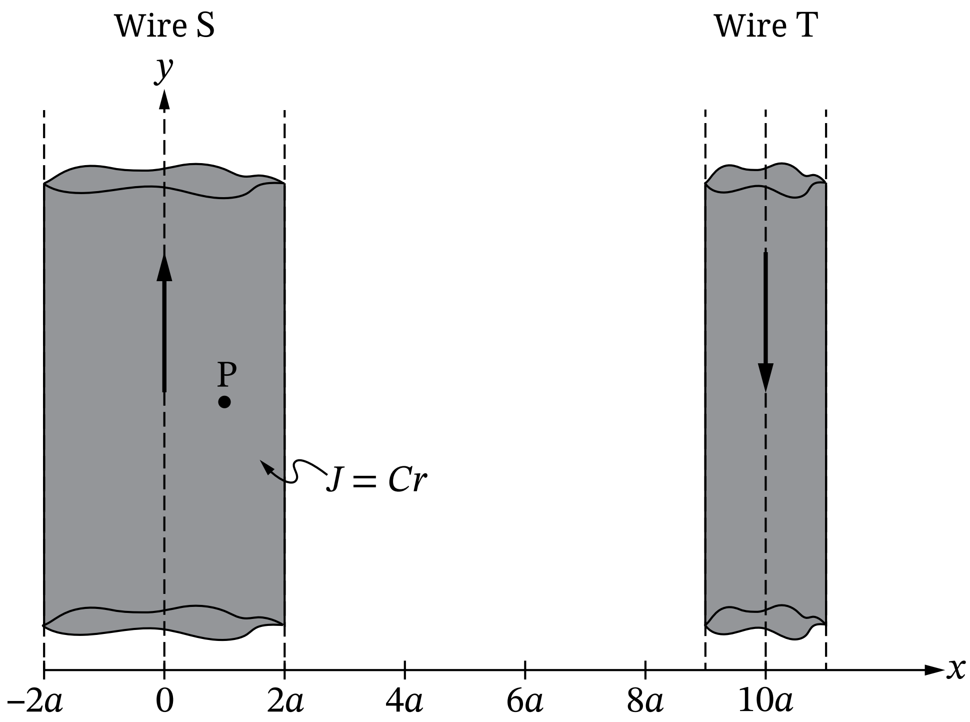
Question 1: Version J

1. Two very long, cylindrical, current-carrying Wires S and T are oriented parallel to the y-axis, as shown in Figure 1. The centers of the wires are in the xy-plane. The wires are described as follows.

- Wire S has radius $2a$ and is centered at $x = 0$. Wire S has nonuniform current density and carries total current I_0 in the $+y$-direction. The magnitude of the current density in Wire S as a function of the radial distance r from the center of the wire is described by $J = Cr$, where C is a positive constant.
- Wire T has radius a and is centered at $x = 10a$. Wire T has uniform current density and carries total current I_0 in the $-y$-direction.

Figure 1

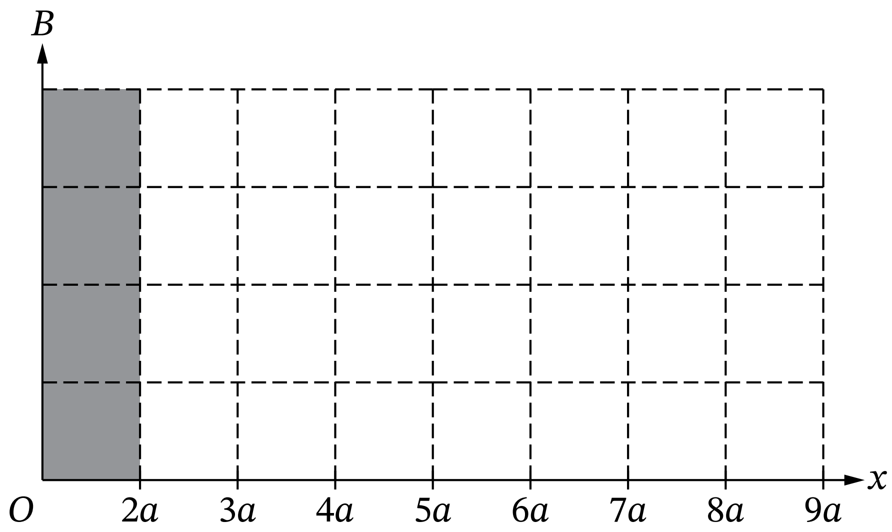


Part A

i. Point P is located inside of Wire S at horizontal position $x = a$. **Derive** an expression for the magnitude of the magnetic field at Point P due only to the current in Wire S in terms of C, a, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

ii. On the axes shown in Figure 2, **sketch** a graph of the magnitude B of the magnetic field due to the currents in both wires as a function of x along the x-axis in the region $2a \le x \le 9a$. Sketches made in the shaded region will not be graded.

