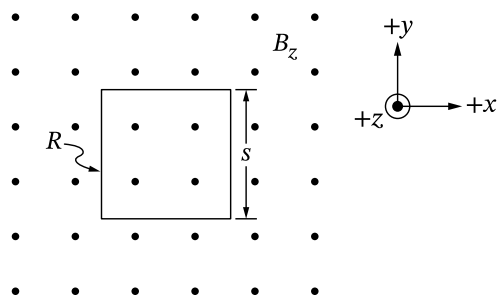


Question 4

4. In Scenario 1, a square conducting loop with side length s has total resistance R . The loop is held fixed in the xy -plane in an external, uniform, time-varying magnetic field that is initially directed in the $+z$ -direction, as shown. The z -component of the magnetic field changes as a function of time t as described by $B_z = B_0 \cos\left(2\pi\frac{t}{t_1}\right)$, where B_0 and t_1 are positive constants.

**Part A**

Indicate whether the induced current I in the loop is clockwise, counterclockwise, or zero during the time interval $0 < t < \frac{t_1}{4}$ by writing one of the following.

- I is clockwise.
- I is counterclockwise.
- I is zero.

Justify your answer without only manipulating equations.

Part B

Derive an expression for I as a function of t in terms of s , R , B_0 , t_1 , t , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

In Scenario 2, a new square conducting loop with side length $2s$ and total resistance $2R$ is placed in the same external, uniform, time-varying magnetic field as in Scenario 1.

Indicate whether the maximum induced current I_2 in Scenario 2 is greater than, less than, or equal to the maximum induced current I_1 in Scenario 1 by writing one of the following.

- $I_2 > I_1$
- $I_2 < I_1$
- $I_2 = I_1$

Briefly **justify** your answer by referencing the expression you derived in part B.

STOP
END OF EXAM

Question 2: Version J

2. A rotating, circular, conducting loop of area A and resistance R is in an external uniform magnetic field of magnitude B that is directed in the $-z$ -direction. At time $t = 0$, the magnetic field is perpendicular to the plane of the loop, as shown in Figure 1. The loop is rotating with constant angular speed ω and period T about the dashed line that is along the diameter of the loop. The value of the magnetic flux through the loop as a function of time t is $\Phi = BA \cos(\omega t)$.

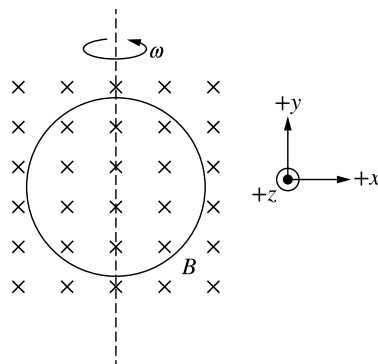


Figure 1

- A. The absolute value of the induced emf in the loop is $|\mathcal{E}|$. The partially completed bar chart in Figure 2 shows a bar that represents $|\mathcal{E}|$ at $t = \frac{3}{4}T$. In Figure 2, **draw** bars to represent $|\mathcal{E}|$ at times $t = 0$, $\frac{1}{4}T$, and $\frac{1}{2}T$ relative to $|\mathcal{E}|$ shown at $\frac{3}{4}T$. If $|\mathcal{E}| = 0$, **write** a “0” in that column.

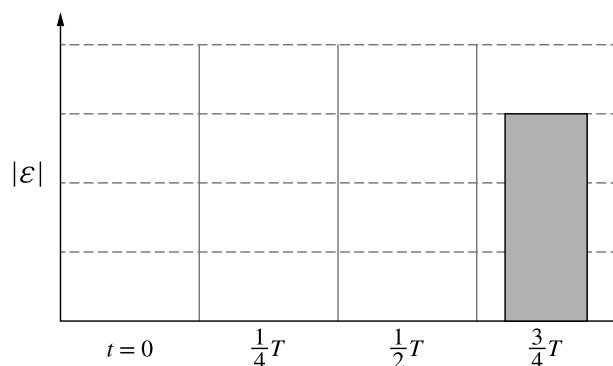


Figure 2

- B. **Derive** an expression for the maximum induced current in the loop in terms of A , R , B , ω , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.
- C. On the axes shown in Figure 3, **sketch** a graph of the instantaneous power P dissipated by the loop as a function of t during the time interval $0 \leq t \leq T$.

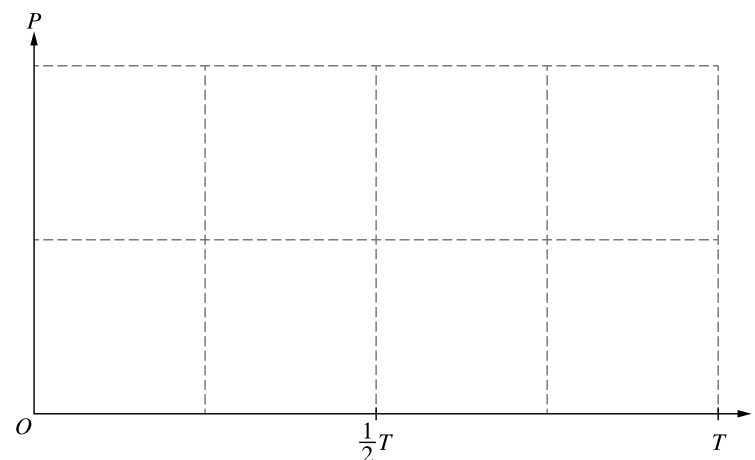


Figure 3

- D. **Indicate** whether the sketch you drew in part C is or is not consistent with the bars that you drew in part A. Briefly **justify** your answer by referencing the functional dependence between P and $|\mathcal{E}|$.

