Question 1: Free-Response Question — 15 points

(a)	For correctly drawing and labeling the electrostatic force directed to the right	**1 point**
	For drawing the force of tension up and to the left and the gravitational force in the downward direction	**1 point**

Scoring Note: A maximum of 1 point may be earned if extraneous forces are included.

Example Response



Total for part (a) **2 points**

(b)	For equating the horizontal component of tension to the electrostatic force	**1 point**

Example Response

$$F_E = F_T \sin(\theta)$$

For equating the vertical component of tension to the gravitational force	**1 point**

Example Response

$$F_g = F_{Ty}$$
$$Mg = F_T \cos\theta$$

For an attempt to simultaneously solve the equations	**1 point**

Example Response

$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}\frac{1}{\sin\theta}\cos\theta = Mg$$

Example Solution

$$\Sigma F_y = 0$$
$$F_{Ty} = F_g$$
$$F_T \cos\theta = Mg$$
$$F_{Tx} = F_E$$
$$F_T = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}\frac{1}{\sin\theta}$$
$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}\frac{1}{\sin\theta}\cos\theta = Mg$$
$$d^2 = \frac{Qq\cos\theta}{4\pi\varepsilon_0 Mg\sin\theta}$$
$$d = \sqrt{\frac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}}$$

Total for part (b) 3 points

(c)	For applying Coulomb's law to determine tension	**1 point**

Scoring Note: This point may be earned if the student used the vertical component of tension.

Example Response

$$F_E = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2} = F_T \sin(\theta)$$

For correct substitution into an expression for tension consistent with part (b) or a correct expression for tension	**1 point**

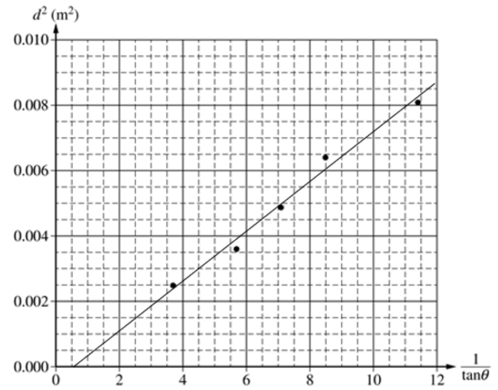
Example Solution

$$\Sigma F = 0$$
$$F_E - F_{Tx} = 0$$
$$F_E = F_{Tx}$$
$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2} = F_T \sin(\theta)$$
$$F_T = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2\sin(\theta)}$$
$$F_T = \frac{1}{4\pi\left(8.85\times10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N\bullet m}^2}\right)}\frac{\left(6.0\times10^{-8}\ \mathrm{C}\right)^2}{(0.057\ \mathrm{m})^2\sin\left(12^\circ\right)}$$
$$F_T = 0.048\ \mathrm{N}$$

Total for part (c) 2 points

(d)(i)	For a line that approximates the trend of the data	**1 point**

Example Response



(d)(ii) For using two points from the trend line drawn by the student to calculate the slope **1 point**

Scoring Note: Points of data may be used only if points of data are located directly on the line.

Example Response

$$\text{Slope} = \frac{\Delta y}{\Delta x}$$

$$\text{Slope} = \frac{\Delta(d^2)}{\Delta\left(\frac{1}{\tan(\theta)}\right)}$$

$$\text{Slope} = \frac{\left(0.0075\ \text{m}^2 - 0.001\ \text{m}^2\right)}{(10.5 - 2)}$$

$$\text{Slope} = 7.647 \times 10^{-4}\ \text{m}^2$$

For correctly relating the slope of the graph to the equation $d = \sqrt{\dfrac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}}$ **1 point**

Example Response

$$d = \sqrt{\frac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}}$$

$$d^2 = \frac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}$$

$$d^2 = \left(\frac{Qq}{4\pi\varepsilon_0 Mg}\right)\frac{1}{\tan\theta}$$

$$\text{slope} = \left(\frac{Qq}{4\pi\varepsilon_0 Mg}\right)$$

For substituting the value of the slope of the graph into the equation $\varepsilon_0 = \dfrac{Qq}{4\pi Mg(\text{slope})}$ to calculate an experimental value of ε_0 **1 point**

Example Solution

$$\text{slope} = \frac{Qq}{4\pi\varepsilon_0 Mg}$$

$$\varepsilon_0 = \frac{Qq}{4\pi Mg(\text{slope})}$$

$$\varepsilon_0 = \frac{\left(6.0 \times 10^{-8}\ \text{C}\right)^2}{4\pi(0.005\ \text{kg})\left(9.8\ \text{m/s}^2\right)\left(7.647 \times 10^{-4}\ \text{m}^2\right)}$$

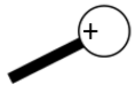
$$\varepsilon_0 = 7.6 \times 10^{-12}\ \frac{\text{C}^2}{\text{N}{\bullet}\text{m}^2}$$

Total for part (d) 4 points

(e)(i) For a "+" drawn on the left side of the sphere **1 point**

Example Response

Sphere 3



(e)(ii) For a statement that indicates correct charge rearrangement on Sphere 3 due to the electric forces from the charges on Sphere 2 **1 point**

Example Response

The negative charges on Sphere 3 move to the right due to the attractive forces from the positive charges on Sphere 2, leaving a net positive charge on the left side of Sphere 3.

(e)(iii) For selecting "$\theta_2 < \theta_1$" with an attempt at a relevant justification **1 point**

For statement that indicates **one** of the following: **1 point**

- The average distance between the repulsive charges is greater.
- The electrostatic or repulsive force is less.

Scoring Note: Points 1 and 2 of part (e)(iii) can be earned with an answer that is consistent with the location of the excess positive charges drawn in part (e)(i).

Example Response

Excess charges on Sphere 3 are now free to move, so excess like charges will be concentrated on the far ends of Sphere 3 when the spheres are in static equilibrium. The excess like charges, located on opposite sides of Sphere 3, repel with less force than if the excess charges were located at the centers of Sphere 3. Thus, the downward force due to gravity on Sphere 2 causes the center of Sphere 2 to hang closer to the center of Sphere 3.

Total for part (e) 4 points

Total for question 1 15 points

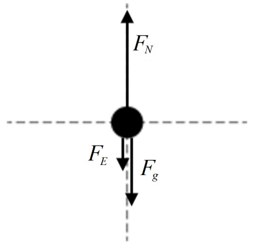
Question 1: Free-Response Question — 15 points

(a)	For correctly drawing and labeling the electrostatic force in the downward direction	**1 point**
	For drawing the normal force in the upward direction and a gravitational force in the downward direction	**1 point**

Scoring Note: F_S is an acceptable label to represent the force that the platform exerts on the sphere

Scoring Note: A maximum of 1 point may be earned if extraneous forces are included.