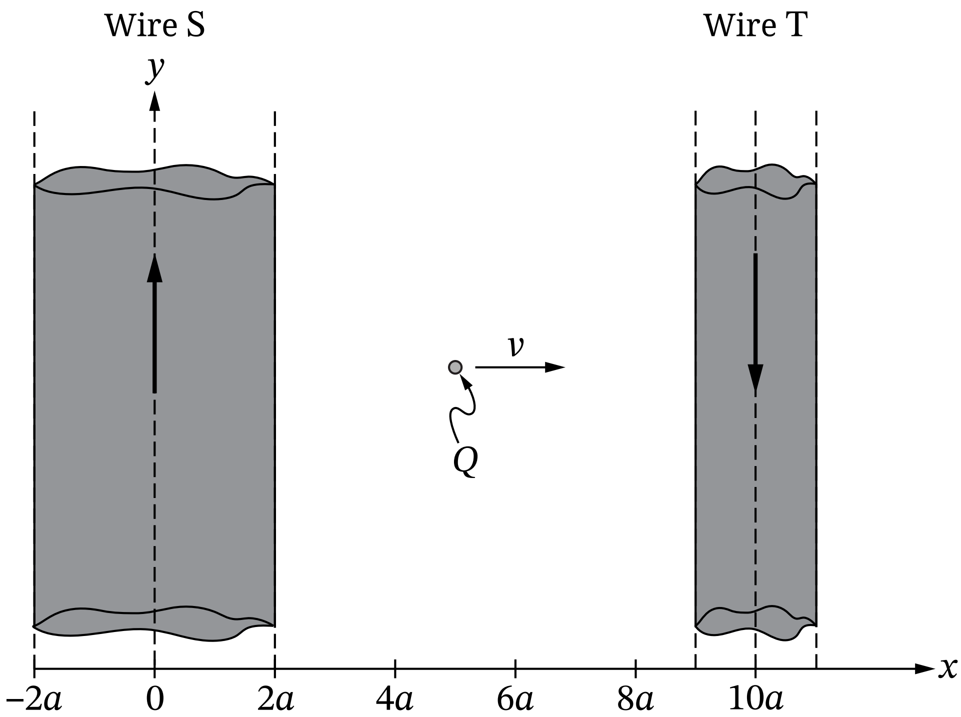
Figure 2



Part B

At the instant shown in Figure 3, a small sphere with charge Q is at horizontal position $x = 5a$ and moving in the xy-plane with speed v in the $+x$-direction. At this instant, the net magnetic force exerted on the sphere due to the currents in Wires S and T has magnitude F.

Figure 3

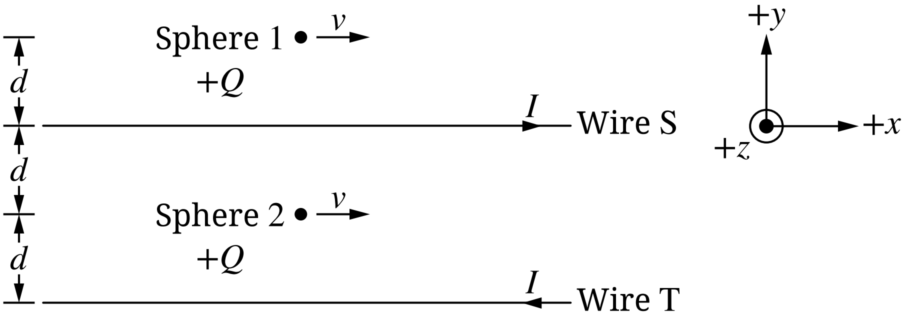


Derive an expression for F in terms of a, Q, I_0, v, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Question 4

4. Long, parallel wires S and T are a distance $2d$ apart. Both wires carry equal currents I, but the currents are in opposite directions. Both wires are parallel to the x-axis. At the instant shown in Figure 1, Sphere 1 is a distance d above Wire S, Sphere 2 is a distance d below Wire S, and both spheres are moving with speed v in the $+x$-direction. Each sphere has positive charge $+Q$. Gravitational effects are negligible.

Figure 1



A. F_1 is the magnitude of the magnetic force exerted on Sphere 1 due to the currents in wires S and T. F_2 is the magnitude of the magnetic force exerted on Sphere 2 due to the currents in wires S and T.

Indicate whether F_2 is greater than, less than, or equal to F_1 by writing one of the following.