Begin your response to **QUESTION 3** on this page.

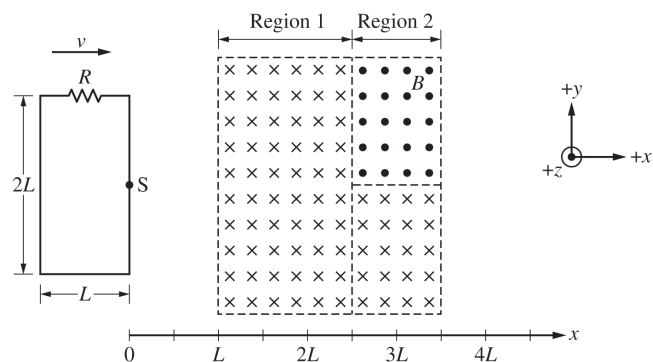


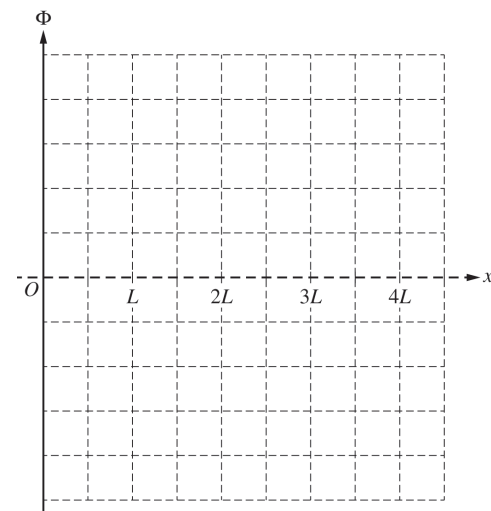
Figure 1

3. A wire is connected to a resistor of resistance R to form a rigid rectangular loop of width L and height $2L$. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$ -direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $-z$ -direction. Region 1 has boundaries $x = L$ and $x = 2.5L$. The loop later enters Region 2 with two external, uniform magnetic fields, each of magnitude B , that are parallel but are directed in opposite z -directions. Region 2 has boundaries $x = 2.5L$ and $x = 3.5L$. Point S is the midpoint of the leading edge of the loop and is aligned with the horizontal boundary in Region 2 that separates the two magnetic fields.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

- (a) On the following axes, **sketch** a graph of the magnetic flux Φ through the rectangular loop as a function of the position x of Point S from $x = 0$ to $x = 4.5L$. The $+z$ -direction indicated in Figure 1 corresponds to $+\Phi$.



- (b) Consider the instant when Point S reaches $x = 1.5L$.

- i. **Indicate** whether the current I_R that is induced in the rectangular loop when Point S reaches $x = 1.5L$ is clockwise, counterclockwise, or zero.

_____ Clockwise _____ Counterclockwise _____ Zero

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

ii. **Derive** an expression for I_R when Point S reaches $x = 1.5L$. If $I_R = 0$, indicate how the derived expression shows that $I_R = 0$. Express your answer in terms of R , L , v , B , and physical constants, as appropriate.

iii. **Derive** an expression for the power P dissipated by the resistor when Point S reaches $x = 1.5L$. Express your answer in terms of R , L , v , B , and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

The total energy dissipated by the resistor in the rectangular loop as Point S moves from $x = 0$ to $x = 4.5L$ is E_{original} .

The vertical boundary between regions 1 and 2 is now shifted to $x = 1.5L$. After the boundary is shifted, the rectangular loop again moves with speed v in the $+x$ -direction, as shown in Figure 2. The total energy dissipated by the resistor as Point S moves from $x = 0$ to $x = 4.5L$ is E_{new} .

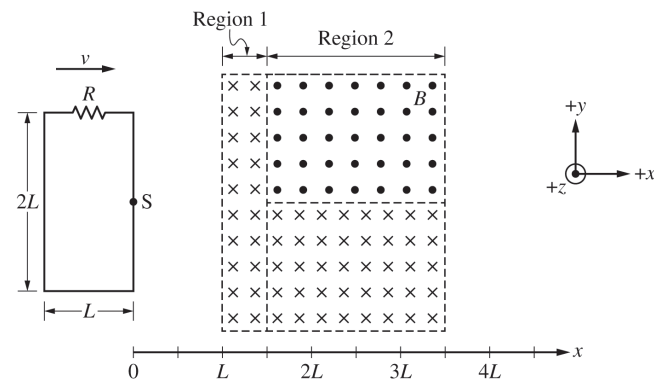


Figure 2

(c) **Indicate** whether E_{new} is greater than, less than, or equal to E_{original} .

☐ $E_{\text{new}} > E_{\text{original}}$ ☐ $E_{\text{new}} < E_{\text{original}}$ ☐ $E_{\text{new}} = E_{\text{original}}$

Briefly **justify** your answer.

