**Question 2: Version J**

**2.** A rotating, circular, conducting loop of area $A$ and resistance $R$ is in an external uniform magnetic field of magnitude $B$ that is directed in the $-z$-direction. At time $t = 0$, the magnetic field is perpendicular to the plane of the loop, as shown in Figure 1. The loop is rotating with constant angular speed $\omega$ and period $T$ about the dashed line that is along the diameter of the loop. The value of the magnetic flux through the loop as a function of time $t$ is $\Phi = BA\cos(\omega t)$.

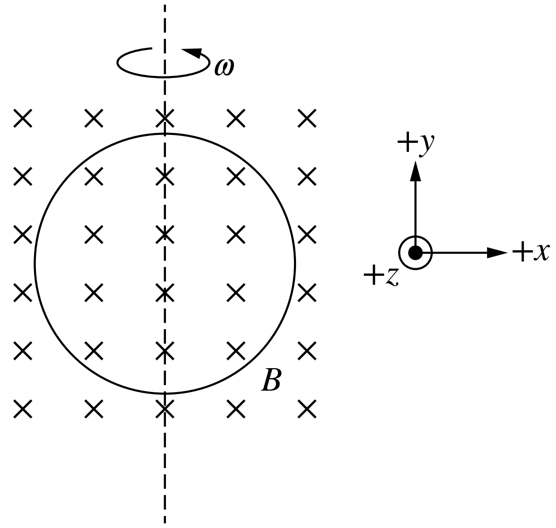



Figure 1

**A.** The absolute value of the induced emf in the loop is $|\varepsilon|$. The partially completed bar chart in Figure 2 shows a bar that represents $|\varepsilon|$ at $t = \frac{3}{4}T$. In Figure 2, **draw** bars to represent $|\varepsilon|$ at times $t = 0$, $\frac{1}{4}T$, and $\frac{1}{2}T$ relative to $|\varepsilon|$ shown at $\frac{3}{4}T$. If $|\varepsilon| = 0$, **write** a “0” in that column.

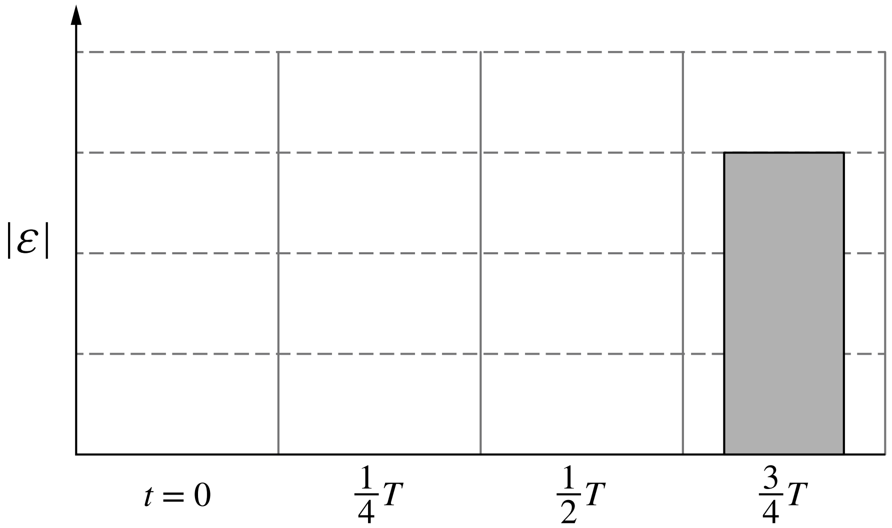



Figure 2

**B. Derive** an expression for the maximum induced current in the loop in terms of $A$, $R$, $B$, $\omega$, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

**C.** On the axes shown in Figure 3, **sketch** a graph of the instantaneous power $P$ dissipated by the loop as a function of $t$ during the time interval $0 \leq t \leq T$.

