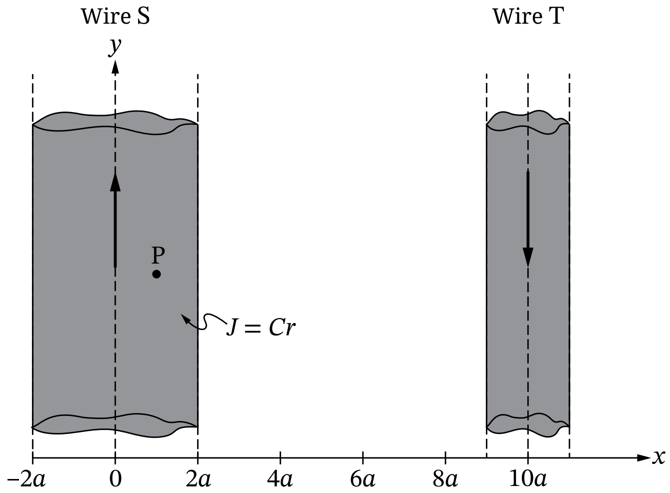
Question 1: Version J

1. Two very long, cylindrical, current-carrying Wires S and T are oriented parallel to the y-axis, as shown in Figure 1. The centers of the wires are in the xy-plane. The wires are described as follows.

- Wire S has radius $2a$ and is centered at $x = 0$. Wire S has nonuniform current density and carries total current I_0 in the $+y$-direction. The magnitude of the current density in Wire S as a function of the radial distance r from the center of the wire is described by $J = Cr$, where C is a positive constant.
- Wire T has radius a and is centered at $x = 10a$. Wire T has uniform current density and carries total current I_0 in the $-y$-direction.

Figure 1

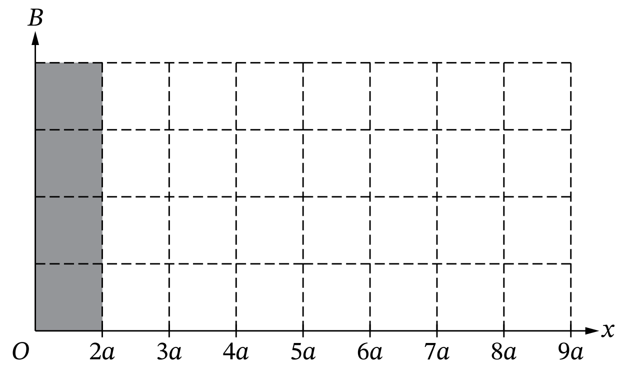


Part A

i. Point P is located inside of Wire S at horizontal position $x = a$. **Derive** an expression for the magnitude of the magnetic field at Point P due only to the current in Wire S in terms of C, a, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

ii. On the axes shown in Figure 2, **sketch** a graph of the magnitude B of the magnetic field due to the currents in both wires as a function of x along the x-axis in the region $2a \leq x \leq 9a$. Sketches made in the shaded region will not be graded.

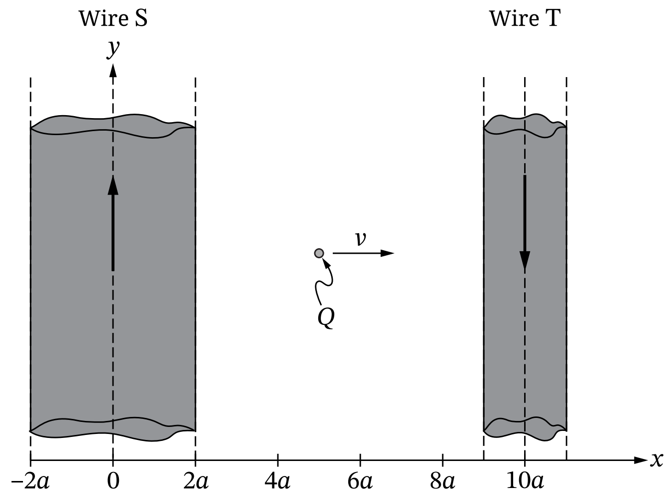
Figure 2



Part B

At the instant shown in Figure 3, a small sphere with charge Q is at horizontal position $x = 5a$ and moving in the xy-plane with speed v in the $+x$-direction. At this instant, the net magnetic force exerted on the sphere due to the currents in Wires S and T has magnitude F.

