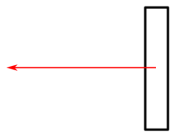
## Question 2: Free-Response Question — 15 points

**(a)** For drawing an arrow pointing to the left with no extraneous arrows — **1 point**

**Example Response**



**Total for part (a)** — **1 point**





**(b)(i)** For using Faraday's law to calculate the value of the induced emf — **1 point**

**Scoring Note:** This point may be earned without the negative sign or a numerical answer.

**Example Response**

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$
$$\mathcal{E} = -\frac{d(B_1 dx)}{dt}$$

For a correct substitution of $v$ for $\frac{dx}{dt}$ — **1 point**

**Scoring Note:** A student can earn points 1 and 2 of part (b)(i) by starting with the expression $\mathcal{E} = B_1 dv$.

**Example Response**

$$\mathcal{E} = -B_1 d\left(\frac{dx}{dt}\right)$$
$$\mathcal{E} = -B_1 dv$$

For substituting the correct resistance into an equation for Ohm's law to solve for the current — **1 point**

**Example Solution**

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$
$$\mathcal{E} = -\frac{d(B_1 dx)}{dt}$$
$$\mathcal{E} = -B_1 d\left(\frac{dx}{dt}\right)$$
$$\mathcal{E} = -B_1 dv$$
$$\mathcal{E} = -(0.40 \text{ T})(0.30 \text{ m})(2.5 \text{ m/s})$$
$$\mathcal{E} = -0.30 \text{ V}$$

$$\Delta V = IR$$
$$I = \frac{\Delta V}{R}$$
$$I = \frac{|\mathcal{E}|}{R}$$
$$I = \frac{|-0.30 \text{ V}|}{0.20\ \Omega} = 1.5 \text{ A}$$

**(b)(ii)** For substituting the current or an expression for the current obtained from part (b)(i) into an appropriate equation for calculating the magnetic force **1 point**

**Example Responses**

$$\vec{F} = \int I d\vec{\ell} \times \vec{B}$$
$$F = IdB_1$$
$$F = (1.5\text{ A})(0.3\text{ m})(0.4\text{ T})$$
$$F = 0.18\text{ N}$$

**OR**

$$\vec{F} = \int I d\vec{\ell} \times \vec{B}$$
$$F = IdB_1$$
$$F = \left(\frac{B_1 dv}{R}\right) dB$$
$$F = \frac{B_1^2 d^2 v}{R}$$
$$F = \frac{(0.4\text{ T})^2 (0.3\text{ m})^2 (2.5\text{ m/s})}{0.2\ \Omega}$$
$$F = 0.18\text{ N}$$

**Total for part (b) 4 points**

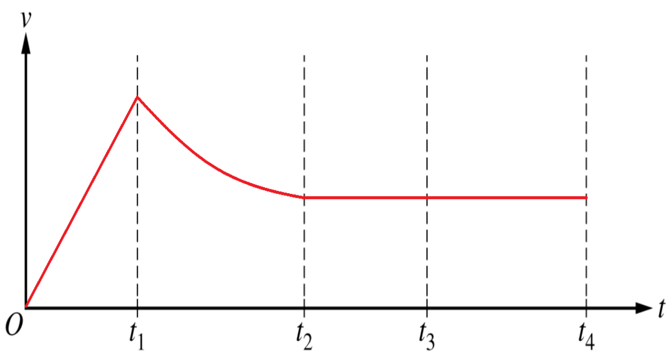
**(c)** For drawing a straight line with a positive slope that starts at the origin from $t = 0$ to $t_1$ **1 point**

For drawing a curve that is decreasing and concave up from $t_1$ to $t_2$ **1 point**

For drawing a nonzero horizontal line from $t_2$ to $t_3$ **1 point**

For drawing a nonzero horizontal line from $t_3$ to $t_4$ **1 point**

**Example Response**




**Total Points for part (c) 4 points**



AP Physics C: **Electricity and Magnetism: 2023 Set 2**

**(d)(i)** For a correct answer with units ( $0.4\ \Omega$ ) **1 point**

**Scoring Note:** This point can be earned without supporting calculations.

**Example Response**

$$R_s = \sum_i R_i$$
$$R_s = (0.2\ \Omega) + (0.2\ \Omega)$$
$$R_s = 0.4\ \Omega$$

**(d)(ii)** For a statement that correctly describes the inverse relation between resistance and current (e.g., as resistance increases, current decreases) **1 point**

For a statement that correctly describes the direct relation between current and force (e.g., as current decreases, magnetic force decreases) **1 point**

For a statement that correctly describes the direct relation between force and acceleration (e.g., as force decreases acceleration decreases) **1 point**

**Scoring Note:** Full credit can be earned with a justification that is consistent with the resistance calculated in part (d)(i).

