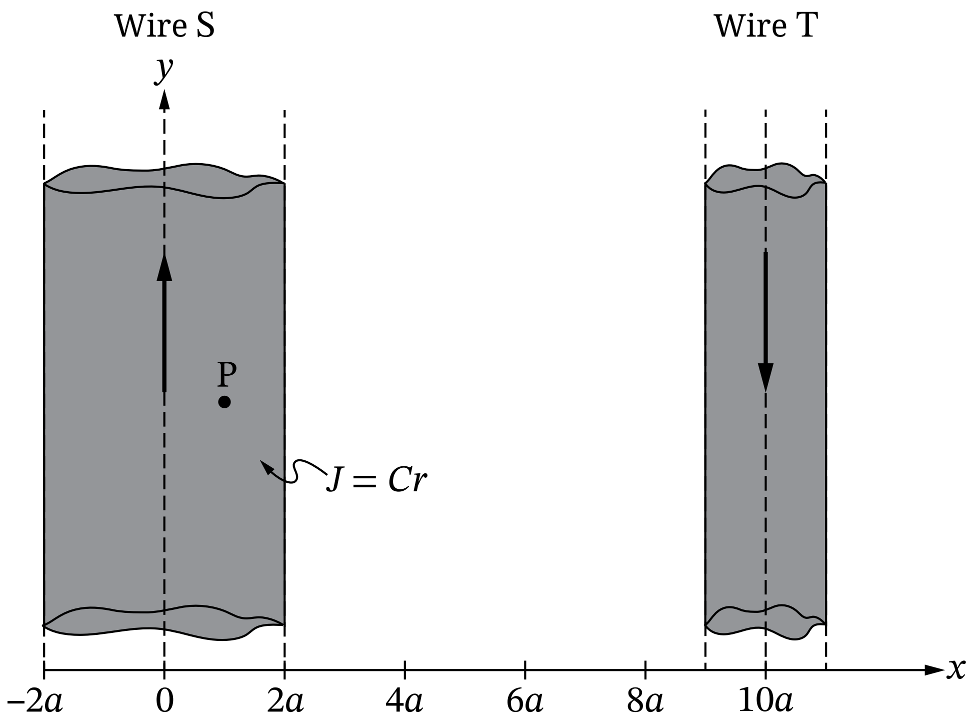
Question 1: Version J

1. Two very long, cylindrical, current-carrying Wires S and T are oriented parallel to the y-axis, as shown in Figure 1. The centers of the wires are in the xy-plane. The wires are described as follows.

- Wire S has radius $2a$ and is centered at $x = 0$. Wire S has nonuniform current density and carries total current I_0 in the $+y$-direction. The magnitude of the current density in Wire S as a function of the radial distance r from the center of the wire is described by $J = Cr$, where C is a positive constant.
- Wire T has radius a and is centered at $x = 10a$. Wire T has uniform current density and carries total current I_0 in the $-y$-direction.

Figure 1

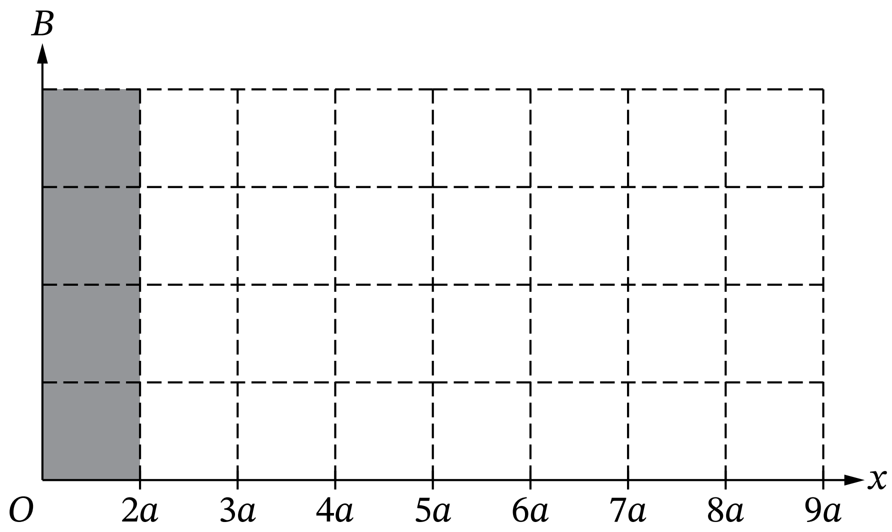


Part A

i. Point P is located inside of Wire S at horizontal position $x = a$. **Derive** an expression for the magnitude of the magnetic field at Point P due only to the current in Wire S in terms of C, a, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

ii. On the axes shown in Figure 2, **sketch** a graph of the magnitude B of the magnetic field due to the currents in both wires as a function of x along the x-axis in the region $2a \leq x \leq 9a$. Sketches made in the shaded region will not be graded.

