

Question 4: Qualitative Quantitative Translation (QQT)

8 points

A	For indicating that $F_2 > F_1$	Point A1
	For correctly relating the magnitude of the magnetic force to the magnitude of the magnetic field	Point A2
	For indicating that the magnitude of the magnetic field at the location of Sphere 2 is greater than the magnitude of the magnetic field at the location of Sphere 1 by referring to either: • The relative distance from the spheres to Wires S and T • The direction of the magnetic fields from Wires S and T at the locations of the spheres	Point A3

Example Response

The magnitude of the magnetic force exerted on a sphere is directly proportional to Q , v , and B . Because Q and v are the same for both spheres, the difference in the magnitudes of the magnetic forces is due to the difference in the magnitudes of the magnetic fields at the locations of the spheres. The magnitude of the magnetic field is greater at the location of Sphere 2 than at the location of Sphere 1 because the fields from the wires are in the same direction. Therefore, $F_2 > F_1$.

B	For a multistep derivation that includes the equation $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$ or	Point B1
	$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I(d\vec{\ell} \times \hat{r})}{r^2}$	
	Scoring Note: Vector notation is not required for this point to be earned. A multistep derivation that includes $B = \frac{\mu_0 I}{2\pi d}$ can earn this point.	
	For a correct expression for the magnitude of the magnetic field due to the current in one wire at the location of Sphere 2	Point B2
	For indicating that the magnitude of the total field is twice that due to one wire	Point B3

Example Response

Determine the magnitude B of the magnetic field due to one wire.

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$$

$$B(2\pi d) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi d}$$

Determine B_{tot} .

$$B_{\text{tot}} = \frac{\mu_0 I}{2\pi d} + \frac{\mu_0 I}{2\pi d}$$

$$B_{\text{tot}} = \frac{\mu_0 I}{\pi d}$$

C	For indicating $F_{\text{new}} = F_2$	Point C1
	For indicating $B_{\text{tot}} = \frac{3\mu_0 I}{2\pi d} - \frac{\mu_0 I}{2\pi d}$	Point C2
	OR	

For indicating that the net magnitude of the magnetic field at the location of Sphere 2 will remain the same even though the field from Wire T will change

Example Response

The magnetic field at the location of Sphere 2 due to the current in Wire T is in the opposite direction to the magnetic field due to the current in Wire S. The current in

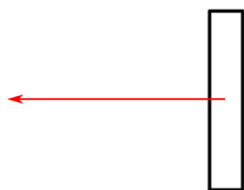
Wire T is increased by a factor of 3. Therefore, $B_{\text{tot}} = \frac{3\mu_0 I}{2\pi d} - \frac{\mu_0 I}{2\pi d} = \frac{\mu_0 I}{\pi d}$ in this scenario. Thus, $F_{\text{new}} = F_2$.

Question 2: Free-Response Question

15 points

(a)	For drawing an arrow pointing to the left with no extraneous arrows	1 point
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Example Response


Total for part (a) 1 point

(b)(i) For using Faraday's law to calculate the value of the induced emf**1 point**

Scoring Note: This point may be earned without the negative sign or a numerical answer.**Example Response**

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -\frac{d(B_1 dx)}{dt}$$

For a correct substitution of v for $\frac{dx}{dt}$ **1 point**

Scoring Note: A student can earn points 1 and 2 of part (b)(i) by starting with the expression $\mathcal{E} = B_1 dv$.**Example Response**

$$\mathcal{E} = -B_1 d\left(\frac{dx}{dt}\right)$$

$$\mathcal{E} = -B_1 dv$$

For substituting the correct resistance into an equation for Ohm's law to solve for the current

1 point

Example Solution

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -\frac{d(B_1 dx)}{dt}$$

$$\mathcal{E} = -B_1 d\left(\frac{dx}{dt}\right)$$

$$\mathcal{E} = -B_1 dv$$

$$\mathcal{E} = -(0.40 \text{ T})(0.30 \text{ m})(2.5 \text{ m/s})$$

$$\mathcal{E} = -0.30 \text{ V}$$

$$\Delta V = IR$$

$$I = \frac{\Delta V}{R}$$

$$I = \frac{|\mathcal{E}|}{R}$$

$$I = \frac{|-0.30 \text{ V}|}{0.20 \Omega} = 1.5 \text{ A}$$

(b)(ii)	For substituting the current or an expression for the current obtained from part (b)(i) into an appropriate equation for calculating the magnetic force	1 point
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Example Responses

$$\vec{F} = \int Id\vec{\ell} \times \vec{B}$$

$$F = IdB_1$$

$$F = (1.5 \text{ A})(0.3 \text{ m})(0.4 \text{ T})$$

$$F = 0.18 \text{ N}$$

OR

$$\vec{F} = \int Id\vec{\ell} \times \vec{B}$$

$$F = IdB_1$$

$$F = \left(\frac{B_1 dv}{R} \right) dB$$

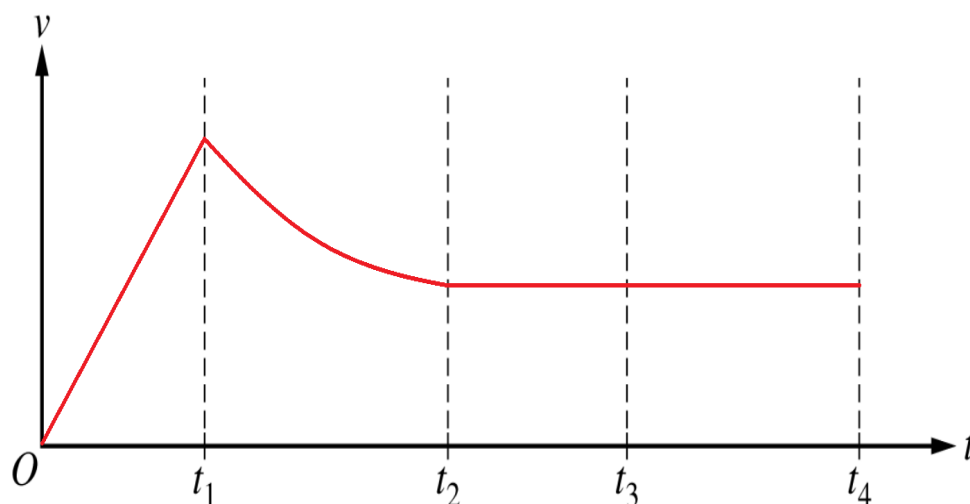
$$F = \frac{B_1^2 d^2 v}{R}$$

$$F = \frac{(0.4 \text{ T})^2 (0.3 \text{ m})^2 (2.5 \text{ m/s})}{0.2 \Omega}$$

$$F = 0.18 \text{ N}$$

Total for part (b) 4 points

(c)	For drawing a straight line with a positive slope that starts at the origin from $t = 0$ to t_1	1 point
	For drawing a curve that is decreasing and concave up from t_1 to t_2	1 point
	For drawing a nonzero horizontal line from t_2 to t_3	1 point
	For drawing a nonzero horizontal line from t_3 to t_4	1 point

Example Response**Total Points for part (c) 4 points**

(d)(i)	For a correct answer with units ($0.4\ \Omega$)	1 point
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Scoring Note: This point can be earned without supporting calculations.