Figure 3

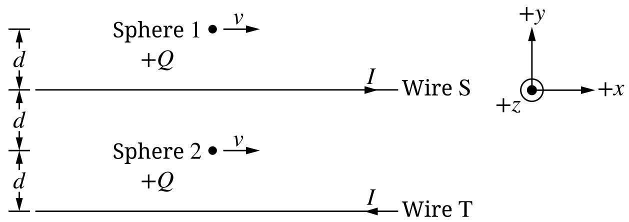


Derive an expression for F in terms of a, Q, I_0, v, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Question 4

4. Long, parallel wires S and T are a distance $2d$ apart. Both wires carry equal currents I, but the currents are in opposite directions. Both wires are parallel to the x-axis. At the instant shown in Figure 1, Sphere 1 is a distance d above Wire S, Sphere 2 is a distance d below Wire S, and both spheres are moving with speed v in the $+x$-direction. Each sphere has positive charge $+Q$. Gravitational effects are negligible.

Figure 1



A. F_1 is the magnitude of the magnetic force exerted on Sphere 1 due to the currents in wires S and T. F_2 is the magnitude of the magnetic force exerted on Sphere 2 due to the currents in wires S and T.

Indicate whether F_2 is greater than, less than, or equal to F_1 by writing one of the following.

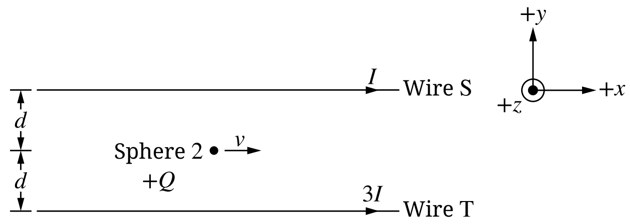
- $F_2 > F_1$
- $F_2 < F_1$
- $F_2 = F_1$

Justify your answer.

B. **Derive** an expression for the magnitude B_{tot} of the magnetic field at the location of Sphere 2 due to the currents in wires S and T in terms of d, I, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

C. Later, Wire T carries current $3I$ in the $+x$-direction. At the instant shown in Figure 2, Sphere 2 is a distance d below Wire S and is moving with speed v in the $+x$-direction. F_{new} is the new magnitude of the magnetic force exerted on Sphere 2 due to the currents in wires S and T.

Figure 2



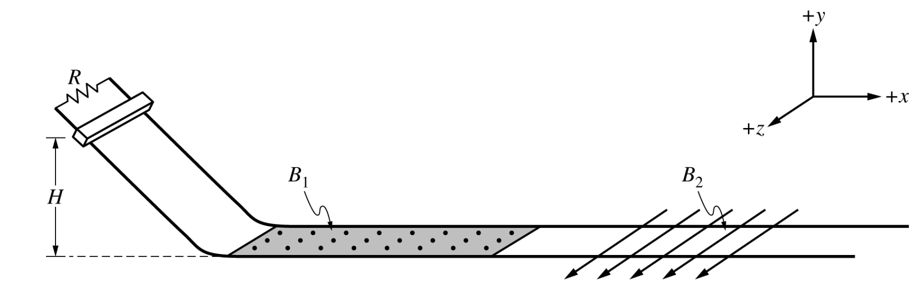
Indicate whether F_{new} is greater than, less than, or equal to F_2 by writing one of the following.

- $F_{new} > F_2$
- $F_{new} < F_2$
- $F_{new} = F_2$

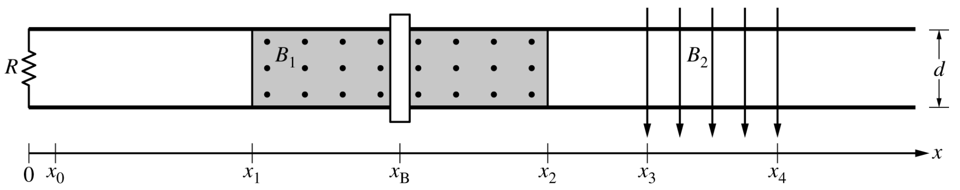
Briefly **justify** your answer by referencing your derivation in part B.