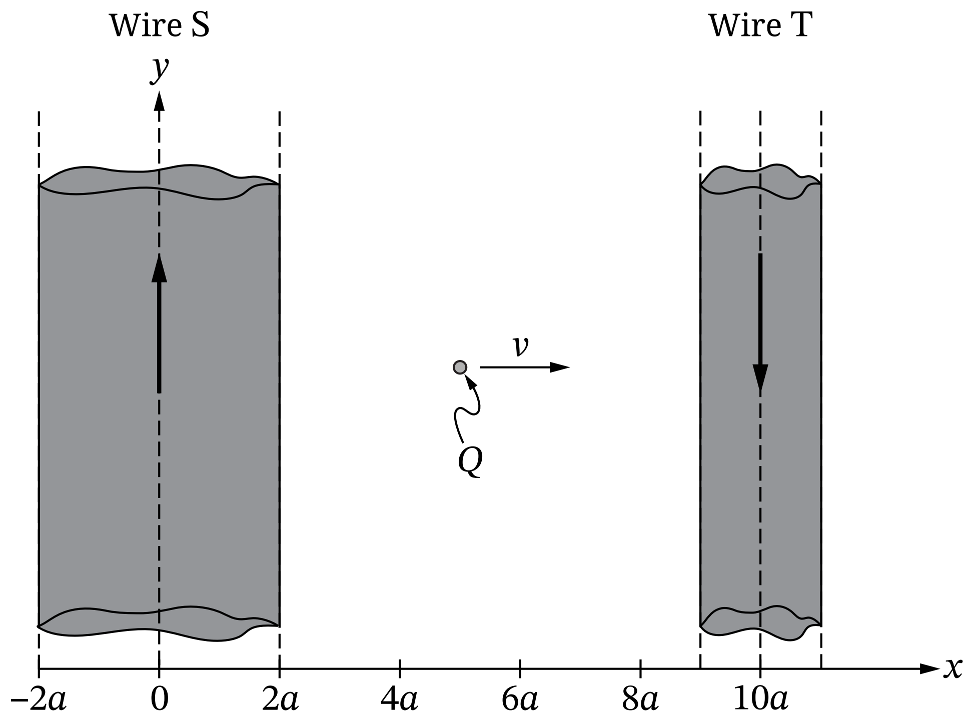
Figure 2



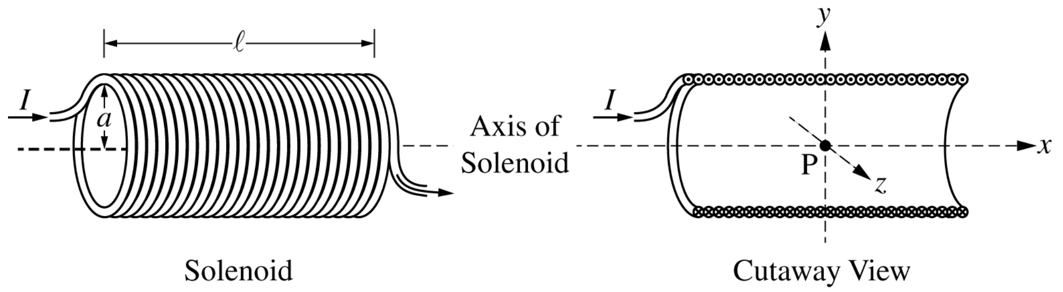
Part B

At the instant shown in Figure 3, a small sphere with charge Q is at horizontal position $x = 5a$ and moving in the xy-plane with speed v in the $+x$-direction. At this instant, the net magnetic force exerted on the sphere due to the currents in Wires S and T has magnitude F.

Figure 3



Derive an expression for F in terms of a, Q, I_0, v, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.



Note: Figures not drawn to scale.

