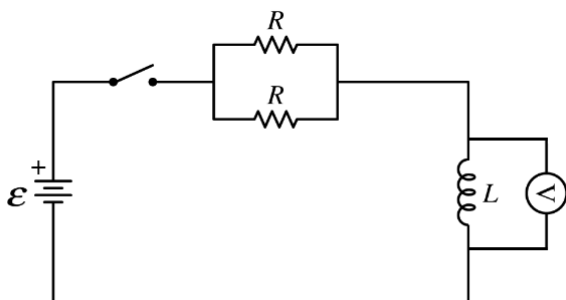


**Question 2: Free-Response Question****15 points**

- (a)(i) For correctly placing the voltmeter in parallel with the inductor **1 point**

**Example Response**

- (a)(ii) For a procedure that indicates that the voltmeter should be used to measure the potential difference for at least one time **1 point**

For measuring the potential difference from immediately after the switch is closed to when steady-state conditions have been established or during a time interval that would allow the time constant to be determined **1 point**

**Example Response**

*Close the switch. Using the voltmeter, record the potential difference as a function of time until steady-state conditions are established.*

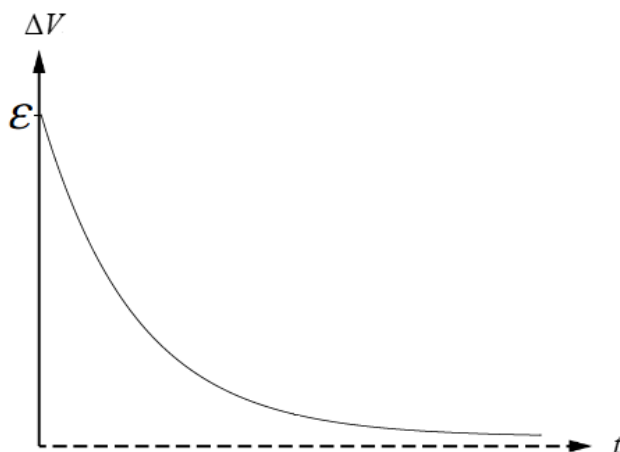
**Total for part (a) 3 points**

- (b)(i) For correctly labeling potential difference on the vertical axis and time on the horizontal axis **1 point**

For a concave-up and decreasing curve **1 point**

For a curve that asymptotically approaches zero, or a curve that starts at the origin and approaches a horizontal asymptote that is consistent with the placement of the voltmeter in the response in part (a)(i) **1 point**

For including a vertical intercept or a horizontal asymptote that is consistent with the placement of the voltmeter in the response in part (a)(i) that is correctly labeled as  $\mathcal{E}$  **1 point**

**Example Response**

- (b)(ii)** For indicating that a curve fit to the graph is that of an exponential function or for correctly relating the area under the curve to the current in the circuit **1 point**

*Alternate Solution*

*For indicating the time on the graph where the potential difference is approximately  $0.37\varepsilon$ , or  $0.63\varepsilon$  for a curve that starts at the origin and approaches a horizontal asymptote that is consistent with the placement of the voltmeter in the response in part (a)(i)*

- For indicating that the coefficient in front of the  $t$  of the curve-fit equation is equal to  $\frac{R}{2L}$  **1 point**  
or for correctly relating  $R$  to  $L$  and the current

*Alternate Solution*

*For indicating that the time constant is equal to  $\frac{2L}{R}$*

**Example Response**

*The data in the graph should be fit with an exponential function for the equation*

$$V_L = \varepsilon \left( e^{-t \frac{R}{2L}} \right). \text{ Because } \varepsilon \text{ and } L \text{ are known, } R \text{ can be calculated.}$$

**Alternate Example Response**

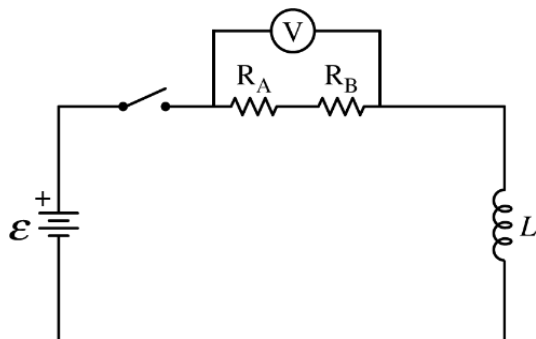
*The potential difference at  $0.37\varepsilon$  along the vertical axis corresponds to the time constant along the horizontal axis. Because the time constant is equal to  $\frac{2L}{R}$ , and  $L$  is known,  $R$  can be calculated.*

**Total for part (b) 6 points**

|                                                                                                                                                                                                                                                                                                                                                                                                            |                                                                                         |                  |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------|------------------|
| (c)                                                                                                                                                                                                                                                                                                                                                                                                        | For a multi-step derivation that begins with an attempt at using Kirchhoff's loop rule  | 1 point          |
| <b>Example Response</b>                                                                                                                                                                                                                                                                                                                                                                                    |                                                                                         |                  |
| $\mathcal{E} - \Delta V_R - \Delta V_L = 0$                                                                                                                                                                                                                                                                                                                                                                |                                                                                         |                  |
| For indicating that the potential difference across the parallel combination of resistors is                                                                                                                                                                                                                                                                                                               |                                                                                         | 1 point          |
| $I\left(\frac{R}{2}\right)$                                                                                                                                                                                                                                                                                                                                                                                |                                                                                         |                  |
| For indicating that the absolute value of the potential difference across the inductor is                                                                                                                                                                                                                                                                                                                  |                                                                                         | 1 point          |
| $L \frac{dI}{dt}$                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                         |                  |
| <b>Example Solution</b>                                                                                                                                                                                                                                                                                                                                                                                    |                                                                                         |                  |
| $\mathcal{E} - \Delta V_R - \Delta V_L = 0$                                                                                                                                                                                                                                                                                                                                                                |                                                                                         |                  |
| $\mathcal{E} - I\left(\frac{R}{2}\right) - L \frac{dI}{dt} = 0$                                                                                                                                                                                                                                                                                                                                            |                                                                                         |                  |
| $\frac{\mathcal{E}}{L} - \frac{IR}{2L} = \frac{dI}{dt}$                                                                                                                                                                                                                                                                                                                                                    |                                                                                         |                  |
| $-\frac{R}{L}\left(\frac{I}{2} - \frac{\mathcal{E}}{R}\right) = \frac{dI}{dt}$                                                                                                                                                                                                                                                                                                                             |                                                                                         |                  |
| <b>Total for part (c)</b>                                                                                                                                                                                                                                                                                                                                                                                  |                                                                                         | <b>3 points</b>  |
| (d)                                                                                                                                                                                                                                                                                                                                                                                                        | For selecting $ \Delta V_2  >  \Delta V_1 $ with an attempt at a relevant justification | 1 point          |
| For indicating that the potential difference across the original inductor is zero when steady-state conditions are established                                                                                                                                                                                                                                                                             |                                                                                         | 1 point          |
| For indicating that the potential difference across the inductor of nonnegligible resistance is nonzero when steady-state conditions are established                                                                                                                                                                                                                                                       |                                                                                         | 1 point          |
| <b>Example Response</b>                                                                                                                                                                                                                                                                                                                                                                                    |                                                                                         |                  |
| <i><math> \Delta V_2  &gt;  \Delta V_1 </math>. The potential difference <math>\Delta V_1</math> across the original, ideal inductor is zero at steady state. The potential difference across the new inductor would be nonzero due to the inductor's nonnegligible resistance. The value of <math> \Delta V_2 </math> would be the product of the steady-state current and the inductor's resistance.</i> |                                                                                         |                  |
| <b>Total for part (d)</b>                                                                                                                                                                                                                                                                                                                                                                                  |                                                                                         | <b>3 points</b>  |
| <b>Total for question 2</b>                                                                                                                                                                                                                                                                                                                                                                                |                                                                                         | <b>15 points</b> |

**Question 2: Free-Response Question****15 points**

- (a)(i)** For correctly placing the voltmeter in parallel with Resistor A, Resistor B, or the combination of resistors A and B **1 point**

**Example Response**

- (a)(ii)** For a procedure that indicates that the voltmeter should be used to measure the potential difference for at least one time **1 point**

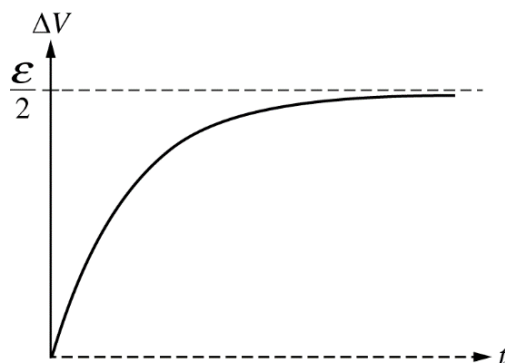
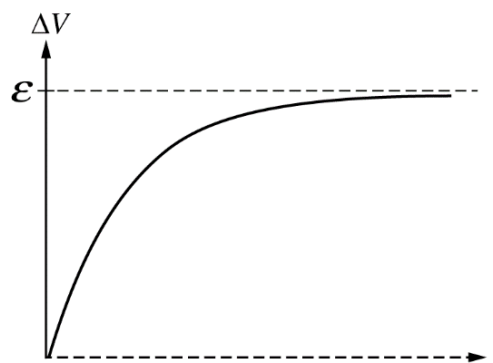
For measuring the potential difference from immediately after the switch is closed to when steady-state conditions have been established or until a time at which the time constant can be determined **1 point**

**Example Response**

*Close the switch. Using the voltmeter, record the potential difference as a function of time until steady-state conditions are established.*