Example Response

$$R_s = \sum_i R_i$$

$$R_s = (0.2\ \Omega) + (0.2\ \Omega)$$

$$R_s = 0.4\ \Omega$$

(d)(ii)	For a statement that correctly describes the inverse relation between resistance and current (e.g., as resistance increases, current decreases)	1 point
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	For a statement that correctly describes the direct relation between current and force (e.g., as current decreases, magnetic force decreases)	1 point
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	For a statement that correctly describes the direct relation between force and acceleration (e.g., as force decreases acceleration decreases)	1 point
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Scoring Note: Full credit can be earned with a justification that is consistent with the resistance calculated in part (d)(i).

Example Response

The new acceleration a_{new} is less than a_{original} . Greater resistance of the bar causes the current to be less than in the original scenario. Less current causes the magnetic force $F = IdB$ on the bar to also be less than the original. By Newton's second law $F = ma$ so less force on a bar of the same mass results in less acceleration.

Total for part (d)	4 points
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- (e) For correctly indicating **one** of the following, with an attempt at a relevant justification: **1 point**
- Increasing H
 - Increasing B_1
 - Increasing d

For correctly justifying the identified modification that will result in a larger induced current in the new bar at position x_B **1 point**

Example Responses

Increasing H will increase the induced current. If the ramp is higher, then the potential energy is greater and this results in greater kinetic energy and greater velocity at the bottom of the ramp. A greater velocity causes a greater rate of change in flux as the bar moves through the field. By Faraday's law the emf is greater and therefore also the current.

OR

Increasing B_1 will increase the induced current. If the magnetic field is stronger this increases the flux through the circuit and therefore also the rate of change in the flux. By Faraday's law the emf is then greater and therefore also the current.

OR

Increasing d will increase the induced current. A larger width results in a greater area encompassed by the circuit. Greater area increases the flux through the circuit and therefore also the rate of change in the flux. By Faraday's law the emf is then greater and therefore also the current.

Total for part (e) 2 points

Total for question 2 15 points

Question 3: Free-Response Question**15 points**

- (a) For a loop rule expression that includes terms for the equivalent resistance $2R$ and the potential difference across the battery **1 point**

For an expression that includes charge Q in the term relating the potential difference across the capacitor and includes charge per unit time $\frac{dQ}{dt}$ in the term relating the potential difference across the pair of resistors **1 point**

Example Response

$$\mathcal{E} - \Delta V_C - \Delta V_{R,eq} = 0$$

$$\mathcal{E} - \frac{Q}{C} - 2R \frac{dQ}{dt} = 0$$

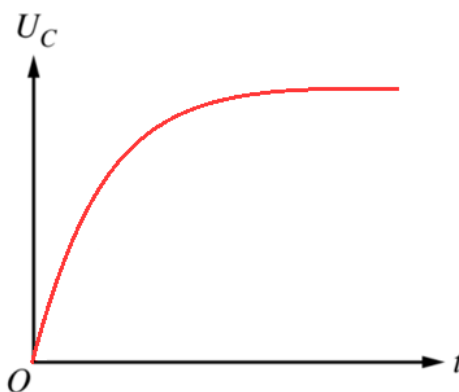
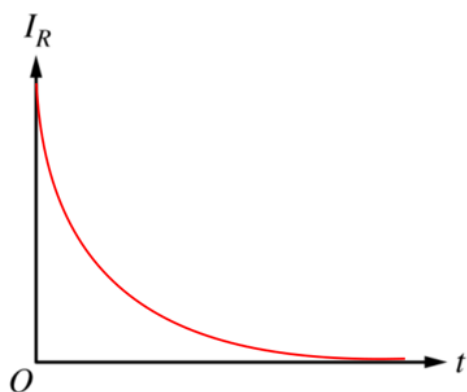
Total for part (a) 2 points

- (b) For sketching a concave up and decreasing curve on the graph of I_R as a function of t **1 point**

For sketching a concave down and increasing curve on the graph of U_C as a function of t **1 point**

For sketching both curves that approach a slope of zero as time increases **1 point**

Scoring Note: The third point can be earned even if the first two points are not earned.

Example Response**Total for part (b) 3 points**

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- (c)(i)** For a correct justification that could include one of the following: **1 point**
- An indication that the current is upwards that includes a statement that indicates that positive charge had accumulated on the top plate of Capacitor 1 and/or negative charge has accumulated on the bottom plate of Capacitor 1 when the switch was closed to Position A
 - An indication that the current is upwards that includes a statement that indicates that the value of the electric potential of the top plate of Capacitor 1 is larger than the electric potential of the bottom plate of Capacitor 1 when the switch was closed to Position A
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Example Responses

When the switch is closed at time t_1 , positive charge has built up on the top plate of the capacitor. This positive charge on the top plate pushes charge up through Resistor 1 and down through Capacitor 2 to charge Capacitor 2.

OR

After a long-time the top plate of Capacitor 1 is at a high potential due to its being charged by the battery. When the switch is closed at time t_1 , the resulting current is up through Resistor 1 as the current goes from high potential on the top plate to low potential clockwise around the circuit through Capacitor 2.

- (c)(ii)** For indicating that the total charge on the positive plate of Capacitor 2 is $\frac{Q_0}{2}$ **1 point**
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Scoring Note: This point can be earned without supporting calculations.

Example Response

The potential difference across Capacitor 1 is equal to the potential difference across Capacitor 2. Capacitor 2 has the same capacitance of Capacitor 1. Therefore, Capacitor 2 stores the same charge that is stored on Capacitor 1. Due to conservation of charge, Capacitor 2 stores half of the original charge equal to $\frac{Q_0}{2}$.