STOP

END OF EXAM



Perspective View (at time $t = 0$)



Top View (at time t_B)

. Two parallel conducting rails are separated by distance $d = 0.30\text{ m}$. A resistor of resistance $R = 0.20\ \Omega$ connects the rails. A conducting bar is placed on a sloped section of the rails at height H above the horizontal section of the rails. Frictional forces and the resistances of the bar and rails are negligible.

- At time $t = 0$, the bar is released from rest from position x_0 and slides down the sloped section of the rails, as shown in the Perspective View.
- At time t_1, the bar reaches position x_1 and smoothly transitions to the horizontal section of the rails and enters a uniform magnetic field of magnitude $B_1 = 0.40\text{ T}$ that is directed in the $+y$-direction.
- At time t_2, the bar reaches position x_2 and enters a region with no magnetic field.
- At time t_3, the bar reaches position x_3 and enters a uniform magnetic field of magnitude $B_2 = 0.60\text{ T}$ that is directed in the $+z$-direction.
- At time t_4, the bar reaches position x_4 and enters a region with no magnetic field.

'he bar is at position x_B (shown in Top View) at time t_B such that $t_1 < t_B < t_2$.

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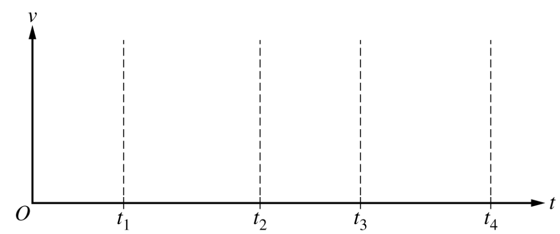
(a) On the following diagram of the bar, as observed from the Top View, draw an arrow indicating the direction of the net force F_{net} exerted on the bar at time t_B. If the net force is zero, write $F_{net} = 0$.

(b) At time t_B, the speed of the bar is $v = 2.5\text{ m/s}$.

i. Calculate the magnitude of the current in the bar at time t_B.

ii. Calculate the magnitude of the net force F_{net} exerted on the bar at time t_B.

(c) On the following axes, sketch a graph of the speed v of the bar as a function of time t between $t = 0$ and t_4.



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Continue your response to **QUESTION 2** on this page.

(d) The original scenario is repeated but with a new bar that has the same mass but with a nonnegligible resistance $R = 0.20\ \Omega$. The new bar is released from rest and smoothly transitions to the horizontal section of the rails and enters the first uniform magnetic field.

i. Determine the total resistance of the closed circuit.

ii. In the original scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{original}. In the new scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{new}. Is a_{new} greater than, less than, or equal to a_{original} ? Justify your answer.

(e) Describe a modification to H, B_1, or d that will result in a larger induced current in the new bar immediately after the bar enters the first uniform magnetic field. Justify your answer.

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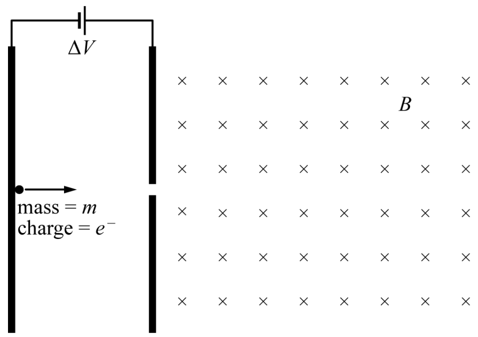


3. Two plates are set up with a potential difference V between them. A small sphere of mass m and charge $-e$ is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude B directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of V, m, e, B, and fundamental constants, as appropriate.

(a)

i. What is the initial direction of the force on the sphere as it enters the magnetic field?