Example Response



Total for part (a) — **2 points**

(b)	For applying an equation of the forces at equilibrium consistent with the diagram drawn in part (a)	**1 point**

Example Response

$$F_N - F_E - F_g = 0$$

For substituting a $k_s x$ or $k_s y$ that represents the normal force — **1 point**

Example Response

$$k_s x - F_E - F_g = 0$$

For substituting a correct expression that represents the electrostatic force — **1 point**

Example Response

$$F_E = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{H^2}$$

For substituting a correct expression that represents the gravitational force — **1 point**

Example Response

$$F_g = Mg$$

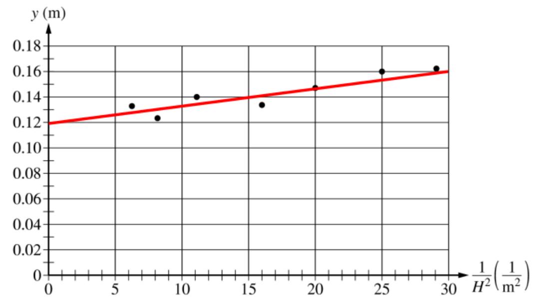
Example Solution

$$F_N - F_E - F_g = 0$$
$$F_N = F_E + F_g$$
$$k_s y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{H^2} + Mg$$
$$y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{k_s H^2} + \frac{Mg}{k_s}$$

Total for part (b) — **4 points**

(c)(i)	For a line that approximates the trend of the data	**1 point**

Example Response



(c)(ii)	For using two points from the trend line drawn by the student to calculate the slope	**1 point**

Scoring Note: Points of data may be used only if points of data are located directly on the line.

Example Response

$$\text{Slope} = \frac{\Delta y}{\Delta x}$$

$$\text{Slope} = \frac{\Delta(y)}{\Delta\left(\frac{1}{H^2}\right)}$$

$$\text{Slope} = \frac{(0.15\text{ m} - 0.12\text{ m})}{\left(20\ \frac{1}{\text{m}^2} - 0\ \frac{1}{\text{m}^2}\right)}$$

$$\text{Slope} = 0.0015\text{ m}^3$$

For indicating the relationship between the slope of the graph to ε_0 in the equation **1 point**

$$y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{k_s H^2} + \frac{Mg}{k_s}$$

Example Response

$$y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{k_s H^2} + \frac{Mg}{k_s}$$

$$\text{slope} = \left(\frac{Qq}{4\pi\varepsilon_0 k_s}\right)$$

For correctly substituting the value of the slope of the graph and other given quantities into the equation to calculate an experimental value of ε_0 **1 point**

Scoring Note: A numerical response without units may earn this point.

Example Solution

$$\text{slope} = \frac{Qq}{4\pi\varepsilon_0 k_s}$$

$$\varepsilon_0 = \frac{Qq}{4\pi k_s(\text{slope})}$$

$$\varepsilon_0 = \frac{\left(2\times10^{-6}\text{ C}\right)^2}{4\pi\left(25\frac{\text{N}}{\text{m}}\right)\left(0.0015\text{ m}^3\right)}$$

$$\varepsilon_0 = 8.5\times10^{-12}\ \frac{\text{C}^2}{\text{N}\bullet\text{m}^2}$$

AP Physics C: **Electricity and Magnetism: 2023 Set 2**

(c)(iii)	For identifying that the y-intercept $= \dfrac{Mg}{k_s}$	**1 point**
	For correctly substituting the value of the y-intercept of the graph and other given quantities and constants into the equation to calculate an experimental value of M	**1 point**

Scoring Note: A numerical response without units may earn this point.

Example Response

$$y\text{-intercept} = 0.12\text{ m}$$

$$y\text{-intercept} = \frac{Mg}{k_s}$$

$$M = \frac{(0.12\text{ m})(25\text{ N/m})}{9.8\text{ m/s}^2}$$

$$M = 0.30\text{ kg}$$

Total for part (c) 6 points

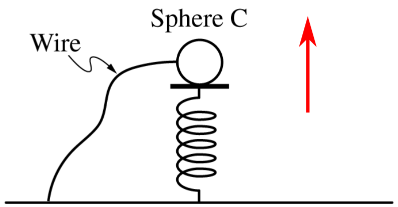
(d)(i)	For selecting "$y_2 < y_1$" with an attempt at a relevant justification	**1 point**
	For a justification that indicates correct charge rearrangement in Sphere C due to electrostatic force from Sphere A	**1 point**

Example Response

$y_2 < y_1$, *Charges are now free to move within Sphere C now that it is a conducting sphere. Electrons on Sphere C will be attracted to the excess positive charges on Sphere A and move closer to Sphere A. This leaves the opposite side of Sphere C more positively charged which results in less electric repulsion between the spheres and as a result less compression in the spring.*

(d)(ii)	For drawing an upward pointing arrow	**1 point**

Example Response



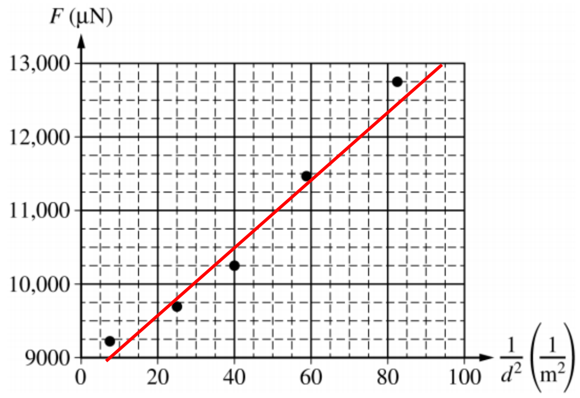
Total for part (d) 3 points

Total for question 1 15 points

Question 2: Free-Response Question — 15 points

(a) **i.** For drawing an appropriate best-fit line — **1 point**

Example response for part (a)(i)



ii. For calculating the slope using two points from the best-fit line — **1 point**

$$slope = m = \frac{\Delta y}{\Delta x} = \frac{(12{,}000 - 10{,}500)\ \mu\text{N}}{(72 - 40)\ \text{m}^{-2}} = 4.7 \times 10^{-5}\ \text{N}\bullet\text{m}^2$$

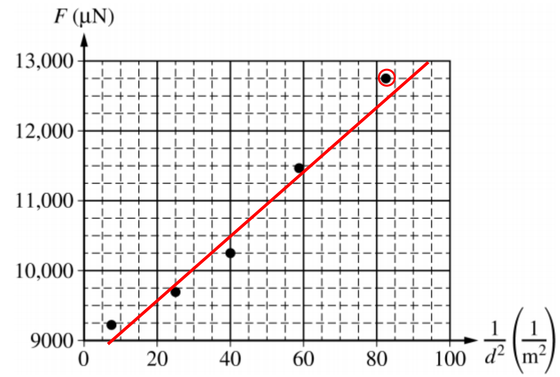
For correctly relating the slope to the charge — **1 point**

$$F = \frac{kQ^2}{r^2}\ \therefore\ \text{slope} = kQ^2$$

For correctly calculating the charge consistent with the slope from the best-fit line — **1 point**

$$Q = \sqrt{\frac{\text{slope}}{k}} = \sqrt{\frac{\left(4.7 \times 10^{-5}\ \text{N}\bullet\text{m}^2\right)}{\left(9 \times 10^{9}\ \text{N}\bullet\text{m}^2/\text{C}^2\right)}} = 7.2 \times 10^{-8}\ \text{C}$$

(a) **iii.** For drawing a circle around the appropriate data point — **1 point**



iv. For a correct answer — **1 point**

From the graph, $\frac{1}{d^2} = 82.5\ \text{m}^{-2}$

$d = 0.11\ \text{m}$

v. For indicating that the y-intercept should be the weight of the conducting sphere and insulating rod — **1 point**

For a correct justification — **1 point**

Example responses for part (a)(v).