- (c)(iii) For indicating that the total energy dissipated by Resistor 1 is the difference between the initial electric potential energy of the system at time $t = t_1$ and the final electric potential energy of the system after the new steady state conditions have been reached **1 point**

Example Response

$$\Delta E_R = U_C - U_{0C}$$

- For indicating that only Capacitor 1 stores nonzero electric potential energy initially and both capacitors store electric potential energy after the new steady-state conditions have been reached, or an alternate response that is consistent with part (c)(ii) **1 point**

Example Response

$$U_{0C} = U_{01} \text{ AND } U_C = U_1 + U_2$$

- For correct substitutions for the charges stored on the capacitors after the new steady state conditions have been reached based on part (c)(ii) **1 point**

Example Solution

$$\Delta E_R = U_C - U_{0C}$$

$$\Delta E_R = \left(\frac{1}{2} \left(\frac{1}{C} \right) \left(\frac{Q_0}{2} \right)^2 + \frac{1}{2} \left(\frac{1}{C} \right) \left(\frac{Q_0}{2} \right)^2 \right) - \frac{1}{2} \frac{Q_0^2}{C}$$

$$\Delta E_R = \frac{Q_0^2}{4C} - \frac{1}{2} \frac{Q_0^2}{C}$$

$$\Delta E_R = -\frac{Q_0^2}{4C}$$

Total for part (c) 5 points

- (d) For the correct expression for charge on the top plate of Capacitor 2 $\left(Q_2 = \frac{2Q_0}{3} \right)$ **1 point**

Example Response

$$C_1 = C \text{ and } C_2 = \kappa C = 2C$$

$$\Delta V_{C1} = \Delta V_{C2}$$

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$Q_1 = \frac{1}{2} Q_2$$

$$Q_1 + Q_2 = Q_0$$

$$\frac{1}{2} Q_2 + Q_2 = Q_0$$

$$Q_2 = \frac{2}{3} Q_0$$

Scoring Note: This point can be earned without supporting calculations.

Total for part (d) 1 point

- (e)(i) For a loop rule equation that includes terms for the potential difference across Resistor 2 and the potential difference across Capacitor 2 with the dielectric **1 point**

Example Response

$$\Delta V_{R,2} - \Delta V_{C,2} = 0$$

$$\Delta V_{R,2} = \Delta V_{C,2}$$

For correct substitution of IR for the potential difference across Resistor 2 **1 point**

Example Response

$$\Delta V_{R,2} = IR$$

For correct substitutions of $2C$ for the new capacitance of Capacitor 2 with the dielectric inserted and of the charge consistent with part (d) into the expression for the potential difference across the capacitor **1 point**

Example Response

$$\Delta V_{C,2} = \frac{Q_2}{C_2}$$

$$\Delta V_{C,2} = \left(\frac{2Q_0}{3} \right) \left(\frac{1}{2C} \right)$$

Example Solution

$$\Delta V_{R,2} - \Delta V_{C,2} = 0$$

$$\Delta V_{R,2} = \Delta V_{C,2}$$

$$IR = \left(\frac{2Q_0}{3} \right) \left(\frac{1}{2C} \right)$$

$$IR = \frac{Q_0}{3C}$$

$$I = \frac{Q_0}{3RC}$$

- (e)(ii) For indicating that the current is zero **1 point**

Total for part (e) 4 points

Total for question 3 15 points

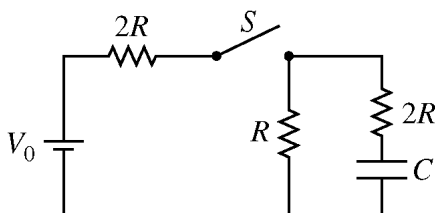
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AP[®] Physics C: Electricity and Magnetism

Scoring Guidelines Set 2

2019 SCORING GUIDELINES**General Notes About 2019 AP Physics Scoring Guidelines**

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.
2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at <https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf>.
3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.
5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.
6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

2019 SCORING GUIDELINES**Question 1****15 points**

The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0 , a capacitor of capacitance C , and a switch S . The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0 , R , C , and fundamental constants, as appropriate.

- (a) LO CNV-7.B.a, SP 5.A, 5.E
2 points

Derive an expression for the steady-state current supplied by the battery.

For using Ohm's law		1 point
$I = \frac{V}{R_{\text{eff}}} = \frac{V_0}{(2R + R)}$		
For correct substitution leading to correct answer		1 point
$I = \frac{V_0}{3R}$		

- (b) LO CNV-7.B.b, SP 5.A, 5.E
2 points

Derive an expression for the charge on the capacitor.

For using the equation relating stored charge to capacitance		1 point
$q = CV = CV_C = CV_R$		
For determining V_R and substituting into the above equation		1 point
$V_R = IR = \left(\frac{V_0}{3R}\right)R = \frac{1}{3}V_0$		
$q = CV_R = C\left(\frac{1}{3}V_0\right) = \frac{1}{3}CV_0$		

2019 SCORING GUIDELINES**Question 1 (continued)**

- (c) LO CNV-7.B.b, SP 5.A, 5.E
2 points

Derive an expression for the energy stored in the capacitor.

For any correct equation for energy stored in a capacitor		1 point
$U = \frac{q^2}{2C}$		
Substitute charge and/or voltage from part (b)		1 point
$U = \frac{(CV_0/3)^2}{2C} = \frac{1}{18}CV_0^2$		

Now the switch is opened at time $t = 0$.

- (d) LO CNV-7.D.a, SP 5.A, 5.E
2 points

Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time t after the switch is opened.

For any correct voltage loop equation		1 point
$V_C - V_R - V_{2R} = 0 \therefore V_C = V_R + V_{2R}$		
$\frac{q(t)}{C} = I(R + 2R)$		
<u>Note:</u> Any correct loop equation for when the switch is open earns the point		
For substituting $-dq/dt$ or dq/dt for the current, consistent with loop equation		1 point
$\frac{q(t)}{C} = -3R \frac{dq}{dt}$		

2019 SCORING GUIDELINES**Question 1 (continued)**

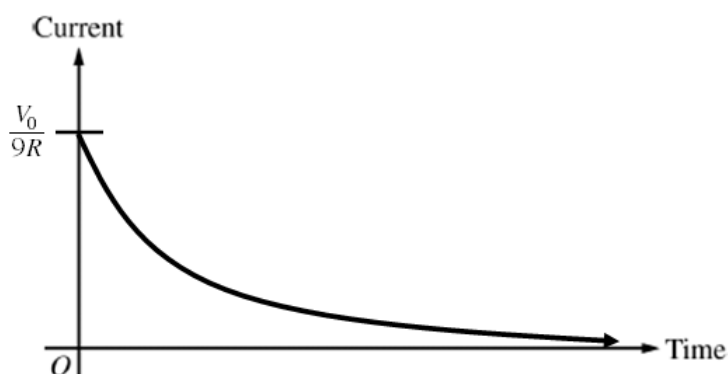
- (e)
- i. LO CNV-7.B.a, SP 5.A, 5.E
2 points

Calculate the current in resistor R immediately after the switch is opened.

For using a voltage consistent with part (b) in $I = \frac{V}{R}$		1 point
For substituting the correct resistance ($3R$) into Ohm's law		1 point
$I = \frac{V_0/3}{3R} = \frac{V_0}{9R}$		

- ii. LO CNV-7.E.b, SP 3.C
3 points

On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



For a curve that is concave up throughout graph		1 point
For having the horizontal axis as an asymptote		1 point
For labeling the maximum current consistent with part (e)(i)		1 point

2019 SCORING GUIDELINES**Question 1 (continued)**

- (f) LO CNV-7.G.a, SP 7.A, 7.C
2 points

Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c)?