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Continue your response to **QUESTION 3** on this page.

The original magnetic fields are modified so that the region $L < x < 3.5L$ contains an external uniform magnetic field of magnitude B that is directed in the $-z$ -direction.

A new wire is connected to a resistor of resistance R to form a rigid triangular loop with base length L and height $2L$. An external force is exerted on the loop so that the loop always moves with speed v in the $+x$ -direction, as shown in Figure 3. Point S represents the lower-leading corner of the loop.

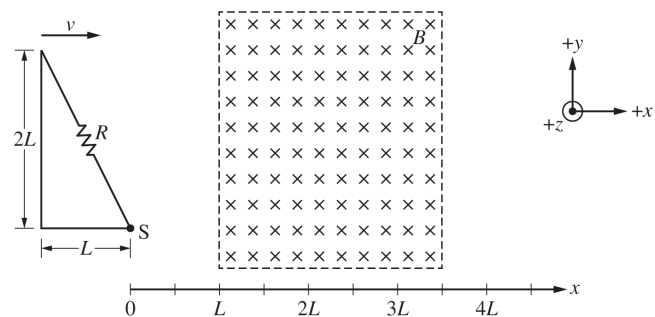
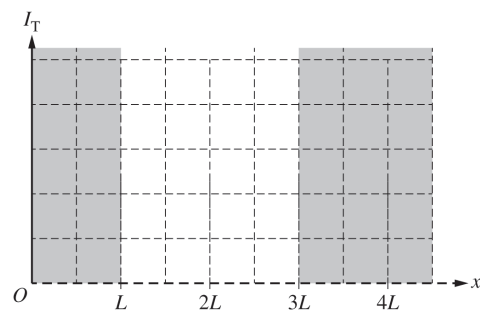


Figure 3

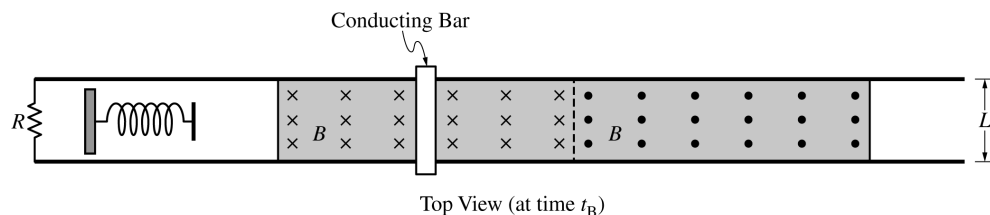
- (d) On the following axes, **sketch** a graph of the induced current I_T in the triangular loop as Point S moves from $x = L$ to $x = 3L$.



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STOP

END OF EXAM



Two horizontal, parallel, conducting rails are separated by distance $L = 0.40$ m. A resistor of resistance $R = 0.30 \, \Omega$ connects the rails. A horizontal ideal spring is located between the rails. The right end of the spring is free to move and the left end is fixed in place. A conducting bar of mass $m = 0.23$ kg is placed on the rails and is in contact with the spring, which is initially compressed. Frictional forces and the resistance of the bar and rails are negligible.

- At time $t = 0$, the bar is released from rest and is pushed to the right by the spring.
- At time t_1 , the bar loses contact with the spring and slides to the right.
- At time t_2 , the bar enters and travels through a uniform magnetic field of magnitude $B = 0.50$ T that is directed into the page, as shown.
- At time t_3 , the bar enters a region where the magnitude of the uniform magnetic field is still $B = 0.50$ T but is directed out of the page.
- At time t_4 , the bar enters a region with no magnetic field.

Consider time t_B such that $t_2 < t_B < t_3$.

- (a) On the following diagram of the bar, draw an arrow indicating the direction of the net force F_{net} exerted on the bar at time t_B . If the net force is zero, write $F_{\text{net}} = 0$.



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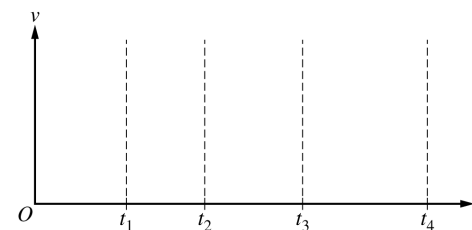
Continue your response to **QUESTION 2** on this page.

- (b) At time t_B , the speed of the bar is $v = 2.5$ m/s.

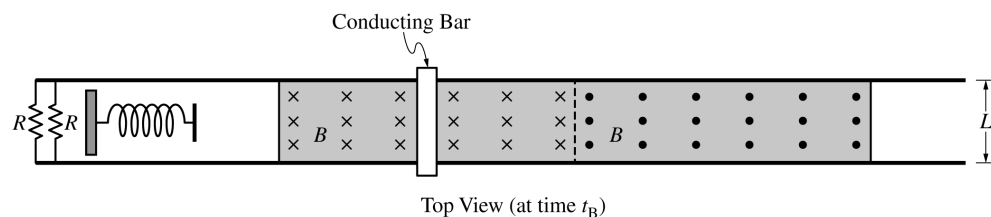
- i. Calculate the magnitude of the current in the bar at time t_B .