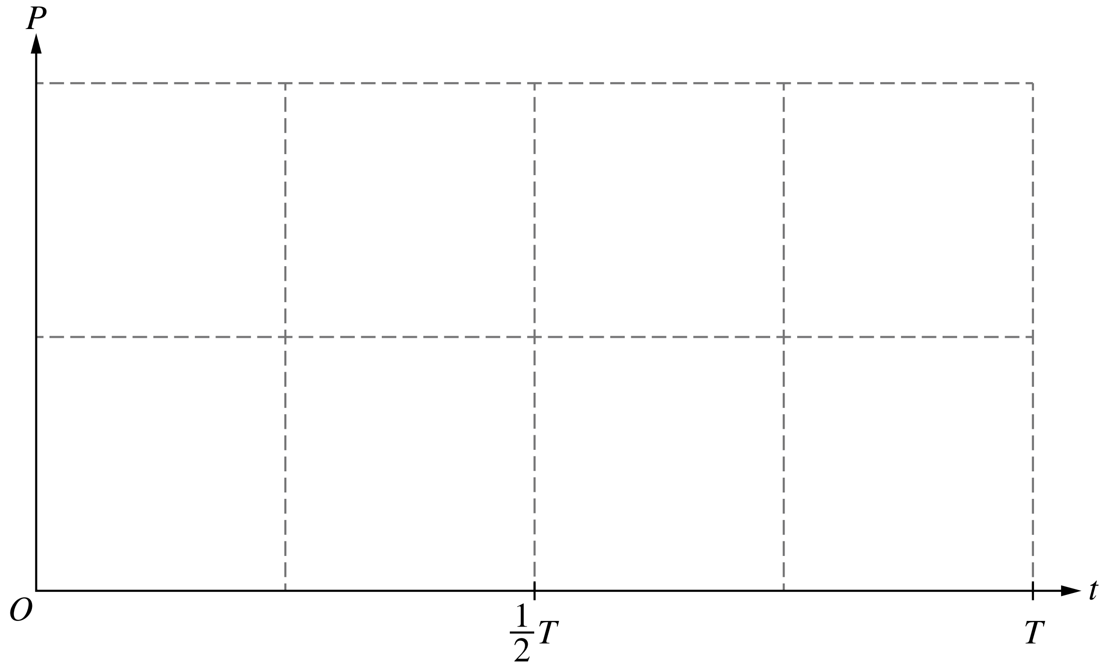



Figure 3

**D. Indicate** whether the sketch you drew in part C is or is not consistent with the bars that you drew in part A. Briefly **justify** your answer by referencing the functional dependence between $P$ and $|\varepsilon|$.

Begin your response to **QUESTION 3** on this page.