___ Greater than ___ Less than ___ Equal to

Justify your answer.

For selecting “Equal to”	1 point
For a correct justification invoking conservation of energy	1 point
Example: After the switch is opened, the capacitor will discharge all of its stored energy and charge. Assuming no energy is lost in the wire, then the only parts of the circuit that will dissipate this energy are the two resistors in series with the capacitor.	
<i>Alternate Solution</i>	<i>Alternate Points</i>
For selecting “Less than”	1 point
For a correct justification invoking conservation of energy	1 point
Example: After the switch is opened, the capacitor will discharge all of its stored energy and charge. Additionally there is energy lost due to resistance in the wire and/or energy loss in the capacitor.	

2019 SCORING GUIDELINES**Question 1 (continued)****Learning Objectives**

CNV-7.B.a: Calculate the potential difference across a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.

CNV-7.B.b: Calculate the stored charge on a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.

CNV-7.D.a: Derive expressions using calculus to describe the time dependence of the stored charge or potential difference across the capacitor, or the current or potential difference across the resistor in an RC circuit when charging or discharging a capacitor.

CNV-7.E.b: Describe the behavior of the voltage or current behavior over time for a circuit that contains resistors and capacitors in a multi-loop arrangement.

CNV-7.G.a: Describe the energy transfer in charging or discharging a capacitor in an RC circuit.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.

5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.

5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

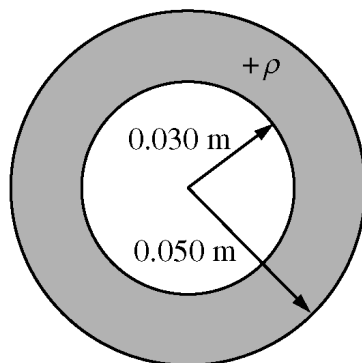
7.A: Make a scientific claim.

7.C: Support a claim with evidence from physical representations.

2019 SCORING GUIDELINES

Question 2

15 points



A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ , as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m: } \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m: } \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

$$r > 0.050 \text{ m: } \rho = 0$$

- (a) LO CNV-2.D.a, SP 6.B, 6.C
3 points

Calculate the total charge of the sphere.

For indicating the need to integrate the expression for charge density to determine the total charge on the sphere		1 point
$Q = \int \rho dV$		
For proper substitutions into the integration		1 point
$Q = \int \frac{(1.6 \times 10^{-6})}{r} (4\pi r^2) dr$		
For using the proper limits of integration		1 point
$Q = 4\pi (1.6 \times 10^{-6}) \int_{r=0.03}^{r=0.05} r dr = 4\pi (1.6 \times 10^{-6}) \left[\frac{r^2}{2} \right]_{r=0.03}^{r=0.05}$		
$Q = (2\pi)(1.6 \times 10^{-6})(0.05^2 - 0.03^2) = 1.61 \times 10^{-8} \text{ C}$		

2019 SCORING GUIDELINES**Question 2 (continued)**

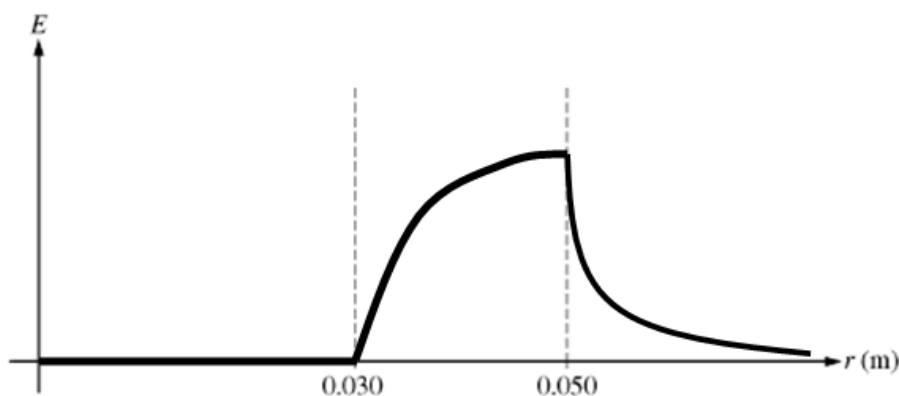
- (b) LO CNV-2.D.a, SP 6.C
3 points

Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.

For correctly evaluating the surface integral in Gauss's law		1 point
$\oint E \cdot dA = E(4\pi r^2)$		
For correctly substituting the answer from part (a) and correct radius into above equation		1 point
$\frac{Q_{enc}}{\epsilon_0} = E(4\pi r^2) \therefore E = \frac{Q_{enc}}{4\pi\epsilon_0 r^2} = \frac{(1.61 \times 10^{-8} \text{ C})}{(4\pi)(8.85 \times 10^{-12})(0.05)^2}$		
For an answer consistent with part (a) with correct units		1 point
$E = 5.79 \times 10^4 \text{ N/C}$		

- (c) LO CNV-2.C, SP 3.C
3 points

On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.



For clearly showing a graph with a value of $E = 0$ for $r < 0.030 \text{ m}$		1 point
For a continuous graph that starts at zero, is concave down, and increases in value from $r = 0.030$ to $r = 0.050$		1 point
For a continuous graph that decreases asymptotically toward the horizontal axis for $r > 0.050 \text{ m}$		1 point

2019 SCORING GUIDELINES

Question 2 (continued)

- (d) LO CNV-1.G.a, SP 6.B, 6.C
2 points

Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

For substituting the total charge from part (a) into a correct expression for electric potential	1 point
For substituting $r = 0.05$ m into a correct expression for electric potential	1 point
$V_R = \frac{Q_{\text{tot}}}{4\pi\epsilon_0 r} = \frac{(9 \times 10^9)(1.61 \times 10^{-8} \text{ C})}{(0.05 \text{ m})} = 2900 \text{ V}$	
<i>Alternate Solution</i>	<i>Alternate Points</i>
For substituting the total charge from part (a) into an integration for electric potential	1 point
$\Delta V_R = V_R - V_\infty = V_R = -\int E dr = -\int \frac{Q_{\text{enc}}}{4\pi\epsilon_0 r^2} dr = -\int \frac{(9 \times 10^9)(1.61 \times 10^{-8} \text{ C})}{r^2} dr$	
For integrating with correct limits of integration	1 point
$V_R = -\int_{r=\infty}^{r=0.05 \text{ m}} \frac{145}{r^2} dr = -145 \left[-\frac{1}{r} \right]_{r=\infty}^{r=0.05 \text{ m}} = 145 \left(\frac{1}{(0.05 \text{ m})} - \frac{1}{\infty} \right) = 2900 \text{ V}$	

A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.

- (e)
i. LO ACT-1.D, SP 6.B, 6.C
2 points

Calculate the magnitude of the initial acceleration of the proton.

