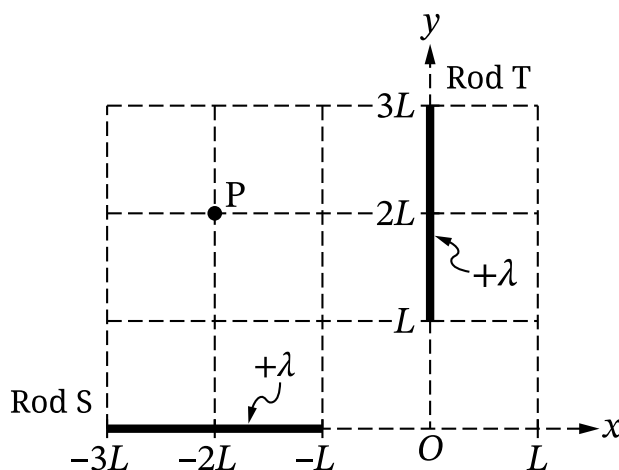


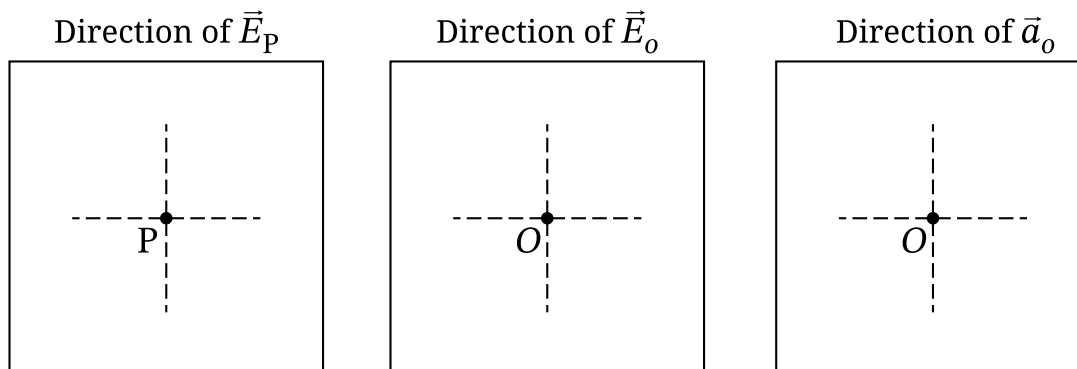
Question 2: Version J

2. Thin, nonconducting, charged Rods S and T are held fixed in the xy -plane, as shown in Figure 1. Rod S is oriented along the x -axis in the region $-3L \leq x \leq -L$. Rod T is oriented along the y -axis in the region $L \leq y \leq 3L$. Each rod has uniform, positive linear charge density $+\lambda$. Point P is located at $(-2L, 2L)$.

Figure 1**Part A**

Complete the following tasks in Figure 2.

- **Draw** an arrow starting on the dot that indicates the direction of the net electric field $\vec{E}_P$ at Point P due to Rods S and T.
- **Draw** an arrow starting on the dot that indicates the direction of the net electric field $\vec{E}_O$ at the origin O due to Rods S and T.
- **Draw** an arrow starting on the dot that indicates the direction of the net acceleration $\vec{a}_O$ of a small, negatively charged sphere (not shown) immediately after the sphere is released from rest at the origin O.

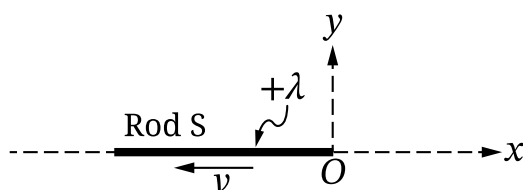
Figure 2

Part B

Derive an expression for the magnitude E_O of the net electric field at the origin O due to Rods S and T in terms of L , λ , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

Rod T is removed. Rod S is then moved such that the right end of Rod S is at the origin O . Rod S is then moved with speed v in the $-x$ -direction, as shown in Figure 3.

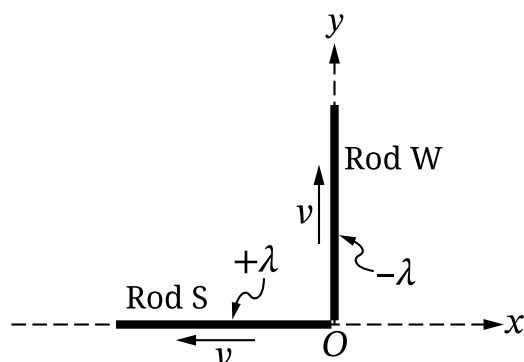
Figure 3

On the axes shown in Figure 4, **sketch** a graph of the electric potential V_O at the origin O due to Rod S as a function of the distance r between the origin and the right end of Rod S.

Figure 4

Part D

A new thin, nonconducting, charged Rod W has the same length as Rod S. Rod W has uniform, negative linear charge density $-\lambda$ and is oriented along the y -axis. Rods S and W are initially held fixed with one end at the origin. Starting at the same time, Rod S is moved with speed v in the $-x$ -direction and Rod W is moved with speed v in the $+y$ -direction, as shown in Figure 5.

Figure 5

Indicate how a sketch of V_O at the origin O due to both Rods S and W as a function of r would be different from the sketch made in part C. Briefly **justify** your answer.