**Example Response**

*The new acceleration $a_{\text{new}}$ is less than $a_{\text{original}}$. Greater resistance of the bar causes the current to be less than in the original scenario. Less current causes the magnetic force $F = IdB$ on the bar to also be less than the original. By Newton's second law $F = ma$ so less force on a bar of the same mass results in less acceleration.*

**Total for part (d) 4 points**

**(e)** For correctly indicating **one** of the following, with an attempt at a relevant justification: **1 point**

- Increasing $H$
- Increasing $B_1$
- Increasing $d$

For correctly justifying the identified modification that will result in a larger induced current in the new bar at position $x_B$ **1 point**

**Example Responses**

*Increasing $H$ will increase the induced current. If the ramp is higher, then the potential energy is greater and this results in greater kinetic energy and greater velocity at the bottom of the ramp. A greater velocity causes a greater rate of change in flux as the bar moves through the field. By Faraday's law the* emf *is greater and therefore also the current.*

**OR**

*Increasing $B_1$ will increase the induced current. If the magnetic field is stronger this increases the flux through the circuit and therefore also the rate of change in the flux. By Faraday's law the* emf *is then greater and therefore also the current.*

**OR**

*Increasing $d$ will increase the induced current. A larger width results in a greater area encompassed by the circuit. Greater area increases the flux through the circuit and therefore also the rate of change in the flux. By Faraday's law the* emf *is then greater and therefore also the current.*

**Total for part (e) 2 points**

**Total for question 2 15 points**





## Question 3: Free-Response Question 15 points

**(a)** For a loop rule expression that includes terms for the equivalent resistance $2R$ and the potential difference across the battery **1 point**

For an expression that includes charge $Q$ in the term relating the potential difference across the capacitor and includes charge per unit time $\frac{dQ}{dt}$ in the term relating the potential difference across the pair of resistors **1 point**

**Example Response**

$$\mathcal{E} - \Delta V_C - \Delta V_{R,eq} = 0$$

$$\mathcal{E} - \frac{Q}{C} - 2R\frac{dQ}{dt} = 0$$

**Total for part (a) 2 points**

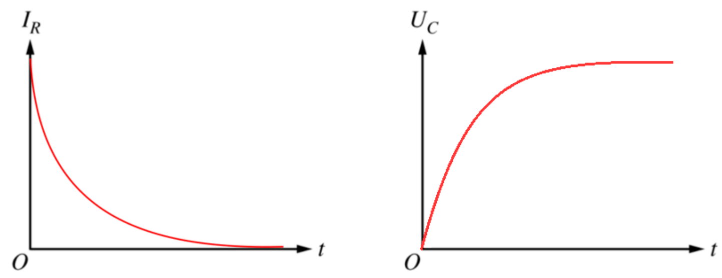
**(b)** For sketching a concave up and decreasing curve on the graph of $I_R$ as a function of $t$ **1 point**

For sketching a concave down and increasing curve on the graph of $U_C$ as a function of $t$ **1 point**

For sketching both curves that approach a slope of zero as time increases **1 point**

**Scoring Note:** The third point can be earned even if the first two points are not earned.

**Example Response**




**Total for part (b) 3 points**

| | | |
|---|---|---|
| **(c)(i)** | For a correct justification that could include one of the following:<br>• An indication that the current is upwards that includes a statement that indicates that positive charge had accumulated on the top plate of Capacitor 1 and/or negative charge has accumulated on the bottom plate of Capacitor 1 when the switch was closed to Position A<br>• An indication that the current is upwards that includes a statement that indicates that the value of the electric potential of the top plate of Capacitor 1 is larger than the electric potential of the bottom plate of Capacitor 1 when the switch was closed to Position A | **1 point** |

**Example Responses**

*When the switch is closed at time $t_1$, positive charge has built up on the top plate of the capacitor. This positive charge on the top plate pushes charge up through Resistor 1 and down through Capacitor 2 to charge Capacitor 2.*

**OR**

*After a long-time the top plate of Capacitor 1 is at a high potential due to its being charged by the battery. When the switch is closed at time $t_1$, the resulting current is up through Resistor 1 as the current goes from high potential on the top plate to low potential clockwise around the circuit through Capacitor 2.*

| | | |
|---|---|---|
| **(c)(ii)** | For indicating that the total charge on the positive plate of Capacitor 2 is $\frac{Q_0}{2}$<br>**Scoring Note:** This point can be earned without supporting calculations. | **1 point** |

**Example Response**

*The potential difference across Capacitor 1 is equal to the potential difference across Capacitor 2. Capacitor 2 has the same capacitance of Capacitor 1. Therefore, Capacitor 2 stores the same charge that is stored on Capacitor 1. Due to conservation of charge, Capacitor 2 stores half of the original charge equal to $\frac{Q_0}{2}$.*



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| | | |
|---|---|---|
| **(c)(iii)** | For indicating that the total energy dissipated by Resistor 1 is the difference between the initial electric potential energy of the system at time $t = t_1$ and the final electric potential energy of the system after the new steady state conditions have been reached | **1 point** |

