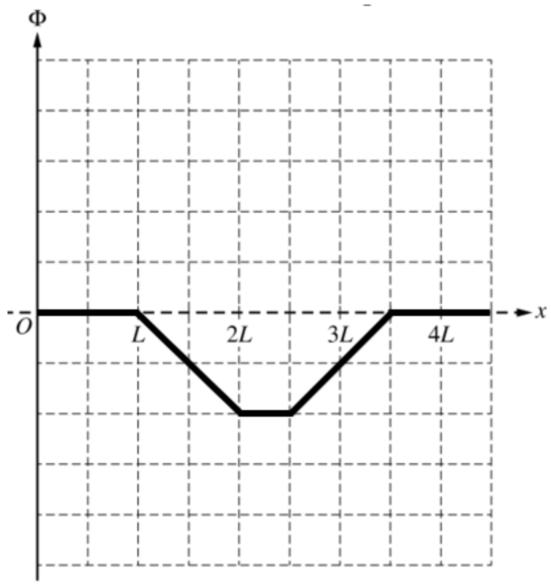
Question 3: Free-Response Question — 15 points

(a)	For a sketch that indicates that the magnetic flux is zero from $x = 0$ to $x = L$ and for the region $x > 3.5L$	**1 point**
	For a sketch that indicates that the absolute value of the magnetic flux increases with a constant slope in the region $L < x < 2L$	**1 point**
	For a sketch that indicates that the magnetic flux is constant and nonzero in the region $2L < x < 2.5L$	**1 point**
	For a sketch that indicates that the absolute value of the magnetic flux decreases from a nonzero value at $x = 2.5L$ to zero at $x = 3.5L$	**1 point**

Example Response



Scoring Note: A sketch that is reflected across the horizontal axis can earn the four points.
Scoring Note: The absolute values of the slopes in regions $L < x < 2D$ and $2.5L < x < 3.5L$ are not considered for earning points in the response in part (a).

Total for part (a) — 4 points

(b)(i)	For selecting Counterclockwise with an attempt at a relevant justification	**1 point**
	For indicating the magnetic flux through the loop is increasing in the $-z$-direction **OR** For indicating the magnetic field due to the induced current will be directed in the $+z$-direction	**1 point**

Example Response

Counterclockwise. The magnetic flux through the loop is increasing in the $-z$-direction. Therefore, a magnetic field is induced by the current in the loop to oppose the increasing magnetic flux. To establish this field, the current must be counterclockwise.

(b)(ii) For a multistep derivation that includes Faraday's law **1 point**

Scoring Note: The point can be earned if a negative sign is not included.

Example Response

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA)$$

For indicating $\frac{dA}{dt}$ is $2Lv$ **1 point**

Scoring Note: The point can be earned if a negative sign is included.

Example Response

$$\mathcal{E} = B\frac{dA}{dt} = 2BLv$$

For using Ohm's law, resulting in an expression for I_R that is consistent with the expression determined for the emf $\mathcal{E}$ **1 point**

Scoring Note: The point can be earned if a negative sign is included.

Example Response

$$I_\mathrm{R} = \frac{2BLv}{R}$$

Example Solution

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -\frac{d}{dt}\left(\int \vec{B} \cdot d\vec{A}\right)$$

$$\mathcal{E} = -\frac{d}{dt}(BA) = -\frac{d}{dt}(-B(2L)x)$$

$$\mathcal{E} = 2BL\frac{dx}{dt}$$

$$\mathcal{E} = 2BLv$$

$$I = \frac{\Delta V}{R}$$

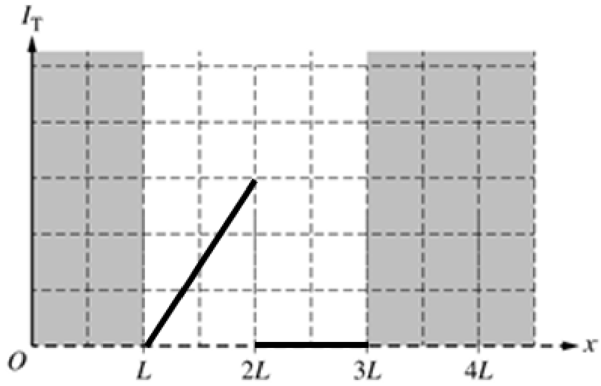
$$I_\mathrm{R} = \frac{\mathcal{E}}{R}$$

$$I_\mathrm{R} = \frac{2BLv}{R}$$

(b)(iii)	For using a correct general expression for P	**1 point**
	Example Responses $P = I^2R$ **OR** $P = \frac{(\Delta V)^2}{R}$ **OR** $P = I\Delta V$	
	For an expression for P that is consistent with the response in part (b)(ii) that is in terms of the provided quantities only	**1 point**
	Example Response $P = \frac{4B^2L^2v^2}{R}$	
	Example Solution $P = I^2R$ $P = \frac{(2BLv)^2}{R}$ $P = \frac{4B^2L^2v^2}{R}$	
	Total for part (b)	**7 points**
(c)	For selecting $E_{new} < E_{original}$ with an attempt at a relevant justification	**1 point**
	For indicating one of the following: • The change in magnetic flux is greater for the original scenario. • The induced current occurs for a greater amount of time in the original scenario. • The induced emf occurs for a greater amount of time in the original scenario.	**1 point**
	Example Response *The change in magnetic flux is greater in the original scenario, which produces an* emf *and current for a longer time. Therefore,* $E_{new} < E_{original}$.	
	Total for part (c)	**2 points**

(d)	For a sketch that has an absolute value that only increases from $x = L$ to $x = 2L$	**1 point**
	For a sketch that is zero from $2L$ to $3L$	**1 point**

Example Response



Scoring Note: A response that is reflected across the horizontal axis earns both points.
Scoring Note: Any portion of the graph before $x = L$ and after $x = 3L$ are not scored.

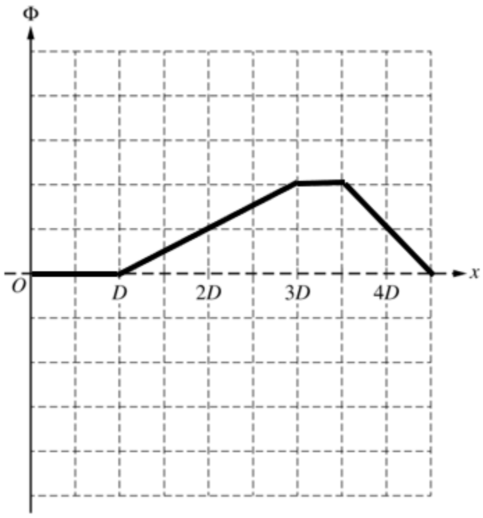
Total for part (d) 2 points

Total for question 3 15 points

Question 3: Free-Response Question — 15 points

(a)	For a sketch that indicates that the absolute value of the magnetic flux increases from zero at $x = D$ with a constant slope until $x = 2D$	**1 point**
	For a sketch that is continuous at $x = 2D$ and the absolute value of the magnetic flux increases until $x = 3D$ with the same slope as the slope in the region $D < x < 2D$	**1 point**
	For a sketch that indicates that the magnetic flux is constant and nonzero in the region $3D < x < 3.5D$	**1 point**
	For a sketch that indicates that the absolute value of the magnetic flux decreases from a nonzero value at $x = 3.5D$ to zero at $x = 4.5D$	**1 point**

Example Response



Scoring Note: A response that is reflected across the horizontal axis can earn the four points.
Scoring Note: The absolute values of the slopes in the entire regions $D < x < 3D$ and $3.5D < x < 4.5D$ are not considered for earning these points.

Total for part (a) — 4 points

(b)(i)	For selecting Clockwise with an attempt at a relevant justification	**1 point**
	For indicating the magnetic flux through the loop is increasing in the $+z$ -direction **OR** For indicating the magnetic field due to the induced current will be directed in the $-z$ -direction	**1 point**

Example Response

Clockwise. The magnetic flux through the loop is increasing in the $+z$ -direction. Therefore, a magnetic field is induced by the current in the loop to oppose the increasing magnetic flux. To establish this field, the current must be clockwise.

(b)(ii) For a multistep derivation that includes Faraday's law **1 point**

Scoring Note: The point can be earned if a negative sign is not included.

Example Response

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA)$$

For indicating $\frac{dA}{dt}$ is Dv **1 point**

Scoring Note: The point can be earned if a negative sign is not included.

Example Response

$$\mathcal{E} = -B\frac{dA}{dt} = -BDv$$

For using Ohm's law, resulting in an expression for I_S that is consistent with the expression determined for emf $\mathcal{E}$ **1 point**

Scoring Note: The point can be earned if a negative sign is not included.