Begin your response to **QUESTION 1** on this page.**PHYSICS C: ELECTRICITY AND MAGNETISM****SECTION II****Time—45 minutes****3 Questions**

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

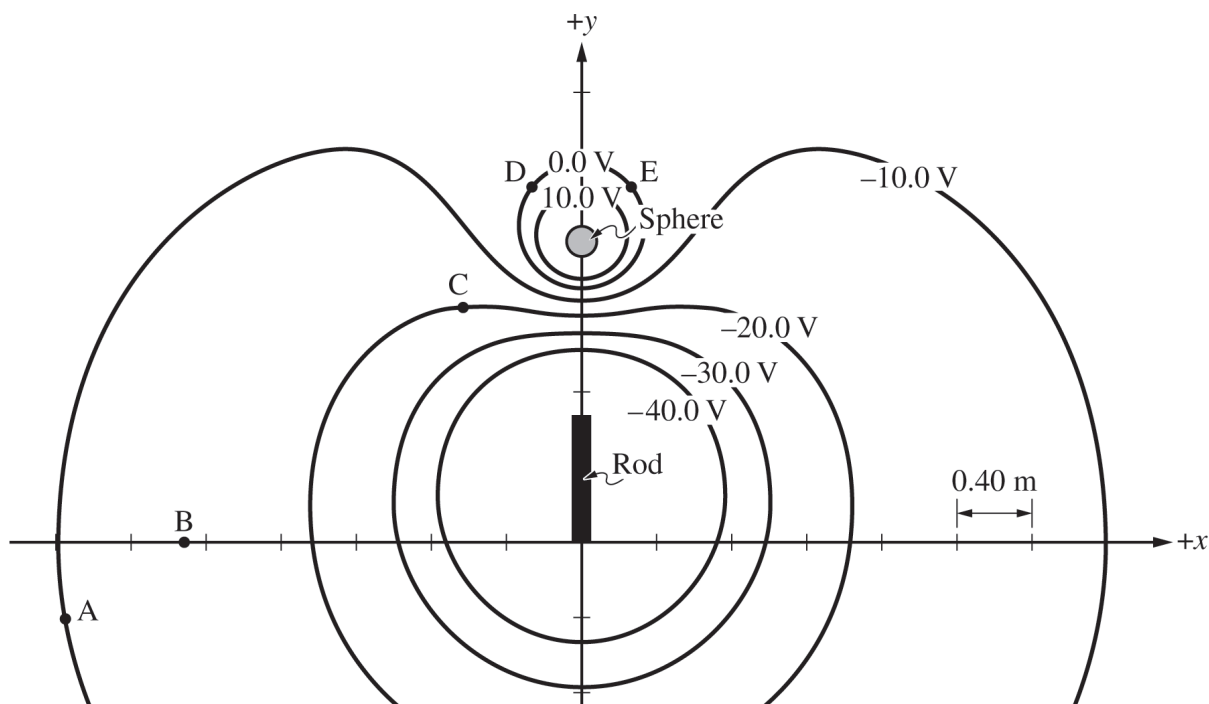


Figure 1

1. A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0 \text{ nC}$. The rod and sphere are held at rest on the y -axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x -axis and on the y -axis are 0.40 m apart.

(a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the 0.0 V equipotential line.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of $C \rightarrow E \rightarrow D \rightarrow A$. The test charge is held momentarily at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE} . If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a “0” in the corresponding columns, as appropriate.

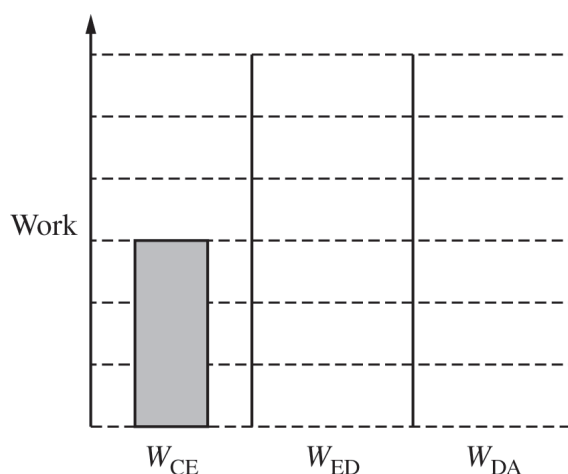


Figure 2

ii. **Calculate** the approximate magnitude of the x -component of the electric field at Position B.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position C. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

GO ON TO THE NEXT PAGE.

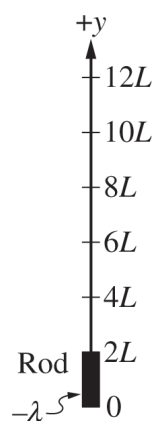
Continue your response to **QUESTION 1** on this page.

Figure 3

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the y -axis in the orientation shown in Figure 3. Position P (not shown) is located on the y -axis a distance y_P from the origin, where $y_P > 2L$.

(d) The electric potential V_P at y_P is $V_P = -k\lambda \ln\left(\frac{y_P}{y_P - 2L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

- ii. On Figure 4, **sketch** a graph of the y -component E_y of the electric field resulting from the rod as a function of y in the region $2L < y < 12L$.