- ii. Calculate the magnitude of the net force F_{net} exerted on the bar at time t_B .

- (c) On the following axes, sketch a graph of the speed v of the bar as a function of time t between $t = 0$ and t_4 .



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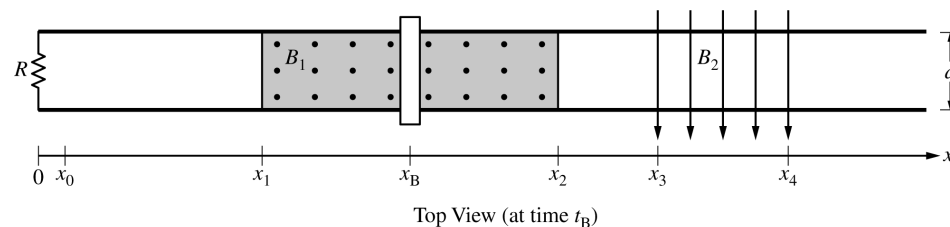
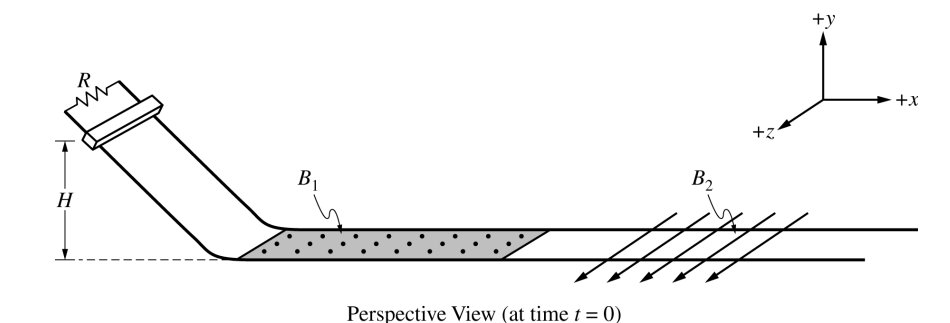
(d) The scenario is repeated but an additional resistor of resistance $R = 0.30 \, \Omega$ is connected, as shown.

i. Determine the total resistance R_{total} of the closed circuit for the new scenario.

ii. In the original scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{original} . In the new scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{new} . Is a_{new} greater than, less than, or equal to a_{original} ? Justify your answer.

(e) Describe a modification to m , B , or L that will result in a smaller induced potential difference across the original resistor immediately after the bar enters the first uniform magnetic field. Justify your answer.

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Two parallel conducting rails are separated by distance $d = 0.30 \, \text{m}$. A resistor of resistance $R = 0.20 \, \Omega$ connects the rails. A conducting bar is placed on a sloped section of the rails at height H above the horizontal section of the rails. Frictional forces and the resistances of the bar and rails are negligible.

- At time $t = 0$, the bar is released from rest from position x_0 and slides down the sloped section of the rails, as shown in the Perspective View.
- At time t_1 , the bar reaches position x_1 and smoothly transitions to the horizontal section of the rails and enters a uniform magnetic field of magnitude $B_1 = 0.40 \, \text{T}$ that is directed in the $+y$ -direction.
- At time t_2 , the bar reaches position x_2 and enters a region with no magnetic field.
- At time t_3 , the bar reaches position x_3 and enters a uniform magnetic field of magnitude $B_2 = 0.60 \, \text{T}$ that is directed in the $+z$ -direction.
- At time t_4 , the bar reaches position x_4 and enters a region with no magnetic field.

The bar is at position x_B (shown in Top View) at time t_B such that $t_1 < t_B < t_2$.

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- (a) On the following diagram of the bar, as observed from the Top View, draw an arrow indicating the direction of the net force F_{net} exerted on the bar at time t_B . If the net force is zero, write $F_{\text{net}} = 0$.

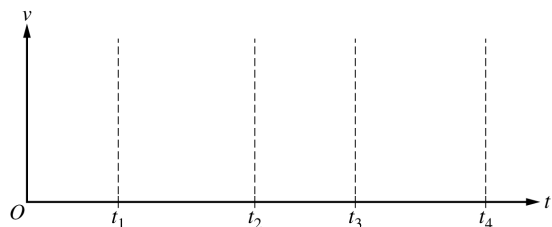


- (b) At time t_B , the speed of the bar is $v = 2.5$ m/s.

i. Calculate the magnitude of the current in the bar at time t_B .

ii. Calculate the magnitude of the net force F_{net} exerted on the bar at time t_B .

- (c) On the following axes, sketch a graph of the speed v of the bar as a function of time t between $t = 0$ and t_4 .



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Continue your response to Question 2 on this page.

- (d) The original scenario is repeated but with a new bar that has the same mass but with a nonnegligible resistance $R = 0.20 \, \Omega$. The new bar is released from rest and smoothly transitions to the horizontal section of the rails and enters the first uniform magnetic field.