**Total for part (a) 3 points**

- (b)(i)** For correctly labeling potential difference on the vertical axis and time on the horizontal axis **1 point**
- For a concave-down and increasing curve **1 point**
- For a curve that starts at the origin and asymptotically approaches a nonzero potential difference value **1 point**
- For correctly labeling the horizontal asymptote that is consistent with the indication of the connection of the voltmeter in the response in part (a)(i) **1 point**

**Example Responses***One Resistor**Combination of Resistors***OR****(b)(ii)** For indicating that a curve fit to the graph is that of an exponential function**1 point***Alternate Solution*

For indicating the time on the graph where the potential difference across Resistor A or Resistor B is approximately  $\frac{0.63\mathcal{E}}{2}$  or that the potential difference across the combination of resistors A and B is approximately  $0.63\mathcal{E}$

For indicating that the coefficient in front of the  $t$  of the curve-fit equation is equal to  $\frac{2R}{L}$

**1 point***Alternate Solution*

For indicating that the time constant is equal to  $\frac{L}{2R}$

**Example Response**

The data in the graph should be fit with an exponential function for the equation

$$V_R = \mathcal{E} \left( 1 - e^{-t \frac{2R}{L}} \right). \text{ Because } \mathcal{E} \text{ and } L \text{ are known, } R \text{ can be calculated.}$$

**Alternate Example Response**

The potential difference at  $0.63\mathcal{E}$  along the vertical axis corresponds to the time constant along the horizontal axis. Because the time constant is equal to  $\frac{L}{2R}$ , and  $L$  is known,  $R$  can be calculated.

**Total for part (b) 6 points**

|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                                       |
|-----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------|
| (c) | <p>For a multi-step derivation that begins with an attempt at using Kirchhoff's loop rule <b>1 point</b></p> <p><b>Example Response</b></p> $\mathcal{E} - \Delta V_R - \Delta V_L = 0$ <p>For indicating that the potential difference across the series combination of resistors is <math>I(2R)</math> <b>1 point</b></p> <p>For indicating that the absolute value of the potential difference across the inductor is <math>L \frac{dI}{dt}</math> <b>1 point</b></p> <p><b>Example Solution</b></p> $\mathcal{E} - \Delta V_R - \Delta V_L = 0$ $\mathcal{E} - I(2R) - L \frac{dI}{dt} = 0$ $\frac{\mathcal{E}}{L} - \frac{2IR}{L} = \frac{dI}{dt}$ $-\frac{R}{L} \left( 2I - \frac{\mathcal{E}}{R} \right) = \frac{dI}{dt}$ |                                       |
|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | <b>Total for part (c) 3 points</b>    |
| (d) | <p>For selecting <math> \Delta V_2  &lt;  \Delta V_1 </math> with an attempt at a relevant justification <b>1 point</b></p> <p>For indicating that the total resistance has increased <b>1 point</b></p> <p>For indicating that the current decreases due to the increased resistance <b>1 point</b></p>                                                                                                                                                                                                                                                                                                                                                                                                                         |                                       |
|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | <b>Total for part (d) 3 points</b>    |
|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | <b>Total for question 2 15 points</b> |

**OR**

For indicating that the potential difference across the inductor will increase, and, therefore, the potential difference across  $R_A$  must decrease

**Example Response**

$|\Delta V_2| < |\Delta V_1|$ , because the inductor has nonnegligible resistance, and the total resistance of the new circuit increases as compared to the original circuit. Therefore, the current in the new circuit is reduced compared to that of the original circuit when steady-state conditions are established.

**OR**

$|\Delta V_2| < |\Delta V_1|$ , because the inductor has nonnegligible resistance, and the total resistance of the new circuit increases as compared to the original circuit. Therefore, the potential difference across the inductor increases, which decreases the potential difference across  $R_A$  compared to that of the original circuit when steady-state conditions are established.

2019

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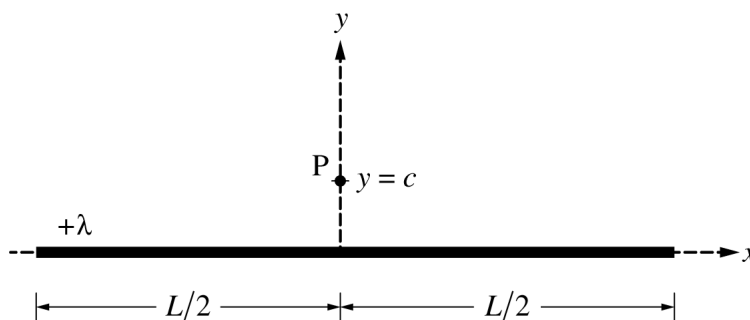
# AP<sup>®</sup> Physics C: Electricity and Magnetism

## Scoring Guidelines Set 1

**2019 SCORING GUIDELINES****General Notes About 2019 AP Physics Scoring Guidelines**

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.
2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at <https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf>.
3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.
5. The scoring guidelines typically show numerical results using the value  $g = 9.8 \text{ m/s}^2$ , but the use of  $10 \text{ m/s}^2$  is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.
6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

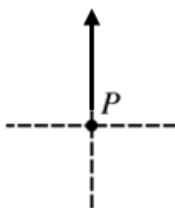


**2019 SCORING GUIDELINES****Question 1****15 points**Note: Figure not drawn to scale.

A very long, thin, nonconducting cylinder of length  $L$  is centered on the  $y$ -axis, as shown above. The cylinder has a uniform linear charge density  $+\lambda$ . Point  $P$  is located on the  $y$ -axis at  $y = c$ , where  $L \gg c$ .

- (a)
- LO CNV-3.B.a, SP 7.A  
1 point

On the figure shown below, draw an arrow to indicate the direction of the electric field at point  $P$  due to the long cylinder. The arrow should start on and point away from the dot.

For drawing an arrow at point  $P$  that points upward

1 point

- LO CNV-2.F, SP 7.C  
1 point

Describe the shape and location of a Gaussian surface that can be used to determine the electric field at point  $P$  due to the long cylinder.

For describing a Gaussian surface that could be used to determine the electric field at point  $P$ 

1 point

Example: Drawing a cylinder that is coaxial with the thin cylinder and whose surface contains point  $P$  can be used to determine the electric field at point  $P$ .

Note: Credit is earned if the student draws the correct surface on the figure.

**2019 SCORING GUIDELINES****Question 1 (continued)**

(a) continued

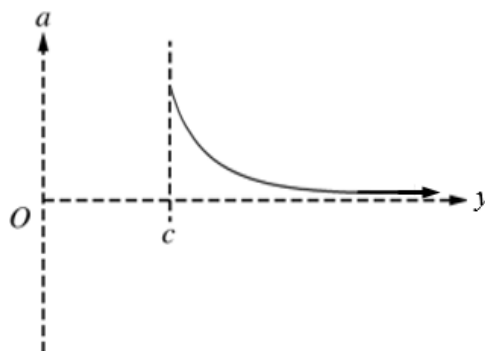
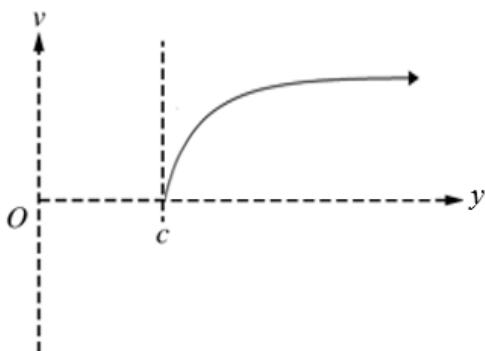
- iii. LO CNV-2.C, SP 5.A, 5.E  
3 points

Use your Gaussian surface to derive an expression for the magnitude of the electric field at point P.  
Express your answer in terms of  $\lambda$ ,  $c$ ,  $L$ , and physical constants, as appropriate.

|                                                                                                                   |  |         |
|-------------------------------------------------------------------------------------------------------------------|--|---------|
| For using Gauss's law to determine the electric field at point P                                                  |  | 1 point |
| $\frac{q_{enc}}{\epsilon_0} = \int E \cdot dA \therefore \frac{Q}{\epsilon_0} = EA$                               |  |         |
| For correctly substituting for the charge into the equation above                                                 |  | 1 point |
| For correctly substituting for the area or into the equation above                                                |  | 1 point |
| $\frac{\lambda L}{\epsilon_0} = E(2\pi cL) \therefore E = \frac{\lambda}{2\pi\epsilon_0 c} = \frac{2k\lambda}{c}$ |  |         |

- (b) LO ACT-1.D, SP 3.C  
2 points

A proton is released from rest at point P. On the axes below, sketch the velocity  $v$  as a function of position  $y$  and the acceleration  $a$  as a function of position  $y$  for the proton.

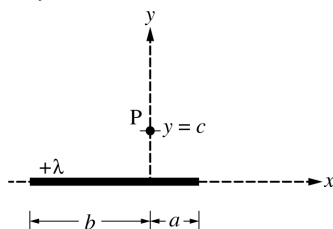


|                                                                                                           |  |         |
|-----------------------------------------------------------------------------------------------------------|--|---------|
| For a concave down graph for $v$ as a function of position $x$ that does not start at the origin          |  | 1 point |
| For a concave up graph for $a$ as a function of position $x$ that has an asymptote at the horizontal axis |  | 1 point |

## 2019 SCORING GUIDELINES

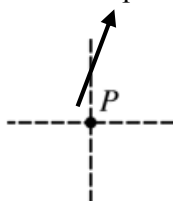
## Question 1 (continued)

The original cylinder is now replaced with a much shorter thin, nonconducting cylinder with the same uniform linear charge density  $+\lambda$ , as shown in the figure below. The length of the cylinder to the right of the  $y$ -axis is  $a$ , and the length of the cylinder to the left of the  $y$ -axis is  $b$ , where  $a < b$ .