3. A solenoid is used to generate a magnetic field. The solenoid has an inner radius a, length ℓ, and N total turns of wire. A power supply, not shown, is connected to the solenoid and generates current I, as shown in the figure on the left above. The x-axis runs along the axis of the solenoid. Point P is in the middle of the solenoid at the origin of the xyz-coordinate system, as shown in the cutaway view on the right above. Assume $\ell >> a$.

(a) Select the correct direction of the magnetic field at point P.

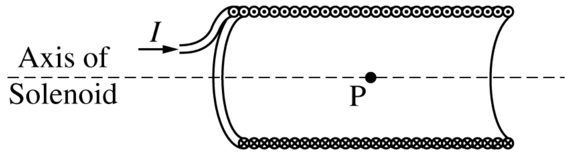
____ $+x$-direction ____ $+y$-direction ____ $+z$-direction

____ $-x$-direction ____ $-y$-direction ____ $-z$-direction

Justify your selection.

(b)

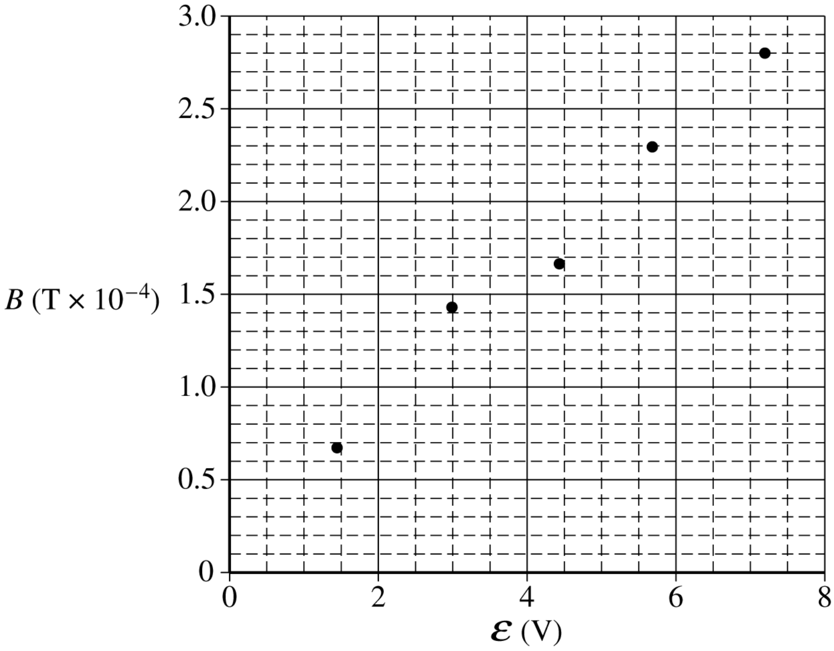
i. On the cutaway view below, clearly draw an Amperian loop that can be used to determine the magnetic field at point P at the center of the solenoid.



Cutaway View

ii. Use Ampere's law to derive an expression for the magnetic field strength at point P. Express your answer in terms of I, ℓ, N, a, and physical constants, as appropriate.

Some physics students conduct an experiment to determine the resistance R_S of a solenoid with radius $a = 0.015$ m, total turns $N = 100$, and total length $\ell = 0.40$ m. The students connect the solenoid to a variable power supply. A magnetic field sensor is used to measure the magnetic field strength along the central axis at the center of the solenoid. The plot of the magnetic field strength B as a function of the emf $\mathcal{E}$ of the power supply is shown below.



(c)

i. On the graph above, draw a best-fit line for the data.

ii. Use the straight line to determine the resistance R_S of the solenoid used in the experiment.

(d) One of the students notes that the horizontal component of the magnetic field of Earth is 2.5×10^{-5} T.

i. Is there evidence from the graph that the horizontal orientation of the solenoid affects the measured values for B?

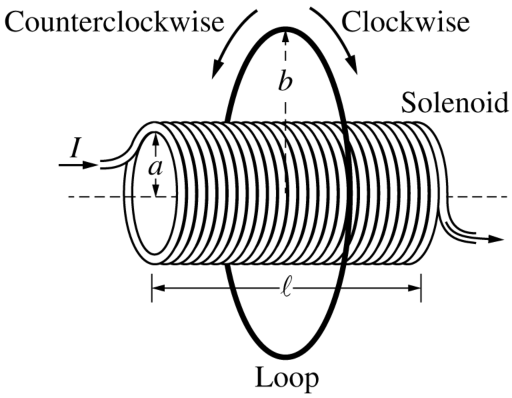
____ Yes ____ No

Justify your answer.

ii. Would the horizontal orientation of the solenoid affect the calculated value for R_S?