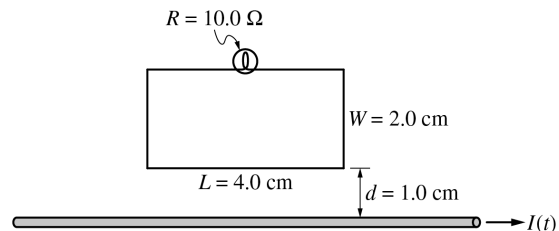
i. Determine the total resistance of the closed circuit.

ii. In the original scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{original} . In the new scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{new} . Is a_{new} greater than, less than, or equal to a_{original} ? Justify your answer.

- (e) Describe a modification to H , B_1 , or d that will result in a larger induced current in the new bar immediately after the bar enters the first uniform magnetic field. Justify your answer.

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Begin your response to QUESTION 3 on this page.



A lightbulb of resistance $R = 10.0 \, \Omega$ is connected to a rectangular loop of wire of negligible resistance near a very long current-carrying wire. The rectangular loop has a length $L = 4.0 \, \text{cm}$ and a width $W = 2.0 \, \text{cm}$ and is positioned so one of the longer sides of the loop is a distance $d = 1.0 \, \text{cm}$ above and parallel to the long wire, as shown. The current in the long wire is initially flowing to the right and is given by $I(t) = C - Dt$, where $C = 10.0 \, \text{A}$ and $D = 2.0 \, \text{A/s}$. At time $t = 5.0 \, \text{s}$, the current in the long wire is instantaneously zero as the current changes direction.

(a) What is the direction, if any, of the magnetic field produced by the induced current in the rectangular loop as the current in the long wire changes direction?

☐ Into the page ☐ Out of the page ☐ No direction, because the field is zero

Justify your answer.

(b) Calculate the magnetic flux through the loop due to only the long wire at time $t = 3.0 \, \text{s}$.

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Continue your response to **QUESTION 3** on this page.

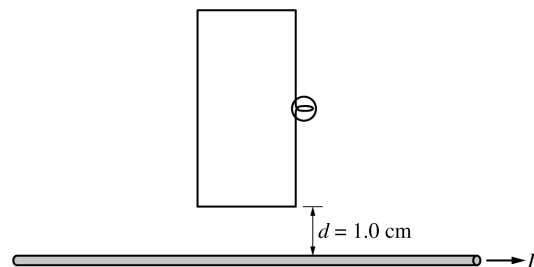
(c) Calculate the current through the lightbulb at time $t = 3.0 \, \text{s}$.

(d) A group of students attempts to experimentally verify whether the current through the lightbulb is consistent with the current calculation from part (c). The current in the rectangular loop is measured to be greater than the current calculated in part (c). Which of the following could explain this discrepancy? Select one answer.

- ☐ The students did not account for Earth's magnetic field.
☐ The rectangular loop is tilted and is not in the same plane as the wire.
☐ The resistance of the lightbulb is greater than the recorded value.
☐ The long side of the rectangular loop is shorter than the recorded value.
☐ The current in the long wire changes at a faster rate than expected.

Briefly justify your answer.

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Continue your response to **QUESTION 3** on this page.

- (e) Later, the same rectangular loop with lightbulb is rotated such that a short side of the loop is 1.0 cm above and parallel to the long current-carrying wire, as shown. The current in the wire is again initially flowing from left to right and given by $I(t) = C - Dt$, where $C = 10.0$ A and $D = 2.0$ A/s. The current through the lightbulb in the loop's new orientation at time $t = 3.0$ s is I_2 . Which of the following correctly relates the current I_2 to I_1 , the current through the lightbulb in part (c)?

☐ $I_2 < I_1$ ☐ $I_2 = I_1$ ☐ $I_2 > I_1$

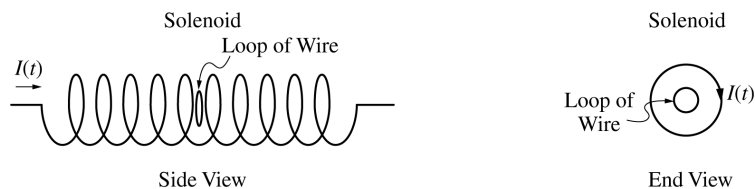
Justify your answer.

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END OF EXAM

Begin your response to QUESTION 3 on this page.

Note: Figures not drawn to scale.

A single loop of wire with resistance $3.0\ \Omega$ and radius $0.10\ \text{m}$ is placed inside a solenoid, with the normal to the loop parallel to the axis of the solenoid. The solenoid has 500 turns, is $0.25\ \text{m}$ long, and is connected to a power supply that is not shown. At time $t = 0$, the power supply is turned on, and the current I in the solenoid as a function of t is given by the equation $I(t) = \beta t$, where $\beta = 5.0\ \text{A/s}$. The direction of the current in the solenoid is clockwise, as shown in the end view.

- (a) At time $t = 2.0\ \text{s}$, is the induced current in the loop, as seen from the end view shown, clockwise, counterclockwise, or zero?

___ Clockwise ___ Counterclockwise ___ Zero

Justify your answer.

- (b) Calculate the current in the loop of wire at time $t = 2.0\ \text{s}$.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

- (c) Calculate the total energy dissipated by the loop of wire from time $t = 0$ to time $t = 2.0\ \text{s}$.

- (d) A group of students attempts to verify experimentally the calculation of the current from part (b). The current in the inner circular loop at time $t = 2.0\ \text{s}$ is measured to be less than the current calculated in part (b). Which of the following could explain this discrepancy? Select one answer.