For using a correct expression of Newton's second law in terms of the electric field	1 point
$F = ma \therefore qE = ma \therefore a = \frac{qE}{m}$	
For correctly substituting into equation above	1 point
$a = \frac{qE}{m} = \frac{(1.6 \times 10^{-19} \text{ C})(5.79 \times 10^4 \text{ N/C})}{1.67 \times 10^{-27} \text{ kg}} = 5.55 \times 10^{12} \text{ m/s}^2$	

2019 SCORING GUIDELINES**Question 2 (continued)**

(e) continued

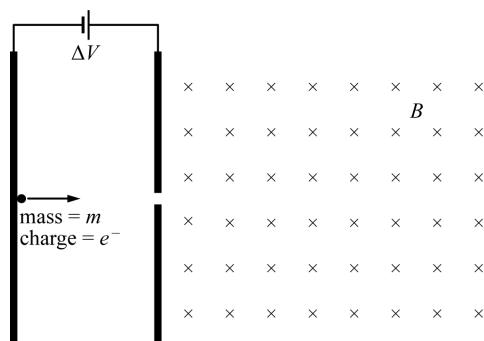
ii. LO CNV-1.E, SP 6.B, 6.C

2 points

Calculate the speed of the proton after a long time.

For a correct expression of kinetic energy in terms of the electric potential difference		1 point
$-q\Delta V = \frac{1}{2}mv^2$		
For correctly substituting into equation above		1 point
$v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{-2(1.6 \times 10^{-19} \text{ C})(0 - 2900 \text{ V})}{(1.67 \times 10^{-27} \text{ kg})}} = 7.45 \times 10^5 \text{ m/s}$		

Learning Objectives**ACT-1.D:** Determine the motion of a charged object of specified charge and mass under the influence of an electrostatic force.**CNV-1.E:** Calculate the work done or changes in kinetic energy (or changes in speed) of a charged particle when it is moved through some known potential difference.**CNV-1.G.a:** Use the general relationship between electric field and electric potential to calculate the relationships between the magnitude of electric field or the potential difference as a function of position.**CNV-2.C:** State and use Gauss's law in integral form to derive unknown electric fields for planar, spherical, or cylindrically symmetrical charge distributions.**CNV-2.D.a:** Using appropriate mathematics (which may involve calculus), calculate the total charge contained in lines, surfaces, or volumes when given a linear-charge density, a surface-charge density, or a volume-charge density of the charge configuration.**Science Practices****3.C:** Sketch a graph that shows a functional relationship between two quantities.**6.B:** Apply an appropriate law, definition, or mathematical relationship to solve a problem.**6.C:** Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

2019 SCORING GUIDELINES**Question 3****15 points**

Two plates are set up with a potential difference V between them. A small sphere of mass m and charge $-e$ is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude B directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of V , m , e , B , and fundamental constants, as appropriate.

(a)

- i. LO CHG-1.A.a, SP 7.A
1 point

What is the initial direction of the force on the sphere as it enters the magnetic field?

☐ Into the page

☐ Out of the page

☐ Toward the top of the page

☐ Toward the bottom of the page

For selecting “Toward the bottom of the page”	
---	--

1 point

- ii. LO CHG-1.B, SP 7.A
1 point

Describe the path taken by the sphere after it enters the magnetic field.

For describing a circular path for the sphere consistent with the selection from part (a)(i)	
--	--

1 point

Example: The sphere will move in a circular path toward the bottom of the page.	
---	--

2019 SCORING GUIDELINES**Question 3 (continued)**

- (b) LO CNV-1.E, SP 5.A, 5.E
2 points

Derive an expression for the speed of the sphere as it passes through the small opening.

For using a valid equation relating potential difference to kinetic energy of the sphere		1 point
$\Delta K = -\Delta U \therefore \frac{1}{2}mv^2 = -q\Delta V$		
$\frac{1}{2}mv^2 = eV$		
For the correct answer with supporting work		1 point
$v = \sqrt{\frac{2eV}{m}}$		

- (c) LO CHG-1.C, SP 5.A, 5.E
3 points

Derive an expression for the radius of the path taken by the sphere as it moves through the magnetic field.

For an expression relating the magnetic force to the centripetal force		1 point
$\frac{mv^2}{r} = Bqv$		
For substituting the charge of the sphere into the above equation and solving for r		1 point
$\frac{mv}{r} = Be \therefore r = \frac{mv}{Be}$		
For substituting the answer from part (b)		1 point
$r = \frac{m}{Be} \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2mV}{eB^2}}$		

An experiment is performed in which a beam of electrons is accelerated across the space between the plates and passes through the small opening. After passing through the opening, the electrons travel in a semicircular path and strike the right-hand plate. The potential difference between the plates is varied in regular increments, as shown in the table below. For each potential difference, the magnetic field is varied in order to cause the beam to strike the right-hand plate at a distance of 0.020 m from the opening.

Potential difference (V)	60	70	100	110	120	140
Magnetic field ($T \times 10^{-3}$)	2.62	2.78	3.39	3.54	3.78	3.99

2019 SCORING GUIDELINES**Question 3 (continued)**

- (d) LO CHG-1.C, SP 4.C
1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the mass-to-charge ratio of an electron.

Vertical axis: _____

Horizontal axis: _____

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

$$r = \sqrt{\frac{2m_e V}{eB^2}} \therefore B^2 = \frac{2m_e}{er^2} V$$

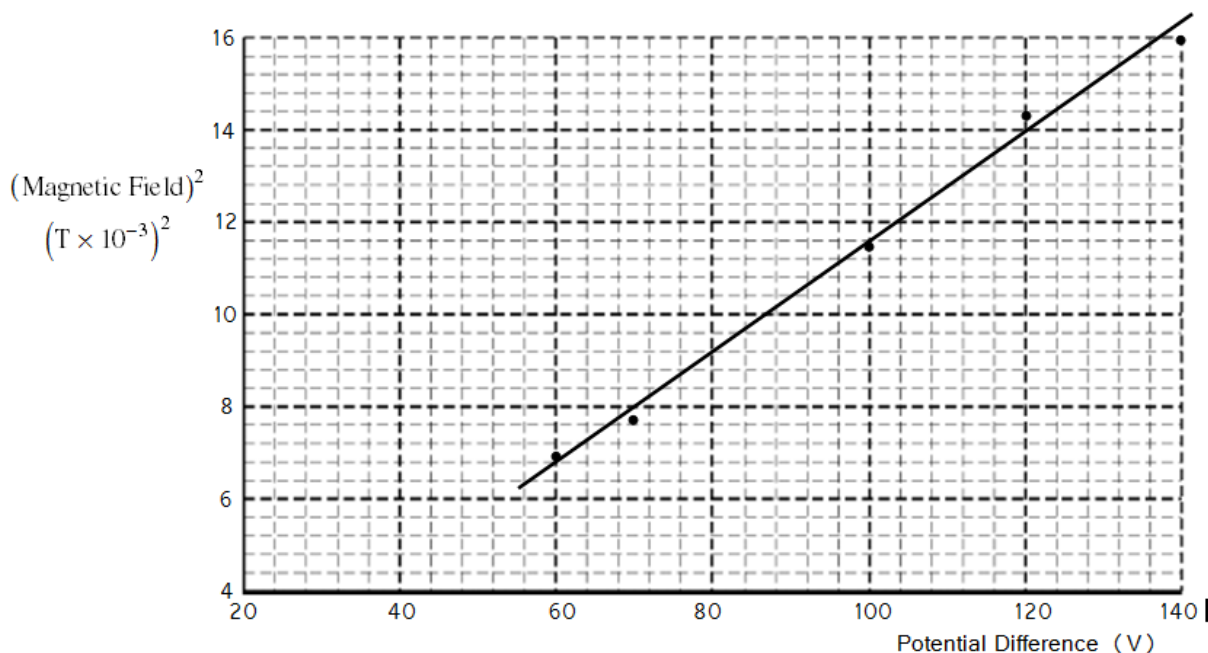
Potential difference (V)	60	70	100	110	120	140
Magnetic field ($T \times 10^{-3}$)	2.62	2.78	3.39	3.54	3.78	3.99
(Magnetic field) ² ($T \times 10^{-3}$) ²	6.86	7.73	11.5	12.5	14.3	15.9