Example Response

$$I_S = -\frac{BDv}{R}$$

Example Solution

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$
$$\mathcal{E} = -\frac{d}{dt}\left(\int \vec{B} \cdot d\vec{A}\right)$$
$$\mathcal{E} = -\frac{d}{dt}(BA) = -\frac{d}{dt}(B(D)x) = -BD\frac{dx}{dt}$$
$$\mathcal{E} = -BDv$$

$$I = \frac{\Delta V}{R}$$
$$I_S = \frac{\mathcal{E}}{R}$$
$$I_S = -\frac{BDv}{R}$$

(b)(iii)	For using a correct general expression for P	**1 point**

Example Responses

$P = I^2R$ **OR** $P = \frac{(\Delta V)^2}{R}$ **OR** $P = I\Delta V$

For an expression for power that is consistent with the response in part (b)(ii) that is in terms of the provided quantities only	**1 point**

Example Response

$$P = \frac{B^2D^2v^2}{R}$$

Example Solution

$$P = I^2R$$

$$P = \left(-\frac{BDv}{R}\right)^2 R$$

$$P = \frac{B^2D^2v^2}{R}$$

Total for part (b) 7 points

(c)	For selecting $E_{new} = E_{original}$ with an attempt at a relevant justification	**1 point**

For indicating one of the following **1 point**

- The change in magnetic flux is the same for both scenarios.
- The induced current occurs for the same amount of time for all transitions.
- The induced emf occurs for the same amount of time for all transitions

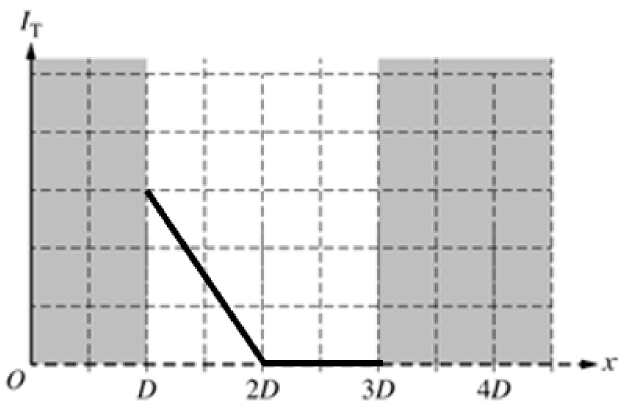
Example Response

The change in magnetic flux in the original scenario is the same as the new scenario, which produces an emf *and current that are the same in both scenarios. Therefore,* $E_{new} = E_{original}$

Total for part (c) 2 points

(d)	For a sketch which has an absolute value that only decreases from $x = D$ to $x = 2D$	**1 point**
	For a sketch that is zero from $x = 2D$ to $x = 3D$	**1 point**

Example Response



Scoring Note: A response that is reflected across the horizontal axis can earn both points.
Scoring Note: Any portion of the graph before $x = D$ and after $x = 3D$ will not be scored.

Total for part (d) 2 points

Total for question 3 15 points

Question 3: Free-Response Question — 15 points

(a) For selecting "Out of the page" and an attempt at a relevant justification — **1 point**

For a justification that correctly relates how the changing current in the long wire changes the flux through the loop with respect to time — **1 point**

For indicating the induced current will oppose the change in magnetic flux — **1 point**

Example Response

Because the current in the straight wire is decreasing, the magnetic field, which is originally pointing out of the page, is decreasing. Hence, the induced current produces a field that is directed out of the page to compensate for the decreasing flux.

Total for part (a) — 3 points

(b) For using an appropriate integral equation, with a substitution of an expression for magnetic field, to calculate magnetic flux — **1 point**

Example Response

$$\Phi_B = \int \vec{B} \bullet d\vec{A}$$

For writing a correct equation for the magnetic field as a function of distance from the wire — **1 point**

Example Response

$$B = \frac{\mu_0 I}{2\pi r}$$

For substituting the value of $t = 3\text{ s}$ to find the electric current — **1 point**

Example Response

$$I(t) = C - Dt$$
$$I(3\text{ s}) = 10\text{ A} - (2\text{ A/s})(3\text{ s})$$
$$I(3\text{ s}) = 4\text{ A}$$

For integrating B with correct limits and a correct substitution for dA, to determine the total flux through the loop — **1 point**

Example Response

$$\Phi_B = \int_d^{d+W} \frac{\mu_0 IL}{2\pi r} dr$$

$$\Phi_B = \frac{\mu_0 IL}{2\pi} \ln[r]_d^{d+W}$$

$$\Phi_B = \frac{\mu_0 IL}{2\pi} \ln\left[\frac{d+W}{d}\right]$$

$$\Phi_B = \frac{(4\pi \times 10^{-7}\text{ T}\bullet\text{m/A})(4\text{ A})(0.04\text{ m})}{2\pi} \ln\left[\frac{0.03\text{ m}}{0.01\text{ m}}\right] = 3.52 \times 10^{-8}\text{ T}\bullet\text{m}^2$$

Total for part (b) — 4 points

(c)	For using Faraday's law to determine the emf across the light bulb	**1 point**

Example Response

$$\varepsilon = \left|\frac{d\Phi}{dt}\right|$$

$$\varepsilon = \frac{d}{dt}\left(\frac{\mu_0 (C - Dt) L}{2\pi}\ln\left[\frac{d + W}{d}\right]\right)$$

$$\varepsilon = \left(\frac{\mu_0 DL}{2\pi}\ln\left[\frac{d + W}{d}\right]\right)$$

For using Ohm's law to find the current in the light bulb consistent with the emf determined from the previous point **1 point**

Example Response

$$I = \frac{\varepsilon}{R}$$

$$I = \left(\frac{\mu_0 DL}{2\pi R}\ln\left[\frac{d + W}{d}\right]\right)$$

For correct substitutions of the values of $\frac{dI}{dt}$ and R **1 point**

Example Response

$$I = \frac{(4\pi \times 10^{-7}\ \text{T•m/A})(2.0\ \text{A/s})(0.04\ \text{m})}{(2\pi•10\Omega)}\ln\left[\frac{0.03\ \text{m}}{0.01\ \text{m}}\right]$$

$$I = 1.8 \times 10^{-9}\ \text{A}$$

Total for part (c) 3 points

(d)	For selecting: "The current in the long wire changes at a faster rate than expected."	**1 point**
	For correctly justifying the selection	**1 point**

Scoring Note: A response cannot earn this point if the incorrect selection is chosen.

Example Response

If the current in the wire changes at a faster rate, there will be a greater rate of change of magnetic flux. So the induced emf and current will be higher.

Total for part (d) 2 points

(e)	For selecting "$I_2 < I_1$" with an attempt at a relevant justification	**1 point**
	For indicating the total flux in the loop is less in the new orientation	**1 point**
	For correctly relating the rate of change of the flux to the total flux inside the loop	**1 point**

Example Response

With the new orientation, some parts of the rectangle are further away from the straight wire, which means that the magnetic flux through the rectangle will be less. The rate of change of the flux has the same dependence on distance and will also decrease, resulting in a smaller current.

Total for part (e) 3 points

Total for question 3 15 points

Question 3: Free-Response Question — 15 points

(a)	For selecting "Counterclockwise" and an attempt at a relevant justification	**1 point**
	For a justification that correctly relates how the changing current in the solenoid changes the flux through the loop with respect to time	**1 point**
	For a justification that has a correct relationship between the change in magnetic flux through the loop of wire and how the current in the loop changes to oppose the change in magnetic flux	**1 point**

Example Response

In the end view, the magnetic field due to the solenoid is directed into the page. The current in the solenoid is increasing; thus, the magnetic flux is increasing. According to Lenz's law, because the magnetic flux is increasing and into the page, the current in the loop must create a magnetic field directed out of the page; thus, the current in the loop must be counterclockwise the counter the change in magnetic flux.