- ___ The experiment did not account for Earth's magnetic field.
 ___ The plane of the loop is not perpendicular to the axis of the solenoid.
 ___ The center of the loop is not on the axis of the solenoid.
 ___ The resistance of the loop is less than the given value.
 ___ The radius of the loop is actually larger than $0.10\ \text{m}$.

Justify your answer.

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Continue your response to **QUESTION 3** on this page.

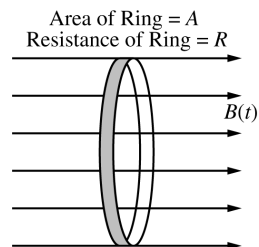
- (e) The power supply is now turned off. The original loop of wire is then replaced with a second loop made from wire that has the same thickness and is made from the same material as the original loop of wire. The second loop has radius 0.20 m, is placed in the same orientation as the original loop, and fits completely inside the solenoid. The power supply is turned on, and the current I in the solenoid as a function of t is again given by the equation $I(t) = \beta t$, where $\beta = 5.0 \text{ A/s}$. Which of the following expressions correctly indicates the ratio $\frac{I_2}{I_1}$ where I_1 represents the current induced in the original loop of wire in part (b) and I_2 represents the current induced in the second loop of wire?

$\frac{I_2}{I_1} = 1$ $1 < \frac{I_2}{I_1} < 2$ $\frac{I_2}{I_1} = 2$ $\frac{I_2}{I_1} > 2$

Justify your answer.

GO ON TO THE NEXT PAGE.**STOP****END OF EXAM**

Begin your response to QUESTION 3 on this page.



3. A thin, conducting ring of area A and resistance R is aligned in a uniform magnetic field directed to the right and perpendicular to the plane of the ring, as shown. At time $t = 0$, the magnitude of the magnetic field is B_0 . At $t = 1$ s, the magnitude of the magnetic field begins to decrease according to the equation $B(t) = \frac{\beta}{t}$, where β has units of T·s.

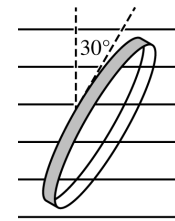
(a) Derive an equation for the magnitude of the induced current I in the ring as a function of t for $t > 1$ s. Express your answer in terms of β , A , R , t , and physical constants, as appropriate.

Assume $A = 0.50 \text{ m}^2$, $R = 2.0 \Omega$, and $\beta = 0.50 \text{ T} \cdot \text{s}$.

- (b) Calculate the electrical energy dissipated in the ring from $t = 1$ s to $t = 2$ s.

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Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.



The ring is then rotated so that the plane of the ring is aligned at a 30° angle to the magnetic field, as shown.

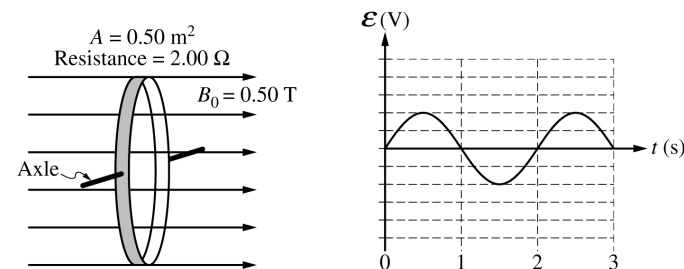
The magnitude of the magnetic field is reset to a magnitude of B_0 at a new time $t = 0$ and again begins to

decrease at $t = 1$ s according to the equation $B(t) = \frac{\beta}{t}$, where β has units of T·s.

- (c) Will the amount of energy dissipated in the ring from $t = 1$ s to $t = 2$ s be greater than, less than, or equal to the energy dissipated in part (b) ?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.



The ring is now mounted on an axle that is perpendicular to the magnetic field. The magnitude of the magnetic field is now held at a constant $B_0 = 0.50 \text{ T}$, as shown. The ring rotates about the axle, and the emf $\mathcal{E}$ induced in the ring as a function of time t is shown on the graph.

- (d) Calculate the angular speed ω of the rotating ring in rad/s.

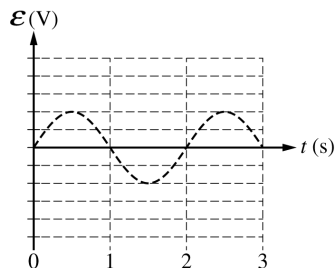
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(e) Calculate the magnitude of the maximum emf $\mathcal{E}_{\text{MAX}}$ induced in the ring.

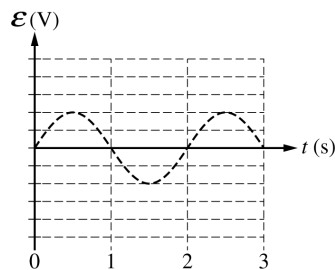
The ring now begins to rotate at an angular speed 2ω .

(f) On the graph below, draw a curve to indicate the new induced emf $\mathcal{E}$ in the ring. The dashed curve shows the emf induced under the original conditions.



Justify your sketch, specifically identifying and addressing any similarities or differences between the sketch and the original graph.