____ Yes ____ No

Justify your answer.



A thin conducting loop of radius b and resistance R_L is placed concentric with the solenoid, as shown above. The current in the solenoid is decreased from I to zero over time Δt.

(e)

i. Is the direction of the induced current in the loop clockwise or counterclockwise during the time period that the current in the solenoid is decreasing?

____ Clockwise ____ Counterclockwise

Justify your answer.

ii. Derive an equation for the average induced current i_{IND} in the loop during the time period that the current in the solenoid is decreasing. Express your answer in terms of I, ℓ, N, a, b, R_L, R_S, Δt, and physical constants, as appropriate.

STOP

END OF EXAM

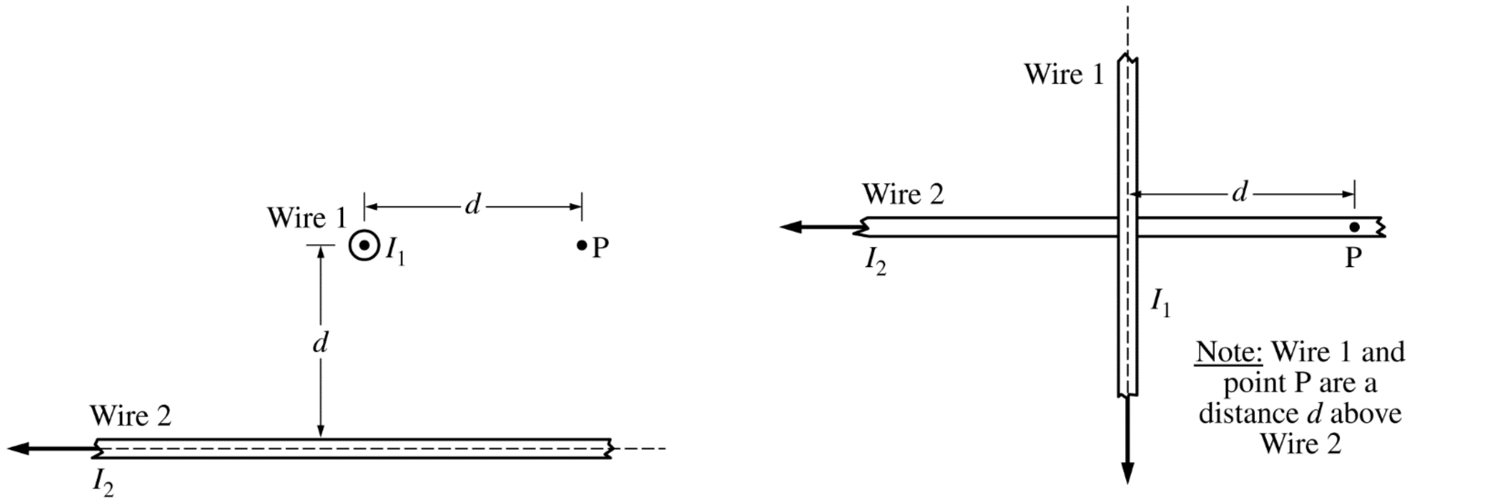


Figure 1. Side view

Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I, d, and physical constants, as appropriate.

   (a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

   (b) Derive an expression for the magnitude of the net magnetic field at point P.

   (c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

   (d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2 ? Assume gravitational effects are negligible.

   ____ Wire 1 will not move.

   ____ Wire 1 will move upward as viewed in Figure 1.

   ____ Wire 1 will move downward as viewed in Figure 1.

   ____ Wire 1 will rotate clockwise as viewed in Figure 2.

   ____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

   Justify your answer.