**Example Response**

$\Delta E_R = U_C - U_{0C}$

For indicating that only Capacitor 1 stores nonzero electric potential energy initially and both capacitors store electric potential energy after the new steady-state conditions have been reached, or an alternate response that is consistent with part (c)(ii) — **1 point**

**Example Response**

$U_{0C} = U_{01}$ ***AND*** $U_C = U_1 + U_2$

For correct substitutions for the charges stored on the capacitors after the new steady state conditions have been reached based on part (c)(ii) — **1 point**

**Example Solution**

$$\Delta E_R = U_C - U_{0C}$$
$$\Delta E_R = \left(\frac{1}{2}\left(\frac{1}{C}\right)\left(\frac{Q_0}{2}\right)^2 + \frac{1}{2}\left(\frac{1}{C}\right)\left(\frac{Q_0}{2}\right)^2\right) - \frac{1}{2}\frac{Q_0^2}{C}$$
$$\Delta E_R = \frac{Q_0^2}{4C} - \frac{1}{2}\frac{Q_0^2}{C}$$
$$\Delta E_R = -\frac{Q_0^2}{4C}$$

**Total for part (c) — 5 points**

| | | |
|---|---|---|
| **(d)** | For the correct expression for charge on the top plate of Capacitor 2 $\left(Q_2 = \frac{2Q_0}{3}\right)$ | **1 point** |

**Example Response**

$$C_1 = C \text{ and } C_2 = \kappa C = 2C$$
$$\Delta V_{C1} = \Delta V_{C2}$$
$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$
$$Q_1 = \frac{1}{2}Q_2$$
$$Q_1 + Q_2 = Q_0$$
$$\frac{1}{2}Q_2 + Q_2 = Q_0$$
$$Q_2 = \frac{2}{3}Q_0$$

**Scoring Note:** This point can be earned without supporting calculations.

**Total for part (d) — 1 point**

| | | |
|---|---|---|
| **(e)(i)** | For a loop rule equation that includes terms for the potential difference across Resistor 2 and the potential difference across Capacitor 2 with the dielectric | **1 point** |

**Example Response**

$\Delta V_{R,2} - \Delta V_{C,2} = 0$

$\Delta V_{R,2} = \Delta V_{C,2}$

For correct substitution of $IR$ for the potential difference across Resistor 2 — **1 point**

**Example Response**

$\Delta V_{R,2} = IR$

For correct substitutions of $2C$ for the new capacitance of Capacitor 2 with the dielectric inserted and of the charge consistent with part (d) into the expression for the potential difference across the capacitor — **1 point**

**Example Response**

$$\Delta V_{C,2} = \frac{Q_2}{C_2}$$

$$\Delta V_{C,2} = \left(\frac{2Q_0}{3}\right)\left(\frac{1}{2C}\right)$$

**Example Solution**

$$\Delta V_{R,2} - \Delta V_{C,2} = 0$$

$$\Delta V_{R,2} = \Delta V_{C,2}$$

$$IR = \left(\frac{2Q_0}{3}\right)\left(\frac{1}{2C}\right)$$

$$IR = \frac{Q_0}{3C}$$

$$I = \frac{Q_0}{3RC}$$

| | | |
|---|---|---|
| **(e)(ii)** | For indicating that the current is zero | **1 point** |
| | **Total for part (e)** | **4 points** |
| | **Total for question 3** | **15 points** |





## Question 1: Free-Response Question — 15 points

**(a)** For an indication that the capacitors act like a short circuit immediately after the switch is closed — **1 point**

$R_{EQ} = R_1 = 100\ \Omega$

For using Ohm's law to calculate the current through $R_1$ — **1 point**

$$I = \frac{\Delta V}{R} = \frac{(10\ \text{V})}{(100\ \Omega)}$$

$I = 0.10\ \text{A}$

**Total for part (a) — 2 points**

**(b)** For correctly determining the equivalent resistance during steady-state — **1 point**

$$\frac{1}{R_P} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{30\ \Omega} + \frac{1}{60\ \Omega} \therefore R_P = 20\ \Omega$$

$R_{EQ} = R_1 + R_P = 100\ \Omega + 20\ \Omega = 120\ \Omega$

For correctly calculating the current through the battery — **1 point**

$$I = \frac{\Delta V}{R} = \frac{(10\ \text{V})}{(120\ \Omega)}$$

$I = 0.083\ \text{A}$

The potential difference across $R_1$

$\Delta V = (0.083\ \text{A})(100\ \Omega) = 8.3\ \text{V}$

**Total for part (b) — 2 points**

**(c) i.** For using a correct equation to calculate the potential difference across $C_3$ — **1 point**

$\Delta V_{C_3} = \Delta V_P = IR_P = (0.083\ \text{A})(20\ \Omega) = 1.67\ \text{V}$

For using the correct equivalent capacitance of the series combination to calculate the charge stored in capacitor $C_2$ — **1 point**