PRACTICE GRAPH - Use the graph below to practice your sketch for part (f). Any work shown on the graph below will NOT be graded.

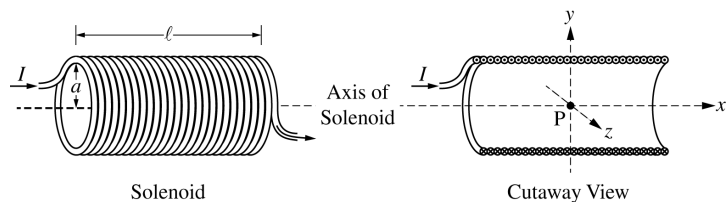


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Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.

STOP

END OF EXAM



Note: Figures not drawn to scale.

3. A solenoid is used to generate a magnetic field. The solenoid has an inner radius a , length ℓ , and N total turns of wire. A power supply, not shown, is connected to the solenoid and generates current I , as shown in the figure on the left above. The x -axis runs along the axis of the solenoid. Point P is in the middle of the solenoid at the origin of the xyz -coordinate system, as shown in the cutaway view on the right above. Assume $\ell \gg a$.

- (a) Select the correct direction of the magnetic field at point P.

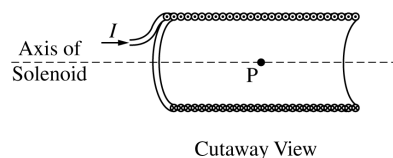
☐ $+x$ -direction ☐ $+y$ -direction ☐ $+z$ -direction

☐ $-x$ -direction ☐ $-y$ -direction ☐ $-z$ -direction

Justify your selection.

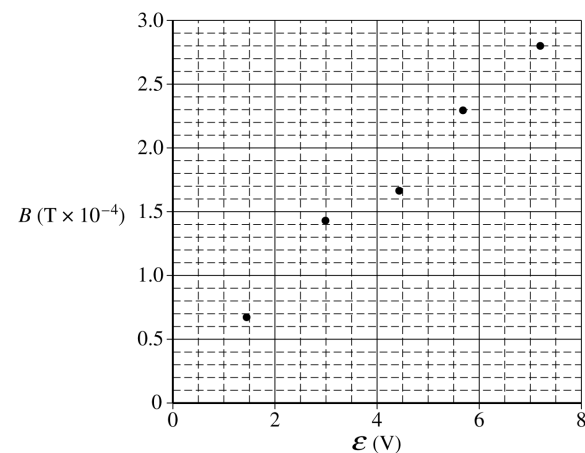
- (b)

- i. On the cutaway view below, clearly draw an Amperian loop that can be used to determine the magnetic field at point P at the center of the solenoid.



- ii. Use Ampere's law to derive an expression for the magnetic field strength at point P. Express your answer in terms of I , ℓ , N , a , and physical constants, as appropriate.

Some physics students conduct an experiment to determine the resistance R_S of a solenoid with radius $a = 0.015$ m, total turns $N = 100$, and total length $\ell = 0.40$ m. The students connect the solenoid to a variable power supply. A magnetic field sensor is used to measure the magnetic field strength along the central axis at the center of the solenoid. The plot of the magnetic field strength B as a function of the emf $\mathcal{E}$ of the power supply is shown below.



- (c)

- i. On the graph above, draw a best-fit line for the data.

- ii. Use the straight line to determine the resistance R_S of the solenoid used in the experiment.

- (d) One of the students notes that the horizontal component of the magnetic field of Earth is 2.5×10^{-5} T.

- i. Is there evidence from the graph that the horizontal orientation of the solenoid affects the measured values for B ?

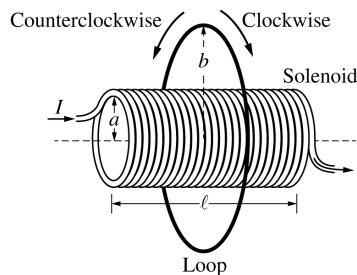
☐ Yes ☐ No

Justify your answer.

- ii. Would the horizontal orientation of the solenoid affect the calculated value for R_S ?

☐ Yes ☐ No

Justify your answer.



A thin conducting loop of radius b and resistance R_L is placed concentric with the solenoid, as shown above. The current in the solenoid is decreased from I to zero over time Δt .

(e)

- i. Is the direction of the induced current in the loop clockwise or counterclockwise during the time period that the current in the solenoid is decreasing?

_____ Clockwise _____ Counterclockwise

Justify your answer.

- ii. Derive an equation for the average induced current i_{IND} in the loop during the time period that the current in the solenoid is decreasing. Express your answer in terms of I , ℓ , N , a , b , R_L , R_S , Δt , and physical constants, as appropriate.

STOP
END OF EXAM

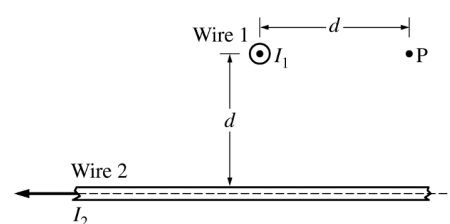


Figure 1. Side view

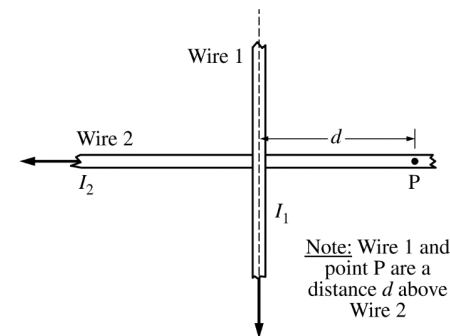


Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I , d , and physical constants, as appropriate.