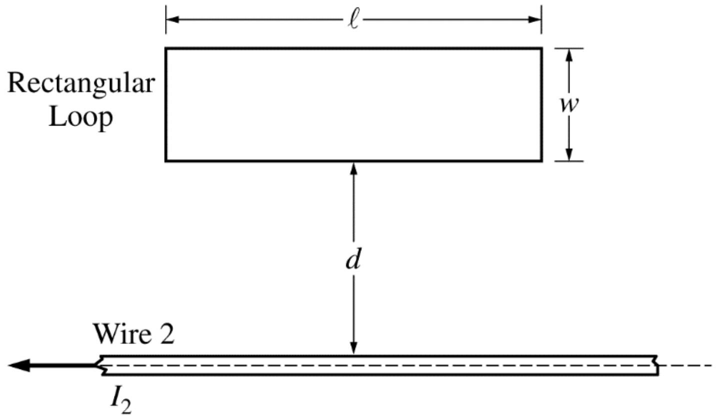


Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1}.

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

(f) Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \frac{\mu_0 I_0 \ell (1-kt)}{\pi} \ln\left(\frac{d+w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

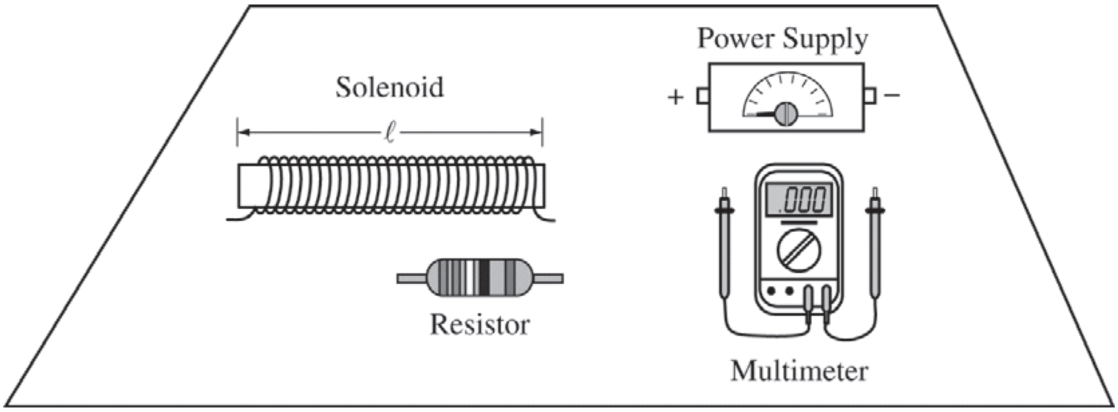
____ Clockwise ____ Counterclockwise

____ Undefined, because there is no current induced in the loop

Justify your answer.

STOP

END OF EXAM



3. When studying Ampere's law, students collect data on the magnetic field of two different solenoids in order to determine the magnetic permeability of free space μ_0. The solenoids are created by wrapping wire around a hollow plastic tube. The solenoids of length ℓ with N turns of wire will be connected in series to a power supply and resistor. A multimeter will be used as an ammeter to measure the magnitude of the current I through the solenoids. The main components for the setup with one of the solenoids are shown in the figure above.

(a)

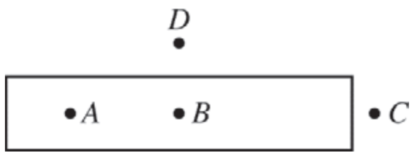
i. On the figure above, draw wire connections between the solenoid, power supply, resistor, and multimeter that will complete the circuit and allow students to measure the magnitude of the current through the solenoid.

ii. Using the connections you made in part (a)i above, what will be the direction of the magnetic field inside the solenoid?

____ Toward the top of the page   ____ To the left   ____ Out of the page

____ Toward the bottom of the page   ____ To the right   ____ Into the page

The rectangle shown below represents the solenoid (the loops of wire are not shown). Points A, B, and C are along the central axis of the solenoid with point B at the middle of the solenoid. Point D is directly above point B.

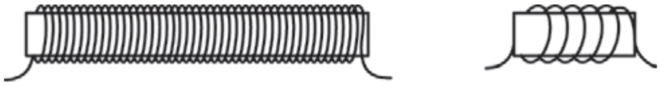


iii. From the choices below, select the point where you would place a magnetic field probe (a probe that can measure the magnitude of the magnetic field) to best measure the strength of the magnetic field of the solenoid in order to determine the magnetic permeability of free space μ_0.