Total for part (a) — 3 points

(b) For using an appropriate equation to calculate emf in the loop — **1 point**

Example Response

$$\varepsilon = \left|\frac{d\Phi}{dt}\right| = \frac{d(BA)}{dt} = A\frac{dB}{dt}$$

For correctly substituting the magnetic field for a solenoid into above equation — **1 point**

Example Response

$$\varepsilon = A\frac{d(\mu_0 nI)}{dt} = \mu_0 nA\frac{dI}{dt}$$

For using Ohm's law to calculate the current in the loop — **1 point**

Example Response

$$I_{\text{loop}} = \frac{\varepsilon}{R} = \frac{\mu_0 nA}{R}\frac{dI}{dt}$$

For correctly substituting into the previous equation for the induced current in the loop — **1 point**

Example Response

$$I_{\text{loop}} = \frac{\left(4\pi\times10^{-7}\ \text{T}\bullet\text{m/A}\right)\left(\dfrac{500\ \text{turns}}{0.25\ \text{m}}\right)\left((\pi)(0.10\ \text{m})^2\right)}{(3\ \Omega)}(5.0\ \text{A/s}) = 1.3\times10^{-4}\ \text{A}$$

Total for part (b) — 4 points

(c)	For using an appropriate expression for the power for a constant current through a resistor **Example Response** $P = I_{\text{loop}}^{\ 2} R$	**1 point**
	For a correct substitution consistent with (b) into a correct expression for electrical power **Example Response** $P = I_{\text{loop}}^{\ 2} R = \left(\dfrac{\mu_0 n A}{R}\dfrac{dI}{dt}\right)^2 R = \left(1.3\times10^{-4}\ \text{A}\right)^2 (3.0\ \Omega) = 5.2\times10^{-8}\ \text{W}$	**1 point**
	For correctly substituting the power dissipated by the loop of wire into a correct energy equation **Example Response** $P = \dfrac{dE}{dt} = \dfrac{\Delta E}{\Delta t} \therefore \Delta E = P\Delta t$ $\Delta E = \left(5.2\times10^{-8} W\right)(2.0\ \text{s}) = 1.0\times10^{-7}\ \text{J}$	**1 point**
	Total for part (c)	**3 points**
(d)	For selecting "The plane of the loop is not perpendicular to the axis of the solenoid."	**1 point**
	For correctly justifying the selection **Scoring Note:** A response cannot earn this point if the incorrect selection is chosen.	**1 point**
	Example Response *If the plane of the loop is not perpendicular to the axis of the solenoid it is not perpendicular to the magnetic field. Therefore, the magnetic flux and emf will be less, so the current will be less.*	
	Total for part (d)	**2 points**
(e)	For selecting "$\dfrac{I_2}{I_1} = 2$" with an attempt at a relevant justification **Scoring Note:** The response can earn this point if selecting "$\dfrac{I_2}{I_1} > 2$" and only referencing the increase in flux through the loop in the justification.	**1 point**
	For a statement that indicates that the resistance of the second loop is double the resistance of the original loop	**1 point**
	For a statement that indicates that the new emf is four times greater than the original emf due to the increased flux through the loop resulting from the quadrupled area	**1 point**

Example Response

The magnetic flux quadruples. This is because of the increase in area of the loop, which quadruples the emf. The resistance of the loop doubles because the length of the wire doubles. Therefore, $\frac{I_2}{I_1} = 2$.

$$\varepsilon = \mu_0 n\left(A_{loop}\right) = \mu_0 n\left(\pi r^2\right)\frac{dI}{dt}$$

The radius of the loop doubles. Therefore, the area of the loop quadruples.

$$R = \rho\frac{2\pi r}{A_{wire}}$$

The radius of the loop doubles. Therefore, the resistance of the loop doubles.

$$\frac{I_2 = \frac{\varepsilon}{R} = \frac{\mu_0 n(4)A}{(2)R}\frac{dI}{dt}}{I_1 = \frac{\varepsilon}{R} = \frac{\mu_0 nA}{R}\frac{dI}{dt}} = 2$$

Total for part (e) 3 points

Total for question 3 15 points

Question 3: Free-Response Question — 15 points

(a) For using Faraday's law to relate emf to change in flux — **1 point**

$$\mathcal{E} = \left|\frac{d\Phi}{dt}\right| = \left|\frac{d(B \cdot A)}{dt}\right|$$

For a correct expression for the emf — **1 point**

$$\mathcal{E} = \left|A\frac{dB}{dt}\right| = \left|A\frac{d}{dt}\left(\frac{\beta}{t}\right)\right| = \left|-\frac{A\beta}{t^2}\right| = \frac{A\beta}{t^2}$$

For correctly substituting into Ohm's law — **1 point**

$$I = \frac{\Delta V}{R} = \frac{\mathcal{E}}{R} = \frac{A\beta}{Rt^2}$$

Scoring note: Full credit is earned if the negative sign is included.

Total for part (a) — 3 points

(b) For using a correct equation to calculate the energy dissipated in the ring — **1 point**

$$E = \int P dt = \int I^2 R dt$$

For correctly substituting into the equation above — **1 point**

$$E = \int \left(\frac{A\beta}{Rt^2}\right)^2 R dt = \int \frac{A^2\beta^2}{R^2 t^4} R dt$$

For indicating the correct limits or constant of integration — **1 point**

$$E = \frac{A^2\beta^2}{R}\int_1^2 \frac{1}{t^4} dt$$

For a correct answer with units — **1 point**

$$E = \frac{A^2\beta^2}{R}\left[-\frac{1}{3t^3}\right]_1^2 = \frac{\left(0.50\ \text{m}^2\right)^2 (0.50\ \text{T}\bullet\text{s})^2}{(3)(2.00\ \Omega)}\left(-\frac{1}{2^3} - \left(-\frac{1}{1^3}\right)\right) = 0.0091\ \text{J}$$

Total for part (b) — 4 points

(c) For selecting "Less than" and attempting a relevant justification — **1 point**

For a correct justification — **1 point**

Example response for part (c)

After the ring rotates, fewer magnetic field lines pass through the ring, and the flux is less than before. Thus, the emf, current, and rate at which energy is dissipated is less.

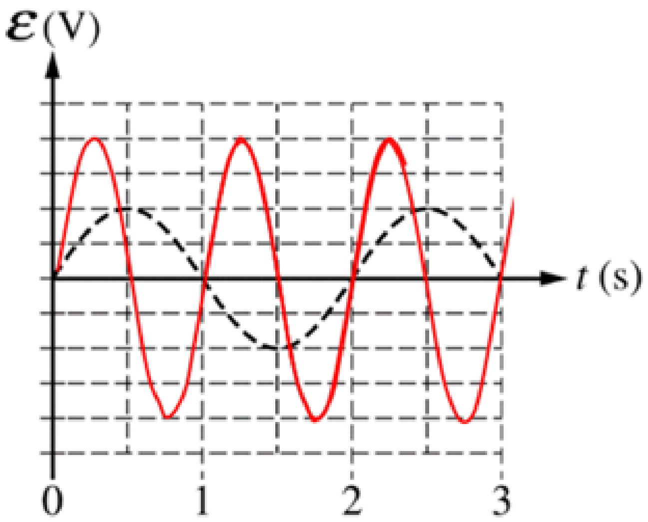
Total for part (c) — 2 points

(d) For using a correct equation to calculate angular speed — **1 point**

$$\omega = \frac{2\pi}{T}$$

	For substituting appropriate values from the graph in the correct equation $\omega = \frac{(2\pi)}{(2\text{ s})} = 3.14\ \frac{\text{rad}}{\text{s}}$	**1 point**
	Total for part (d)	**2 points**
(e)	For using a correct equation for solving for the emf in terms of the angular speed $\mathcal{E} = \frac{d\Phi}{dt} = \frac{d(B \cdot A)}{dt} = BA\frac{d}{dt}(\cos(\omega t)) = -BA\omega\sin(\omega t)$	**1 point**
	For correctly substituting into the correct equation $\mathcal{E}_{\text{MAX}} = BA\omega = (0.50\text{ T})\left(0.50\text{ m}^2\right)\left(3.14\ \frac{\text{rad}}{\text{s}}\right) = 0.79\text{ V}$	**1 point**
	Total for part (e)	**2 points**
(f)	For a sine curve with half the period of the dashed curve and a correct justification	**1 point**
	For a sine curve with twice the amplitude of the dashed curve and a correct justification **OR** a sine curve with an amplitude consistent with work shown in part (e)	**1 point**

Example response for part (f)



Because the rotational speed is twice the original speed, then, according to the equation $|\mathcal{E}| = B_0 A\omega\sin(\omega t)$ *this will double the amplitude of the curve and cut the period in half.*

Total for part (f) **2 points**

Total for question 3 **15 points**

2019

AP® CollegeBoard

AP® Physics C: Electricity and Magnetism

Scoring Guidelines Set 1

2019 SCORING GUIDELINES

General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

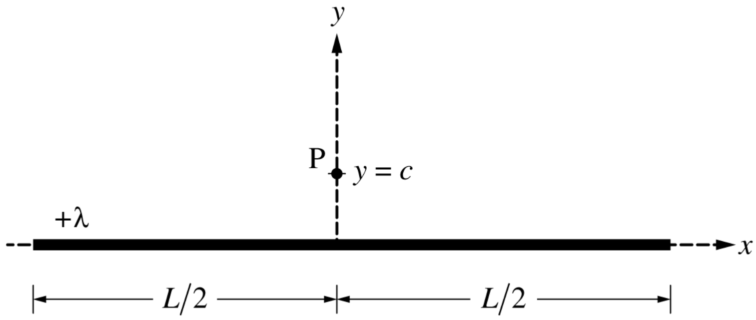
5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

2019 SCORING GUIDELINES

Question 1

15 points



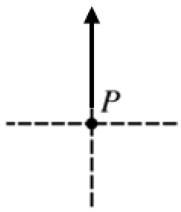
Note: Figure not drawn to scale.