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{10\ \mu\text{F}} + \frac{1}{15\ \mu\text{F}} \therefore C_S = 6.0\ \mu\text{F}$$

$Q_2 = Q_S = C_S \Delta V_{C_{12}} = (1.67\ \text{V})(6.0\ \mu\text{F}) = 10\ \mu C$

**ii.** For selecting "Greater than" and attempting a relevant justification — **1 point**

For a correct justification — **1 point**

**Example response for part (c)(ii)**

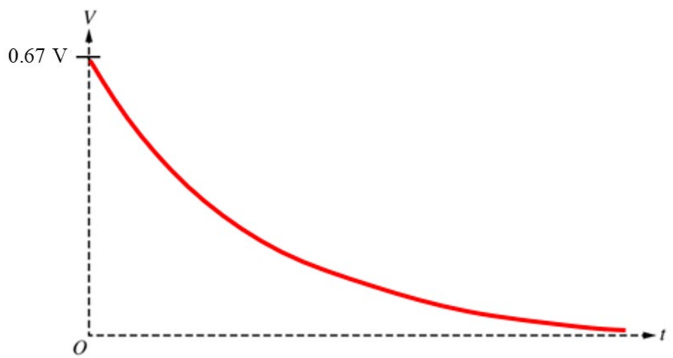
*The potential difference across the series combination of $C_1$ and $C_2$ is the same as the potential difference across $C_3$; thus, capacitor $C_3$ has a greater potential difference across its plates than $C_2$ and has a greater capacitance. Because $Q = C\Delta V$, $C_3$ stores a greater charge.*

**Scoring note:** A justification showing a mathematical proof earns full credit.

**Total for part (c) — 4 points**

| | | |
|---|---|---|
| **(d) i.** | For indicating that the current through $R_1$ is zero when the switch is opened<br>$I_1 = 0$ | **1 point** |
| **ii.** | For indicating that the potential difference across $R_2$ is equal to the potential difference across the capacitors before the switch was opened<br>$\Delta V_{R_2} = \Delta V_{C_{12}} = 1.67\ \text{V}$ | **1 point** |
| | For using Ohm's law to calculate the current through $R_2$<br>$I = \frac{\Delta V_{R_2}}{R_2} = \frac{(1.67\ \text{V})}{(30\ \Omega)}$<br>$I = 0.056\ \text{A}$ | **1 point** |
| | **Total for part (d)** | **3 points** |
| **(e)** | For a curve starting at a nonzero, labeled maximum value | **1 point** |
| | For a concave-up curve with the horizontal axis as an asymptote | **1 point** |

**Example response for part (e)**




| | | |
|---|---|---|
| | **Total for part (e)** | **2 points** |
| **(f)** | For selecting "Below" and attempting a relevant justification | **1 point** |
| | For a correct justification | **1 point** |

**Example response for part (f)**

*Because the equivalent capacitance of the circuit would decrease, the time constant would also decrease. Thus the capacitors would discharge more rapidly, and the new curve would be below the original curve.*

| | |
|---|---|
| **Total for part (f)** | **2 points** |
| **Total for question 1** | **15 points** |





2019

AP® CollegeBoard

# AP® Physics C: Electricity and Magnetism

## Scoring Guidelines Set 2

## General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8\ \text{m/s}^2$ , but the use of $10\ \text{m/s}^2$ is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

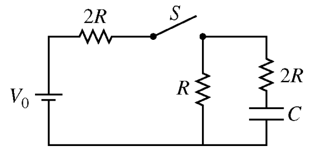
6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.





## Question 1

**15 points**




The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage $V_0$, a capacitor of capacitance $C$, and a switch $S$. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of $V_0$, $R$, $C$, and fundamental constants, as appropriate.

(a) LO CNV-7.B.a, SP 5.A, 5.E
2 points

Derive an expression for the steady-state current supplied by the battery.

| | | |
|---|---|---|
| For using Ohm's law | | 1 point |
| $I = \dfrac{V}{R_{eff}} = \dfrac{V_0}{(2R + R)}$ | | |
| For correct substitution leading to correct answer | | 1 point |
| $I = \dfrac{V_0}{3R}$ | | |

(b) LO CNV-7.B.b, SP 5.A, 5.E
2 points

Derive an expression for the charge on the capacitor.

| | | |
|---|---|---|
| For using the equation relating stored charge to capacitance | | 1 point |
| $q = CV = CV_C = CV_R$ | | |
| For determining $V_R$ and substituting into the above equation | | 1 point |
| $V_R = IR = \left(\dfrac{V_0}{3R}\right)R = \dfrac{1}{3}V_0$ | | |
| $q = CV_R = C\left(\dfrac{1}{3}V_0\right) = \dfrac{1}{3}CV_0$ | | |

## Question 1 (continued)

(c) LO CNV-7.B.b, SP 5.A, 5.E
2 points

Derive an expression for the energy stored in the capacitor.

| | | |
|---|---|---|
| For any correct equation for energy stored in a capacitor | | 1 point |
| $U = \dfrac{q^2}{2C}$ | | |
| Substitute charge and/or voltage from part (b) | | 1 point |
| $U = \dfrac{(CV_0/3)^2}{2C} = \dfrac{1}{18}CV_0^2$ | | |

Now the switch is opened at time $t = 0$.