- (c) LO CNV-3.B.a, SP 7.A  
2 points

On the figure shown below, draw an arrow to indicate the direction of the electric field at point  $P$  due to the shorter cylinder. The arrow should start on and point away from the dot.



|                                                                   |         |
|-------------------------------------------------------------------|---------|
| For drawing an arrow at point $P$ that points to the right        | 1 point |
| For drawing an arrow at point $P$ that points up and to the right | 1 point |

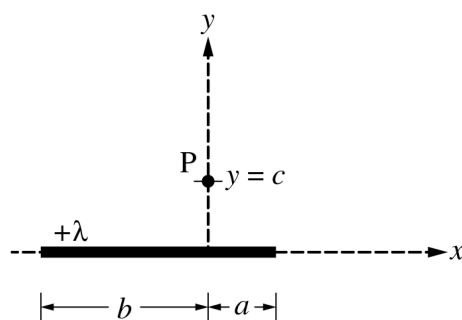
- (d)  
i. and ii. LO CNV-2.F, SP 7.A  
1 point

Is there a single Gaussian surface that can be used with Gauss's law to derive an expression for the electric field at point  $P$ ?

\_\_\_\_ Yes    \_\_\_\_ No

If your answer to part (d)(i) is yes, explain how you can use Gauss's law to derive an expression for the field at point  $P$ . If your answer to part (d)(i) is no, explain why Gauss's law cannot be applied to derive an expression for the electric field in this case.

|                                                                                                                                                      |         |
|------------------------------------------------------------------------------------------------------------------------------------------------------|---------|
| For selecting "No" with a valid explanation                                                                                                          | 1 point |
| Claim: Select "No."                                                                                                                                  |         |
| Evidence: The length of the cylinder is not much greater than the distance from the cylinder to point $P$ and the charge distribution is asymmetric. |         |
| Reasoning: Therefore, cannot use the approximation of the constant magnitude of electric field over a cylindrical surface.                           |         |

**2019 SCORING GUIDELINES****Question 1 (continued)**

Note: This figure is shown again for reference.

A student in class argues that using the integral shown below might be a useful approach for determining the electric field at point P.

$$E = \int \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} dq$$

The student uses this approach and writes the following two integrals for the magnitude of the horizontal and vertical components of the electric field at point P.

$$\text{Horizontal component: } |E_x| = \frac{\lambda}{4\pi\epsilon_0} \int_{-b}^a \frac{x}{(c^2 + x^2)^{3/2}} dx$$

$$\text{Vertical component: } |E_y| = \frac{\lambda}{4\pi\epsilon_0} \int_{-b}^a \frac{y}{(c^2 + x^2)} dy$$

- (e)
- i. LO CNV-3.A, SP 7.A  
1 point

One of the two expressions above is not correct. Which expression is not correct?

\_\_\_\_\_ Horizontal component      \_\_\_\_\_ Vertical component

|                                              |         |
|----------------------------------------------|---------|
| For correctly selecting “Vertical component” | 1 point |
|----------------------------------------------|---------|

**2019 SCORING GUIDELINES****Question 1 (continued)**

(e) continued

- ii. LO CNV-3.A, SP 7.D  
4 points

Identify two mistakes in the incorrect expression, and explain how to correct the mistakes.

|                                                                                                                                                                                   |         |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|
| For indicating the integral is not along the length of the cylinder                                                                                                               | 1 point |
| For an appropriate correction                                                                                                                                                     | 1 point |
| Claim: Change $dy$ to $dx$ .<br>Evidence: The integral is not along the cylinder.<br>Reasoning: This change will make the integral valid.                                         |         |
| For indicating the power on the denominator term for the vertical component is incorrect                                                                                          | 1 point |
| For an appropriate correction                                                                                                                                                     | 1 point |
| Claim: The power on the term in the denominator should be $3/2$ .<br>Evidence: The units of the integrand are not valid.<br>Reasoning: This change will make the integrand valid. |         |

**Learning Objectives**

**ACT-1.D:** Determine the motion of a charged object of specified charge and mass under the influence of an electrostatic force.

**CNV-2.C:** State and use Gauss's law in integral form to derive unknown electric fields for planar, spherical, or cylindrically symmetrical charge distributions.

**CNV-2.F:** Describe the general features of an unknown charge distribution, given other features of the system.

**CNV-3.A:** Derive expressions for the electric field of specified charge distributions using integration and the principle of superposition. Examples of such charge distributions include a uniformly charged wire, a thin ring of charge (along the axis of the ring), and a semicircular or part of a semicircular arc.

**CNV-3.B.a:** Identify and qualitatively describe situations in which the direction and magnitude of the electric field can be deduced from symmetry considerations and understanding the general behavior of certain charge distributions.

**Science Practices**

**3.C:** Sketch a graph that shows a functional relationship between two quantities.

**5.A:** Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.

**5.E:** Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

**7.A:** Make a scientific claim.

**7.C:** Support a claim with evidence from physical representations.

**7.D:** Provide reasoning to justify a claim using physical principles or laws.

## 2019 SCORING GUIDELINES

## Question 2

15 points

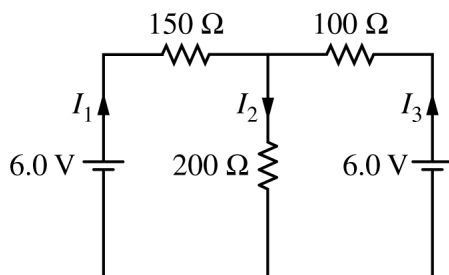


Figure 1

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents  $I_1$ ,  $I_2$ , and  $I_3$  in each branch of the circuit are indicated.

- (a)
- LO CNV-6.F.b, SP 6.A  
3 points

Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

|                                                                                                               |  |         |
|---------------------------------------------------------------------------------------------------------------|--|---------|
| For an equation representing the sum of the currents at one of the junctions                                  |  | 1 point |
| $I_1 - I_2 + I_3 = 0$                                                                                         |  |         |
| For an equation representing the sum of the potential differences around one of the loops                     |  | 1 point |
| For an equation representing the sum of the potential differences around a loop different from the above loop |  | 1 point |
| $6 - 150I_1 - 200I_2 = 0$                                                                                     |  |         |
| $6 - 100I_3 - 200I_2 = 0$                                                                                     |  |         |
| $6 - 150I_1 + 100I_3 - 6 = 0$                                                                                 |  |         |
| <u>Note:</u> Full credit is earned for two correct loop equations using loop currents.                        |  |         |

- LO CNV-6.F.b, SP 6.C  
2 points

Calculate the current in the  $200\ \Omega$  resistor.

|                                                                                                                      |  |         |
|----------------------------------------------------------------------------------------------------------------------|--|---------|
| For combining the equations from part (a)(i)                                                                         |  | 1 point |
| $I_1 - I_2 + I_3 = 0$<br>$-I_1 - 1.33I_2 = -.04 \therefore -4.33I_2 = -.10$<br>$-I_3 - 2I_2 = -.06$                  |  |         |
| <u>Note:</u> Credit is earned if students indicate they used a calculator function to solve the system of equations. |  |         |
| For a correct answer with correct units                                                                              |  | 1 point |
| $-4.33I_2 = -.10 \therefore I_2 = 0.023\text{ A}$                                                                    |  |         |

**2019 SCORING GUIDELINES****Question 2 (continued)**

(a) continued

iii. LO CNV-5.A.a, SP 6.C

1 point

Calculate the power dissipated by the  $200\ \Omega$  resistor.

|                                                                                   |         |
|-----------------------------------------------------------------------------------|---------|
| For using a correct equation to calculate the power in the $200\ \Omega$ resistor | 1 point |
| $P = I^2 R = (0.023\ \text{A})^2 (200\ \Omega) \therefore P = 0.107\ \text{W}$    |         |

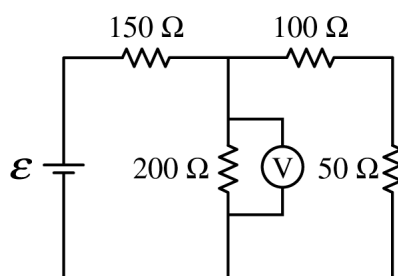


Figure 2

The two  $6.0\ \text{V}$  batteries are replaced with a battery with voltage  $\mathcal{E}$  and a resistor of resistance  $50\ \Omega$ , as shown above. The voltmeter V shows that the voltage across the  $200\ \Omega$  resistor is  $4.4\ \text{V}$ .

(b) LO CNV-6.C.a, SP 6.B, 6.C

2 points

Calculate the current through the  $50\ \Omega$  resistor.

|                                                                                                               |         |
|---------------------------------------------------------------------------------------------------------------|---------|
| For correctly calculating the equivalent resistance of the branch with the $50\ \Omega$ resistor              | 1 point |
| $R = 100\ \Omega + 50\ \Omega = 150\ \Omega$                                                                  |         |
| For using the correct potential difference in Ohm's law to calculate the current in the $50\ \Omega$ resistor | 1 point |
| $I = \frac{V}{R} = \frac{(4.4\ \text{V})}{(100\ \Omega + 50\ \Omega)} \therefore I = 0.029\ \text{A}$         |         |

**2019 SCORING GUIDELINES****Question 2 (continued)**

- (c) LO CNV-6.C.a, SP 6.A, 6.C  
3 points

Calculate the voltage  $\mathcal{E}$  of the battery.