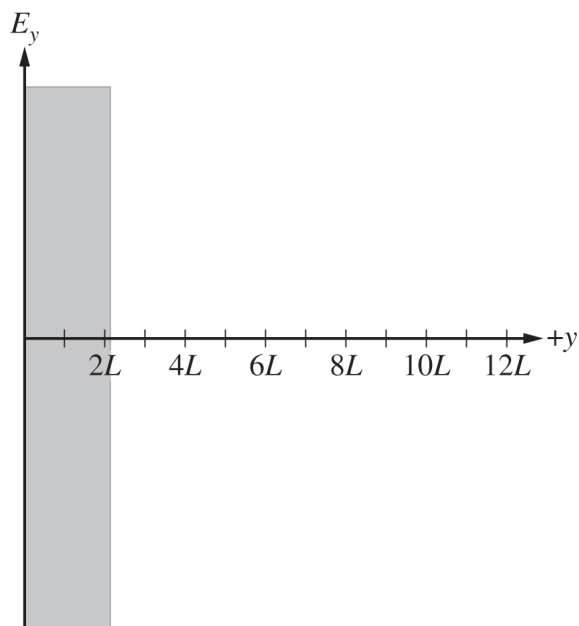
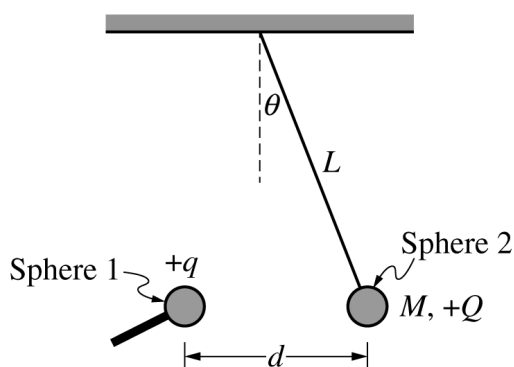


Figure 4

GO ON TO THE NEXT PAGE.

PHYSICS C: ELECTRICITY AND MAGNETISM**SECTION II****Time—45 minutes****3 Questions**

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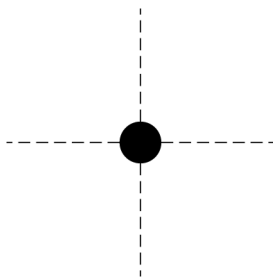


Students perform an experiment to determine the value of vacuum permittivity ϵ_0 . Sphere 1 is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere 2 is nonconducting with charge $+Q$ and has mass M . Sphere 2 is hung from a string of negligible mass and length L . Sphere 1 is brought near, without touching, Sphere 2, as shown. Equilibrium is established when the centers of the two spheres have the same vertical position, are a horizontal distance d apart, and the string is at an angle θ from the vertical.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

- (a) On the following dot that represents Sphere 2 at the position shown in the previous figure, draw and label the forces (not components) that act on Sphere 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

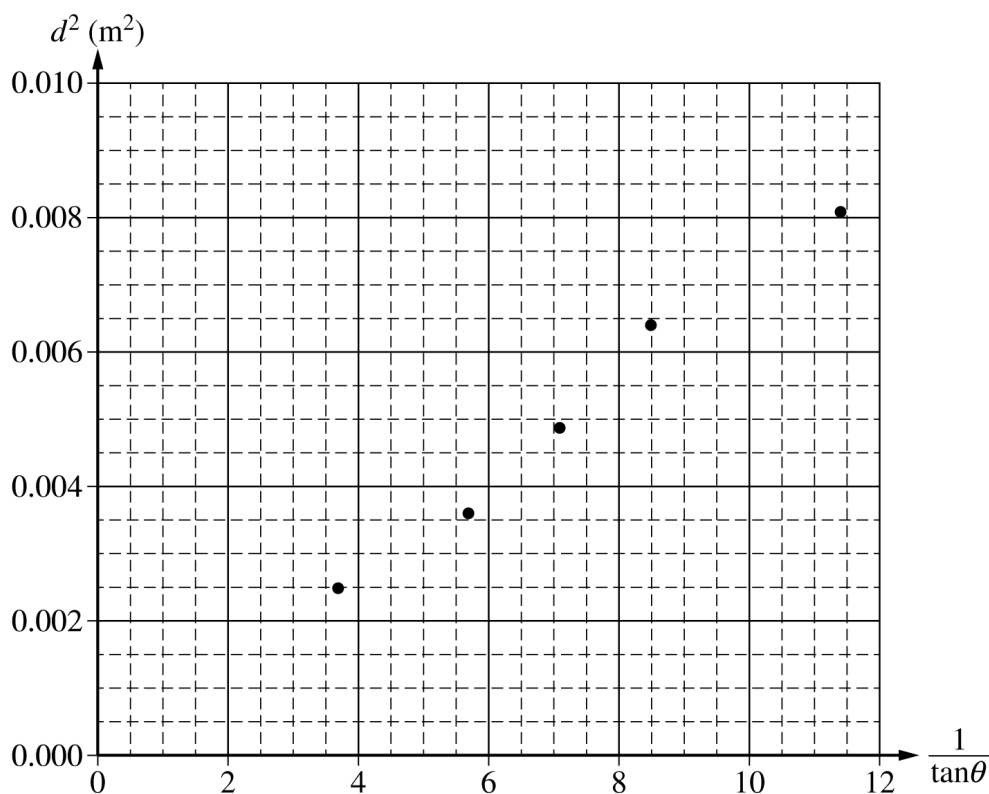


- (b) Derive the relationship between the distance d and the angle θ to show that $d = \sqrt{\frac{Qq}{4\pi\epsilon_0 Mg \tan\theta}}$.