When the spheres are infinitely far apart, there is no electric force between them. The only force recorded by the balance must be the weight of the lower sphere and insulating rod.

OR

The scale is zeroed before the insulating rod, and lower sphere are placed on it, so the nonzero y-intercept is their weight.

Total for part (a) — **8 points**

(b) For selecting "Yes" and providing a correct justification — **1 point**

Example response for part (b)

Because the later force measurements are lower than the expected values on the best-fit line, then a decrease in charge could explain the discrepancy seen in the data.

(c)	**i.**	For correctly indicating the position of the excess positive charges	**1 point**
		For correctly indicating that the net charge is positive	**1 point**
		Example response for part (c)(i) 	
	ii.	For a correct justification	**1 point**
		Example response for part (c)(ii) *On a conductor, excess charges are free to move and will repel each other. The excess charges in the sphere will rearrange themselves until they are as far apart from each other as possible; thus, the repelling force decreases.*	
		Total for part (c)	**3 points**
(d)	**i.**	For appropriately indicating the position of the excess charges on both spheres 	**1 point**
	ii.	For selecting “Less than” and attempting a relevant justification	**1 point**
		For a correct justification	**1 point**
		Example response for part (d) *Because the opposite charged spheres will attract each other, this will decrease the upward normal force exerted by the scale on the lower sphere; thus, the reading on the scale will be less than when the spheres had the same charge.*	
		Total for part (d)	**3 points**
		Total for question 2	**15 points**

2019

AP® Physics C: Electricity and Magnetism

Scoring Guidelines Set 1

General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

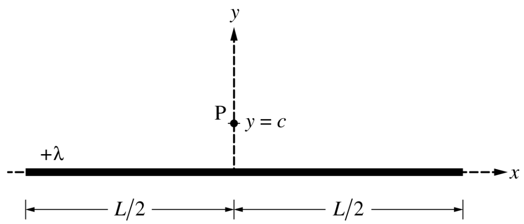
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 1

15 points

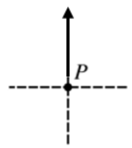


Note: Figure not drawn to scale.

A very long, thin, nonconducting cylinder of length L is centered on the y-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y-axis at $y = c$, where $L >> c$.

(a)
i. LO CNV-3.B.a, SP 7.A
1 point

On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the long cylinder. The arrow should start on and point away from the dot.



For drawing an arrow at point P that points upward		1 point

ii. LO CNV-2.F, SP 7.C
1 point

Describe the shape and location of a Gaussian surface that can be used to determine the electric field at point P due to the long cylinder.

For describing a Gaussian surface that could be used to determine the electric field at point P		1 point
Example: Drawing a cylinder that is coaxial with the thin cylinder and whose surface contains point P can be used to determine the electric field at point P.		
Note: Credit is earned if the student draws the correct surface on the figure.		

Question 1 (continued)

(a) continued

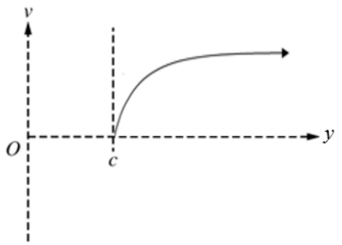
iii. LO CNV-2.C, SP 5.A, 5.E
3 points

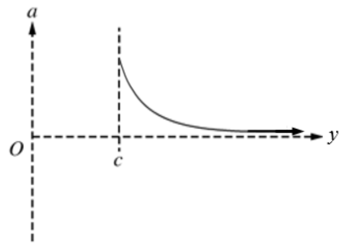
Use your Gaussian surface to derive an expression for the magnitude of the electric field at point P. Express your answer in terms of λ, c, L, and physical constants, as appropriate.

For using Gauss's law to determine the electric field at point *P*		1 point
$\frac{q_{enc}}{\varepsilon_0} = \int E \bullet dA \therefore \frac{Q}{\varepsilon_0} = EA$		
For correctly substituting for the charge into the equation above		1 point
For correctly substituting for the area or into the equation above		1 point
$\frac{\lambda L}{\varepsilon_0} = E(2\pi cL) \therefore E = \frac{\lambda}{2\pi\varepsilon_0 c} = \frac{2k\lambda}{c}$		

(b) LO ACT-1.D, SP 3.C
2 points

A proton is released from rest at point P. On the axes below, sketch the velocity v as a function of <u>position</u> y and the acceleration a as a function of <u>position</u> y for the proton.

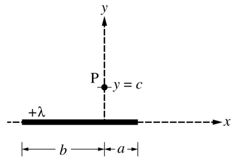




For a concave down graph for v as a function of position x that does not start at the origin		1 point
For a concave up graph for a as a function of position x that has an asymptote at the horizontal axis		1 point

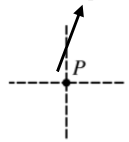
Question 1 (continued)

The original cylinder is now replaced with a much shorter thin, nonconducting cylinder with the same uniform linear charge density $+\lambda$, as shown in the figure below. The length of the cylinder to the right of the y-axis is a, and the length of the cylinder to the left of the y-axis is b, where $a < b$.



(c) LO CNV-3.B.a, SP 7.A
2 points

On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the shorter cylinder. The arrow should start on and point away from the dot.



For drawing an arrow at point *P* that points to the right		1 point
For drawing an arrow at point *P* that points up and to the right		1 point

(d)

i. and ii. LO CNV-2.F, SP 7.A
1 point

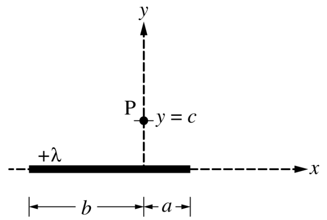
Is there a single Gaussian surface that can be used with Gauss's law to derive an expression for the electric field at point P?

____ Yes ____ No

If your answer to part (d)(i) is yes, explain how you can use Gauss's law to derive an expression for the field at point P. If your answer to part (d)(i) is no, explain why Gauss's law cannot be applied to derive an expression for the electric field in this case.

For selecting "No" with a valid explanation		1 point
Claim: Select "No." Evidence: The length of the cylinder is not much greater than the distance from the cylinder to point P and the charge distribution is asymmetric. Reasoning: Therefore, cannot use the approximation of the constant magnitude of electric field over a cylindrical surface.		

Question 1 (continued)



Note: This figure is shown again for reference.

A student in class argues that using the integral shown below might be a useful approach for determining the electric field at point P.

$$E = \int \frac{1}{4\pi\varepsilon_0} \frac{1}{r^2}\, dq$$

The student uses this approach and writes the following two integrals for the magnitude of the horizontal and vertical components of the electric field at point P.

$$\text{Horizontal component: } \left|E_x\right| = \frac{\lambda}{4\pi\varepsilon_0} \int_{-b}^{a} \frac{x}{\left(c^2 + x^2\right)^{3/2}}\, dx$$

$$\text{Vertical component: } \left|E_y\right| = \frac{\lambda}{4\pi\varepsilon_0} \int_{-b}^{a} \frac{y}{\left(c^2 + x^2\right)}\, dy$$

(e)

i. LO CNV-3.A, SP 7.A
1 point

One of the two expressions above is not correct. Which expression is not correct?