For indicating axis choices with appropriate quantities to produce a straight line		1 point
Note: There are several possible choices.		

2019 SCORING GUIDELINES**Question 3 (continued)**

- (e) LO CHG-1.C, SP 3.A, 4.C
4 points

On the graph below, plot the relationship determined in part (d). Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



For correctly labeling both axes with quantities and units	1 point
For correctly scaling and labeling the axes so that the points use at least half the grid	1 point
For correctly plotting the data	1 point
For drawing a straight line that represents the data	1 point

- (f) LO CHG-1.C, SP 4.D, 5.E
3 points

Using the straight line from part (e), determine the mass-to-charge ratio of an electron.

For correctly calculating the slope from the best-fit straight line and not from the data points unless the points fall on the best-fit line	1 point
$\text{slope} = \frac{14 - 9.2}{120 - 80} = 0.12 \frac{(\text{T} \times 10^{-3})^2}{\text{V}} = 1.2 \times 10^{-7} \text{ T}^2/\text{V}$	
For a correct expression relating the slope to the mass to charge ratio of an electron	1 point
$B^2 = \frac{2m_e}{er^2} V \therefore \text{slope} = \frac{2m_e}{er^2} \therefore \frac{m_e}{e} = \frac{r^2 \times \text{slope}}{2}$	
For substituting correct values into the above equation	1 point
$\frac{m_e}{e} = \frac{(0.01 \text{ m})^2 \times (1.2 \times 10^{-7} \text{ T}^2/\text{V})}{2} = 6.0 \times 10^{-12} \text{ kg/C}$	

2019 SCORING GUIDELINES**Question 3 (continued)****Learning Objectives**

CHG-1.A.a: Calculate the magnitude and direction of the magnetic force of interaction between a moving charged particle of specified charge and velocity moving in a region of a uniform magnetic field.

CHG-1.B: Describe the path of different moving charged particles (i.e., of different type of charge or mass) in a uniform magnetic field.

CHG-1.C: Derive an expression for the radius of a circular path for a charged particle of specified characteristics moving in a specified magnetic field.

CNV-1.E: Calculate the work done or changes in kinetic energy (or changes in speed) of a charged particle when it is moved through some known potential difference.

Science Practices

3.A: Select and plot appropriate data.

4.C: Linearize data and/or determine a best-fit line or curve.

4.D: Select relevant features of a graph to describe a physical situation or solve problems.

5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.

5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

7.A: Make a scientific claim.

2018**AP[®]** **CollegeBoard**

AP Physics C: Electricity and Magnetism

Scoring Guidelines

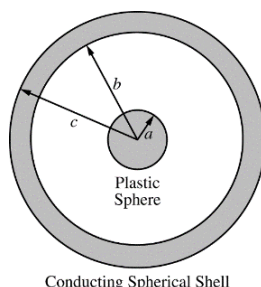
2018 SCORING GUIDELINES**General Notes About 2018 AP Physics Scoring Guidelines**

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.
2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at <https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf>.
3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as “derive” and “calculate” on the exams, and what is expected for each, see “The Free-Response Sections — Student Presentation” in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or “Terms Defined” in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.
5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.
6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

2018 SCORING GUIDELINES

Question 1

15 points total

Distribution
of points

A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β , r , a , and physical constants, as appropriate.

- (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

- i. 3 points

$$r < a$$

For using the integral to determine the charge enclosed	1 point
$q_{enc} = \int \rho dV = \int \beta r dV$	
For correctly substituting for dV into the integration to determine the charge enclosed	1 point
$q_{enc} = \int (4\pi r'^2)(\beta r') dr' = 4\beta\pi \int_{r'=0}^{r'=r} r'^3 dr' = 4\beta\pi \left[\frac{r'^4}{4} \right]_{r'=0}^{r'=r}$	
For a correct answer	1 point
$q_{enc} = \beta\pi r^4$	

- ii. 1 points

$$a < r < b$$

For an answer consistent with (a)(i)	1 point
$q_{enc} = \beta\pi a^4$	

2018 SCORING GUIDELINES

Question 1 (continued)

**Distribution
of points**

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. 2 points

$$r < a$$

For correctly substituting the area of a Gaussian sphere into Gauss's law		1 point
$\frac{q_{enc}}{\epsilon_0} = \oint \vec{E} \cdot d\vec{A}$		
$\frac{q(r)}{\epsilon_0} = EA_{sphere} = E(4\pi r^2)$		
For correctly substituting the charge from part (a)(i) into the equation above		1 point
$\frac{\beta\pi r^4}{\epsilon_0} = E(4\pi r^2)$		
$E = \frac{\beta r^2}{4\epsilon_0}$		

ii. 1 point

$$a < r < b$$

For correctly substituting the charge from part (a)(ii) into Gauss's law to calculate the electric field		1 point
$\frac{\beta\pi a^4}{\epsilon_0} = E(4\pi r^2)$		
$E = \frac{\beta a^4}{4\epsilon_0 r^2}$		

2018 SCORING GUIDELINES

Question 1 (continued)

**Distribution
of points**

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. 2 points

Determine the charge on the inner surface of the conducting shell.
Justify your answer.