A very long, thin, nonconducting cylinder of length L is centered on the y-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y-axis at $y = c$, where $L >> c$.

(a)

i. LO CNV-3.B.a, SP 7.A
1 point

On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the long cylinder. The arrow should start on and point away from the dot.



For drawing an arrow at point P that points upward		1 point

ii. LO CNV-2.F, SP 7.C
1 point

Describe the shape and location of a Gaussian surface that can be used to determine the electric field at point P due to the long cylinder.

For describing a Gaussian surface that could be used to determine the electric field at point P		1 point
Example: Drawing a cylinder that is coaxial with the thin cylinder and whose surface contains point P can be used to determine the electric field at point P.		
Note: Credit is earned if the student draws the correct surface on the figure.		

2019 SCORING GUIDELINES

Question 1 (continued)

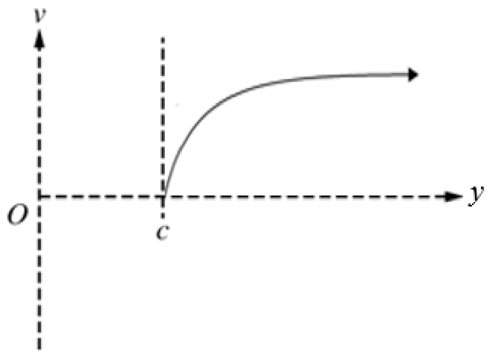
(a) continued
iii. LO CNV-2.C, SP 5.A, 5.E
3 points

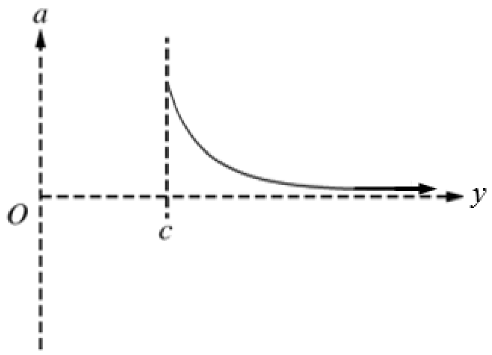
Use your Gaussian surface to derive an expression for the magnitude of the electric field at point P. Express your answer in terms of λ, c, L, and physical constants, as appropriate.

For using Gauss's law to determine the electric field at point P		1 point
$\frac{q_{enc}}{\varepsilon_0} = \int E \bullet dA \therefore \frac{Q}{\varepsilon_0} = EA$		
For correctly substituting for the charge into the equation above		1 point
For correctly substituting for the area or into the equation above		1 point
$\frac{\lambda L}{\varepsilon_0} = E(2\pi c L) \therefore E = \frac{\lambda}{2\pi\varepsilon_0 c} = \frac{2k\lambda}{c}$		

(b) LO ACT-1.D, SP 3.C
2 points

A proton is released from rest at point P. On the axes below, sketch the velocity v as a function of <u>position</u> y and the acceleration a as a function of <u>position</u> y for the proton.

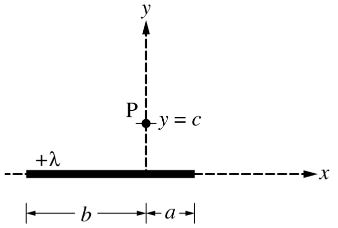




For a concave down graph for v as a function of position x that does not start at the origin		1 point
For a concave up graph for a as a function of position x that has an asymptote at the horizontal axis		1 point

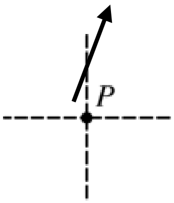
Question 1 (continued)

The original cylinder is now replaced with a much shorter thin, nonconducting cylinder with the same uniform linear charge density $+\lambda$, as shown in the figure below. The length of the cylinder to the right of the y-axis is a, and the length of the cylinder to the left of the y-axis is b, where $a < b$.



(c) LO CNV-3.B.a, SP 7.A
2 points

On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the shorter cylinder. The arrow should start on and point away from the dot.



For drawing an arrow at point P that points to the right		1 point
For drawing an arrow at point P that points up and to the right		1 point

(d)
i. and ii. LO CNV-2.F, SP 7.A
1 point

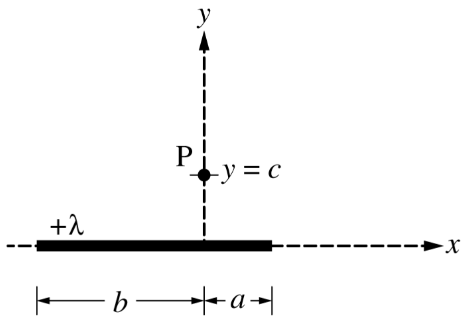
Is there a single Gaussian surface that can be used with Gauss's law to derive an expression for the electric field at point P?

____ Yes   ____ No

If your answer to part (d)(i) is yes, explain how you can use Gauss's law to derive an expression for the field at point P. If your answer to part (d)(i) is no, explain why Gauss's law cannot be applied to derive an expression for the electric field in this case.

For selecting "No" with a valid explanation		1 point
Claim: Select "No." Evidence: The length of the cylinder is not much greater than the distance from the cylinder to point P and the charge distribution is asymmetric. Reasoning: Therefore, cannot use the approximation of the constant magnitude of electric field over a cylindrical surface.		

Question 1 (continued)



Note: This figure is shown again for reference.

A student in class argues that using the integral shown below might be a useful approach for determining the electric field at point P.

$$E = \int \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\,dq$$

The student uses this approach and writes the following two integrals for the magnitude of the horizontal and vertical components of the electric field at point P.

$$\text{Horizontal component: } \left|E_x\right| = \frac{\lambda}{4\pi\varepsilon_0}\int_{-b}^{a}\frac{x}{\left(c^2+x^2\right)^{3/2}}\,dx$$

$$\text{Vertical component: } \left|E_y\right| = \frac{\lambda}{4\pi\varepsilon_0}\int_{-b}^{a}\frac{y}{\left(c^2+x^2\right)}\,dy$$

(e)

i. LO CNV-3.A, SP 7.A
1 point

One of the two expressions above is not correct. Which expression is not correct?

____ Horizontal component ____ Vertical component

For correctly selecting "Vertical component"		1 point

Question 1 (continued)

(e) continued

ii. LO CNV-3.A, SP 7.D

4 points

Identify two mistakes in the incorrect expression, and explain how to correct the mistakes.

For indicating the integral is not along the length of the cylinder		1 point
For an appropriate correction		1 point
Claim: Change *dy* to *dx*. Evidence: The integral is not along the cylinder. Reasoning: This change will make the integral valid.		
For indicating the power on the denominator term for the vertical component is incorrect		1 point
For an appropriate correction		1 point
Claim: The power on the term in the denominator should be $3/2$. Evidence: The units of the integrand are not valid. Reasoning: This change will make the integrand valid.		

Learning Objectives

ACT-1.D: Determine the motion of a charged object of specified charge and mass under the influence of an electrostatic force.
CNV-2.C: State and use Gauss's law in integral form to derive unknown electric fields for planar, spherical, or cylindrically symmetrical charge distributions.
CNV-2.F: Describe the general features of an unknown charge distribution, given other features of the system.
CNV-3.A: Derive expressions for the electric field of specified charge distributions using integration and the principle of superposition. Examples of such charge distributions include a uniformly charged wire, a thin ring of charge (along the axis of the ring), and a semicircular or part of a semicircular arc.
CNV-3.B.a: Identify and qualitatively describe situations in which the direction and magnitude of the electric field can be deduced from symmetry considerations and understanding the general behavior of certain charge distributions.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.
7.D: Provide reasoning to justify a claim using physical principles or laws.

2019 SCORING GUIDELINES

Question 2

15 points

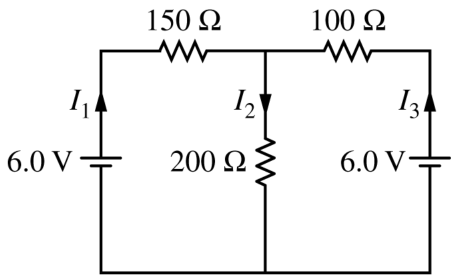


Figure 1

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(a)

i. LO CNV-6.F.b, SP 6.A
3 points

Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

For an equation representing the sum of the currents at one of the junctions		1 point
$I_1 - I_2 + I_3 = 0$		
For an equation representing the sum of the potential differences around one of the loops		1 point
For an equation representing the sum of the potential differences around a loop different from the above loop		1 point
$6 - 150I_1 - 200I_2 = 0$		
$6 - 100I_3 - 200I_2 = 0$		
$6 - 150I_1 + 100I_3 - 6 = 0$		
Note: Full credit is earned for two correct loop equations using loop currents.		

ii. LO CNV-6.F.b, SP 6.C
2 points

Calculate the current in the $200\ \Omega$ resistor.