____ Horizontal component ____ Vertical component

For correctly selecting "Vertical component"		1 point

Question 1 (continued)

(e) continued

ii. LO CNV-3.A, SP 7.D
4 points

Identify two mistakes in the incorrect expression, and explain how to correct the mistakes.

For indicating the integral is not along the length of the cylinder		1 point
For an appropriate correction		1 point
Claim: Change *dy* to *dx*. Evidence: The integral is not along the cylinder. Reasoning: This change will make the integral valid.		
For indicating the power on the denominator term for the vertical component is incorrect		1 point
For an appropriate correction		1 point
Claim: The power on the term in the denominator should be $3/2$. Evidence: The units of the integrand are not valid. Reasoning: This change will make the integrand valid.		

Learning Objectives

ACT-1.D: Determine the motion of a charged object of specified charge and mass under the influence of an electrostatic force.
CNV-2.C: State and use Gauss's law in integral form to derive unknown electric fields for planar, spherical, or cylindrically symmetrical charge distributions.
CNV-2.F: Describe the general features of an unknown charge distribution, given other features of the system.
CNV-3.A: Derive expressions for the electric field of specified charge distributions using integration and the principle of superposition. Examples of such charge distributions include a uniformly charged wire, a thin ring of charge (along the axis of the ring), and a semicircular or part of a semicircular arc.
CNV-3.B.a: Identify and qualitatively describe situations in which the direction and magnitude of the electric field can be deduced from symmetry considerations and understanding the general behavior of certain charge distributions.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.
7.D: Provide reasoning to justify a claim using physical principles or laws.

Question 2

15 points

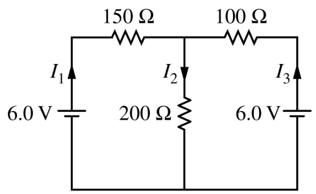


Figure 1

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(a)

i. LO CNV-6.F.b, SP 6.A
3 points

Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

For an equation representing the sum of the currents at one of the junctions		1 point
$I_1 - I_2 + I_3 = 0$		
For an equation representing the sum of the potential differences around one of the loops		1 point
For an equation representing the sum of the potential differences around a loop different from the above loop		1 point
$6 - 150I_1 - 200I_2 = 0$		
$6 - 100I_3 - 200I_2 = 0$		
$6 - 150I_1 + 100I_3 - 6 = 0$		
Note: Full credit is earned for two correct loop equations using loop currents.		

ii. LO CNV-6.F.b, SP 6.C
2 points

Calculate the current in the $200\ \Omega$ resistor.

For combining the equations from part (a)(i)		1 point
$I_1 - I_2 + I_3 = 0$ $-I_1 - 1.33I_2 = -.04 \therefore -4.33I_2 = -0.10$ $-I_3 - 2I_2 = -.06$		
Note: Credit is earned if students indicate they used a calculator function to solve the system of equations.		
For a correct answer with correct units		1 point
$-4.33I_2 = -0.10 \therefore I_2 = 0.023\text{ A}$		

Question 2 (continued)

(a) continued

iii. LO CNV-5.A.a, SP 6.C
1 point

Calculate the power dissipated by the $200\ \Omega$ resistor.

For using a correct equation to calculate the power in the $200\ \Omega$ resistor		1 point
$P = I^2R = (0.023\text{ A})^2(200\ \Omega) \therefore P = 0.107\text{ W}$		

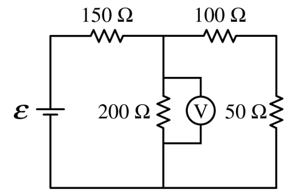


Figure 2

The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

(b) LO CNV-6.C.a, SP 6.B, 6.C
2 points

Calculate the current through the $50\ \Omega$ resistor.

For correctly calculating the equivalent resistance of the branch with the $50\ \Omega$ resistor		1 point
$R = 100\ \Omega + 50\ \Omega = 150\ \Omega$		
For using the correct potential difference in Ohm's law to calculate the current in the $50\ \Omega$ resistor		1 point
$I = \frac{V}{R} = \frac{(4.4\text{ V})}{(100\ \Omega + 50\ \Omega)} \therefore I = 0.029\text{ A}$		

Question 2 (continued)

(c) LO CNV-6.C.a, SP 6.A, 6.C
3 points

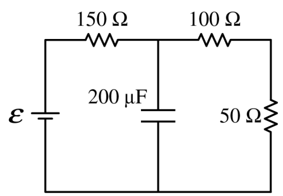
Calculate the voltage $\mathcal{E}$ of the battery.

For using a correct equation to determine the current through the $150\ \Omega$ resistor		1 point
For correctly substituting the current from part (b) as I_3		1 point
$I_1 = I_2 + I_3 = \dfrac{(4.4\text{ V})}{(200\ \Omega)} + 0.029\text{ A} = 0.051\text{ A}$		
For using a correct equation to determine the emf of the battery		1 point
$\mathcal{E} = I_1 R_1 + V_2 = (0.051\text{ A})(150\ \Omega) + 4.4\text{ V} = 12.1\text{ V}$		
Alternate Third Point		
For calculating the equivalent resistance of the circuit and substituting this resistance into a correct equation to determine the emf of the battery		*1 point*
$R_T = 150\ \Omega + \dfrac{1}{\dfrac{1}{200\ \Omega} + \dfrac{1}{150\ \Omega}} = 236\ \Omega$		
$\mathcal{E} = I_1 R_T = (0.051\text{ A})(236\ \Omega) = 12.0\text{ V}$		
Alternate Solution		*Alternate Points*
For using a correct equation to determine the equivalent resistance of the parallel resistors		*1 point*
$\dfrac{1}{R_P} = \dfrac{1}{200\ \Omega} + \dfrac{1}{150\ \Omega}\ \therefore R_P = 86\ \Omega$		
For correctly substituting the given potential difference and the calculated equivalent resistance to determine the total current of the circuit		*1 point*
$I_T = \dfrac{(4.4\text{ V})}{(86\ \Omega)} = 0.051\text{ A}$		
For calculating the equivalent resistance of the circuit and substituting into a correct equation to determine the emf of the battery		*1 point*
$R_T = 150\ \Omega + 86\ \Omega = 236\ \Omega$		
$\mathcal{E} = I_1 R_T = (0.051\text{ A})(236\ \Omega) = 12.0\text{ V}$		

Question 2 (continued)

(d)
i. LO CNV-7.B.a, SP 6.A, 6.C
2 points

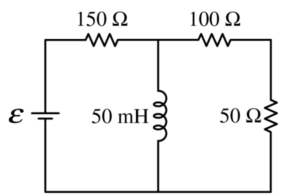
The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu\text{F}$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.



For substituting the voltage consistent with part (c) into Ohm's law		1 point
For correctly calculating the equivalent resistance of the circuit		1 point
$I = \dfrac{\mathcal{E}}{R_{tot}} = \dfrac{(12.1\text{ V})}{(150\ \Omega + 100\ \Omega + 50\ \Omega)} = 40.3\text{ mA}$		

ii. LO CNV-10.C.a, SP 7.A, 7.C
2 points

The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b)?



____ Greater than ____ Less than ____ Equal to

Justify your answer.