For an answer consistent with part (a)(ii) (must have opposite sign)	1 point
$q_{\text{inner}} = -\beta\pi a^4$	
For a correct justification	1 point
$E_{\text{shell}} = 0 = \frac{q_{\text{enc}}}{\epsilon_0} \therefore q_{\text{enc}} = 0 = q_{\text{sphere}} + q_{\text{inner}} \therefore q_{\text{inner}} = -q_{\text{sphere}}$	

ii. 1 point

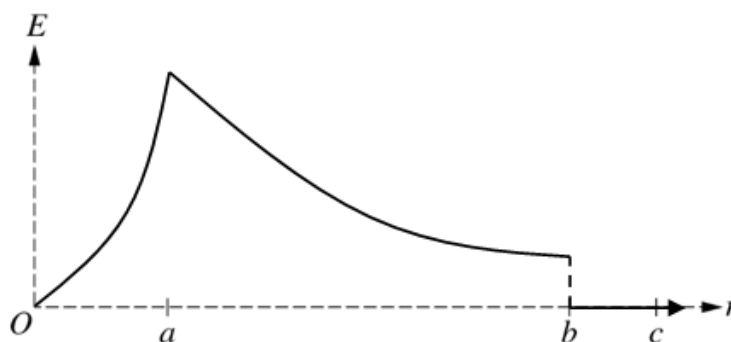
Determine the charge on the outer surface of the conducting shell.

A correct answer	1 point
$q_{\text{outer}} = 0$	

(d)

i. 3 points

On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



For concave up graph that begins at the origin and increases to $r = a$	1 point
For a concave up graph that is continuous with the first section and decreases from $r = a$ to nonzero value at $r = b$	1 point
For a graph with a value of $E = 0$ for $r > b$	1 point
Note: Due to scale, discontinuity at point b is not required.	

2018 SCORING GUIDELINES

Question 1 (continued)

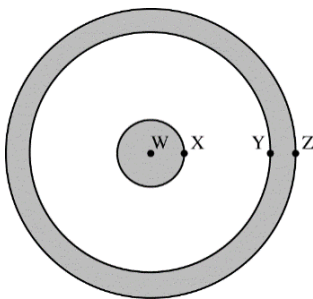
**Distribution
of points**

(d) (continued)

ii. 2 points

The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

__1__ W __2__ X __3__ Y __3__ Z

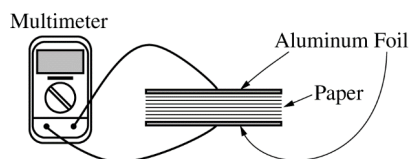


For any ranking that has Y and Z at the same potential	1 point
For ranking the electric potentials: $W > X > Y \text{ \& } Z$ (because $Y = Z$ is the first point)	1 point

2018 SCORING GUIDELINES

Question 2

15 points total

Distribution
of points

An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060 \text{ m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)		
500	0.045	6.5×10^{-11}		
400	0.036	7.4×10^{-11}		
300	0.027	8.9×10^{-11}		
200	0.018	11.9×10^{-11}		
100	0.010	21.0×10^{-11}		

(a) 1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: _____

Horizontal axis: _____

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

For indicating variables that will create a straight line whose slope can be used to determine the dielectric constant of the paper		1 point
Example: Vertical axis: C Horizontal axis: $\frac{1}{d}$		
Note: Student earns full credit if the axes are reversed or if they use another acceptable combination.		

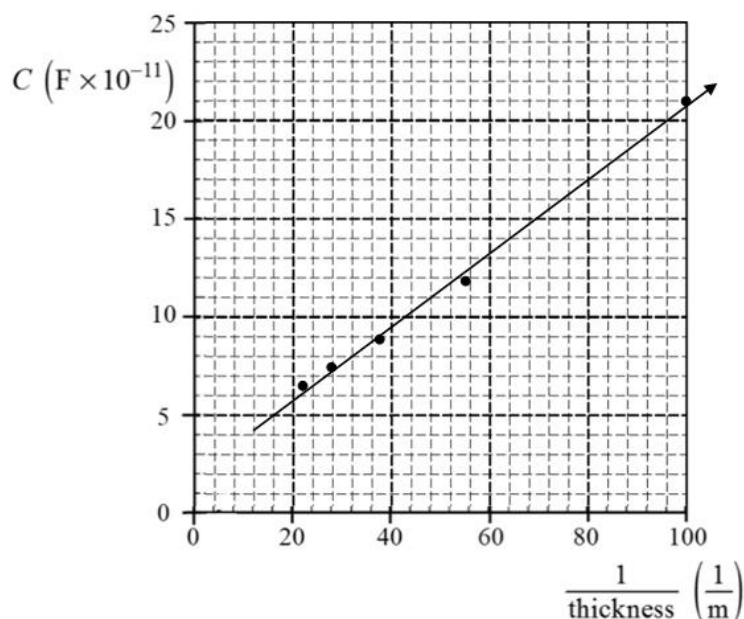
2018 SCORING GUIDELINES

Question 2 (continued)

Distribution
of points

(b) 4 points

Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



For a correct scale that uses more than half the grid	1 point
For correctly labeling the axes including units	1 point
For correctly plotting the data	1 point
For drawing a straight line consistent with the plotted data	1 point

(c) 3 points

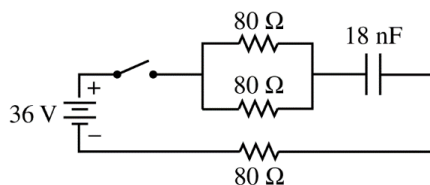
Using the straight line, calculate a dielectric constant for the paper.

For correctly calculating the slope from the best-fit line and not the data points unless the points fall on the best-fit line	1 point
$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{(17 - 8)(\text{F} \times 10^{-11})}{(80 - 32)(1/\text{m})} = 0.19 \text{ F} \cdot \text{m} \quad (\text{Linear regression} = 0.187 \text{ F} \cdot \text{m})$	
For correctly relating the slope to the dielectric constant	1 point
$\text{slope} = \kappa \epsilon_0 A$	
$\kappa = \frac{\text{slope}}{\epsilon_0 A} = \frac{(0.19 \text{ F} \cdot \text{m})}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.060 \text{ m}^2)}$	
For a correct answer	1 point
$\kappa = 3.58 \quad (\text{Linear regression} = 3.52)$	

2018 SCORING GUIDELINES

Question 2 (continued)

Distribution
of points



The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical 80 Ω resistors, and an open switch, as shown in the figure above.

(d) 3 points

Calculate the current in the battery immediately after the switch is closed.

For calculating the equivalent resistance for the parallel resistors	1 point
$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{80\ \Omega} + \frac{1}{80\ \Omega} = \frac{2}{80\ \Omega} = \frac{1}{40\ \Omega}$	
$R_p = 40\ \Omega$	
For using Ohm's law with the potential difference across the capacitor equal to zero	1 point
$I = \frac{V}{R} = \frac{V}{R_p + R_3} = \frac{(36\ \text{V})}{R_p + R_3}$	
For substitution of values for resistance including the value for combined resistance above	1 point
$I = \frac{V}{R_p + R_3} = \frac{(36\ \text{V})}{(40\ \Omega + 80\ \Omega)} = 0.30\ \text{A}$	

(e) 2 points

Determine the time constant for this circuit.