For combining the equations from part (a)(i)		1 point
$I_1 - I_2 + I_3 = 0$ $-I_1 - 1.33I_2 = -.04 \therefore -4.33I_2 = -0.10$ $-I_3 - 2I_2 = -.06$		
Note: Credit is earned if students indicate they used a calculator function to solve the system of equations.		
For a correct answer with correct units		1 point
$-4.33I_2 = -0.10 \therefore I_2 = 0.023\ \text{A}$		

Question 2 (continued)

(a) continued

iii. LO CNV-5.A.a, SP 6.C
1 point

Calculate the power dissipated by the $200\ \Omega$ resistor.

For using a correct equation to calculate the power in the $200\ \Omega$ resistor		1 point
$P = I^2R = (0.023\ \text{A})^2(200\ \Omega)\ \therefore\ P = 0.107\ \text{W}$		

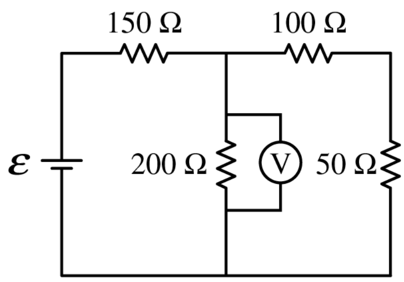


Figure 2

The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

(b) LO CNV-6.C.a, SP 6.B, 6.C
2 points

Calculate the current through the $50\ \Omega$ resistor.

For correctly calculating the equivalent resistance of the branch with the $50\ \Omega$ resistor		1 point
$R = 100\ \Omega + 50\ \Omega = 150\ \Omega$		
For using the correct potential difference in Ohm's law to calculate the current in the $50\ \Omega$ resistor		1 point
$I = \dfrac{V}{R} = \dfrac{(4.4\ \text{V})}{(100\ \Omega + 50\ \Omega)}\ \therefore\ I = 0.029\ \text{A}$		

Question 2 (continued)

(c) LO CNV-6.C.a, SP 6.A, 6.C
3 points

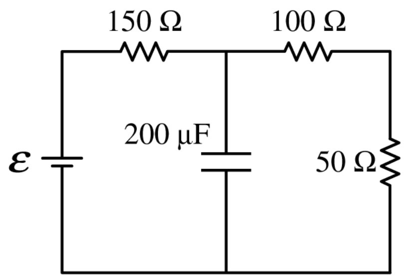
Calculate the voltage ε of the battery.

For using a correct equation to determine the current through the $150\ \Omega$ resistor		1 point
For correctly substituting the current from part (b) as I_3		1 point
$I_1 = I_2 + I_3 = \dfrac{(4.4\text{ V})}{(200\ \Omega)} + 0.029\text{ A} = 0.051\text{ A}$		
For using a correct equation to determine the emf of the battery		1 point
$\varepsilon = I_1R_1 + V_2 = (0.051\text{ A})(150\ \Omega) + 4.4\text{ V} = 12.1\text{ V}$		
Alternate Third Point		
For calculating the equivalent resistance of the circuit and substituting this resistance into a correct equation to determine the emf of the battery		*1 point*
$R_T = 150\ \Omega + \dfrac{1}{\dfrac{1}{200\ \Omega} + \dfrac{1}{150\ \Omega}} = 236\ \Omega$		
$\varepsilon = I_1R_T = (0.051\text{ A})(236\ \Omega) = 12.0\text{ V}$		
Alternate Solution		*Alternate Points*
For using a correct equation to determine the equivalent resistance of the parallel resistors		*1 point*
$\dfrac{1}{R_P} = \dfrac{1}{200\ \Omega} + \dfrac{1}{150\ \Omega}\ \therefore R_P = 86\ \Omega$		
For correctly substituting the given potential difference and the calculated equivalent resistance to determine the total current of the circuit		*1 point*
$I_T = \dfrac{(4.4\text{ V})}{(86\ \Omega)} = 0.051\text{ A}$		
For calculating the equivalent resistance of the circuit and substituting into a correct equation to determine the emf of the battery		*1 point*
$R_T = 150\ \Omega + 86\ \Omega = 236\ \Omega$		
$\varepsilon = I_1R_T = (0.051\text{ A})(236\ \Omega) = 12.0\text{ V}$		

Question 2 (continued)

(d)

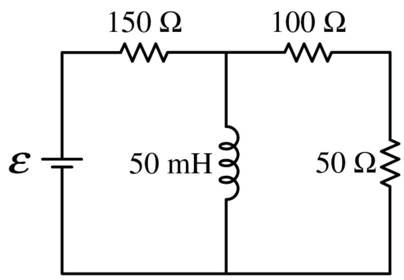
i. LO CNV-7.B.a, SP 6.A, 6.C
2 points



The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu F$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.

For substituting the voltage consistent with part (c) into Ohm's law		1 point
For correctly calculating the equivalent resistance of the circuit		1 point
$I = \dfrac{\mathcal{E}}{R_{tot}} = \dfrac{(12.1\ \text{V})}{(150\ \Omega + 100\ \Omega + 50\ \Omega)} = 40.3\ \text{mA}$		

ii. LO CNV-10.C.a, SP 7.A, 7.C
2 points



The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b)?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

For correctly selecting "Less than" with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because steady state is reached, the inductor will act as a short circuit. So all the current will pass through the inductor and no current will pass through the $50\ \Omega$ resistor.		

Question 2 (continued)

Learning Objectives

CNV-5.A.a: Derive expressions that relate current, voltage, and resistance to the rate at which heat is produced in a resistor.
CNV-6.C.a: Calculate voltage, current, and power dissipation for any resistor in a circuit containing a network of known resistors with a single battery or energy source.
CNV-6.F.b: Set up simultaneous equations to calculate at least two unknowns (currents or resistance values) in a multi-loop circuit.
CNV-7.B.a: Calculate the potential difference across a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.
CNV-10.C.a: Calculate initial transient currents and final steady-state currents through any part of a series or parallel circuit containing an inductor and one or more resistors.

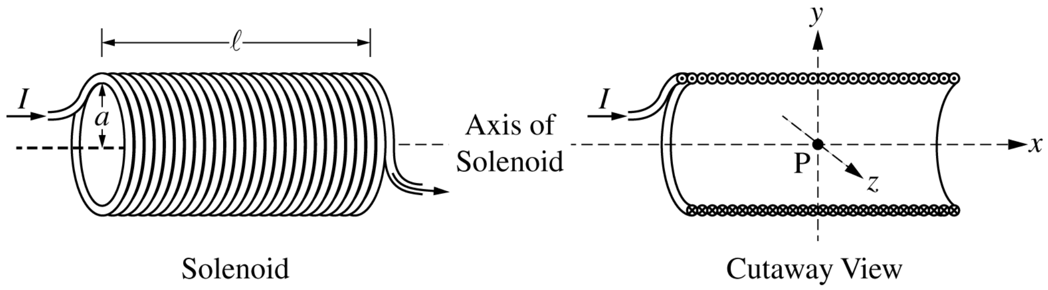
Science Practices

6.A: Extract quantities from narratives or mathematical relationships to solve problems.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

2019 SCORING GUIDELINES

Question 3

15 points



Note: Figures not drawn to scale.

A solenoid is used to generate a magnetic field. The solenoid has an inner radius a, length ℓ, and N total turns of wire. A power supply, not shown, is connected to the solenoid and generates current I, as shown in the figure on the left above. The x-axis runs along the axis of the solenoid. Point P is in the middle of the solenoid at the origin of the xyz-coordinate system, as shown in the cutaway view on the right above. Assume $\ell >> a$.

(a) LO CNV-8.E.a, SP 7.A, 7.C
2 points

Select the correct direction of the magnetic field at point P.

____ $+x$-direction ____ $+y$-direction ____ $+z$-direction
____ $-x$-direction ____ $-y$-direction ____ $-z$-direction

Justify your selection.