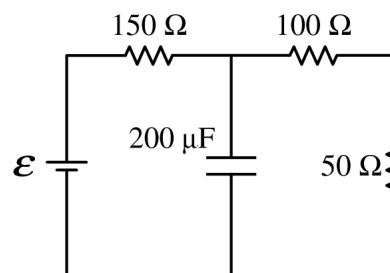
|                                                                                                                                                              |                         |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------|
| For using a correct equation to determine the current through the $150\ \Omega$ resistor                                                                     | 1 point                 |
| For correctly substituting the current from part (b) as $I_3$                                                                                                | 1 point                 |
| $I_1 = I_2 + I_3 = \frac{(4.4\ \text{V})}{(200\ \Omega)} + 0.029\ \text{A} = 0.051\ \text{A}$                                                                |                         |
| For using a correct equation to determine the emf of the battery                                                                                             | 1 point                 |
| $\mathcal{E} = I_1 R_1 + V_2 = (0.051\ \text{A})(150\ \Omega) + 4.4\ \text{V} = 12.1\ \text{V}$                                                              |                         |
|                                                                                                                                                              |                         |
| <i>Alternate Third Point</i>                                                                                                                                 |                         |
| <i>For calculating the equivalent resistance of the circuit and substituting this resistance into a correct equation to determine the emf of the battery</i> | <i>1 point</i>          |
| $R_T = 150\ \Omega + \frac{1}{\frac{1}{200\ \Omega} + \frac{1}{150\ \Omega}} = 236\ \Omega$                                                                  |                         |
| $\mathcal{E} = I_1 R_T = (0.051\ \text{A})(236\ \Omega) = 12.0\ \text{V}$                                                                                    |                         |
|                                                                                                                                                              |                         |
| <i>Alternate Solution</i>                                                                                                                                    | <i>Alternate Points</i> |
| <i>For using a correct equation to determine the equivalent resistance of the parallel resistors</i>                                                         | <i>1 point</i>          |
| $\frac{1}{R_P} = \frac{1}{200\ \Omega} + \frac{1}{150\ \Omega} \therefore R_P = 86\ \Omega$                                                                  |                         |
| <i>For correctly substituting the given potential difference and the calculated equivalent resistance to determine the total current of the circuit</i>      | <i>1 point</i>          |
| $I_T = \frac{(4.4\ \text{V})}{(86\ \Omega)} = 0.051\ \text{A}$                                                                                               |                         |
| <i>For calculating the equivalent resistance of the circuit and substituting into a correct equation to determine the emf of the battery</i>                 | <i>1 point</i>          |
| $R_T = 150\ \Omega + 86\ \Omega = 236\ \Omega$                                                                                                               |                         |
| $\mathcal{E} = I_1 R_T = (0.051\ \text{A})(236\ \Omega) = 12.0\ \text{V}$                                                                                    |                         |



**2019 SCORING GUIDELINES****Question 2 (continued)**

- (d)  
i. LO CNV-7.B.a, SP 6.A, 6.C  
2 points

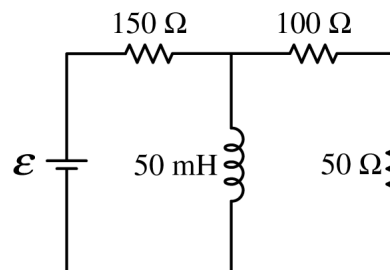
The  $200\ \Omega$  resistor in the circuit in Figure 2 is replaced with a  $200\ \mu\text{F}$  capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the  $50\ \Omega$  resistor.



|                                                                                                                         |  |         |
|-------------------------------------------------------------------------------------------------------------------------|--|---------|
| For substituting the voltage consistent with part (c) into Ohm's law                                                    |  | 1 point |
| For correctly calculating the equivalent resistance of the circuit                                                      |  | 1 point |
| $I = \frac{\mathcal{E}}{R_{tot}} = \frac{(12.1\ \text{V})}{(150\ \Omega + 100\ \Omega + 50\ \Omega)} = 40.3\ \text{mA}$ |  |         |

- ii. LO CNV-10.C.a, SP 7.A, 7.C  
2 points

The  $200\ \Omega$  resistor in the circuit in Figure 2 is replaced with an ideal  $50\ \text{mH}$  inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the  $50\ \Omega$  resistor greater than, less than, or equal to the current calculated in part (b)?



\_\_\_\_ Greater than      \_\_\_\_ Less than      \_\_\_\_ Equal to

Justify your answer.

|                                                                                                                                                                                                   |  |         |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---------|
| For correctly selecting "Less than" with an attempt at a relevant justification                                                                                                                   |  | 1 point |
| For a correct justification                                                                                                                                                                       |  | 1 point |
| Example: Because steady state is reached, the inductor will act as a short circuit. So all the current will pass through the inductor and no current will pass through the $50\ \Omega$ resistor. |  |         |

**2019 SCORING GUIDELINES****Question 2 (continued)****Learning Objectives**

**CNV-5.A.a:** Derive expressions that relate current, voltage, and resistance to the rate at which heat is produced in a resistor.

**CNV-6.C.a:** Calculate voltage, current, and power dissipation for any resistor in a circuit containing a network of known resistors with a single battery or energy source.

**CNV-6.F.b:** Set up simultaneous equations to calculate at least two unknowns (currents or resistance values) in a multi-loop circuit.

**CNV-7.B.a:** Calculate the potential difference across a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.

**CNV-10.C.a:** Calculate initial transient currents and final steady-state currents through any part of a series or parallel circuit containing an inductor and one or more resistors.

**Science Practices**

**6.A:** Extract quantities from narratives or mathematical relationships to solve problems.

**6.B:** Apply an appropriate law, definition, or mathematical relationship to solve a problem.

**6.C:** Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

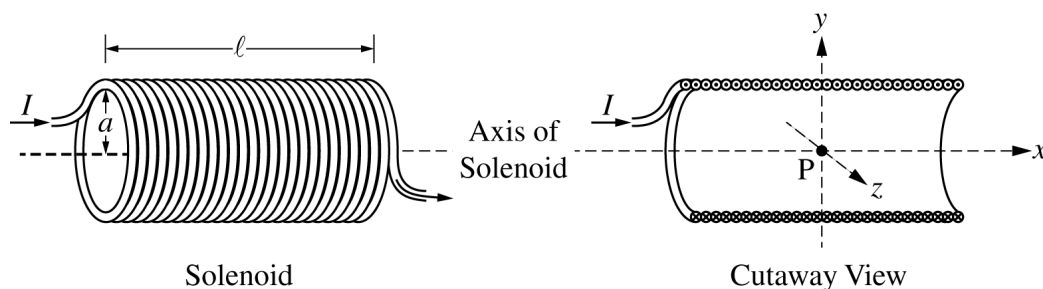
**7.A:** Make a scientific claim.

**7.C:** Support a claim with evidence from physical representations.

## 2019 SCORING GUIDELINES

## Question 3

15 points



Note: Figures not drawn to scale.

A solenoid is used to generate a magnetic field. The solenoid has an inner radius  $a$ , length  $\ell$ , and  $N$  total turns of wire. A power supply, not shown, is connected to the solenoid and generates current  $I$ , as shown in the figure on the left above. The  $x$ -axis runs along the axis of the solenoid. Point P is in the middle of the solenoid at the origin of the  $xyz$ -coordinate system, as shown in the cutaway view on the right above. Assume  $\ell \gg a$ .

- (a) LO CNV-8.E.a, SP 7.A, 7.C  
2 points

Select the correct direction of the magnetic field at point P.

☐ + $x$ -direction    ☐ + $y$ -direction    ☐ + $z$ -direction  
☐ - $x$ -direction    ☐ - $y$ -direction    ☐ - $z$ -direction

Justify your selection.

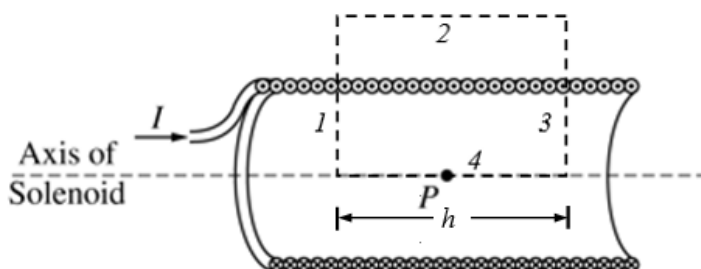
|                                                                                                                                                                                                                                       |         |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|
| For choosing the “+ $x$ -direction” and providing a justification                                                                                                                                                                     | 1 point |
| For a correct justification                                                                                                                                                                                                           | 1 point |
| Example: Using the right-hand rule for current on the left side of the solenoid, the fingers curl into the loop, so the magnetic field points to the right, or in the + $x$ -direction.                                               |         |
| Example: Using the right-hand rule for solenoids, when the fingers curl around the solenoid in the direction of the current, the thumb points to the right, therefore the magnetic field is to the right, or in the + $x$ -direction. |         |

## 2019 SCORING GUIDELINES

## Question 3 (continued)

- (b)
- i. LO CNV-8.C.c, SP 3.D  
1 point

On the cutaway view below, clearly draw an Amperian loop that can be used to determine the magnetic field at point P at the center of the solenoid.



|                                                                                                                                                                      |         |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|
| For drawing a rectangle with one side along the central axis of the solenoid and another side outside the solenoid and whose edges do not extend beyond the solenoid | 1 point |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|

- ii. LO CNV-8.C.c, SP 5.A, 5.E  
2 points

Use Ampere's law to derive an expression for the magnetic field strength at point P. Express your answer in terms of  $I$ ,  $\ell$ ,  $N$ ,  $a$ , and physical constants, as appropriate.