For correctly selecting "Less than" with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because steady state is reached, the inductor will act as a short circuit. So all the current will pass through the inductor and no current will pass through the $50\ \Omega$ resistor.		

Question 2 (continued)

Learning Objectives

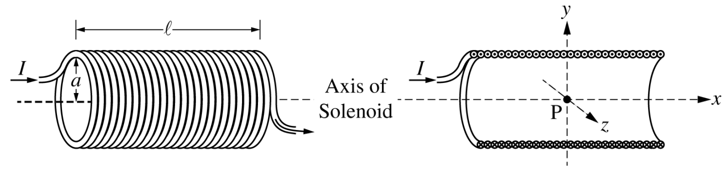
CNV-5.A.a: Derive expressions that relate current, voltage, and resistance to the rate at which heat is produced in a resistor.
CNV-6.C.a: Calculate voltage, current, and power dissipation for any resistor in a circuit containing a network of known resistors with a single battery or energy source.
CNV-6.F.b: Set up simultaneous equations to calculate at least two unknowns (currents or resistance values) in a multi-loop circuit.
CNV-7.B.a: Calculate the potential difference across a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.
CNV-10.C.a: Calculate initial transient currents and final steady-state currents through any part of a series or parallel circuit containing an inductor and one or more resistors.

Science Practices

6.A: Extract quantities from narratives or mathematical relationships to solve problems.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

Question 3

15 points



Solenoid   Cutaway View

Note: Figures not drawn to scale.

A solenoid is used to generate a magnetic field. The solenoid has an inner radius a, length ℓ, and N total turns of wire. A power supply, not shown, is connected to the solenoid and generates current I, as shown in the figure on the left above. The x-axis runs along the axis of the solenoid. Point P is in the middle of the solenoid at the origin of the xyz-coordinate system, as shown in the cutaway view on the right above. Assume $\ell >> a$.

(a) LO CNV-8.E.a, SP 7.A, 7.C
2 points

Select the correct direction of the magnetic field at point P.

____ $+x$-direction   ____ $+y$-direction   ____ $+z$-direction
____ $-x$-direction   ____ $-y$-direction   ____ $-z$-direction

Justify your selection.

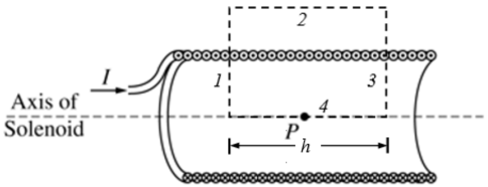
For choosing the "$+x$-direction" and providing a justification		1 point
For a correct justification		1 point
Example: Using the right-hand rule for current on the left side of the solenoid, the fingers curl into the loop, so the magnetic field points to the right, or in the $+x$-direction.		
Example: Using the right-hand rule for solenoids, when the fingers curl around the solenoid in the direction of the current, the thumb points to the right, therefore the magnetic field is to the right, or in the $+x$-direction.		

Question 3 (continued)

(b)

i. LO CNV-8.C.c, SP 3.D
1 point

On the cutaway view below, clearly draw an Amperian loop that can be used to determine the magnetic field at point P at the center of the solenoid.



For drawing a rectangle with one side along the central axis of the solenoid and another side outside the solenoid and whose edges do not extend beyond the solenoid		1 point

ii. LO CNV-8.C.c, SP 5.A, 5.E
2 points

Use Ampere's law to derive an expression for the magnetic field strength at point P. Express your answer in terms of I, ℓ, N, a, and physical constants, as appropriate.

For using Ampere's law to calculate the magnetic field along the axis of the solenoid		1 point
$\int B\bullet d\ell = \mu_0 I_{enc} \therefore \left(\int B\bullet d\ell\right)_1 + \left(\int B\bullet d\ell\right)_2 + \left(\int B\bullet d\ell\right)_3 + \left(\int B\bullet d\ell\right)_4 = \mu_0 I_{enc}$		
$\left(\int B\bullet d\ell\right)_1 + 0 + \left(-\left(\int B\bullet d\ell\right)_1\right) + Bh = \mu_0 \frac{N}{\ell} hI$		
$Bh = \mu_0 \frac{N}{\ell} hI$		
For a correct answer		1 point
$B = \frac{\mu_0 NI}{\ell}$		

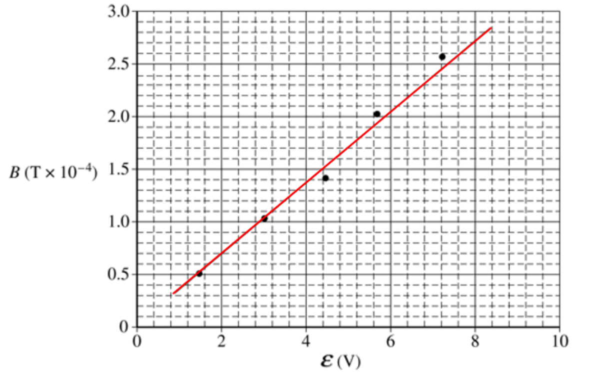
Some physics students conduct an experiment to determine the resistance R_S of a solenoid with radius $a = 0.015$ m, total turns $N = 100$, and total length $\ell = 0.40$ m. The students connect the solenoid to a variable power supply. A magnetic field sensor is used to measure the magnetic field strength along the central axis at the center of the solenoid. The plot of the magnetic field strength B as a function of the emf $\mathcal{E}$ of the power supply is shown below.

Question 3 (continued)

(c) LO CNV-8.C.c, SP 4.C

i. 1 point

On the graph above, draw a best-fit line for the data.



For drawing a best-fit line with at least one point above and one point below the line		1 point

ii. LO CNV-8.C.c, SP 6.B, 6.C
2 points

Use the straight line to determine the resistance R_S of the solenoid used in the experiment.

For calculating the slope using the best-fit line and not the data points unless they fall on the best-fit line		1 point
$slope = \frac{\Delta y}{\Delta x} = \frac{(2.5-0.9)(\times 10^{-4}\text{ T})}{(6.4-2.0)\text{ V}} = 0.36\times 10^{-4} = 3.6\times 10^{-5}\text{ T/V}$		
For correctly giving the expression that relates the slope to the resistance of the solenoid		1 point
$B = \frac{\mu_0 NI}{\ell} = \frac{\mu_0 N\mathcal{E}}{\ell R} \therefore slope = \frac{\mu_0 N}{\ell R_S} \therefore R_S = \frac{\mu_0 N}{\ell \times slope}$ $R_S = \frac{(4\pi\times 10^{-7}\text{ (T}\bullet\text{m)/A})(100\text{ turns})}{(0.40\text{ m})(3.6\times 10^{-5}\text{ T/V})} = 8.7\ \Omega$		

One of the students notes that the horizontal component of the magnetic field of Earth is 2.5×10^{-5} T.

Question 3 (continued)

(d)

i. LO CNV-8.E.a, SP 2.D
1 point

Is there evidence from the graph that the horizontal orientation of the solenoid affects the measured values for B?

____ Yes ____ No

Justify your answer.