- (c) These values are collected in one trial: $Q = q = 6.0 \times 10^{-8} \text{ C}$, $\theta = 12^\circ$, and $d = 0.057 \text{ m}$. Calculate the expected force of tension exerted on Sphere 2 by the string.

GO ON TO THE NEXT PAGE.

- (d) The students vary d and measure θ after equilibrium is reached. The students use the collected data to plot the following graph of d^2 vs. $\frac{1}{\tan\theta}$.

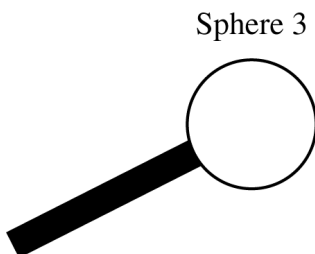


- Draw the best-fit line for the data.
- Using the best-fit line, calculate an experimental value for the vacuum permittivity ϵ_0 when $M = 0.0050$ kg and $Q = q = 6.0 \times 10^{-8}$ C.

GO ON TO THE NEXT PAGE.

(e) The students modify the experiment by replacing Sphere 1 with a conducting Sphere 3 that has the same size and charge $+q$. The experiment is repeated.

i. The circle in the following figure represents Sphere 3 when spheres 2 and 3 are at equilibrium. On the circle, draw a single “+” sign to represent the location of highest concentration of the excess positive charges.



ii. Briefly explain your reasoning for the sketch drawn in part (e)(i).

iii. In the original experiment, when the centers of the two spheres are a horizontal distance d_1 apart, the string makes an angle θ_1 from the vertical. In the modified experiment, when the centers of the two spheres are a horizontal distance d_1 apart, the string makes an angle θ_2 from the vertical.

Is θ_2 greater than, less than, or equal to θ_1 ?

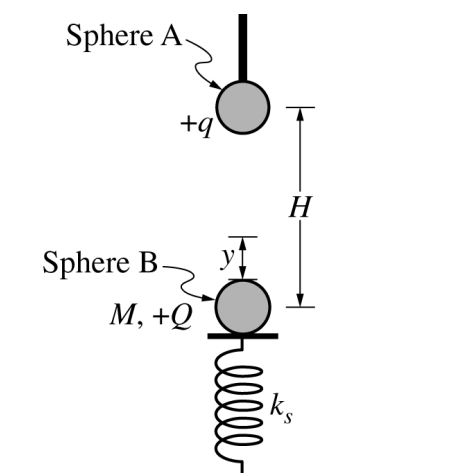
_____ $\theta_2 > \theta_1$ _____ $\theta_2 < \theta_1$ _____ $\theta_2 = \theta_1$

Briefly justify your answer.

GO ON TO THE NEXT PAGE.

PHYSICS C: ELECTRICITY AND MAGNETISM**SECTION II****Time—45 minutes****3 Questions**

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Note: Figure not drawn to scale.

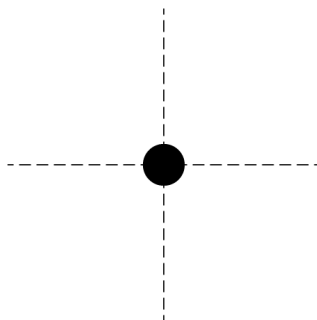
Students perform an experiment to determine the value of vacuum permittivity ϵ_0 . Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M . Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s . Sphere B and the spring are initially at rest.

Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H , the spring has been compressed a distance y , as shown in the figure. The students measure y for different values of H .

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Continue your response to **QUESTION 1** on this page.

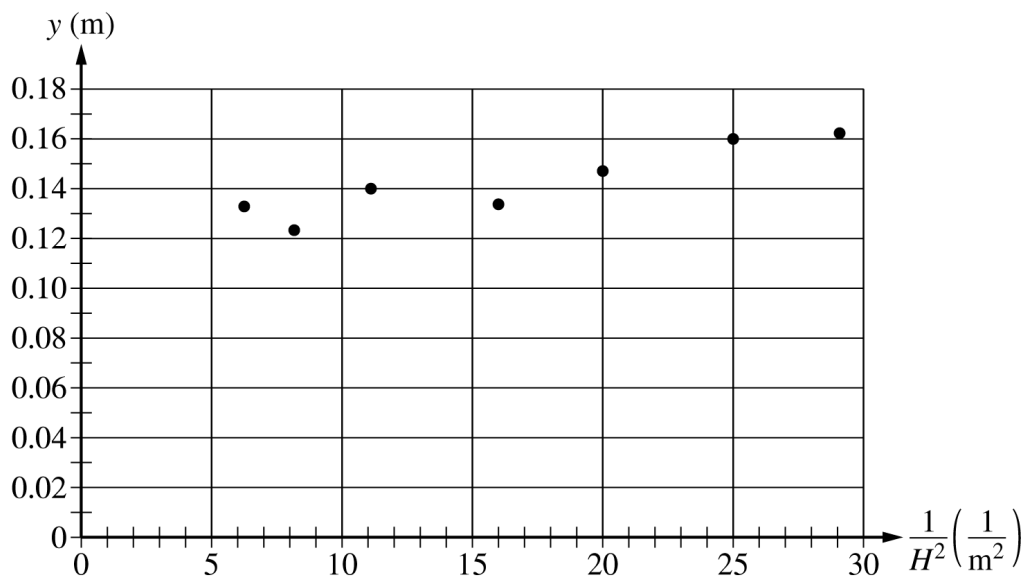
- (a) On the following dot that represents Sphere B in the figure on the previous page, draw and label the forces (not components) that are exerted on Sphere B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