For using the equation for the time constant with the equivalent resistance from above	1 point
$\tau = R_{eq}C = (120\ \Omega)(18\ \text{nF})$	
For an answer with units consistent with part (d)	1 point
$t = 2.16\ \mu\text{s}$	

AP PHYSICS C: ELECTRICITY AND MAGNETISM

2018 SCORING GUIDELINES

Question 2 (continued)

**Distribution
of points**

- (f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. 1 point

Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

____ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

Select “Larger than measured”		
For an appropriate explanation		1 point
<i>Example: The battery is not ideal and has internal resistance, so the actual resistance for the circuit is larger than the measured resistance.</i>		

ii. 1 points

Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

____ Larger than measured ____ Smaller than measured

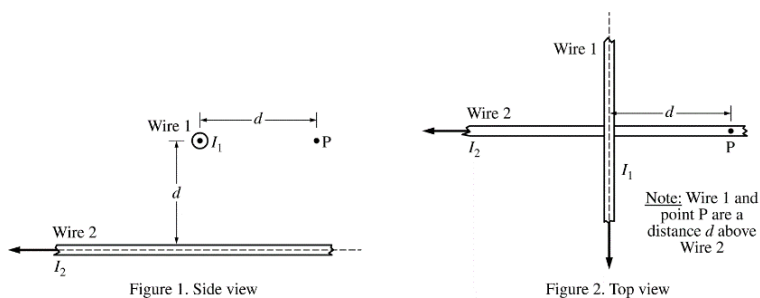
Explain experimentally what could account for this.

Select “Larger than measured”		
For an appropriate explanation		1 point
<i>Example: Some of the sheets of paper may be thinner than expected, so the actual capacitance for the circuit is larger than the measured capacitance.</i>		

2018 SCORING GUIDELINES

Question 3

15 points total

Distribution
of points

The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I , d , and physical constants, as appropriate.

(a) 2 points

Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

For attempting to use Ampere's law to calculate the magnetic field at point P	1 point
$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$	
$B(2\pi d) = \mu_0 I$	
For a correct answer	1 point
$B = \frac{\mu_0 I}{2\pi d}$	

(b) 2 points

Derive an expression for the magnitude of the net magnetic field at point P.

For indicating $B_2 = 2B_1$	1 point
$B_1 = \frac{\mu_0 I}{2\pi d}$ & $B_2 = \frac{\mu_0 (2I)}{2\pi d} = \frac{\mu_0 I}{\pi d}$	
For an indication that the magnitude of the magnetic field is the vector sum of B_1 and B_2	1 point
$B_{\text{net}} = \sqrt{B_1^2 + B_2^2} = \sqrt{B^2 + B^2} = \sqrt{\left(\frac{\mu_0 I}{2\pi d}\right)^2 + \left(\frac{\mu_0 (2I)}{2\pi d}\right)^2} = \frac{\sqrt{5}\mu_0 I}{2\pi d}$	

2018 SCORING GUIDELINES

Question 3 (continued)

Distribution
of points

(c) 2 points

Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

For correctly relating the angle to the individual magnetic fields	1 point
$\theta = \tan^{-1}\left(\frac{B_1}{B_2}\right)$	
For correctly substituting B_1 and B_2 into the equation	1 point
$\theta = \tan^{-1}\left(\frac{B_1}{B_2}\right) = \tan^{-1}\left(\frac{\left(\frac{\mu_0 I}{2\pi d}\right)}{\left(\frac{\mu_0 (2I)}{2\pi d}\right)}\right) = \tan^{-1}\left(\frac{1}{2}\right) = 26.6^\circ$	

(d) 2 points

Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2? Assume gravitational effects are negligible.

___ Wire 1 will not move.

___ Wire 1 will move upward as viewed in Figure 1.

___ Wire 1 will move downward as viewed in Figure 1.

___ Wire 1 will rotate clockwise as viewed in Figure 2.

___ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

For stating there is no translational motion since the magnetic forces on the wire cancel	1 point
For stating there is a net torque which causes rotation	1 point
<i>Example: In Figure 2, the top portion of wire 1 will be in a magnetic field into the page from wire 2 and, thus, will experience a force to the right. The bottom portion of wire 1 will be in a magnetic field out of the page from wire 2 and, thus, will experience a force to the left. So the net force will be zero, but there will be a net clockwise torque, so the wire will rotate clockwise.</i>	

2018 SCORING GUIDELINES

Question 3 (continued)

Distribution
of points

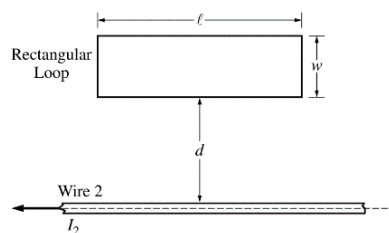


Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ , width w , and resistance R . The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1} .

(e) 1 point

Of the following, select the integration that will give an expression for the flux Φ as a function of time t .

_____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

_____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

___X___ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

_____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

For selecting $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$	1 point
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Question 3 (continued)

Distribution
of points

(f) 3 points

Given that the flux through the rectangular loop as a function of time t is given by the equation

$\Phi = \frac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\frac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0 , d , r , R , w , k , ℓ , and physical constants, as appropriate.

For attempting to take the time derivative of the magnetic flux to calculate the emf	1 point
$\mathcal{E} = \left -\frac{d\Phi}{dt} \right = \frac{\mu_0 I_0 \ell}{\pi} \ln\left(\frac{d + w}{d}\right) \left \frac{d}{dt} [(1 - kt)] \right $	
$\mathcal{E} = \frac{\mu_0 I_0 k \ell}{\pi} \ln\left(\frac{d + w}{d}\right)$	
For dividing the emf by the resistance to calculate the current	1 point
$I = \frac{\mathcal{E}}{R} = \frac{\mu_0 I_0 k \ell}{\pi R} \ln\left(\frac{d + w}{d}\right)$	
For a correct answer	1 point
$I = \frac{\mu_0 I_0 k \ell}{\pi R} \ln\left(\frac{d + w}{d}\right)$	

(g) 3 points

What is the direction of the current, if any, induced in the loop as seen in Figure 3?

___ Clockwise ___ Counterclockwise

___ Undefined because there is no current induced in the loop

Justify your answer.

For selecting “Clockwise” with an attempt at a relevant justification	1 point
For indicating that the flux inside the loop will decrease	1 point
For using Lenz’s law to relate the decrease in the flux to the clockwise current	1 point
<i>Example: Because the current in the wire is decreasing, the flux in the loop will decrease. According to Lenz’s law, the induced current should create a magnetic field to oppose this decrease. Thus the induced magnetic field must be into the page, and the current in the loop must be clockwise.</i>	