If the line on the graph does not go through the origin, select "Yes"		
For a correct justification		1 point
Example: The horizontal component of Earth's magnetic field will add or subtract from the magnetic field of the solenoid depending on the orientation of the solenoid.		
Alternate Solution		*Alternate Points*
If the line on the graph does pass through the origin, select "No"		
For a correct justification		*1 point*
Example: Based on the graph, the line passes through the origin, so the magnetic field is zero when the emf of the power supply is zero, therefore Earth's magnetic field is not affecting the values of B.		

ii. LO CNV-8.E.a, SP 2.E
1 point

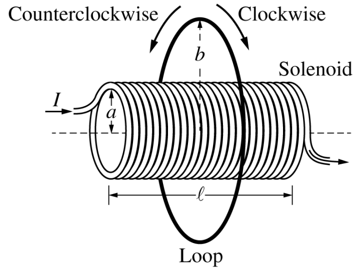
Would the horizontal orientation of the solenoid affect the calculated value for R_S ?

____ Yes ____ No

Justify your answer.

Select "No"		
For a correct justification		1 point
Example: The horizontal component of Earth's magnetic field will not affect the change in magnetic field as the emf is changed. Therefore, the value for the resistance of the solenoid will not change.		

Question 3 (continued)



A thin conducting loop of radius b and resistance R_L is placed concentric with the solenoid, as shown above. The current in the solenoid is decreased from I to zero over time Δt.

(e)

i. LO FIE-6.A.b, SP 7.A, 7.C
2 points

Is the direction of the induced current in the loop clockwise or counterclockwise during the time period that the current in the solenoid is decreasing?

____ Clockwise ____ Counterclockwise

Justify your answer.

Select "Clockwise"		
For a justification indicating that the magnetic field inside the solenoid, and therefore the loop, will decrease		1 point
For a justification using Lenz's law to relate the change in magnetic field to the direction of the current		1 point
Example: As the current in the solenoid decreases, the magnetic field inside the solenoid decreases. As the solenoid's magnetic field decreases, the induced current in the loop will create a magnetic field to oppose this change. Because the solenoid's magnetic field is toward the right and decreasing, the magnetic field due to the current in the loop must be toward the right. Therefore, the current in the loop must be clockwise.		

Question 3 (continued)

(e) continued
ii. LO FIE-6.A.d, SP 5.A, 5.E
3 points

Derive an equation for the average induced current i_{IND} in the loop during the time period that the current in the solenoid is decreasing. Express your answer in terms of I, ℓ, N, a, b, R_L, R_S, Δt, and physical constants, as appropriate.

For using Faraday's law to calculate the emf in the loop		1 point
$\varepsilon = \left\lvert \frac{d\Phi}{dt} \right\rvert = \frac{d(BA)}{dt} = A\frac{\Delta B}{\Delta t}$		
For using Ohm's law to calculate the current in the loop		1 point
$I_{\text{IND}} = \frac{V}{R} = \frac{A\Delta B/\Delta t}{R_L} = \frac{A\Delta B}{R_L \Delta t}$		
For using the correct radius for the area in the equation above		1 point
$I_{\text{IND}} = \frac{\pi a^2 \mu_0 N \Delta I/\ell}{R_L \Delta t} = \frac{\pi a^2 \mu_0 N I}{R_L \ell \Delta t}$		

Learning Objectives

CNV-8.C.c: Derive the expression for the magnetic field of an ideal solenoid (length dimension is much larger than the radius of the solenoid) using Ampère's law.
CNV-8.E.a: Describe the direction of a magnetic field at a point in space due to various combinations of conductors, wires, cylindrical conductors, or loops.
FIE-6.A.b: Describe the direction of an induced current in a conductive loop that is placed in a changing magnetic field.
FIE-6.A.d: Calculate the magnitude and direction of induced EMF and induced current in a conductive loop (or conductive bar) when the magnitude of either the field or area of loop is changing at a constant rate.

Science Practices

2.D: Make observations or collect data from representations of laboratory setups or results.
2.E: Identify or describe potential sources of experimental error.
3.D: Create appropriate diagrams to represent physical situations.
4.C: Linearize data and/or determine a best-fit line or curve.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

2019

AP® CollegeBoard

AP® Physics C: Electricity and Magnetism

Scoring Guidelines Set 2

General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

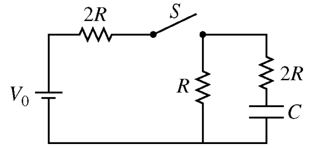
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8\ \text{m/s}^2$, but the use of $10\ \text{m/s}^2$ is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 1

15 points



The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(a) LO CNV-7.B.a, SP 5.A, 5.E
2 points

Derive an expression for the steady-state current supplied by the battery.

For using Ohm's law		1 point
$I = \dfrac{V}{R_{eff}} = \dfrac{V_0}{(2R + R)}$		
For correct substitution leading to correct answer		1 point
$I = \dfrac{V_0}{3R}$		

(b) LO CNV-7.B.b, SP 5.A, 5.E
2 points

Derive an expression for the charge on the capacitor.

For using the equation relating stored charge to capacitance		1 point
$q = CV = CV_C = CV_R$		
For determining V_R and substituting into the above equation		1 point
$V_R = IR = \left(\dfrac{V_0}{3R}\right)R = \dfrac{1}{3}V_0$		
$q = CV_R = C\left(\dfrac{1}{3}V_0\right) = \dfrac{1}{3}CV_0$		

Question 1 (continued)

(c) LO CNV-7.B.b, SP 5.A, 5.E
2 points

Derive an expression for the energy stored in the capacitor.

For any correct equation for energy stored in a capacitor		1 point
$U = \dfrac{q^2}{2C}$		
Substitute charge and/or voltage from part (b)		1 point
$U = \dfrac{(CV_0/3)^2}{2C} = \dfrac{1}{18}CV_0^2$		

Now the switch is opened at time $t = 0$.

(d) LO CNV-7.D.a, SP 5.A, 5.E
2 points

Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time t after the switch is opened.

For any correct voltage loop equation		1 point
$V_C - V_R - V_{2R} = 0 \therefore V_C = V_R + V_{2R}$		
$\dfrac{q(t)}{C} = I(R + 2R)$		
Note: Any correct loop equation for when the switch is open earns the point		
For substituting $-dq/dt$ or dq/dt for the current, consistent with loop equation		1 point
$\dfrac{q(t)}{C} = -3R\dfrac{dq}{dt}$		

Question 1 (continued)

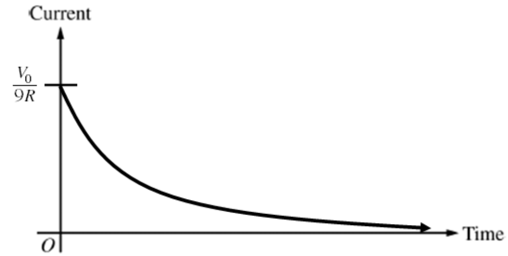
(e)
i. LO CNV-7.B.a, SP 5.A, 5.E
2 points

Calculate the current in resistor R immediately after the switch is opened.