- (b) Derive the relationship between y and H to show that $y = \frac{1}{4\pi\epsilon_0} \frac{Qq}{k_s H^2} + \frac{Mg}{k_s}$.

GO ON TO THE NEXT PAGE.

- (c) The students plot collected data of y as a function of $\frac{1}{H^2}$, as shown in the graph.



- i. Draw the best-fit line for the data.
- ii. Using the best-fit line, calculate an experimental value for the vacuum permittivity ϵ_0 when $Q = q = 2.00 \times 10^{-6} \text{ C}$ and $k_s = 25 \text{ N/m}$.

- iii. Using the best-fit line, calculate an experimental value for the mass of Sphere B.

GO ON TO THE NEXT PAGE.

(d) The students modify the experiment by replacing nonconducting Sphere B with conducting Sphere C that has the same charge $+Q$ and mass M . Sphere A is brought near Sphere C without touching, compressing the spring. Sphere C comes to rest.

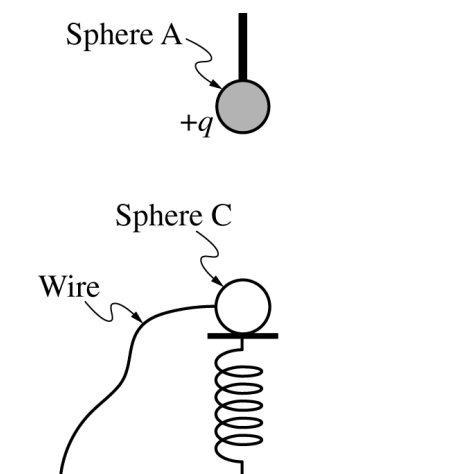
i. In the original experiment, when the centers of spheres A and B are a vertical distance H_1 apart, the spring is compressed a distance y_1 . In the modified experiment, when the centers of spheres A and C are a vertical distance H_1 apart, the spring is compressed a distance y_2 .

Is y_2 greater than, less than, or equal to y_1 ?

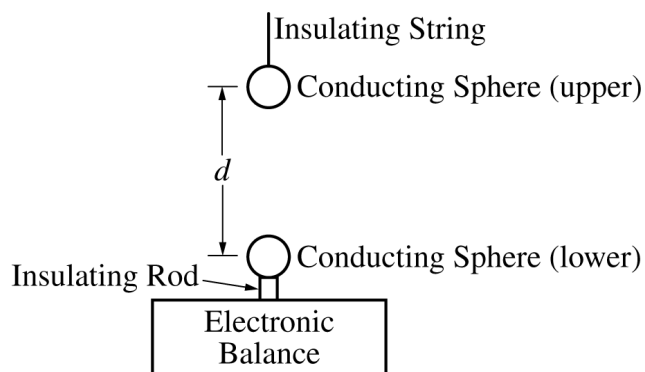
_____ $y_2 > y_1$ _____ $y_2 < y_1$ _____ $y_2 = y_1$

Justify your answer.

ii. Sphere C is then grounded with a wire. On the following figure, draw an arrow indicating the direction that the platform will move immediately after being grounded. If the platform remains stationary, write “does not move.”

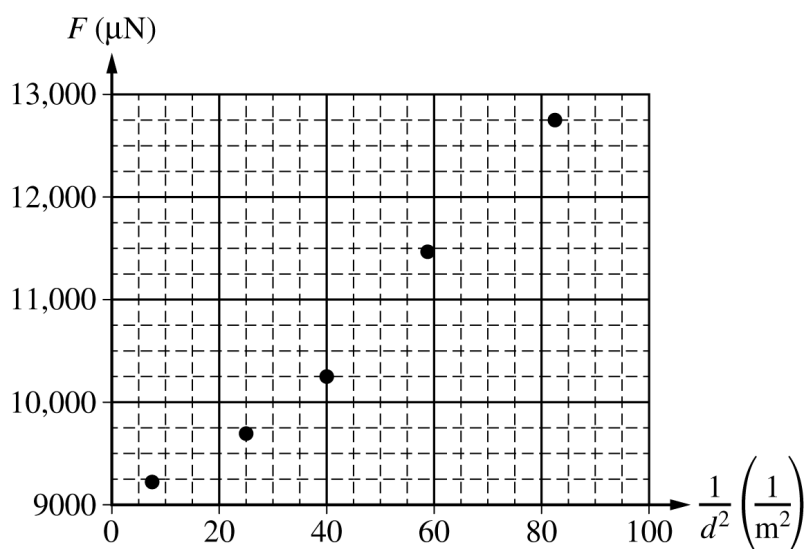


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Students perform an experiment to study the force between two charged objects using the apparatus shown above, which contains two identical conducting spheres. The upper sphere is attached to an insulating string, which can be used to move the sphere downward. The lower sphere sits on an insulating rod, which is on an electronic balance. The electronic balance is zeroed before the lower sphere and insulating rod are in place.