____ A ____ B ____ C ____ D

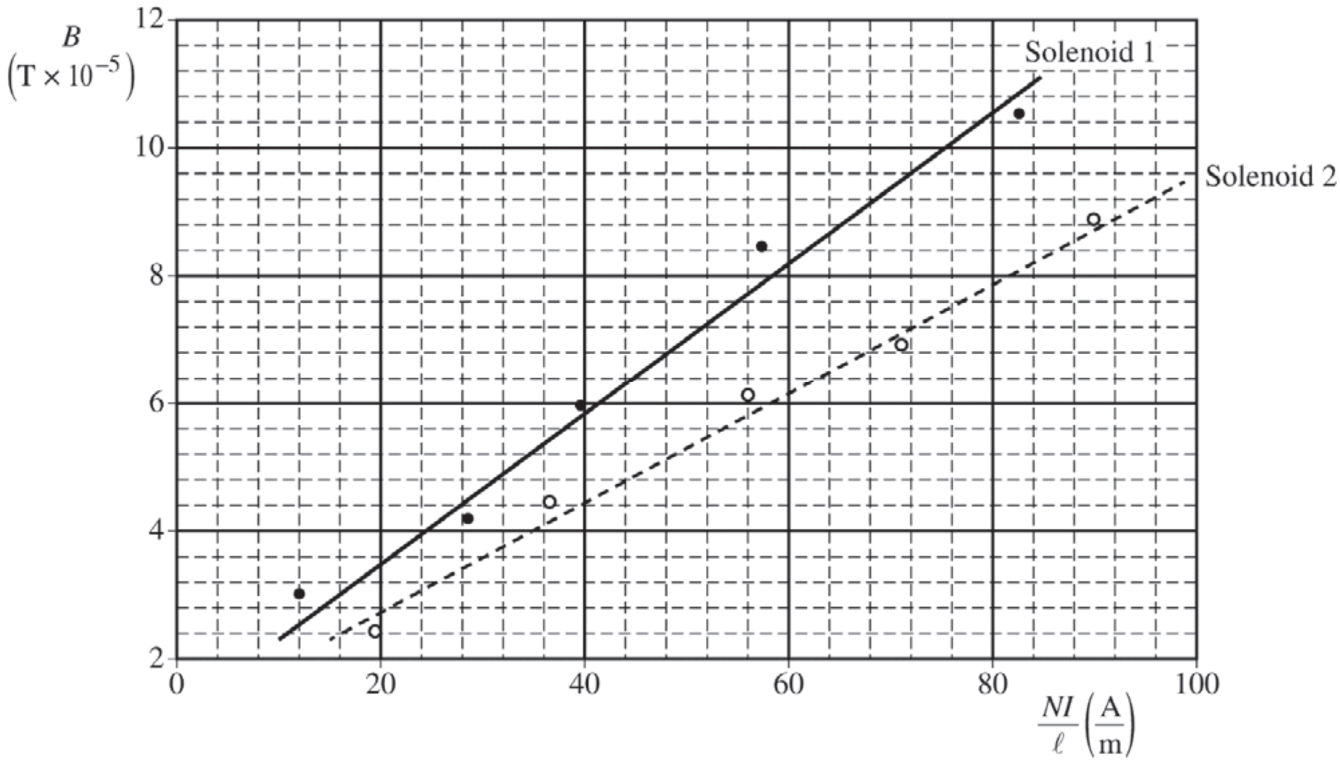
Justify your answer based on the model for a simple solenoid.

The figures below show two different solenoids that will be connected in the circuit above. Solenoid 1 has a length $\ell = 25$ cm with $N = 100$ turns. Solenoid 2 has a length $\ell = 5.0$ cm with $N = 5$ turns.



Note: Figures not drawn to scale.

A graph of the magnitude of the magnetic field B as a function of NI/ℓ is shown below. The best-fit lines for the data are shown as a solid line for solenoid 1 and as a dashed line for solenoid 2.



(b) Which solenoid's best-fit line would give the best results for determining a value for the magnetic permeability of free space μ_0?

____ Solenoid 1 ____ Solenoid 2

Justify your answer.

(c)

i. Use the slope of the best-fit line for the solenoid chosen in part (b) to calculate the magnetic permeability of free space μ_0.

ii. Calculate the percent error for the experimental value of the magnetic permeability of free space μ_0 determined in part (c)i.

(d)

i. What is a reasonable physical explanation for a best-fit line that does not pass through the origin?

ii. Suppose a student connects the solenoid in a closed circuit similar to the circuit in part (a)i but without the resistor. The student notices the multimeter stops functioning after the power supply is turned on. Explain what causes the failure of the multimeter.

STOP

END OF EXAM