(d) LO CNV-7.D.a, SP 5.A, 5.E
2 points

Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time $t$ after the switch is opened.

| | | |
|---|---|---|
| For any correct voltage loop equation | | 1 point |
| $V_C - V_R - V_{2R} = 0 \therefore V_C = V_R + V_{2R}$ | | |
| $\dfrac{q(t)}{C} = I(R + 2R)$ | | |
| Note: Any correct loop equation for when the switch is open earns the point | | |
| For substituting $-dq/dt$ or $dq/dt$ for the current, consistent with loop equation | | 1 point |
| $\dfrac{q(t)}{C} = -3R\dfrac{dq}{dt}$ | | |





## Question 1 (continued)

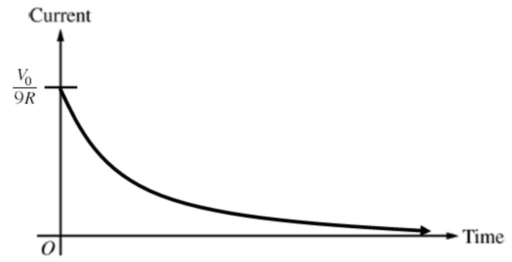
(e)
i. LO CNV-7.B.a, SP 5.A, 5.E
2 points

Calculate the current in resistor $R$ immediately after the switch is opened.

| | | |
|---|---|---|
| For using a voltage consistent with part (b) in $I = \dfrac{V}{R}$ | | 1 point |
| For substituting the correct resistance ($3R$) into Ohm's law | | 1 point |
| $I = \dfrac{V_0/3}{3R} = \dfrac{V_0}{9R}$ | | |

ii. LO CNV-7.E.b, SP 3.C
3 points

On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.




| | | |
|---|---|---|
| For a curve that is concave up throughout graph | | 1 point |
| For having the horizontal axis as an asymptote | | 1 point |
| For labeling the maximum current consistent with part (e)(i) | | 1 point |

## Question 1 (continued)

(f) LO CNV-7.G.a, SP 7.A, 7.C
2 points

Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c)?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

| | | |
|---|---|---|
| For selecting "Equal to" | | 1 point |
| For a correct justification invoking conservation of energy | | 1 point |
| Example: After the switch is opened, the capacitor will discharge all of its stored energy and charge. Assuming no energy is lost in the wire, then the only parts of the circuit that will dissipate this energy are the two resistors in series with the capacitor. | | |
| | | |
| *Alternate Solution* | | *Alternate Points* |
| *For selecting "Less than"* | | *1 point* |
| *For a correct justification invoking conservation of energy* | | *1 point* |
| *Example: After the switch is opened, the capacitor will discharge all of its stored energy and charge. Additionally there is energy lost due to resistance in the wire and/or energy loss in the capacitor.* | | |

2019 SCORING GUIDELINES

## Question 1 (continued)

**Learning Objectives**

**CNV-7.B.a:** Calculate the potential difference across a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.
**CNV-7.B.b:** Calculate the stored charge on a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.
**CNV-7.D.a:** Derive expressions using calculus to describe the time dependence of the stored charge or potential difference across the capacitor, or the current or potential difference across the resistor in an RC circuit when charging or discharging a capacitor.
**CNV-7.E.b:** Describe the behavior of the voltage or current behavior over time for a circuit that contains resistors and capacitors in a multi-loop arrangement.
**CNV-7.G.a:** Describe the energy transfer in charging or discharging a capacitor in an RC circuit.

**Science Practices**

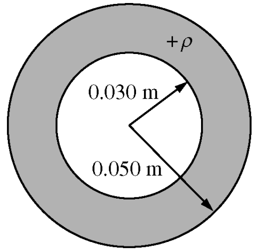
**3.C:** Sketch a graph that shows a functional relationship between two quantities.
**5.A:** Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
**5.E:** Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
**7.A:** Make a scientific claim.
**7.C:** Support a claim with evidence from physical representations.