|                                                                                                                                                                                                                                                                                                                                     |         |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|
| For using Ampere's law to calculate the magnetic field along the axis of the solenoid                                                                                                                                                                                                                                               | 1 point |
| $\int \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I_{enc} \therefore \left( \int \mathbf{B} \cdot d\boldsymbol{\ell} \right)_1 + \left( \int \mathbf{B} \cdot d\boldsymbol{\ell} \right)_2 + \left( \int \mathbf{B} \cdot d\boldsymbol{\ell} \right)_3 + \left( \int \mathbf{B} \cdot d\boldsymbol{\ell} \right)_4 = \mu_0 I_{enc}$ |         |
| $\left( \int \mathbf{B} \cdot d\boldsymbol{\ell} \right)_1 + 0 + \left( - \left( \int \mathbf{B} \cdot d\boldsymbol{\ell} \right)_1 \right) + Bh = \mu_0 \frac{N}{\ell} hI$                                                                                                                                                         |         |
| $Bh = \mu_0 \frac{N}{\ell} hI$                                                                                                                                                                                                                                                                                                      |         |
| For a correct answer                                                                                                                                                                                                                                                                                                                | 1 point |
| $B = \frac{\mu_0 NI}{\ell}$                                                                                                                                                                                                                                                                                                         |         |

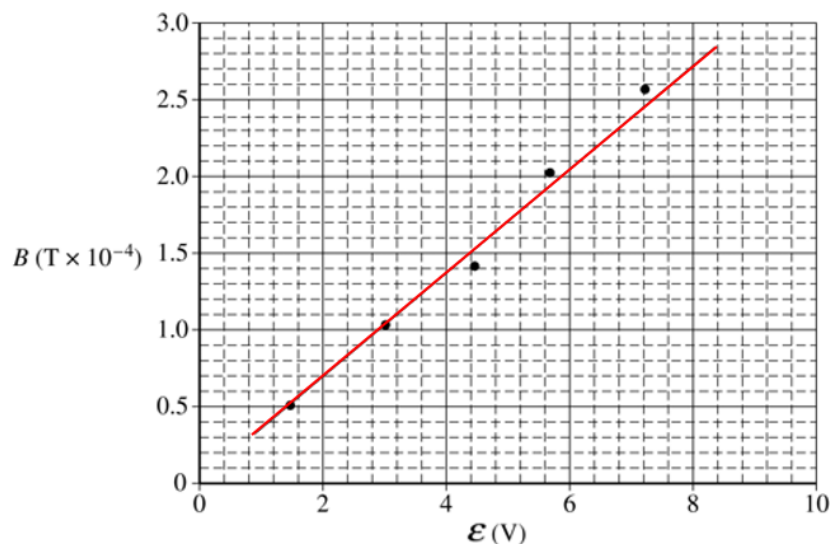
Some physics students conduct an experiment to determine the resistance  $R_S$  of a solenoid with radius  $a = 0.015$  m, total turns  $N = 100$ , and total length  $\ell = 0.40$  m. The students connect the solenoid to a variable power supply. A magnetic field sensor is used to measure the magnetic field strength along the central axis at the center of the solenoid. The plot of the magnetic field strength  $B$  as a function of the emf  $\mathcal{E}$  of the power supply is shown below.

## 2019 SCORING GUIDELINES

## Question 3 (continued)

- (c) LO CNV-8.C.c, SP 4.C  
i. 1 point

On the graph above, draw a best-fit line for the data.



|                                                                                        |         |
|----------------------------------------------------------------------------------------|---------|
| For drawing a best-fit line with at least one point above and one point below the line | 1 point |
|----------------------------------------------------------------------------------------|---------|

- ii. LO CNV-8.C.c, SP 6.B, 6.C  
2 points

Use the straight line to determine the resistance  $R_S$  of the solenoid used in the experiment.

|                                                                                                                                                                                                                                                                                                                                  |         |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|
| For calculating the slope using the best-fit line and not the data points unless they fall on the best-fit line                                                                                                                                                                                                                  | 1 point |
| $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{(2.5 - 0.9)(\times 10^{-4} \text{ T})}{(6.4 - 2.0) \text{ V}} = 0.36 \times 10^{-4} = 3.6 \times 10^{-5} \text{ T/V}$                                                                                                                                                          |         |
| For correctly giving the expression that relates the slope to the resistance of the solenoid                                                                                                                                                                                                                                     | 1 point |
| $B = \frac{\mu_0 N I}{\ell} = \frac{\mu_0 N \mathcal{E}}{\ell R} \therefore \text{slope} = \frac{\mu_0 N}{\ell R_S} \therefore R_S = \frac{\mu_0 N}{\ell \times \text{slope}}$ $R_S = \frac{(4\pi \times 10^{-7} \text{ (T}\cdot\text{m)/A})(100 \text{ turns})}{(0.40 \text{ m})(3.6 \times 10^{-5} \text{ T/V})} = 8.7 \Omega$ |         |

One of the students notes that the horizontal component of the magnetic field of Earth is  $2.5 \times 10^{-5} \text{ T}$ .

**2019 SCORING GUIDELINES****Question 3 (continued)**

- (d)
- i. LO CNV-8.E.a, SP 2.D  
1 point

Is there evidence from the graph that the horizontal orientation of the solenoid affects the measured values for  $B$ ?

\_\_\_\_ Yes      \_\_\_\_ No

Justify your answer.

|                                                                                                                                                                                                                |                         |         |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------|---------|
| If the line on the graph does not go through the origin, select “Yes”                                                                                                                                          |                         |         |
| For a correct justification                                                                                                                                                                                    |                         | 1 point |
| Example: The horizontal component of Earth’s magnetic field will add or subtract from the magnetic field of the solenoid depending on the orientation of the solenoid.                                         |                         |         |
|                                                                                                                                                                                                                |                         |         |
| <i>Alternate Solution</i>                                                                                                                                                                                      | <i>Alternate Points</i> |         |
| If the line on the graph does pass through the origin, select “No”                                                                                                                                             |                         |         |
| For a correct justification                                                                                                                                                                                    |                         | 1 point |
| Example: Based on the graph, the line passes through the origin, so the magnetic field is zero when the emf of the power supply is zero, therefore Earth’s magnetic field is not affecting the values of $B$ . |                         |         |

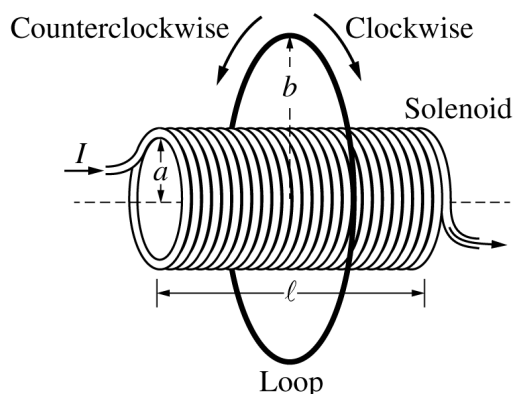
- ii. LO CNV-8.E.a, SP 2.E  
1 point

Would the horizontal orientation of the solenoid affect the calculated value for  $R_S$  ?

\_\_\_\_ Yes      \_\_\_\_ No

Justify your answer.

|                                                                                                                                                                                                          |  |         |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---------|
| Select “No”                                                                                                                                                                                              |  |         |
| For a correct justification                                                                                                                                                                              |  | 1 point |
| Example: The horizontal component of Earth’s magnetic field will not affect the change in magnetic field as the emf is changed. Therefore, the value for the resistance of the solenoid will not change. |  |         |

**2019 SCORING GUIDELINES****Question 3 (continued)**

A thin conducting loop of radius  $b$  and resistance  $R_L$  is placed concentric with the solenoid, as shown above. The current in the solenoid is decreased from  $I$  to zero over time  $\Delta t$ .

- (e)
- i. LO FIE-6.A.b, SP 7.A, 7.C  
2 points

Is the direction of the induced current in the loop clockwise or counterclockwise during the time period that the current in the solenoid is decreasing?

\_\_\_\_\_ Clockwise      \_\_\_\_\_ Counterclockwise

Justify your answer.

|                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |         |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---------|
| Select "Clockwise"                                                                                                                                                                                                                                                                                                                                                                                                                               |  |         |
| For a justification indicating that the magnetic field inside the solenoid, and therefore the loop, will decrease                                                                                                                                                                                                                                                                                                                                |  | 1 point |
| For a justification using Lenz's law to relate the change in magnetic field to the direction of the current                                                                                                                                                                                                                                                                                                                                      |  | 1 point |
| Example: As the current in the solenoid decreases, the magnetic field inside the solenoid decreases. As the solenoid's magnetic field decreases, the induced current in the loop will create a magnetic field to oppose this change. Because the solenoid's magnetic field is toward the right and decreasing, the magnetic field due to the current in the loop must be toward the right. Therefore, the current in the loop must be clockwise. |  |         |

**2019 SCORING GUIDELINES****Question 3 (continued)**

(e) continued

- ii. LO FIE-6.A.d, SP 5.A, 5.E  
3 points

Derive an equation for the average induced current  $i_{\text{IND}}$  in the loop during the time period that the current in the solenoid is decreasing. Express your answer in terms of  $I$ ,  $\ell$ ,  $N$ ,  $a$ ,  $b$ ,  $R_L$ ,  $R_S$ ,  $\Delta t$ , and physical constants, as appropriate.

|                                                                                                                       |         |
|-----------------------------------------------------------------------------------------------------------------------|---------|
| For using Faraday's law to calculate the emf in the loop                                                              | 1 point |
| $\mathcal{E} = \left  \frac{d\Phi}{dt} \right  = \frac{d(BA)}{dt} = A \frac{\Delta B}{\Delta t}$                      |         |
| For using Ohm's law to calculate the current in the loop                                                              | 1 point |
| $I_{\text{IND}} = \frac{V}{R} = \frac{A\Delta B/\Delta t}{R_L} = \frac{A\Delta B}{R_L\Delta t}$                       |         |
| For using the correct radius for the area in the equation above                                                       | 1 point |
| $I_{\text{IND}} = \frac{\pi a^2 \mu_0 N \Delta I / \ell}{R_L \Delta t} = \frac{\pi a^2 \mu_0 N I}{R_L \ell \Delta t}$ |         |

**Learning Objectives**

**CNV-8.C.c:** Derive the expression for the magnetic field of an ideal solenoid (length dimension is much larger than the radius of the solenoid) using Ampère's law.