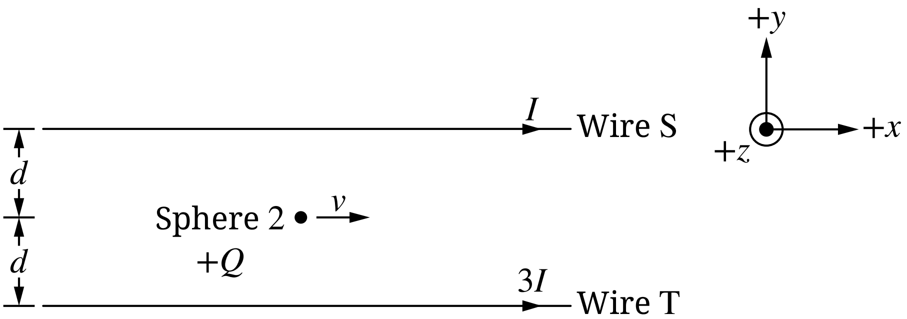
- $F_2 > F_1$
- $F_2 < F_1$
- $F_2 = F_1$

Justify your answer.

B. **Derive** an expression for the magnitude B_{tot} of the magnetic field at the location of Sphere 2 due to the currents in wires S and T in terms of d, I, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

C. Later, Wire T carries current $3I$ in the $+x$-direction. At the instant shown in Figure 2, Sphere 2 is a distance d below Wire S and is moving with speed v in the $+x$-direction. F_{new} is the new magnitude of the magnetic force exerted on Sphere 2 due to the currents in wires S and T.

Figure 2



Indicate whether F_{new} is greater than, less than, or equal to F_2 by writing one of the following.

- $F_{\text{new}} > F_2$
- $F_{\text{new}} < F_2$
- $F_{\text{new}} = F_2$

Briefly **justify** your answer by referencing your derivation in part B.

STOP

END OF EXAM

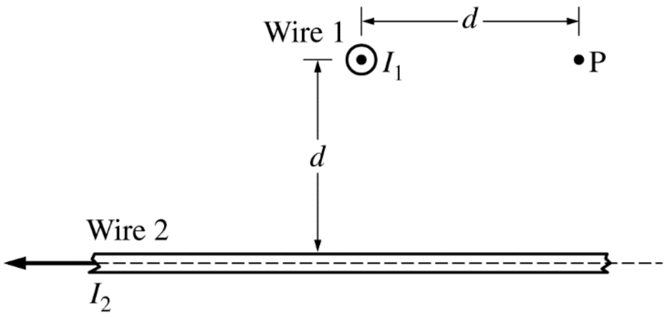


Figure 1. Side view

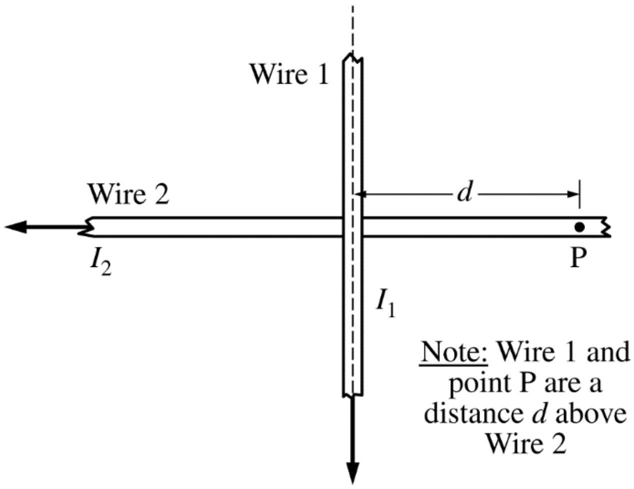


Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I, d, and physical constants, as appropriate.

(a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

(b) Derive an expression for the magnitude of the net magnetic field at point P.

(c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

(d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2 ? Assume gravitational effects are negligible.

____ Wire 1 will not move.

____ Wire 1 will move upward as viewed in Figure 1.

____ Wire 1 will move downward as viewed in Figure 1.

____ Wire 1 will rotate clockwise as viewed in Figure 2.

____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

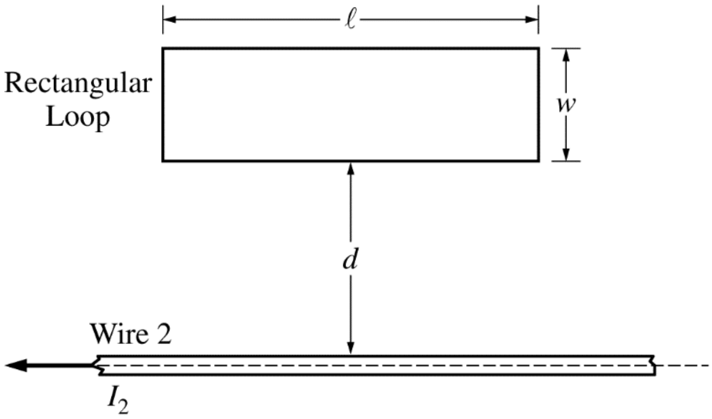


Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1}.

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

(f) Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \frac{\mu_0 I_0 \ell (1-kt)}{\pi} \ln\left(\frac{d+w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

____ Clockwise   ____ Counterclockwise

____ Undefined, because there is no current induced in the loop

Justify your answer.

STOP

END OF EXAM