## Question 2

**15 points**




A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density $\rho$, as shown in the figure above. The charge density $\rho$ of the sphere is given as a function of the distance $r$ from the center of the sphere, in meters, by the following.

$$r < 0.030\text{ m}: \ \rho = 0$$

$$0.030\text{ m} < r < 0.050\text{ m}: \ \rho = b/r\text{, where } b = 1.6 \times 10^{-6}\ \text{C/m}^2$$

$$r > 0.050\text{ m}: \ \rho = 0$$

(a) LO CNV-2.D.a, SP 6.B, 6.C
3 points

Calculate the total charge of the sphere.

| | | |
|---|---|---|
| For indicating the need to integrate the expression for charge density to determine the total charge on the sphere | | 1 point |
| $Q = \int \rho dV$ | | |
| For proper substitutions into the integration | | 1 point |
| $Q = \int \frac{\left(1.6 \times 10^{-6}\right)}{r}\left(4\pi r^2\right)dr$ | | |
| For using the proper limits of integration | | 1 point |
| $Q = 4\pi\left(1.6 \times 10^{-6}\right)\int_{r=0.03}^{r=0.05} r dr = 4\pi\left(1.6 \times 10^{-6}\right)\left[\frac{r^2}{2}\right]_{r=0.03}^{r=0.05}$ | | |
| $Q = (2\pi)\left(1.6 \times 10^{-6}\right)\left(0.05^2 - 0.03^2\right) = 1.61 \times 10^{-8}\ \text{C}$ | | |

## Question 2 (continued)

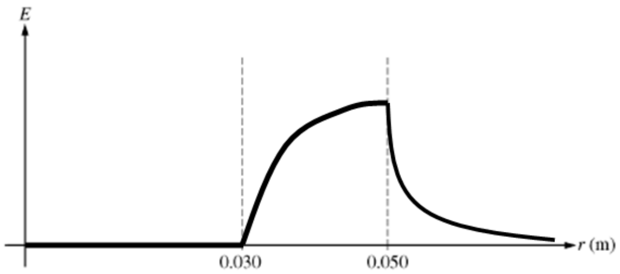
(b) LO CNV-2.D.a, SP 6.C
3 points

Using Gauss's law, calculate the magnitude of the electric field $E$ at the outer surface of the sphere.

| | | |
|---|---|---|
| For correctly evaluating the surface integral in Gauss's law | | 1 point |
| $\oint E \bullet dA = E\left(4\pi r^2\right)$ | | |
| For correctly substituting the answer from part (a) and correct radius into above equation | | 1 point |
| $\frac{Q_{enc}}{\varepsilon_0} = E\left(4\pi r^2\right) \therefore E = \frac{Q_{enc}}{4\pi\varepsilon_0 r^2} = \frac{\left(1.61\times10^{-8}\text{ C}\right)}{(4\pi)\left(8.85\times10^{-12}\right)(0.05)^2}$ | | |
| For an answer consistent with part (a) with correct units | | 1 point |
| $E = 5.79\times10^4\text{ N/C}$ | | |

(c) LO CNV-2.C, SP 3.C
3 points

On the axes below, sketch the magnitude of the electric field $E$ as a function of distance $r$ from the center of the sphere.




| | | |
|---|---|---|
| For clearly showing a graph with a value of $E = 0$ for $r < 0.030$ m | | 1 point |
| For a continuous graph that starts at zero, is concave down, and increases in value from $r = 0.030$ to $r = 0.050$ | | 1 point |
| For a continuous graph that decreases asymptotically toward the horizontal axis for $r > 0.050$ m | | 1 point |





## Question 2 (continued)

(d) LO CNV-1.G.a, SP 6.B, 6.C
2 points

Calculate the electric potential $V$ at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

| | | |
|---|---|---|
| For substituting the total charge from part (a) into a correct expression for electric potential | | 1 point |
| For substituting $r = 0.05$ m into a correct expression for electric potential | | 1 point |
| $V_R = \frac{Q_{tot}}{4\pi\varepsilon_0 r} = \frac{\left(9\times10^9\right)\left(1.61\times10^{-8}\text{ C}\right)}{(0.05\text{ m})} = 2900\text{ V}$ | | |
| | | |
| *Alternate Solution* | | *Alternate Points* |
| *For substituting the total charge from part (a) into an integration for electric potential* | | *1 point* |
| $\Delta V_R = V_R - V_\infty = V_R = -\int E dr = -\int \frac{Q_{enc}}{4\pi\varepsilon_0 r^2} dr = -\int \frac{\left(9\times10^9\right)\left(1.61\times10^{-8}\text{ C}\right)}{r^2} dr$ | | |
| *For integrating with correct limits of integration* | | *1 point* |
| $V_R = -\int_{r=\infty}^{r=0.05\text{ m}} \frac{145}{r^2} dr = -145\left[-\frac{1}{r}\right]_{r=\infty}^{r=0.05\text{ m}} = 145\left(\frac{1}{(0.05\text{ m})} - \frac{1}{\infty}\right) = 2900\text{ V}$ | | |

A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.