**CNV-8.E.a:** Describe the direction of a magnetic field at a point in space due to various combinations of conductors, wires, cylindrical conductors, or loops.

**FIE-6.A.b:** Describe the direction of an induced current in a conductive loop that is placed in a changing magnetic field.

**FIE-6.A.d:** Calculate the magnitude and direction of induced EMF and induced current in a conductive loop (or conductive bar) when the magnitude of either the field or area of loop is changing at a constant rate.

**Science Practices**

**2.D:** Make observations or collect data from representations of laboratory setups or results.

**2.E:** Identify or describe potential sources of experimental error.

**3.D:** Create appropriate diagrams to represent physical situations.

**4.C:** Linearize data and/or determine a best-fit line or curve.

**5.A:** Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.

**5.E:** Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

**6.B:** Apply an appropriate law, definition, or mathematical relationship to solve a problem.

**6.C:** Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

**7.A:** Make a scientific claim.

**7.C:** Support a claim with evidence from physical representations.



**2019****AP<sup>®</sup>** **CollegeBoard**

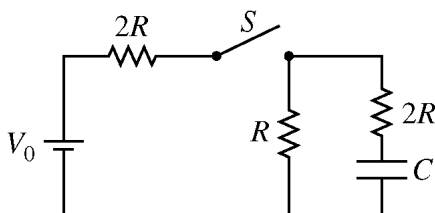
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# **AP<sup>®</sup> Physics C: Electricity and Magnetism**

## **Scoring Guidelines Set 2**

**2019 SCORING GUIDELINES****General Notes About 2019 AP Physics Scoring Guidelines**

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.
2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at <https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf>.
3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as “derive” and “calculate” on the exams, and what is expected for each, see “The Free-Response Sections — Student Presentation” in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or “Terms Defined” in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.
5. The scoring guidelines typically show numerical results using the value  $g = 9.8 \text{ m/s}^2$ , but the use of  $10 \text{ m/s}^2$  is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.
6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

**2019 SCORING GUIDELINES****Question 1****15 points**

The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage  $V_0$ , a capacitor of capacitance  $C$ , and a switch  $S$ . The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of  $V_0$ ,  $R$ ,  $C$ , and fundamental constants, as appropriate.

- (a) LO CNV-7.B.a, SP 5.A, 5.E  
2 points

Derive an expression for the steady-state current supplied by the battery.

|                                                       |  |         |
|-------------------------------------------------------|--|---------|
| For using Ohm's law                                   |  | 1 point |
| $I = \frac{V}{R_{\text{eff}}} = \frac{V_0}{(2R + R)}$ |  |         |
| For correct substitution leading to correct answer    |  | 1 point |
| $I = \frac{V_0}{3R}$                                  |  |         |

- (b) LO CNV-7.B.b, SP 5.A, 5.E  
2 points

Derive an expression for the charge on the capacitor.

|                                                                |  |         |
|----------------------------------------------------------------|--|---------|
| For using the equation relating stored charge to capacitance   |  | 1 point |
| $q = CV = CV_C = CV_R$                                         |  |         |
| For determining $V_R$ and substituting into the above equation |  | 1 point |
| $V_R = IR = \left(\frac{V_0}{3R}\right)R = \frac{1}{3}V_0$     |  |         |
| $q = CV_R = C\left(\frac{1}{3}V_0\right) = \frac{1}{3}CV_0$    |  |         |

**2019 SCORING GUIDELINES****Question 1 (continued)**

- (c) LO CNV-7.B.b, SP 5.A, 5.E  
2 points

Derive an expression for the energy stored in the capacitor.

|                                                           |  |         |
|-----------------------------------------------------------|--|---------|
| For any correct equation for energy stored in a capacitor |  | 1 point |
| $U = \frac{q^2}{2C}$                                      |  |         |
| Substitute charge and/or voltage from part (b)            |  | 1 point |
| $U = \frac{(CV_0/3)^2}{2C} = \frac{1}{18}CV_0^2$          |  |         |

Now the switch is opened at time  $t = 0$ .

- (d) LO CNV-7.D.a, SP 5.A, 5.E  
2 points

Write, but do NOT solve, a differential equation that could be used to solve for the charge  $q(t)$  on the capacitor as a function of the time  $t$  after the switch is opened.

|                                                                                     |  |         |
|-------------------------------------------------------------------------------------|--|---------|
| For any correct voltage loop equation                                               |  | 1 point |
| $V_C - V_R - V_{2R} = 0 \therefore V_C = V_R + V_{2R}$                              |  |         |
| $\frac{q(t)}{C} = I(R + 2R)$                                                        |  |         |
| <u>Note:</u> Any correct loop equation for when the switch is open earns the point  |  |         |
| For substituting $-dq/dt$ or $dq/dt$ for the current, consistent with loop equation |  | 1 point |
| $\frac{q(t)}{C} = -3R \frac{dq}{dt}$                                                |  |         |

**2019 SCORING GUIDELINES****Question 1 (continued)**

(e)

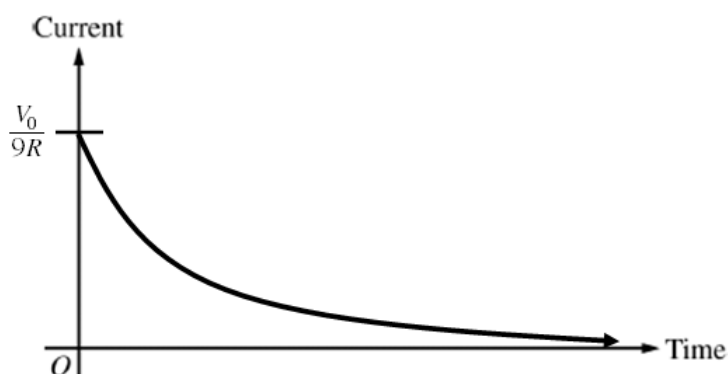
- i. LO CNV-7.B.a, SP 5.A, 5.E  
2 points

Calculate the current in resistor  $R$  immediately after the switch is opened.

|                                                                   |  |         |
|-------------------------------------------------------------------|--|---------|
| For using a voltage consistent with part (b) in $I = \frac{V}{R}$ |  | 1 point |
| For substituting the correct resistance ( $3R$ ) into Ohm's law   |  | 1 point |
| $I = \frac{V_0/3}{3R} = \frac{V_0}{9R}$                           |  |         |

- ii. LO CNV-7.E.b, SP 3.C  
3 points

On the axes below, sketch the current in the circuit as a function of time from time  $t = 0$  to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



|                                                              |  |         |
|--------------------------------------------------------------|--|---------|
| For a curve that is concave up throughout graph              |  | 1 point |
| For having the horizontal axis as an asymptote               |  | 1 point |
| For labeling the maximum current consistent with part (e)(i) |  | 1 point |

**2019 SCORING GUIDELINES****Question 1 (continued)**

- (f) LO CNV-7.G.a, SP 7.A, 7.C  
2 points

Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c)?

\_\_\_ Greater than    \_\_\_ Less than    \_\_\_ Equal to

Justify your answer.

|                                                                                                                                                                                                                                                                       |                         |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------|
| For selecting “Equal to”                                                                                                                                                                                                                                              | 1 point                 |
| For a correct justification invoking conservation of energy                                                                                                                                                                                                           | 1 point                 |
| Example: After the switch is opened, the capacitor will discharge all of its stored energy and charge. Assuming no energy is lost in the wire, then the only parts of the circuit that will dissipate this energy are the two resistors in series with the capacitor. |                         |
|                                                                                                                                                                                                                                                                       |                         |
| <i>Alternate Solution</i>                                                                                                                                                                                                                                             | <i>Alternate Points</i> |
| For selecting “Less than”                                                                                                                                                                                                                                             | 1 point                 |
| For a correct justification invoking conservation of energy                                                                                                                                                                                                           | 1 point                 |
| Example: After the switch is opened, the capacitor will discharge all of its stored energy and charge. Additionally there is energy lost due to resistance in the wire and/or energy loss in the capacitor.                                                           |                         |

**2019 SCORING GUIDELINES****Question 1 (continued)****Learning Objectives**

**CNV-7.B.a:** Calculate the potential difference across a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.

**CNV-7.B.b:** Calculate the stored charge on a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.

**CNV-7.D.a:** Derive expressions using calculus to describe the time dependence of the stored charge or potential difference across the capacitor, or the current or potential difference across the resistor in an RC circuit when charging or discharging a capacitor.

**CNV-7.E.b:** Describe the behavior of the voltage or current behavior over time for a circuit that contains resistors and capacitors in a multi-loop arrangement.

**CNV-7.G.a:** Describe the energy transfer in charging or discharging a capacitor in an RC circuit.

**Science Practices**

**3.C:** Sketch a graph that shows a functional relationship between two quantities.

**5.A:** Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.

**5.E:** Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

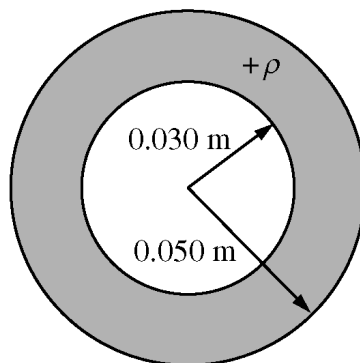
**7.A:** Make a scientific claim.