- (a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.
(b) Derive an expression for the magnitude of the net magnetic field at point P.
(c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.
(d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2? Assume gravitational effects are negligible.

- _____ Wire 1 will not move.
_____ Wire 1 will move upward as viewed in Figure 1.
_____ Wire 1 will move downward as viewed in Figure 1.
_____ Wire 1 will rotate clockwise as viewed in Figure 2.
_____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

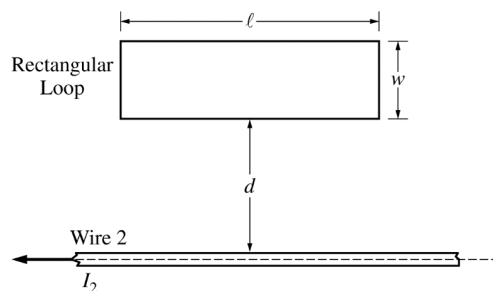


Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ , width w , and resistance R . The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1} .

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t .

_____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$ _____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

_____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$ _____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

(f) Given that the flux through the rectangular loop as a function of time t is given by the equation

$$\Phi = \frac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\frac{d + w}{d}\right),$$

derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0 , d , r , R , w , k , ℓ , and physical constants, as appropriate.

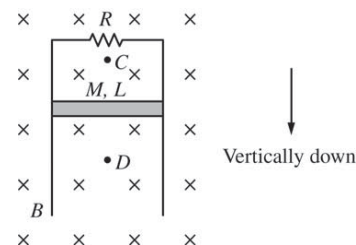
(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3?

_____ Clockwise _____ Counterclockwise

_____ Undefined, because there is no current induced in the loop

Justify your answer.

STOP
END OF EXAM



E&M.3.

A conducting bar of mass M , length L , and negligible resistance is connected to two long vertical conducting rails of negligible resistance. The two rails are connected by a resistor of resistance R at the top. The entire apparatus is located in a magnetic field of magnitude B directed into the page, as shown in the figure above. The bar is released from rest and slides without friction down the rails.

(a) What is the direction of the current in the resistor?

_____ Left _____ Right

(b)

i. Is the magnitude of the net magnetic field above the bar at point C greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

ii. While the bar is above point D , is the magnitude of the net magnetic field at point D greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Express your answers to parts (c) and (d) in terms of M , L , R , B , and physical constants, as appropriate.

(c) Write, but do NOT solve, a differential equation that could be used to determine the velocity of the falling bar as a function of time t .

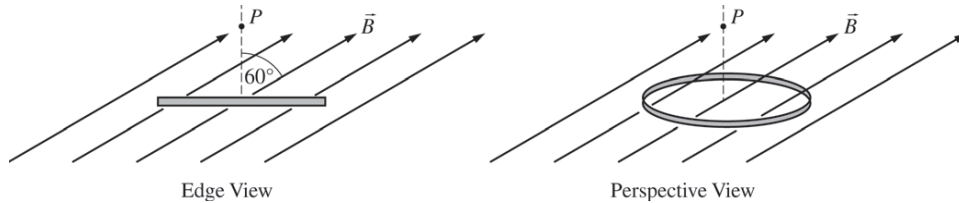
(d) Determine an expression for the terminal velocity v_T of the bar.

Express your answers to parts (e) and (f) in terms of v_T , M , L , R , B , and physical constants, as appropriate.

(e) Derive an expression for the power dissipated in the resistor when the bar is falling at terminal velocity.

(f) Using your differential equation from part (c), derive an expression for the speed of the falling bar $v(t)$ as a function of time t .

STOP
END OF EXAM



E&M. 3.

A circular wire loop with radius 0.10 m and resistance $50\ \Omega$ is held in place horizontally in a magnetic field $\vec{B}$ directed upward at an angle of 60° with the vertical, as shown in the figure above. The magnetic field in the direction shown is given as a function of time t by $B(t) = a(1 - bt)$, where $a = 4.0\text{ T}$ and $b = 0.20\text{ s}^{-1}$.

- (a) Derive an expression for the magnetic flux through the loop as a function of time t .
- (b) Calculate the numerical value of the induced emf in the loop.
- (c)
 - i. Calculate the numerical value of the induced current in the loop.
 - ii. What is the direction of the induced current in the loop as viewed from point P ?
 ___ Clockwise ___ Counterclockwise
 Justify your answer.
- (d) Assuming the loop stays in its current position, calculate the energy dissipated in the loop in 4.0 seconds.
- (e) Indicate whether the net magnetic force and net magnetic torque on the loop are zero or nonzero while the loop is in the magnetic field.
 Net magnetic force: ___ Zero ___ Nonzero
 Net magnetic torque: ___ Zero ___ Nonzero
 Justify both of your answers.

STOP
END OF EXAM