(e)

i. LO ACT-1.D, SP 6.B, 6.C
2 points

Calculate the magnitude of the initial acceleration of the proton.

| | | |
|---|---|---|
| For using a correct expression of Newton's second law in terms of the electric field | | 1 point |
| $F = ma \therefore qE = ma \therefore a = \frac{qE}{m}$ | | |
| For correctly substituting into equation above | | 1 point |
| $a = \frac{qE}{m} = \frac{\left(1.6\times10^{-19}\text{ C}\right)\left(5.79\times10^4\text{ N/C}\right)}{1.67\times10^{-27}\text{ kg}} = 5.55\times10^{12}\text{ m/s}^2$ | | |

## Question 2 (continued)

(e) continued
ii. LO CNV-1.E, SP 6.B, 6.C
2 points

Calculate the speed of the proton after a long time.

| | | |
|---|---|---|
| For a correct expression of kinetic energy in terms of the electric potential difference | | 1 point |
| $-q\Delta V = \frac{1}{2}mv^2$ | | |
| For correctly substituting into equation above | | 1 point |
| $v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{-2\left(1.6\times10^{-19}\ \text{C}\right)(0-2900\ \text{V})}{\left(1.67\times10^{-27}\ \text{kg}\right)}} = 7.45\times10^{5}\ \text{m/s}$ | | |

**Learning Objectives**

**ACT-1.D:** Determine the motion of a charged object of specified charge and mass under the influence of an electrostatic force.
**CNV-1.E:** Calculate the work done or changes in kinetic energy (or changes in speed) of a charged particle when it is moved through some known potential difference.
**CNV-1.G.a:** Use the general relationship between electric field and electric potential to calculate the relationships between the magnitude of electric field or the potential difference as a function of position.
**CNV-2.C:** State and use Gauss's law in integral form to derive unknown electric fields for planar, spherical, or cylindrically symmetrical charge distributions.
**CNV-2.D.a:** Using appropriate mathematics (which may involve calculus), calculate the total charge contained in lines, surfaces, or volumes when given a linear-charge density, a surface-charge density, or a volume-charge density of the charge configuration.

**Science Practices**

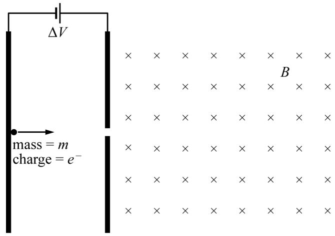
**3.C:** Sketch a graph that shows a functional relationship between two quantities.
**6.B:** Apply an appropriate law, definition, or mathematical relationship to solve a problem.
**6.C:** Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.





## Question 3

**15 points**




Two plates are set up with a potential difference $V$ between them. A small sphere of mass $m$ and charge $-e$ is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude $B$ directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of $V$, $m$, $e$, $B$, and fundamental constants, as appropriate.

(a)
i. LO CHG-1.A.a, SP 7.A
1 point

What is the initial direction of the force on the sphere as it enters the magnetic field?

____ Into the page ____ Out of the page
____ Toward the top of the page ____ Toward the bottom of the page

| | | |
|---|---|---|
| For selecting "Toward the bottom of the page" | | 1 point |

ii. LO CHG-1.B, SP 7.A
1 point

Describe the path taken by the sphere after it enters the magnetic field.

| | | |
|---|---|---|
| For describing a circular path for the sphere consistent with the selection from part (a)(i) | | 1 point |
| Example: The sphere will move in a circular path toward the bottom of the page. | | |

## Question 3 (continued)

(b) LO CNV-1.E, SP 5.A, 5.E
2 points

Derive an expression for the speed of the sphere as it passes through the small opening.

| | | |
|---|---|---|
| For using a valid equation relating potential difference to kinetic energy of the sphere | | 1 point |
| $\Delta K = -\Delta U \therefore \frac{1}{2}mv^2 = -q\Delta V$ | | |
| $\frac{1}{2}mv^2 = eV$ | | |
| For the correct answer with supporting work | | 1 point |
| $v = \sqrt{\frac{2eV}{m}}$ | | |

(c) LO CHG-1.C, SP 5.A, 5.E
3 points

Derive an expression for the radius of the path taken by the sphere as it moves through the magnetic field.

| | | |
|---|---|---|
| For an expression relating the magnetic force to the centripetal force | | 1 point |
| $\frac{mv^2}{r} = Bqv$ | | |
| For substituting the charge of the sphere into the above equation and solving for $r$ | | 1 point |
| $\frac{mv}{r} = Be \therefore r = \frac{mv}{Be}$ | | |
| For substituting the answer from part (b) | | 1 point |
| $r = \frac{m}{Be}\sqrt{\frac{2eV}{m}} = \sqrt{\frac{2mV}{eB^2}}$ | | |

An experiment is performed in which a beam of electrons is accelerated across the space between the plates and passes through the small opening. After passing through the opening, the electrons travel in a semicircular path and strike the right-hand plate. The potential difference between the plates is varied in regular increments, as shown in the table below. For each potential difference, the magnetic field is varied in order to cause the beam to strike the right-hand plate at a distance of 0.020 m from the opening.

| Potential difference (V) | 60 | 70 | 100 | 110 | 120 | 140 |
|---|---|---|---|---|---|---|
| Magnetic field $(\mathrm{T} \times 10^{-3})$ | 2.62 | 2.78 | 3.39 | 3.54 | 3.78 | 3.99 |
| | | | | | | |
| | | | | | | |





## Question 3 (continued)

(d) LO CHG-1.C, SP 4.C
1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the mass-to-charge ratio of an electron.