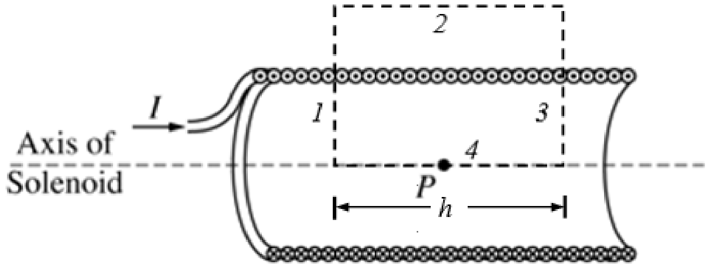
For choosing the "$+x$-direction" and providing a justification		1 point
For a correct justification		1 point
Example: Using the right-hand rule for current on the left side of the solenoid, the fingers curl into the loop, so the magnetic field points to the right, or in the $+x$-direction.		
Example: Using the right-hand rule for solenoids, when the fingers curl around the solenoid in the direction of the current, the thumb points to the right, therefore the magnetic field is to the right, or in the $+x$-direction.		

Question 3 (continued)

(b)

i. LO CNV-8.C.c, SP 3.D
1 point

On the cutaway view below, clearly draw an Amperian loop that can be used to determine the magnetic field at point P at the center of the solenoid.



For drawing a rectangle with one side along the central axis of the solenoid and another side outside the solenoid and whose edges do not extend beyond the solenoid		1 point

ii. LO CNV-8.C.c, SP 5.A, 5.E
2 points

Use Ampere's law to derive an expression for the magnetic field strength at point P. Express your answer in terms of I, ℓ, N, a, and physical constants, as appropriate.

For using Ampere's law to calculate the magnetic field along the axis of the solenoid		1 point
$\int B \bullet d\ell = \mu_0 I_{enc} \therefore \left(\int B \bullet d\ell\right)_1 + \left(\int B \bullet d\ell\right)_2 + \left(\int B \bullet d\ell\right)_3 + \left(\int B \bullet d\ell\right)_4 = \mu_0 I_{enc}$		
$\left(\int B \bullet d\ell\right)_1 + 0 + \left(-\left(\int B \bullet d\ell\right)_1\right) + Bh = \mu_0 \frac{N}{\ell} hI$		
$Bh = \mu_0 \frac{N}{\ell} hI$		
For a correct answer		1 point
$B = \frac{\mu_0 NI}{\ell}$		

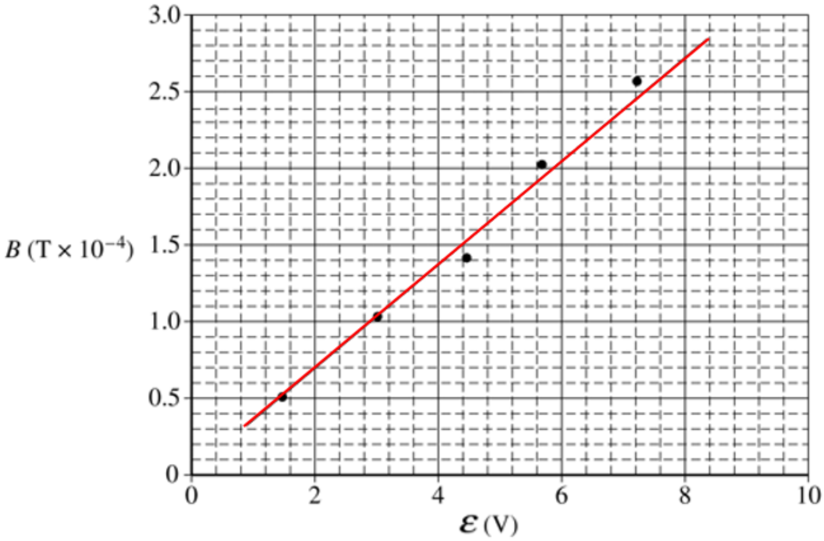
Some physics students conduct an experiment to determine the resistance R_S of a solenoid with radius $a = 0.015$ m, total turns $N = 100$, and total length $\ell = 0.40$ m. The students connect the solenoid to a variable power supply. A magnetic field sensor is used to measure the magnetic field strength along the central axis at the center of the solenoid. The plot of the magnetic field strength B as a function of the emf $\mathcal{E}$ of the power supply is shown below.

Question 3 (continued)

(c) LO CNV-8.C.c, SP 4.C

i. 1 point

On the graph above, draw a best-fit line for the data.



For drawing a best-fit line with at least one point above and one point below the line		1 point

ii. LO CNV-8.C.c, SP 6.B, 6.C

2 points

Use the straight line to determine the resistance R_S of the solenoid used in the experiment.

For calculating the slope using the best-fit line and not the data points unless they fall on the best-fit line		1 point
$slope = \dfrac{\Delta y}{\Delta x} = \dfrac{(2.5 - 0.9)\left(\times 10^{-4}\ \text{T}\right)}{(6.4 - 2.0)\ \text{V}} = 0.36 \times 10^{-4} = 3.6 \times 10^{-5}\ \text{T/V}$		
For correctly giving the expression that relates the slope to the resistance of the solenoid		1 point
$B = \dfrac{\mu_0 NI}{\ell} = \dfrac{\mu_0 N\mathcal{E}}{\ell R} \therefore slope = \dfrac{\mu_0 N}{\ell R_S} \therefore R_S = \dfrac{\mu_0 N}{\ell \times slope}$ $R_S = \dfrac{\left(4\pi \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}\right)(100\ \text{turns})}{(0.40\ \text{m})\left(3.6 \times 10^{-5}\ \text{T/V}\right)} = 8.7\ \Omega$		

One of the students notes that the horizontal component of the magnetic field of Earth is 2.5×10^{-5} T.

Question 3 (continued)

(d)

i. LO CNV-8.E.a, SP 2.D
1 point

Is there evidence from the graph that the horizontal orientation of the solenoid affects the measured values for B?

____ Yes ____ No

Justify your answer.

If the line on the graph does not go through the origin, select "Yes"		
For a correct justification		1 point
Example: The horizontal component of Earth's magnetic field will add or subtract from the magnetic field of the solenoid depending on the orientation of the solenoid.		
Alternate Solution		*Alternate Points*
If the line on the graph does pass through the origin, select "No"		
For a correct justification		*1 point*
Example: Based on the graph, the line passes through the origin, so the magnetic field is zero when the emf of the power supply is zero, therefore Earth's magnetic field is not affecting the values of B.		

ii. LO CNV-8.E.a, SP 2.E
1 point

Would the horizontal orientation of the solenoid affect the calculated value for R_S ?

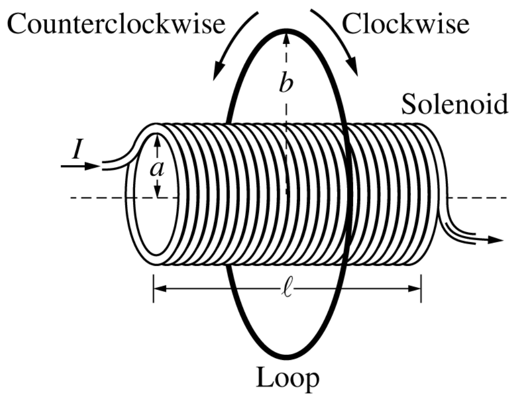
____ Yes ____ No

Justify your answer.

Select "No"		
For a correct justification		1 point
Example: The horizontal component of Earth's magnetic field will not affect the change in magnetic field as the emf is changed. Therefore, the value for the resistance of the solenoid will not change.		

2019 SCORING GUIDELINES

Question 3 (continued)



A thin conducting loop of radius b and resistance R_L is placed concentric with the solenoid, as shown above. The current in the solenoid is decreased from I to zero over time Δt.

(e)

i. LO FIE-6.A.b, SP 7.A, 7.C
2 points

Is the direction of the induced current in the loop clockwise or counterclockwise during the time period that the current in the solenoid is decreasing?

____ Clockwise ____ Counterclockwise

Justify your answer.

Select "Clockwise"		
For a justification indicating that the magnetic field inside the solenoid, and therefore the loop, will decrease		1 point
For a justification using Lenz's law to relate the change in magnetic field to the direction of the current		1 point
Example: As the current in the solenoid decreases, the magnetic field inside the solenoid decreases. As the solenoid's magnetic field decreases, the induced current in the loop will create a magnetic field to oppose this change. Because the solenoid's magnetic field is toward the right and decreasing, the magnetic field due to the current in the loop must be toward the right. Therefore, the current in the loop must be clockwise.		

Question 3 (continued)

(e) continued

ii. LO FIE-6.A.d, SP 5.A, 5.E

3 points

Derive an equation for the average induced current i_{IND} in the loop during the time period that the current in the solenoid is decreasing. Express your answer in terms of I, ℓ, N, a, b, R_L, R_S, Δt, and physical constants, as appropriate.