____ Into the page ____ Out of the page

____ Toward the top of the page ____ Toward the bottom of the page

ii. Describe the path taken by the sphere after it enters the magnetic field.

(b) Derive an expression for the speed of the sphere as it passes through the small opening.

(c) Derive an expression for the radius of the path taken by the sphere as it moves through the magnetic field.

An experiment is performed in which a beam of electrons is accelerated across the space between the plates and passes through the small opening. After passing through the opening, the electrons travel in a semicircular path and strike the right-hand plate. The potential difference between the plates is varied in regular increments, as shown in the table below. For each potential difference, the magnetic field is varied in order to cause the beam to strike the right-hand plate at a distance of 0.020 m from the opening.

Potential difference (V)	60	70	100	110	120	140
Magnetic field $\left(\mathrm{T} \times 10^{-3}\right)$	2.62	2.78	3.39	3.54	3.78	3.99

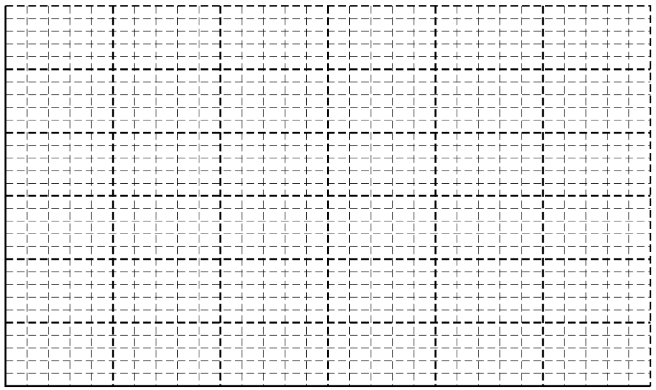
(d) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the mass-to-charge ratio of an electron.

Vertical axis: ____________

Horizontal axis: ____________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

(e) On the graph below, plot the relationship determined in part (d). Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



(f) Using the straight line from part (e), determine the mass-to-charge ratio of an electron.

STOP

END OF EXAM

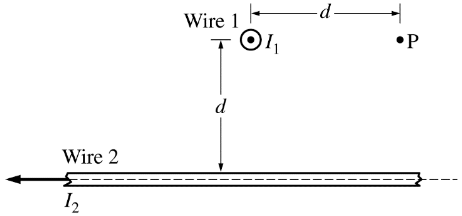


Figure 1. Side view

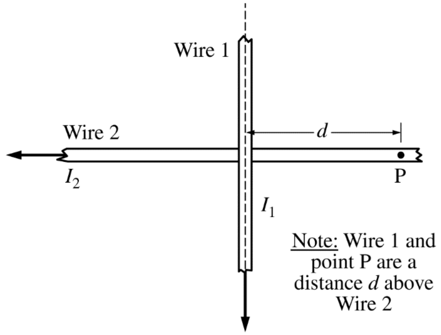


Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I, d, and physical constants, as appropriate.

(a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

(b) Derive an expression for the magnitude of the net magnetic field at point P.

(c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

(d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2 ? Assume gravitational effects are negligible.

____ Wire 1 will not move.

____ Wire 1 will move upward as viewed in Figure 1.

____ Wire 1 will move downward as viewed in Figure 1.

____ Wire 1 will rotate clockwise as viewed in Figure 2.

____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

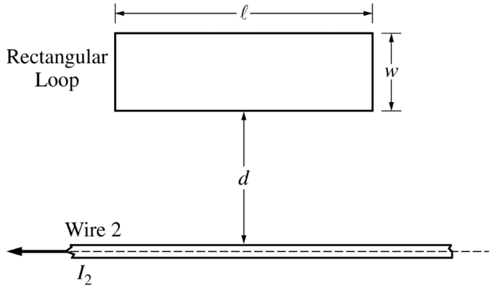


Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1}.

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

(f) Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \frac{\mu_0 I_0 \ell (1-kt)}{\pi} \ln\left(\frac{d+w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

____ Clockwise ____ Counterclockwise

____ Undefined, because there is no current induced in the loop

Justify your answer.

STOP

END OF EXAM