Vertical axis:____________
Horizontal axis:____________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

$$r = \sqrt{\frac{2m_eV}{eB^2}} \therefore B^2 = \frac{2m_e}{er^2}V$$

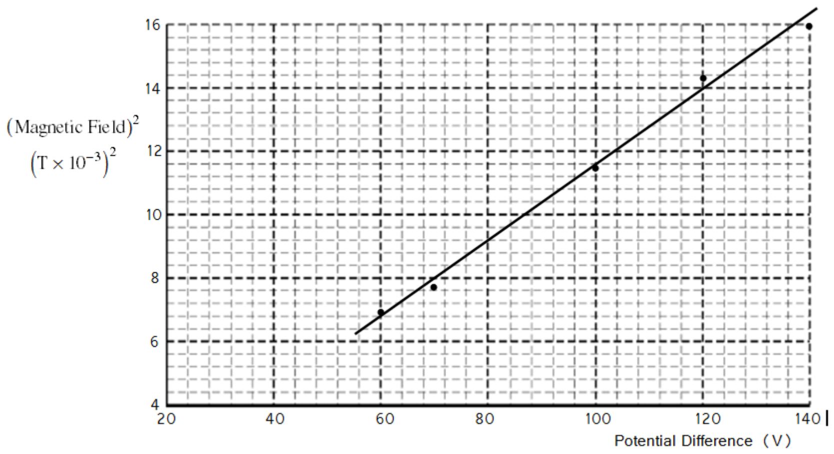
| Potential difference (V) | 60 | 70 | 100 | 110 | 120 | 140 |
|---|---|---|---|---|---|---|
| Magnetic field $(\mathrm{T} \times 10^{-3})$ | 2.62 | 2.78 | 3.39 | 3.54 | 3.78 | 3.99 |
| (Magnetic field)$^2$ $(\mathrm{T} \times 10^{-3})^2$ | 6.86 | 7.73 | 11.5 | 12.5 | 14.3 | 15.9 |
| | | | | | | |

| | | |
|---|---|---|
| For indicating axis choices with appropriate quantities to produce a straight line | | 1 point |
| <u>Note:</u> There are several possible choices. | | |

## Question 3 (continued)

(e) LO CHG-1.C, SP 3.A, 4.C
4 points

On the graph below, plot the relationship determined in part (d). Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.




| | | |
|---|---|---|
| For correctly labeling both axes with quantities and units | | 1 point |
| For correctly scaling and labeling the axes so that the points use at least half the grid | | 1 point |
| For correctly plotting the data | | 1 point |
| For drawing a straight line that represents the data | | 1 point |

(f) LO CHG-1.C, SP 4.D, 5.E
3 points

Using the straight line from part (e), determine the mass-to-charge ratio of an electron.

| | | |
|---|---|---|
| For correctly calculating the slope from the best-fit straight line and not from the data points unless the points fall on the best-fit line | | 1 point |
| slope $= \frac{14-9.2}{120-80} = 0.12 \frac{\left(\text{T}\times 10^{-3}\right)^2}{V} = 1.2\times 10^{-7}\ \text{T}^2/\text{V}$ | | |
| For a correct expression relating the slope to the mass to charge ratio of an electron | | 1 point |
| $B^2 = \frac{2m_e}{er^2}V \therefore \text{slope} = \frac{2m_e}{er^2} \therefore \frac{m_e}{e} = \frac{r^2 \times \text{slope}}{2}$ | | |
| For substituting correct values into the above equation | | 1 point |
| $\frac{m_e}{e} = \frac{(0.01\ \text{m})^2 \times \left(1.2\times 10^{-7}\ \text{T}^2/\text{V}\right)}{2} = 6.0\times 10^{-12}\ \text{kg/C}$ | | |





## Question 3 (continued)

**Learning Objectives**

**CHG-1.A.a:** Calculate the magnitude and direction of the magnetic force of interaction between a moving charged particle of specified charge and velocity moving in a region of a uniform magnetic field.
**CHG-1.B:** Describe the path of different moving charged particles (i.e., of different type of charge or mass) in a uniform magnetic field.
**CHG-1.C:** Derive an expression for the radius of a circular path for a charged particle of specified characteristics moving in a specified magnetic field.
**CNV-1.E:** Calculate the work done or changes in kinetic energy (or changes in speed) of a charged particle when it is moved through some known potential difference.

**Science Practices**

**3.A:** Select and plot appropriate data.
**4.C:** Linearize data and/or determine a best-fit line or curve.
**4.D:** Select relevant features of a graph to describe a physical situation or solve problems.
**5.A:** Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
**5.E:** Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
**7.A:** Make a scientific claim.