For using a voltage consistent with part (b) in $I = \dfrac{V}{R}$		1 point
For substituting the correct resistance ($3R$) into Ohm's law		1 point
$I = \dfrac{V_0/3}{3R} = \dfrac{V_0}{9R}$		

ii. LO CNV-7.E.b, SP 3.C
3 points

On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



For a curve that is concave up throughout graph		1 point
For having the horizontal axis as an asymptote		1 point
For labeling the maximum current consistent with part (e)(i)		1 point

Question 1 (continued)

(f) LO CNV-7.G.a, SP 7.A, 7.C
2 points

Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c)?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

For selecting "Equal to"		1 point
For a correct justification invoking conservation of energy		1 point
Example: After the switch is opened, the capacitor will discharge all of its stored energy and charge. Assuming no energy is lost in the wire, then the only parts of the circuit that will dissipate this energy are the two resistors in series with the capacitor.		
Alternate Solution		*Alternate Points*
For selecting "Less than"		*1 point*
For a correct justification invoking conservation of energy		*1 point*
Example: After the switch is opened, the capacitor will discharge all of its stored energy and charge. Additionally there is energy lost due to resistance in the wire and/or energy loss in the capacitor.		

2019 SCORING GUIDELINES

Question 1 (continued)

Learning Objectives

CNV-7.B.a: Calculate the potential difference across a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.
CNV-7.B.b: Calculate the stored charge on a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.
CNV-7.D.a: Derive expressions using calculus to describe the time dependence of the stored charge or potential difference across the capacitor, or the current or potential difference across the resistor in an RC circuit when charging or discharging a capacitor.
CNV-7.E.b: Describe the behavior of the voltage or current behavior over time for a circuit that contains resistors and capacitors in a multi-loop arrangement.
CNV-7.G.a: Describe the energy transfer in charging or discharging a capacitor in an RC circuit.

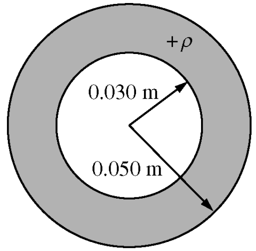
Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

AP Physics C: **Electricity and Magnetism: 2019 Set 2**
2019 SCORING GUIDELINES

Question 2

15 points



A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m: } \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m: } \rho = b/r \text{, where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

$$r > 0.050 \text{ m: } \rho = 0$$

(a) LO CNV-2.D.a, SP 6.B, 6.C
3 points

Calculate the total charge of the sphere.

For indicating the need to integrate the expression for charge density to determine the total charge on the sphere		1 point
$Q = \int \rho dV$		
For proper substitutions into the integration		1 point
$Q = \int \frac{(1.6 \times 10^{-6})}{r}(4\pi r^2)dr$		
For using the proper limits of integration		1 point
$Q = 4\pi(1.6 \times 10^{-6}) \int_{r=0.03}^{r=0.05} rdr = 4\pi(1.6 \times 10^{-6})\left[\frac{r^2}{2}\right]_{r=0.03}^{r=0.05}$		
$Q = (2\pi)(1.6 \times 10^{-6})(0.05^2 - 0.03^2) = 1.61 \times 10^{-8}$ C		

Question 2 (continued)

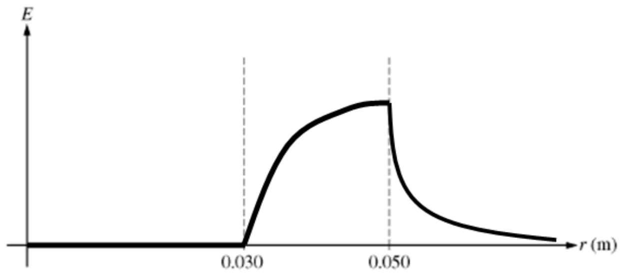
(b) LO CNV-2.D.a, SP 6.C
3 points

Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.

For correctly evaluating the surface integral in Gauss's law		1 point
$\oint E \bullet dA = E\left(4\pi r^2\right)$		
For correctly substituting the answer from part (a) and correct radius into above equation		1 point
$\frac{Q_{enc}}{\varepsilon_0} = E\left(4\pi r^2\right) \therefore E = \frac{Q_{enc}}{4\pi\varepsilon_0 r^2} = \frac{\left(1.61\times10^{-8}\text{ C}\right)}{(4\pi)\left(8.85\times10^{-12}\right)(0.05)^2}$		
For an answer consistent with part (a) with correct units		1 point
$E = 5.79\times10^4 \text{ N/C}$		

(c) LO CNV-2.C, SP 3.C
3 points

On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.



For clearly showing a graph with a value of $E = 0$ for $r < 0.030$ m		1 point
For a continuous graph that starts at zero, is concave down, and increases in value from $r = 0.030$ to $r = 0.050$		1 point
For a continuous graph that decreases asymptotically toward the horizontal axis for $r > 0.050$ m		1 point

Question 2 (continued)

(d) LO CNV-1.G.a, SP 6.B, 6.C
2 points

Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

For substituting the total charge from part (a) into a correct expression for electric potential		1 point
For substituting $r = 0.05$ m into a correct expression for electric potential		1 point
$V_R = \frac{Q_{tot}}{4\pi\varepsilon_0 r} = \frac{\left(9\times10^9\right)\left(1.61\times10^{-8}\text{ C}\right)}{(0.05\text{ m})} = 2900\text{ V}$		
Alternate Solution		*Alternate Points*
For substituting the total charge from part (a) into an integration for electric potential		*1 point*
$\Delta V_R = V_R - V_\infty = V_R = -\int E dr = -\int \frac{Q_{enc}}{4\pi\varepsilon_0 r^2} dr = -\int \frac{\left(9\times10^9\right)\left(1.61\times10^{-8}\text{ C}\right)}{r^2} dr$		
For integrating with correct limits of integration		*1 point*
$V_R = -\int_{r=\infty}^{r=0.05\text{ m}} \frac{145}{r^2} dr = -145\left[-\frac{1}{r}\right]_{r=\infty}^{r=0.05\text{ m}} = 145\left(\frac{1}{(0.05\text{ m})} - \frac{1}{\infty}\right) = 2900\text{ V}$		

A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.

(e)
i. LO ACT-1.D, SP 6.B, 6.C
2 points

Calculate the magnitude of the initial acceleration of the proton.

For using a correct expression of Newton's second law in terms of the electric field		1 point
$F = ma \therefore qE = ma \therefore a = \frac{qE}{m}$		
For correctly substituting into equation above		1 point
$a = \frac{qE}{m} = \frac{\left(1.6\times10^{-19}\text{ C}\right)\left(5.79\times10^4\text{ N/C}\right)}{1.67\times10^{-27}\text{ kg}} = 5.55\times10^{12}\text{ m/s}^2$		

Question 2 (continued)

(e) continued
ii. LO CNV-1.E, SP 6.B, 6.C
2 points

Calculate the speed of the proton after a long time.

For a correct expression of kinetic energy in terms of the electric potential difference		1 point
$-q\Delta V = \frac{1}{2}mv^2$		
For correctly substituting into equation above		1 point
$v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{-2\left(1.6\times10^{-19}\ \text{C}\right)(0-2900\ \text{V})}{\left(1.67\times10^{-27}\ \text{kg}\right)}} = 7.45\times10^{5}\ \text{m/s}$		

Learning Objectives

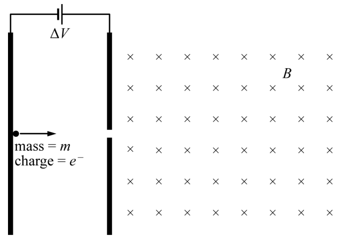
ACT-1.D: Determine the motion of a charged object of specified charge and mass under the influence of an electrostatic force.
CNV-1.E: Calculate the work done or changes in kinetic energy (or changes in speed) of a charged particle when it is moved through some known potential difference.
CNV-1.G.a: Use the general relationship between electric field and electric potential to calculate the relationships between the magnitude of electric field or the potential difference as a function of position.
CNV-2.C: State and use Gauss's law in integral form to derive unknown electric fields for planar, spherical, or cylindrically symmetrical charge distributions.
CNV-2.D.a: Using appropriate mathematics (which may involve calculus), calculate the total charge contained in lines, surfaces, or volumes when given a linear-charge density, a surface-charge density, or a volume-charge density of the charge configuration.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

Question 3

15 points



Two plates are set up with a potential difference V between them. A small sphere of mass m and charge $-e$ is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude B directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of V, m, e, B, and fundamental constants, as appropriate.