For using Faraday's law to calculate the emf in the loop		1 point
$\varepsilon = \left\lvert \frac{d\Phi}{dt} \right\rvert = \frac{d(BA)}{dt} = A\frac{\Delta B}{\Delta t}$		
For using Ohm's law to calculate the current in the loop		1 point
$I_{\text{IND}} = \frac{V}{R} = \frac{A\Delta B/\Delta t}{R_L} = \frac{A\Delta B}{R_L \Delta t}$		
For using the correct radius for the area in the equation above		1 point
$I_{\text{IND}} = \frac{\pi a^2 \mu_0 N \Delta I/\ell}{R_L \Delta t} = \frac{\pi a^2 \mu_0 N I}{R_L \ell \Delta t}$		

Learning Objectives

CNV-8.C.c: Derive the expression for the magnetic field of an ideal solenoid (length dimension is much larger than the radius of the solenoid) using Ampère's law.
CNV-8.E.a: Describe the direction of a magnetic field at a point in space due to various combinations of conductors, wires, cylindrical conductors, or loops.
FIE-6.A.b: Describe the direction of an induced current in a conductive loop that is placed in a changing magnetic field.
FIE-6.A.d: Calculate the magnitude and direction of induced EMF and induced current in a conductive loop (or conductive bar) when the magnitude of either the field or area of loop is changing at a constant rate.

Science Practices

2.D: Make observations or collect data from representations of laboratory setups or results.
2.E: Identify or describe potential sources of experimental error.
3.D: Create appropriate diagrams to represent physical situations.
4.C: Linearize data and/or determine a best-fit line or curve.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

2018

AP Physics C: Electricity and Magnetism

Scoring Guidelines

2018 SCORING GUIDELINES

General Notes About 2018 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

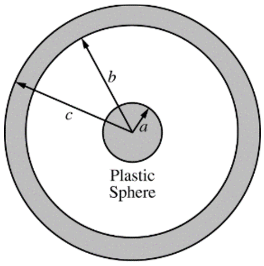
5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

AP PHYSICS C: ELECTRICITY AND MAGNETISM
2018 SCORING GUIDELINES

Question 1

15 points total **Distribution of points**



Conducting Spherical Shell

A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

(a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

i. 3 points

$r < a$

For using the integral to determine the charge enclosed		1 point
$q_{enc} = \int \rho dV = \int \beta r dV$		
For correctly substituting for dV into the integration to determine the charge enclosed		1 point
$q_{enc} = \int \left(4\pi r'^2\right)(\beta r')dr' = 4\beta\pi \int_{r'=0}^{r'=r} r'^3 dr' = 4\beta\pi \left[\frac{r'^4}{4}\right]_{r'=0}^{r'=r}$		
For a correct answer		1 point
$q_{enc} = \beta\pi r^4$		

ii. 1 points

$a < r < b$

For an answer consistent with (a)(i)		1 point
$q_{enc} = \beta\pi a^4$		

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Question 1 (continued)

Distribution of points

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. 2 points

$r < a$

For correctly substituting the area of a Gaussian sphere into Gauss's law		1 point
$\frac{q_{enc}}{\varepsilon_0} = \oint \vec{E} \cdot d\vec{A}$		
$\frac{q(r)}{\varepsilon_0} = EA_{sphere} = E\left(4\pi r^2\right)$		
For correctly substituting the charge from part (a)(i) into the equation above		1 point
$\frac{\beta \pi r^4}{\varepsilon_0} = E\left(4\pi r^2\right)$		
$E = \frac{\beta r^2}{4\varepsilon_0}$		

ii. 1 point

$a < r < b$

For correctly substituting the charge from part (a)(ii) into Gauss's law to calculate the electric field		1 point
$\frac{\beta \pi a^4}{\varepsilon_0} = E\left(4\pi r^2\right)$		
$E = \frac{\beta a^4}{4\varepsilon_0 r^2}$		

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Question 1 (continued)

Distribution of points

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. 2 points

Determine the charge on the inner surface of the conducting shell.
Justify your answer.

For an answer consistent with part (a)(ii) (must have opposite sign)		1 point
$q_{inner} = -\beta\pi a^4$		
For a correct justification		1 point
$E_{shell} = 0 = \frac{q_{enc}}{\varepsilon_0} \therefore q_{enc} = 0 = q_{sphere} + q_{inner} \therefore q_{inner} = -q_{sphere}$		

ii. 1 point

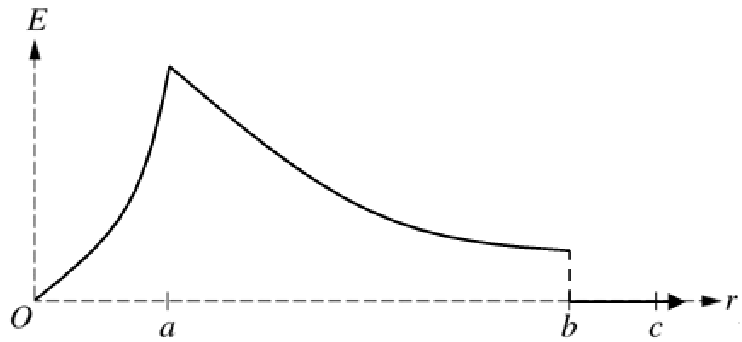
Determine the charge on the outer surface of the conducting shell.

A correct answer		1 point
$q_{outer} = 0$		

(d)

i. 3 points

On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



For concave up graph that begins at the origin and increases to $r = a$		1 point
For a concave up graph that is continuous with the first section and decreases from $r = a$ to nonzero value at $r = b$		1 point
For a graph with a value of $E = 0$ for $r > b$		1 point
Note: Due to scale, discontinuity at point b is not required.		

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Question 1 (continued)

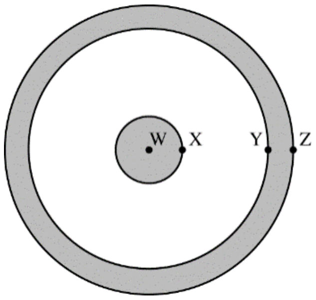
Distribution of points

(d) (continued)

ii. 2 points

The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

__1__ W   __2__ X   __3__ Y   __3__ Z



For any ranking that has Y and Z at the same potential		1 point
For ranking the electric potentials: W > X > Y & Z (because Y = Z is the first point)		1 point

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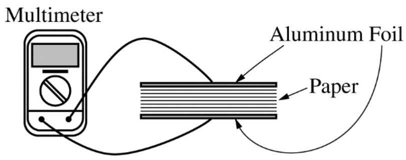
Question 2

15 points total **Distribution of points**



An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \text{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)		
500	0.045	6.5×10^{-11}		
400	0.036	7.4×10^{-11}		
300	0.027	8.9×10^{-11}		
200	0.018	11.9×10^{-11}		
100	0.010	21.0×10^{-11}		

(a) 1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: ____________

Horizontal axis: ____________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

For indicating variables that will create a straight line whose slope can be used to determine the dielectric constant of the paper		1 point
Example: Vertical axis: C *Horizontal axis:* $\dfrac{1}{d}$		
Note: Student earns full credit if the axes are reversed or if they use another acceptable combination.		

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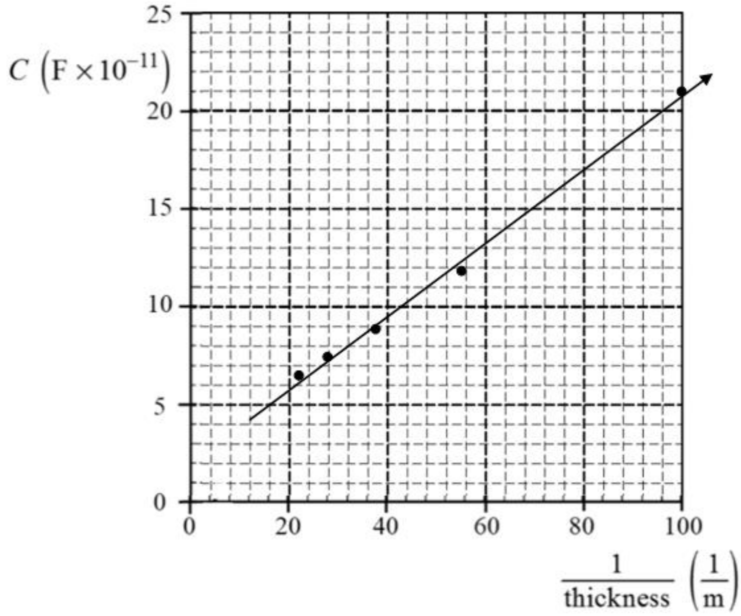
Question 2 (continued)

Distribution of points

(b) 4 points

Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



For a correct scale that uses more than half the grid		1 point
For correctly labeling the axes including units		1 point
For correctly plotting the data		1 point
For drawing a straight line consistent with the plotted data		1 point

(c) 3 points

Using the straight line, calculate a dielectric constant for the paper.