**7.C:** Support a claim with evidence from physical representations.

## 2019 SCORING GUIDELINES

## Question 2

15 points



A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density  $\rho$ , as shown in the figure above. The charge density  $\rho$  of the sphere is given as a function of the distance  $r$  from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m: } \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m: } \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

$$r > 0.050 \text{ m: } \rho = 0$$

- (a) LO CNV-2.D.a, SP 6.B, 6.C  
3 points

Calculate the total charge of the sphere.

|                                                                                                                                        |  |         |
|----------------------------------------------------------------------------------------------------------------------------------------|--|---------|
| For indicating the need to integrate the expression for charge density to determine the total charge on the sphere                     |  | 1 point |
| $Q = \int \rho dV$                                                                                                                     |  |         |
| For proper substitutions into the integration                                                                                          |  | 1 point |
| $Q = \int \frac{(1.6 \times 10^{-6})}{r} (4\pi r^2) dr$                                                                                |  |         |
| For using the proper limits of integration                                                                                             |  | 1 point |
| $Q = 4\pi (1.6 \times 10^{-6}) \int_{r=0.03}^{r=0.05} r dr = 4\pi (1.6 \times 10^{-6}) \left[ \frac{r^2}{2} \right]_{r=0.03}^{r=0.05}$ |  |         |
| $Q = (2\pi)(1.6 \times 10^{-6})(0.05^2 - 0.03^2) = 1.61 \times 10^{-8} \text{ C}$                                                      |  |         |

**2019 SCORING GUIDELINES****Question 2 (continued)**

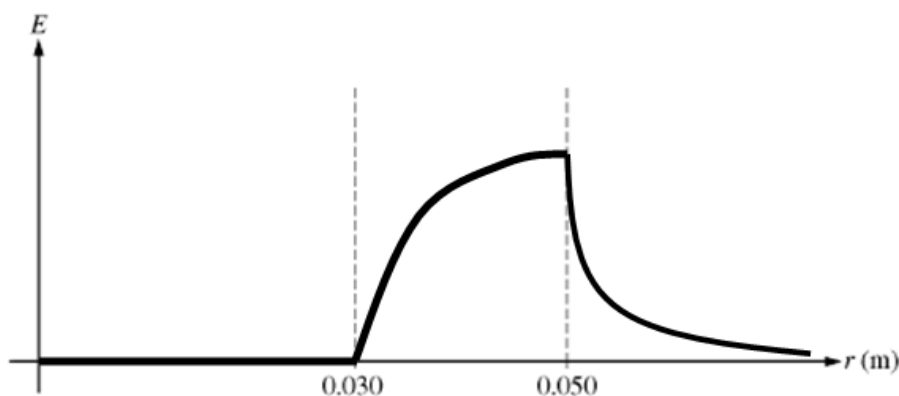
- (b) LO CNV-2.D.a, SP 6.C  
3 points

Using Gauss's law, calculate the magnitude of the electric field  $E$  at the outer surface of the sphere.

|                                                                                                                                                                             |  |         |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---------|
| For correctly evaluating the surface integral in Gauss's law                                                                                                                |  | 1 point |
| $\oint E \cdot dA = E(4\pi r^2)$                                                                                                                                            |  |         |
| For correctly substituting the answer from part (a) and correct radius into above equation                                                                                  |  | 1 point |
| $\frac{Q_{enc}}{\epsilon_0} = E(4\pi r^2) \therefore E = \frac{Q_{enc}}{4\pi\epsilon_0 r^2} = \frac{(1.61 \times 10^{-8} \text{ C})}{(4\pi)(8.85 \times 10^{-12})(0.05)^2}$ |  |         |
| For an answer consistent with part (a) with correct units                                                                                                                   |  | 1 point |
| $E = 5.79 \times 10^4 \text{ N/C}$                                                                                                                                          |  |         |

- (c) LO CNV-2.C, SP 3.C  
3 points

On the axes below, sketch the magnitude of the electric field  $E$  as a function of distance  $r$  from the center of the sphere.



|                                                                                                                     |  |         |
|---------------------------------------------------------------------------------------------------------------------|--|---------|
| For clearly showing a graph with a value of $E = 0$ for $r < 0.030 \text{ m}$                                       |  | 1 point |
| For a continuous graph that starts at zero, is concave down, and increases in value from $r = 0.030$ to $r = 0.050$ |  | 1 point |
| For a continuous graph that decreases asymptotically toward the horizontal axis for $r > 0.050 \text{ m}$           |  | 1 point |



## 2019 SCORING GUIDELINES

## Question 2 (continued)

- (d) LO CNV-1.G.a, SP 6.B, 6.C  
2 points

Calculate the electric potential  $V$  at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

|                                                                                                                                                                                                                     |                         |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------|
| For substituting the total charge from part (a) into a correct expression for electric potential                                                                                                                    | 1 point                 |
| For substituting $r = 0.05$ m into a correct expression for electric potential                                                                                                                                      | 1 point                 |
| $V_R = \frac{Q_{\text{tot}}}{4\pi\epsilon_0 r} = \frac{(9 \times 10^9)(1.61 \times 10^{-8} \text{ C})}{(0.05 \text{ m})} = 2900 \text{ V}$                                                                          |                         |
| <i>Alternate Solution</i>                                                                                                                                                                                           | <i>Alternate Points</i> |
| For substituting the total charge from part (a) into an integration for electric potential                                                                                                                          | 1 point                 |
| $\Delta V_R = V_R - V_\infty = V_R = -\int E dr = -\int \frac{Q_{\text{enc}}}{4\pi\epsilon_0 r^2} dr = -\int \frac{(9 \times 10^9)(1.61 \times 10^{-8} \text{ C})}{r^2} dr$                                         |                         |
| For integrating with correct limits of integration                                                                                                                                                                  | 1 point                 |
| $V_R = -\int_{r=\infty}^{r=0.05 \text{ m}} \frac{145}{r^2} dr = -145 \left[ -\frac{1}{r} \right]_{r=\infty}^{r=0.05 \text{ m}} = 145 \left( \frac{1}{(0.05 \text{ m})} - \frac{1}{\infty} \right) = 2900 \text{ V}$ |                         |

A proton is released from rest at the outer surface of the sphere at time  $t = 0$  s.

- (e)  
i. LO ACT-1.D, SP 6.B, 6.C  
2 points

Calculate the magnitude of the initial acceleration of the proton.

|                                                                                                                                                                |         |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|
| For using a correct expression of Newton's second law in terms of the electric field                                                                           | 1 point |
| $F = ma \therefore qE = ma \therefore a = \frac{qE}{m}$                                                                                                        |         |
| For correctly substituting into equation above                                                                                                                 | 1 point |
| $a = \frac{qE}{m} = \frac{(1.6 \times 10^{-19} \text{ C})(5.79 \times 10^4 \text{ N/C})}{1.67 \times 10^{-27} \text{ kg}} = 5.55 \times 10^{12} \text{ m/s}^2$ |         |

**2019 SCORING GUIDELINES****Question 2 (continued)**

(e) continued

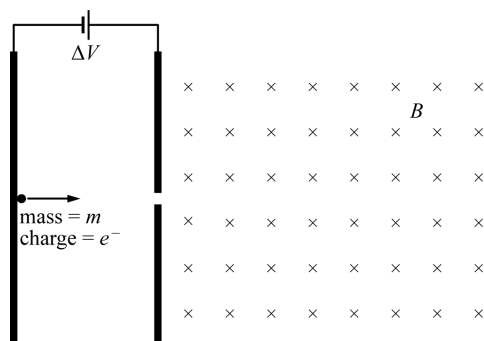
ii. LO CNV-1.E, SP 6.B, 6.C

2 points

Calculate the speed of the proton after a long time.

|                                                                                                                                                                           |  |         |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---------|
| For a correct expression of kinetic energy in terms of the electric potential difference                                                                                  |  | 1 point |
| $-q\Delta V = \frac{1}{2}mv^2$                                                                                                                                            |  |         |
| For correctly substituting into equation above                                                                                                                            |  | 1 point |
| $v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{-2(1.6 \times 10^{-19} \text{ C})(0 - 2900 \text{ V})}{(1.67 \times 10^{-27} \text{ kg})}} = 7.45 \times 10^5 \text{ m/s}$ |  |         |

**Learning Objectives****ACT-1.D:** Determine the motion of a charged object of specified charge and mass under the influence of an electrostatic force.**CNV-1.E:** Calculate the work done or changes in kinetic energy (or changes in speed) of a charged particle when it is moved through some known potential difference.**CNV-1.G.a:** Use the general relationship between electric field and electric potential to calculate the relationships between the magnitude of electric field or the potential difference as a function of position.**CNV-2.C:** State and use Gauss's law in integral form to derive unknown electric fields for planar, spherical, or cylindrically symmetrical charge distributions.**CNV-2.D.a:** Using appropriate mathematics (which may involve calculus), calculate the total charge contained in lines, surfaces, or volumes when given a linear-charge density, a surface-charge density, or a volume-charge density of the charge configuration.**Science Practices****3.C:** Sketch a graph that shows a functional relationship between two quantities.**6.B:** Apply an appropriate law, definition, or mathematical relationship to solve a problem.**6.C:** Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

**2019 SCORING GUIDELINES****Question 3****15 points**

Two plates are set up with a potential difference  $V$  between them. A small sphere of mass  $m$  and charge  $-e$  is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude  $B$  directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of  $V$ ,  $m$ ,  $e$ ,  $B$ , and fundamental constants, as appropriate.

(a)

- i. LO CHG-1.A.a, SP 7.A  
1 point

What is the initial direction of the force on the sphere as it enters the magnetic field?