For the first trial, a charge of Q is placed on each sphere and then the upper sphere is slowly moved downward. The students measure the distance d between the centers of the spheres and the magnitude F of the force that appears on the electronic balance. The recorded data are shown on the graph of F as a function of $\frac{1}{d^2}$ shown below.



(a)

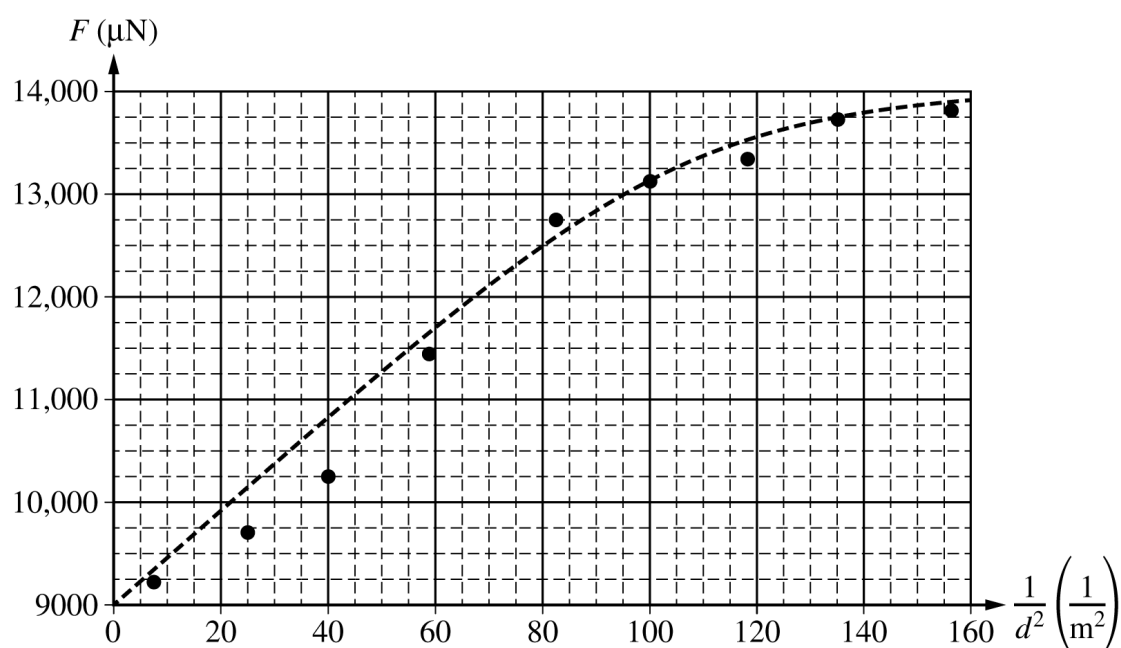
- i. Draw a line that represents the best fit to the points shown.
- ii. Use the graph to calculate the charge Q .

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Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.

- iii. On the graph on the previous page, draw a circle around the data point that was taken when the distance between the centers of the spheres was the least.
- iv. Determine the distance between the centers of the spheres for the data point indicated above.
- v. What physical quantity does the vertical intercept represent?

Justify your answer.



The experiment is extended by collecting additional data points, which appear on the right side of the graph shown above. The new data points do not follow the linear pattern seen with the first points. The group of students tries to explain this discrepancy.

- (b) One student suspects that charge is slowly leaking off the top sphere. Could this explain the discrepancy?

☐ Yes ☐ No

Justify your answer.

GO ON TO THE NEXT PAGE.

Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.

) A second student suspects that the excess charges have rearranged themselves, polarizing the spheres.

i. On the circles representing the spheres below, use a single “+” sign on each sphere to represent the locations of highest concentration of the excess positive charges.



ii. Explain how this rearrangement could be responsible for the discrepancy.

(d) A third student suggests that the experiment be modified so that the top sphere is given a negative charge that is equal in magnitude to the positive charge given to the bottom sphere.

i. On the circles representing the spheres below, use a single “+” sign on the bottom sphere to represent the location of highest concentration of the excess positive charges. Use a single “−” sign on the top sphere to represent the location of the highest concentration of the excess negative charges.



ii. For a separation distance equal to that of the data point indicated in part (a)(iii), would the magnitude of the force reading with spheres of opposite charges be greater than, less than, or equal to the magnitude of the force reading with spheres of the same charges?

_____ Greater than _____ Less than _____ Equal to

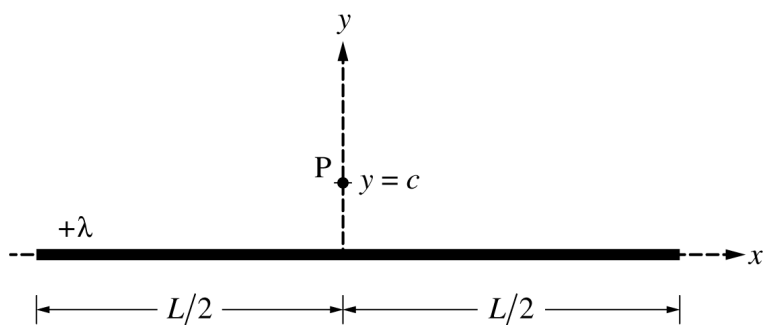
Justify your answer.