For correctly calculating the slope from the best-fit line and not the data points unless the points fall on the best-fit line		1 point
$\text{slope} = \dfrac{\Delta y}{\Delta x} = \dfrac{(17-8)\left(\mathrm{F}\times 10^{-11}\right)}{(80-32)(1/\mathrm{m})} = 0.19\ \mathrm{F{\bullet}m}$ (Linear regression = $0.187\ \mathrm{F{\bullet}m}$)		
For correctly relating the slope to the dielectric constant		1 point
$\text{slope} = \kappa\varepsilon_0 A$		
$\kappa = \dfrac{\text{slope}}{\varepsilon_0 A} = \dfrac{(0.19\ \mathrm{F{\bullet}m})}{\left(8.85\times 10^{-12}\ \mathrm{C^2/N{\bullet}m^2}\right)\left(0.060\ \mathrm{m^2}\right)}$		
For a correct answer		1 point
$\kappa = 3.58$ (Linear regression = 3.52)		

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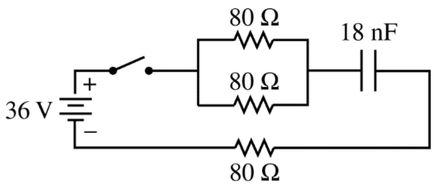
Question 2 (continued)

Distribution of points



The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical $80\ \Omega$ resistors, and an open switch, as shown in the figure above.

(d) 3 points

Calculate the current in the battery immediately after the switch is closed.

For calculating the equivalent resistance for the parallel resistors		1 point
$\dfrac{1}{R_P} = \dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{1}{80\ \Omega} + \dfrac{1}{80\ \Omega} = \dfrac{2}{80\ \Omega} = \dfrac{1}{40\ \Omega}$		
$R_P = 40\ \Omega$		
For using Ohm's law with the potential difference across the capacitor equal to zero		1 point
$I = \dfrac{V}{R} = \dfrac{V}{R_P + R_3} = \dfrac{(36\ \text{V})}{R_P + R_3}$		
For substitution of values for resistance including the value for combined resistance above		1 point
$I = \dfrac{V}{R_P + R_3} = \dfrac{(36\ \text{V})}{(40\ \Omega + 80\ \Omega)} = 0.30\ \text{A}$		

(e) 2 points

Determine the time constant for this circuit.

For using the equation for the time constant with the equivalent resistance from above		1 point
$\tau = R_{eq}C = (120\ \Omega)(18\ \text{nF})$		
For an answer with units consistent with part (d)		1 point
$t = 2.16\ \mu\text{s}$		

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Question 2 (continued)

Distribution of points

(f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. 1 point

Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

____ Larger than measured   ____ Smaller than measured

Explain experimentally what could account for this.

Select “Larger than measured”		
For an appropriate explanation		1 point
Example: The battery is not ideal and has internal resistance, so the actual resistance for the circuit is larger than the measured resistance.		

ii. 1 points

Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

____ Larger than measured   ____ Smaller than measured

Explain experimentally what could account for this.

Select “Larger than measured”		
For an appropriate explanation		1 point
Example: Some of the sheets of paper may be thinner than expected, so the actual capacitance for the circuit is larger than the measured capacitance.		

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Question 3

15 points total **Distribution of points**

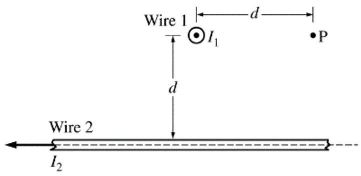

Figure 1. Side view

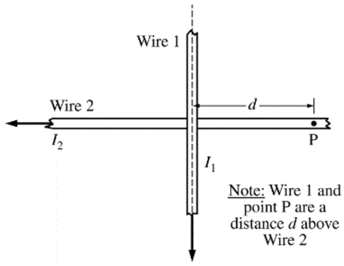

Figure 2. Top view

The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance *d* above wire 2. Point P is a horizontal distance *d* from wire 1 and a distance *d* directly above wire 2. Express your answers to parts (a) and (b) in terms of I, *d*, and physical constants, as appropriate.

(a) 2 points

Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

For attempting to use Ampere's law to calculate the magnetic field at point *P*		1 point
$\oint \vec{B} \bullet d\vec{\ell} = \mu_0 I$		
$B(2\pi d) = \mu_0 I$		
For a correct answer		1 point
$B = \dfrac{\mu_0 I}{2\pi d}$		

(b) 2 points

Derive an expression for the magnitude of the net magnetic field at point P.

For indicating $B_2 = 2B_1$		1 point
$B_1 = \dfrac{\mu_0 I}{2\pi d}$ & $B_2 = \dfrac{\mu_0 (2I)}{2\pi d} = \dfrac{\mu_0 I}{\pi d}$		
For an indication that the magnitude of the magnetic field is the vector sum of B_1 and B_2		1 point
$B_{net} = \sqrt{B_1^2 + B_2^2} = \sqrt{B^2 + B^2} = \sqrt{\left(\dfrac{\mu_0 I}{2\pi d}\right)^2 + \left(\dfrac{\mu_0 (2I)}{2\pi d}\right)^2} = \dfrac{\sqrt{5}\mu_0 I}{2\pi d}$		

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Question 3 (continued)

Distribution of points

(c) 2 points

Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

For correctly relating the angle to the individual magnetic fields		1 point
$\theta = \tan^{-1}\left(\frac{B_1}{B_2}\right)$		
For correctly substituting B_1 and B_2 into the equation		1 point
$\theta = \tan^{-1}\left(\frac{B_1}{B_2}\right) = \tan^{-1}\left(\frac{\left(\frac{\mu_0 I}{2\pi d}\right)}{\left(\frac{\mu_0 (2I)}{2\pi d}\right)}\right) == \tan^{-1}\left(\frac{1}{2}\right) = 26.6°$		

(d) 2 points

Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2? Assume gravitational effects are negligible.

____ Wire 1 will not move.

____ Wire 1 will move upward as viewed in Figure 1.

____ Wire 1 will move downward as viewed in Figure 1.

____ Wire 1 will rotate clockwise as viewed in Figure 2.

____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

For stating there is no translational motion since the magnetic forces on the wire cancel		1 point
For stating there is a net torque which causes rotation		1 point
Example: In Figure 2, the top portion of wire 1 will be in a magnetic field into the page from wire 2 and, thus, will experience a force to the right. The bottom portion of wire 1 will be in a magnetic field out of the page from wire 2 and, thus, will experience a force to the left. So the net force will be zero, but there will be a net clockwise torque, so the wire will rotate clockwise.		

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Question 3 (continued)

Distribution of points

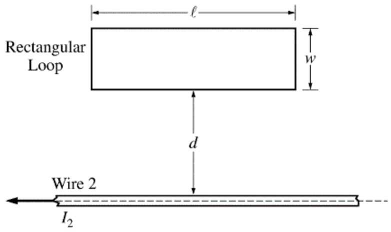

Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1}.

(e) 1 point

Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

____ $\Phi = \displaystyle\int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$ ____ $\Phi = \displaystyle\int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

__X__ $\Phi = \displaystyle\int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$ ____ $\Phi = \displaystyle\int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

For selecting $\Phi = \displaystyle\int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$		1 point

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Question 3 (continued)

Distribution of points

(f) 3 points

Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \dfrac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\dfrac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

For attempting to take the time derivative of the magnetic flux to calculate the emf		1 point
$\varepsilon = \left\lvert -\dfrac{d\Phi}{dt} \right\rvert = \dfrac{\mu_0 I_0 \ell}{\pi} \ln\left(\dfrac{d + w}{d}\right) \left\lvert \dfrac{d}{dt}[(1 - kt)] \right\rvert$		
$\varepsilon = \dfrac{\mu_0 I_0 k \ell}{\pi} \ln\left(\dfrac{d + w}{d}\right)$		
For dividing the emf by the resistance to calculate the current		1 point
$I = \dfrac{\varepsilon}{R} = \dfrac{\dfrac{\mu_0 I_0 k \ell}{\pi} \ln\left(\dfrac{d + w}{d}\right)}{R}$		
For a correct answer		1 point
$I = \dfrac{\mu_0 I_0 k \ell}{\pi R} \ln\left(\dfrac{d + w}{d}\right)$		

(g) 3 points

What is the direction of the current, if any, induced in the loop as seen in Figure 3?

____ Clockwise ____ Counterclockwise

____ Undefined because there is no current induced in the loop

Justify your answer.

For selecting “Clockwise” with an attempt at a relevant justification		1 point
For indicating that the flux inside the loop will decrease		1 point
For using Lenz’s law to relate the decrease in the flux to the clockwise current		1 point
Example: Because the current in the wire is decreasing, the flux in the loop will decrease. According to Lenz’s law, the induced current should create a magnetic field to oppose this decrease. Thus the induced magnetic field must be into the page, and the current in the loop must be clockwise.		