☐ Into the page

☐ Out of the page

☐ Toward the top of the page

☐ Toward the bottom of the page

|                                               |  |
|-----------------------------------------------|--|
| For selecting “Toward the bottom of the page” |  |
|-----------------------------------------------|--|

|         |
|---------|
| 1 point |
|---------|

- ii. LO CHG-1.B, SP 7.A  
1 point

Describe the path taken by the sphere after it enters the magnetic field.

|                                                                                              |  |
|----------------------------------------------------------------------------------------------|--|
| For describing a circular path for the sphere consistent with the selection from part (a)(i) |  |
|----------------------------------------------------------------------------------------------|--|

|         |
|---------|
| 1 point |
|---------|

|                                                                                 |  |
|---------------------------------------------------------------------------------|--|
| Example: The sphere will move in a circular path toward the bottom of the page. |  |
|---------------------------------------------------------------------------------|--|

**2019 SCORING GUIDELINES****Question 3 (continued)**

- (b) LO CNV-1.E, SP 5.A, 5.E  
2 points

Derive an expression for the speed of the sphere as it passes through the small opening.

|                                                                                          |  |         |
|------------------------------------------------------------------------------------------|--|---------|
| For using a valid equation relating potential difference to kinetic energy of the sphere |  | 1 point |
| $\Delta K = -\Delta U \therefore \frac{1}{2}mv^2 = -q\Delta V$                           |  |         |
| $\frac{1}{2}mv^2 = eV$                                                                   |  |         |
| For the correct answer with supporting work                                              |  | 1 point |
| $v = \sqrt{\frac{2eV}{m}}$                                                               |  |         |

- (c) LO CHG-1.C, SP 5.A, 5.E  
3 points

Derive an expression for the radius of the path taken by the sphere as it moves through the magnetic field.

|                                                                                       |  |         |
|---------------------------------------------------------------------------------------|--|---------|
| For an expression relating the magnetic force to the centripetal force                |  | 1 point |
| $\frac{mv^2}{r} = Bqv$                                                                |  |         |
| For substituting the charge of the sphere into the above equation and solving for $r$ |  | 1 point |
| $\frac{mv}{r} = Be \therefore r = \frac{mv}{Be}$                                      |  |         |
| For substituting the answer from part (b)                                             |  | 1 point |
| $r = \frac{m}{Be} \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2mV}{eB^2}}$                     |  |         |

An experiment is performed in which a beam of electrons is accelerated across the space between the plates and passes through the small opening. After passing through the opening, the electrons travel in a semicircular path and strike the right-hand plate. The potential difference between the plates is varied in regular increments, as shown in the table below. For each potential difference, the magnetic field is varied in order to cause the beam to strike the right-hand plate at a distance of 0.020 m from the opening.

|                                       |      |      |      |      |      |      |
|---------------------------------------|------|------|------|------|------|------|
| Potential difference (V)              | 60   | 70   | 100  | 110  | 120  | 140  |
| Magnetic field ( $T \times 10^{-3}$ ) | 2.62 | 2.78 | 3.39 | 3.54 | 3.78 | 3.99 |
|                                       |      |      |      |      |      |      |
|                                       |      |      |      |      |      |      |

**2019 SCORING GUIDELINES****Question 3 (continued)**

- (d) LO CHG-1.C, SP 4.C  
1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the mass-to-charge ratio of an electron.

Vertical axis: \_\_\_\_\_

Horizontal axis: \_\_\_\_\_

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

$$r = \sqrt{\frac{2m_e V}{eB^2}} \therefore B^2 = \frac{2m_e}{er^2} V$$

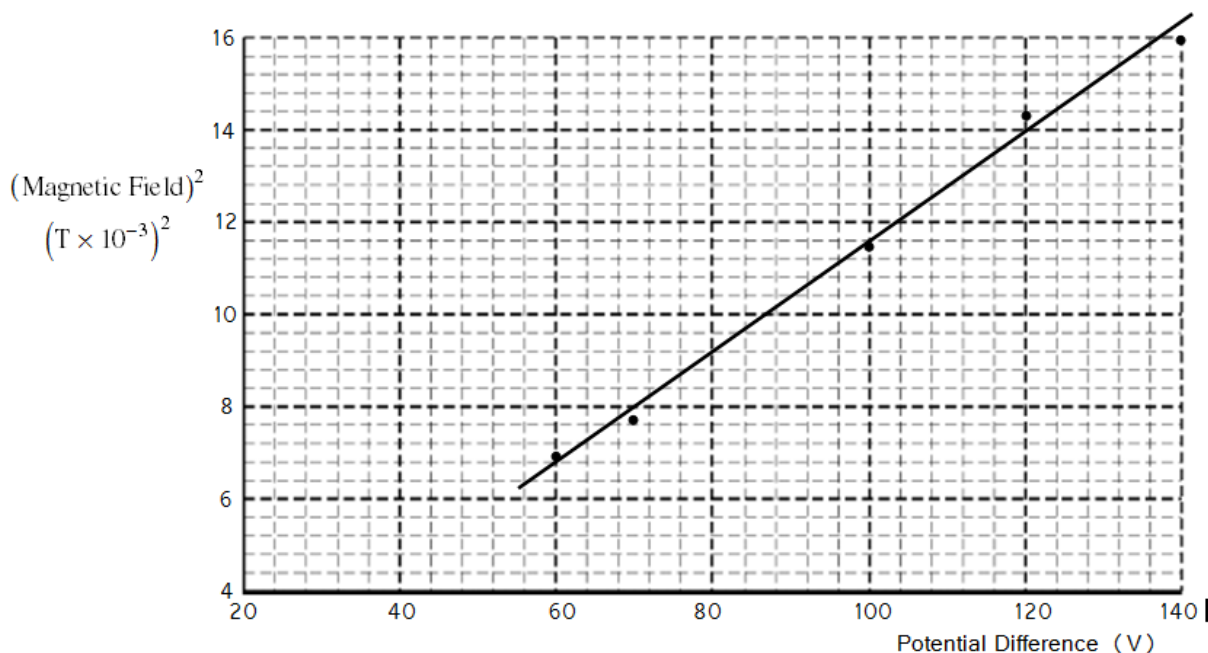
|                                                                   |      |      |      |      |      |      |
|-------------------------------------------------------------------|------|------|------|------|------|------|
| Potential difference (V)                                          | 60   | 70   | 100  | 110  | 120  | 140  |
| Magnetic field ( $T \times 10^{-3}$ )                             | 2.62 | 2.78 | 3.39 | 3.54 | 3.78 | 3.99 |
| (Magnetic field) <sup>2</sup> ( $T \times 10^{-3}$ ) <sup>2</sup> | 6.86 | 7.73 | 11.5 | 12.5 | 14.3 | 15.9 |
|                                                                   |      |      |      |      |      |      |

|                                                                                    |  |         |
|------------------------------------------------------------------------------------|--|---------|
| For indicating axis choices with appropriate quantities to produce a straight line |  | 1 point |
| <u>Note:</u> There are several possible choices.                                   |  |         |

**2019 SCORING GUIDELINES****Question 3 (continued)**

- (e) LO CHG-1.C, SP 3.A, 4.C  
4 points

On the graph below, plot the relationship determined in part (d). Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



|                                                                                           |         |
|-------------------------------------------------------------------------------------------|---------|
| For correctly labeling both axes with quantities and units                                | 1 point |
| For correctly scaling and labeling the axes so that the points use at least half the grid | 1 point |
| For correctly plotting the data                                                           | 1 point |
| For drawing a straight line that represents the data                                      | 1 point |

- (f) LO CHG-1.C, SP 4.D, 5.E  
3 points

Using the straight line from part (e), determine the mass-to-charge ratio of an electron.

|                                                                                                                                              |         |
|----------------------------------------------------------------------------------------------------------------------------------------------|---------|
| For correctly calculating the slope from the best-fit straight line and not from the data points unless the points fall on the best-fit line | 1 point |
| $\text{slope} = \frac{14 - 9.2}{120 - 80} = 0.12 \frac{(\text{T} \times 10^{-3})^2}{\text{V}} = 1.2 \times 10^{-7} \text{ T}^2/\text{V}$     |         |
| For a correct expression relating the slope to the mass to charge ratio of an electron                                                       | 1 point |
| $B^2 = \frac{2m_e}{er^2} V \therefore \text{slope} = \frac{2m_e}{er^2} \therefore \frac{m_e}{e} = \frac{r^2 \times \text{slope}}{2}$         |         |
| For substituting correct values into the above equation                                                                                      | 1 point |
| $\frac{m_e}{e} = \frac{(0.01 \text{ m})^2 \times (1.2 \times 10^{-7} \text{ T}^2/\text{V})}{2} = 6.0 \times 10^{-12} \text{ kg/C}$           |         |

**2019 SCORING GUIDELINES****Question 3 (continued)****Learning Objectives**

**CHG-1.A.a:** Calculate the magnitude and direction of the magnetic force of interaction between a moving charged particle of specified charge and velocity moving in a region of a uniform magnetic field.

**CHG-1.B:** Describe the path of different moving charged particles (i.e., of different type of charge or mass) in a uniform magnetic field.

**CHG-1.C:** Derive an expression for the radius of a circular path for a charged particle of specified characteristics moving in a specified magnetic field.

**CNV-1.E:** Calculate the work done or changes in kinetic energy (or changes in speed) of a charged particle when it is moved through some known potential difference.

**Science Practices**

**3.A:** Select and plot appropriate data.

**4.C:** Linearize data and/or determine a best-fit line or curve.

**4.D:** Select relevant features of a graph to describe a physical situation or solve problems.

**5.A:** Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.

**5.E:** Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

**7.A:** Make a scientific claim.