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PHYSICS C: ELECTRICITY AND MAGNETISM**SECTION II****Time—45 minutes****3 Questions**

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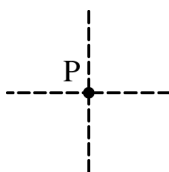


Note: Figure not drawn to scale.

1. A very long, thin, nonconducting cylinder of length L is centered on the y -axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y -axis at $y = c$, where $L \gg c$.

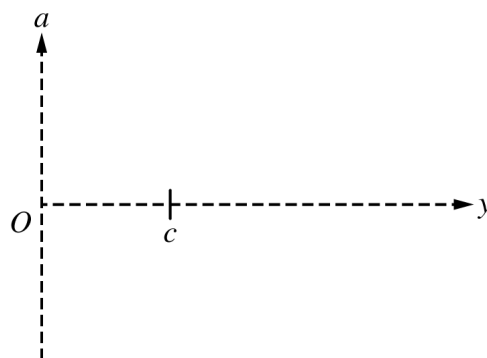
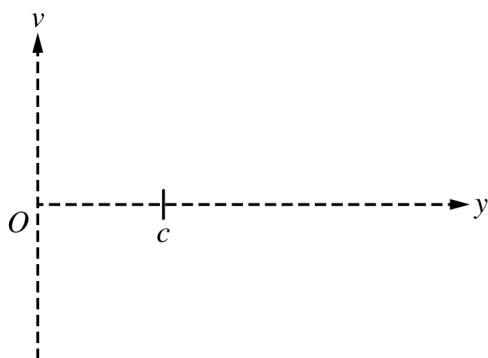
(a)

- i. On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the long cylinder. The arrow should start on and point away from the dot.

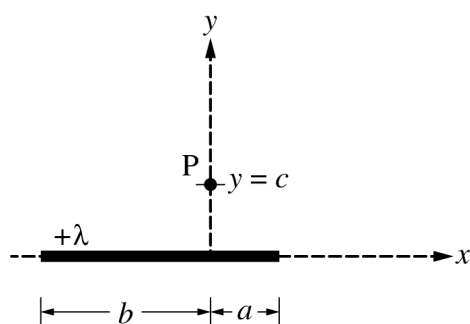


- ii. Describe the shape and location of a Gaussian surface that can be used to determine the electric field at point P due to the long cylinder.
- iii. Use your Gaussian surface to derive an expression for the magnitude of the electric field at point P. Express your answer in terms of λ , c , L , and physical constants, as appropriate.

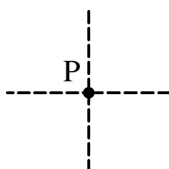
- (b) A proton is released from rest at point P. On the axes below, sketch the velocity v as a function of position y and the acceleration a as a function of position y for the proton.



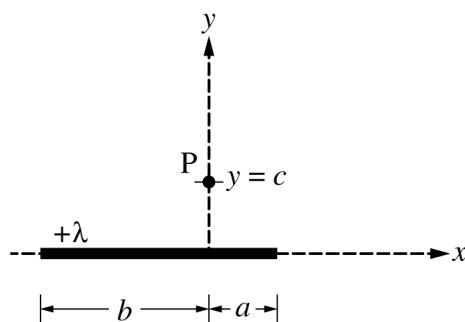
The original cylinder is now replaced with a much shorter thin, nonconducting cylinder with the same uniform linear charge density $+\lambda$, as shown in the figure below. The length of the cylinder to the right of the y -axis is a , and the length of the cylinder to the left of the y -axis is b , where $a < b$.



- (c) On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the shorter cylinder. The arrow should start on and point away from the dot.



- (d)
- Is there a single Gaussian surface that can be used with Gauss's law to derive an expression for the electric field at point P?
 ____ Yes ____ No
 - If your answer to part (d)(i) is yes, explain how you can use Gauss's law to derive an expression for the field at point P. If your answer to part (d)(i) is no, explain why Gauss's law cannot be applied to derive an expression for the electric field in this case.



Note: This figure is shown again for reference.

A student in class argues that using the integral shown below might be a useful approach for determining the electric field at point P.

$$E = \int \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} dq$$

The student uses this approach and writes the following two integrals for the magnitude of the horizontal and vertical components of the electric field at point P.

Horizontal component: $|E_x| = \frac{\lambda}{4\pi\epsilon_0} \int_{-b}^a \frac{x}{(c^2 + x^2)^{3/2}} dx$

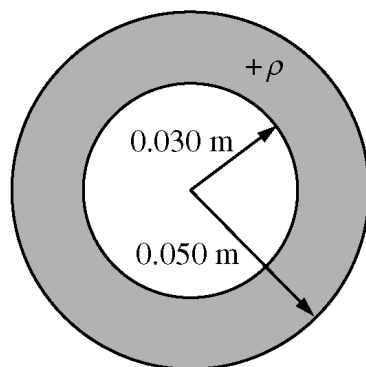
Vertical component: $|E_y| = \frac{\lambda}{4\pi\epsilon_0} \int_{-b}^a \frac{y}{(c^2 + x^2)} dy$

(e)

- i. One of the two expressions above is not correct. Which expression is not correct?