(a)
i. LO CHG-1.A.a, SP 7.A
1 point

What is the initial direction of the force on the sphere as it enters the magnetic field?

____ Into the page ____ Out of the page
____ Toward the top of the page ____ Toward the bottom of the page

For selecting "Toward the bottom of the page"		1 point

ii. LO CHG-1.B, SP 7.A
1 point

Describe the path taken by the sphere after it enters the magnetic field.

For describing a circular path for the sphere consistent with the selection from part (a)(i)		1 point
Example: The sphere will move in a circular path toward the bottom of the page.		

Question 3 (continued)

(b) LO CNV-1.E, SP 5.A, 5.E
2 points

Derive an expression for the speed of the sphere as it passes through the small opening.

For using a valid equation relating potential difference to kinetic energy of the sphere		1 point
$\Delta K = -\Delta U \therefore \frac{1}{2}mv^2 = -q\Delta V$		
$\frac{1}{2}mv^2 = eV$		
For the correct answer with supporting work		1 point
$v = \sqrt{\frac{2eV}{m}}$		

(c) LO CHG-1.C, SP 5.A, 5.E
3 points

Derive an expression for the radius of the path taken by the sphere as it moves through the magnetic field.

For an expression relating the magnetic force to the centripetal force		1 point
$\frac{mv^2}{r} = Bqv$		
For substituting the charge of the sphere into the above equation and solving for r		1 point
$\frac{mv}{r} = Be \therefore r = \frac{mv}{Be}$		
For substituting the answer from part (b)		1 point
$r = \frac{m}{Be}\sqrt{\frac{2eV}{m}} = \sqrt{\frac{2mV}{eB^2}}$		

An experiment is performed in which a beam of electrons is accelerated across the space between the plates and passes through the small opening. After passing through the opening, the electrons travel in a semicircular path and strike the right-hand plate. The potential difference between the plates is varied in regular increments, as shown in the table below. For each potential difference, the magnetic field is varied in order to cause the beam to strike the right-hand plate at a distance of 0.020 m from the opening.

Potential difference (V)	60	70	100	110	120	140
Magnetic field $(\mathrm{T} \times 10^{-3})$	2.62	2.78	3.39	3.54	3.78	3.99

Question 3 (continued)

(d) LO CHG-1.C, SP 4.C
1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the mass-to-charge ratio of an electron.

Vertical axis:____________
Horizontal axis:____________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

$$r = \sqrt{\frac{2m_eV}{eB^2}} \therefore B^2 = \frac{2m_e}{er^2}V$$

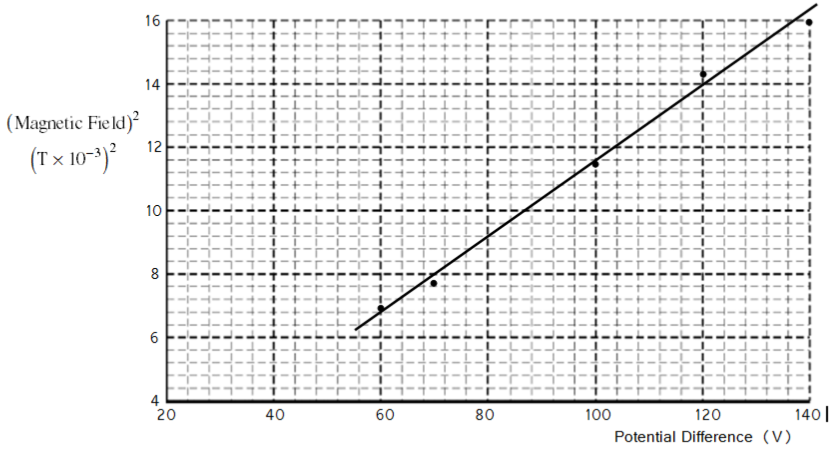
Potential difference (V)	60	70	100	110	120	140
Magnetic field $(\mathrm{T} \times 10^{-3})$	2.62	2.78	3.39	3.54	3.78	3.99
$(\text{Magnetic field})^2$ $(\mathrm{T} \times 10^{-3})^2$	6.86	7.73	11.5	12.5	14.3	15.9

For indicating axis choices with appropriate quantities to produce a straight line		1 point
Note: There are several possible choices.		

Question 3 (continued)

(e) LO CHG-1.C, SP 3.A, 4.C
4 points

On the graph below, plot the relationship determined in part (d). Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



For correctly labeling both axes with quantities and units		1 point
For correctly scaling and labeling the axes so that the points use at least half the grid		1 point
For correctly plotting the data		1 point
For drawing a straight line that represents the data		1 point

(f) LO CHG-1.C, SP 4.D, 5.E
3 points

Using the straight line from part (e), determine the mass-to-charge ratio of an electron.

For correctly calculating the slope from the best-fit straight line and not from the data points unless the points fall on the best-fit line		1 point
$\text{slope} = \dfrac{14 - 9.2}{120 - 80} = 0.12\ \dfrac{(\text{T} \times 10^{-3})^2}{V} = 1.2 \times 10^{-7}\ \text{T}^2/\text{V}$		
For a correct expression relating the slope to the mass to charge ratio of an electron		1 point
$B^2 = \dfrac{2m_e}{er^2}V \therefore \text{slope} = \dfrac{2m_e}{er^2} \therefore \dfrac{m_e}{e} = \dfrac{r^2 \times \text{slope}}{2}$		
For substituting correct values into the above equation		1 point
$\dfrac{m_e}{e} = \dfrac{(0.01\ \text{m})^2 \times \left(1.2 \times 10^{-7}\ \text{T}^2/\text{V}\right)}{2} = 6.0 \times 10^{-12}\ \text{kg/C}$		

Question 3 (continued)

Learning Objectives

CHG-1.A.a: Calculate the magnitude and direction of the magnetic force of interaction between a moving charged particle of specified charge and velocity moving in a region of a uniform magnetic field.
CHG-1.B: Describe the path of different moving charged particles (i.e., of different type of charge or mass) in a uniform magnetic field.
CHG-1.C: Derive an expression for the radius of a circular path for a charged particle of specified characteristics moving in a specified magnetic field.
CNV-1.E: Calculate the work done or changes in kinetic energy (or changes in speed) of a charged particle when it is moved through some known potential difference.

Science Practices

3.A: Select and plot appropriate data.
4.C: Linearize data and/or determine a best-fit line or curve.
4.D: Select relevant features of a graph to describe a physical situation or solve problems.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
7.A: Make a scientific claim.