_____ Horizontal component _____ Vertical component

- ii. Identify two mistakes in the incorrect expression, and explain how to correct the mistakes.



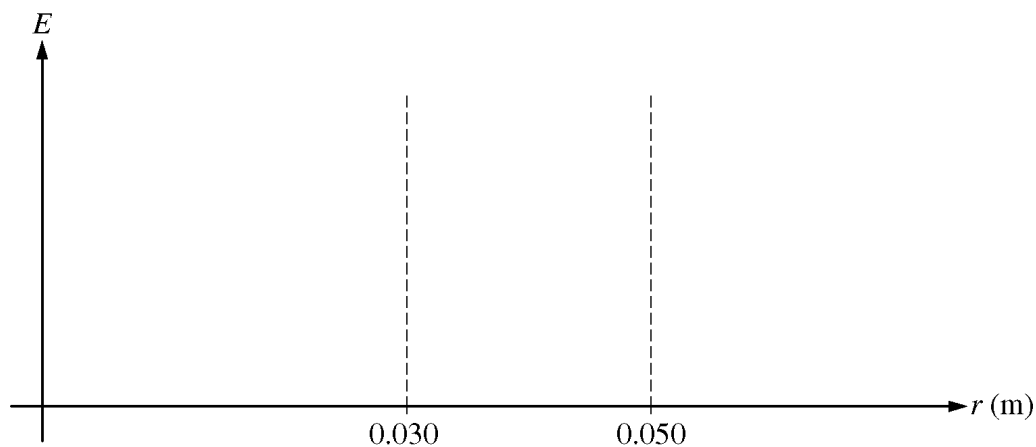
2. A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ , as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m: } \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m: } \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

$$r > 0.050 \text{ m: } \rho = 0$$

- (a) Calculate the total charge of the sphere.
- (b) Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.
- (c) On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.



- (d) Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.
- (e) A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.
- Calculate the magnitude of the initial acceleration of the proton.
 - Calculate the speed of the proton after a long time.

E&M.3.

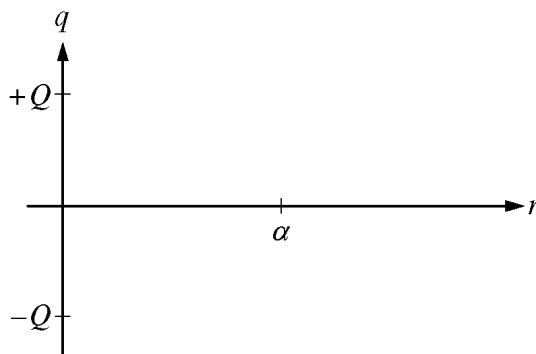
A scientist describes an electrically neutral atom with a model that consists of a nucleus that is a point particle with positive charge $+Q$ at the center of the atom and an electron volume charge density of the form

$$\rho(r) = \begin{cases} -\frac{\beta}{r^2} e^{-r/\alpha} & r < \alpha \\ 0 & r > \alpha \end{cases}$$

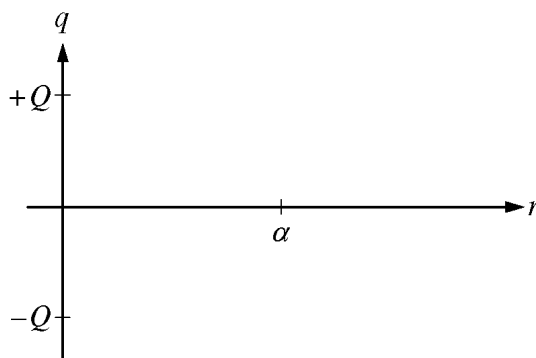
where α and β are positive constants and r is the distance from the center of the atom.

- (a) On the axes below, let r stand for the radius of a Gaussian sphere. Sketch the graph for each of the following charges enclosed by the Gaussian sphere as a function of r . Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

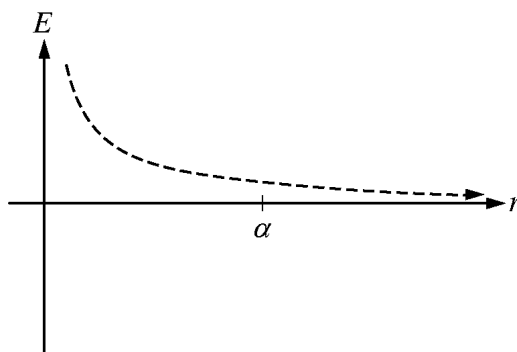
- i. The nuclear charge only



- ii. The electron charge only



- (b) The dashed curve on the graph below represents the electric field as a function of distance r due to the positive nucleus of the atom without any electrons. The nucleus is modeled as a point particle of charge $+Q$. On the same graph, sketch the electric field as a function of distance r for the neutral atom as defined by the scientist's model, which includes the nucleus and the negative electrons surrounding it.



- (c) Use Gauss's law to derive an expression for the electric field strength due to the neutral atom for the following positions in terms of Q , α , β , r , and fundamental constants, as appropriate.
- $r > \alpha$
 - $r < \alpha$
- (d) Based on the model proposed by the scientist, what is the physical meaning of the constant α ?

STOP
END OF EXAM