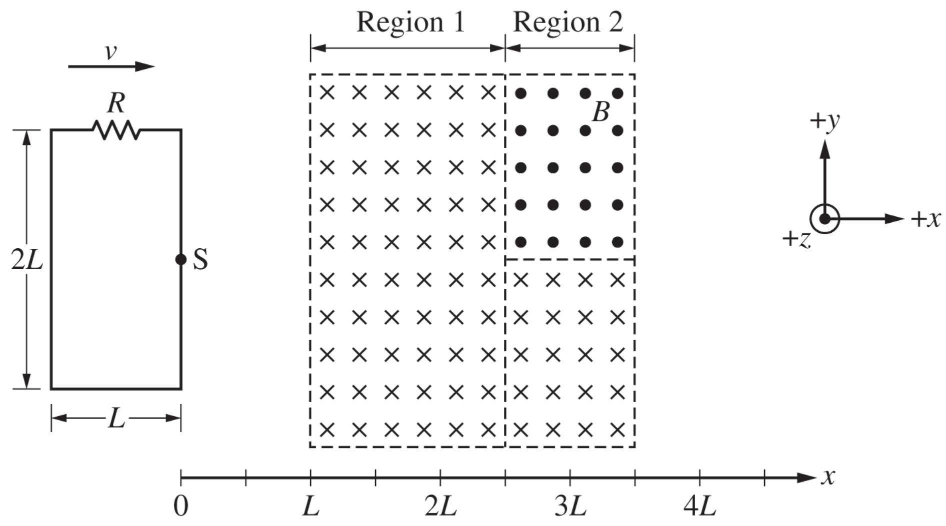


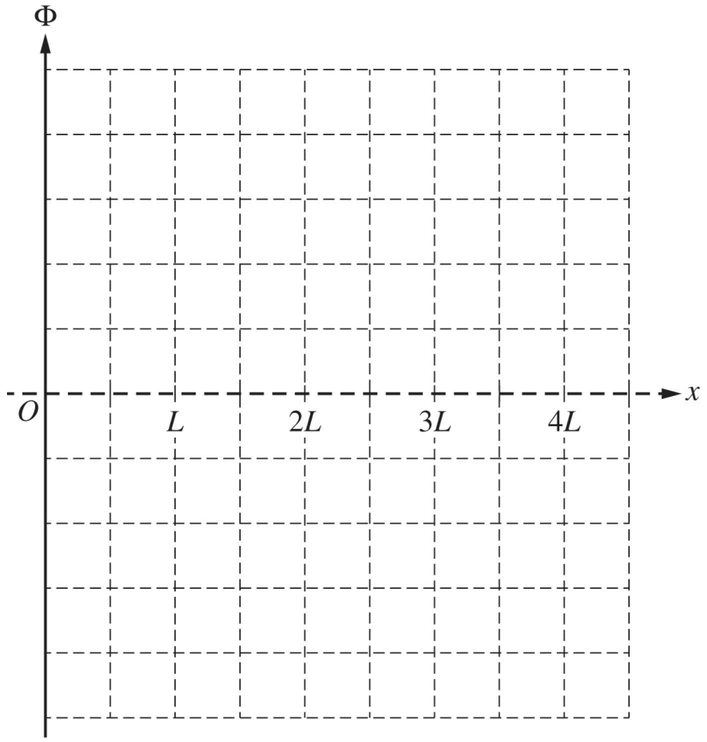

Figure 1

3. A wire is connected to a resistor of resistance $R$ to form a rigid rectangular loop of width $L$ and height $2L$. An external force is exerted on the loop so that the loop always moves with constant speed $v$ in the $+x$-direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude $B$ that is directed in the $-z$-direction. Region 1 has boundaries $x = L$ and $x = 2.5L$. The loop later enters Region 2 with two external, uniform magnetic fields, each of magnitude $B$, that are parallel but are directed in opposite $z$-directions. Region 2 has boundaries $x = 2.5L$ and $x = 3.5L$. Point S is the midpoint of the leading edge of the loop and is aligned with the horizontal boundary in Region 2 that separates the two magnetic fields.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 3** on this page.

(a) On the following axes, **sketch** a graph of the magnetic flux $\Phi$ through the rectangular loop as a function of the position $x$ of Point S from $x = 0$ to $x = 4.5L$. The $+z$-direction indicated in Figure 1 corresponds to $+\Phi$.




(b) Consider the instant when Point S reaches $x = 1.5L$.

i. **Indicate** whether the current $I_R$ that is induced in the rectangular loop when Point S reaches $x = 1.5L$ is clockwise, counterclockwise, or zero.

_____ Clockwise _____ Counterclockwise _____ Zero

Briefly **justify** your answer.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 3** on this page.

ii. **Derive** an expression for $I_R$ when Point S reaches $x = 1.5L$. If $I_R = 0$, indicate how the derived expression shows that $I_R = 0$. Express your answer in terms of $R$, $L$, $v$, $B$, and physical constants, as appropriate.

iii. **Derive** an expression for the power $P$ dissipated by the resistor when Point S reaches $x = 1.5L$. Express your answer in terms of $R$, $L$, $v$, $B$, and physical constants, as appropriate.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 3** on this page.

The total energy dissipated by the resistor in the rectangular loop as Point S moves from $x = 0$ to $x = 4.5L$ is $E_{\text{original}}$.

The vertical boundary between regions 1 and 2 is now shifted to $x = 1.5L$. After the boundary is shifted, the rectangular loop again moves with speed $v$ in the $+x$-direction, as shown in Figure 2. The total energy dissipated by the resistor as Point S moves from $x = 0$ to $x = 4.5L$ is $E_{\text{new}}$.

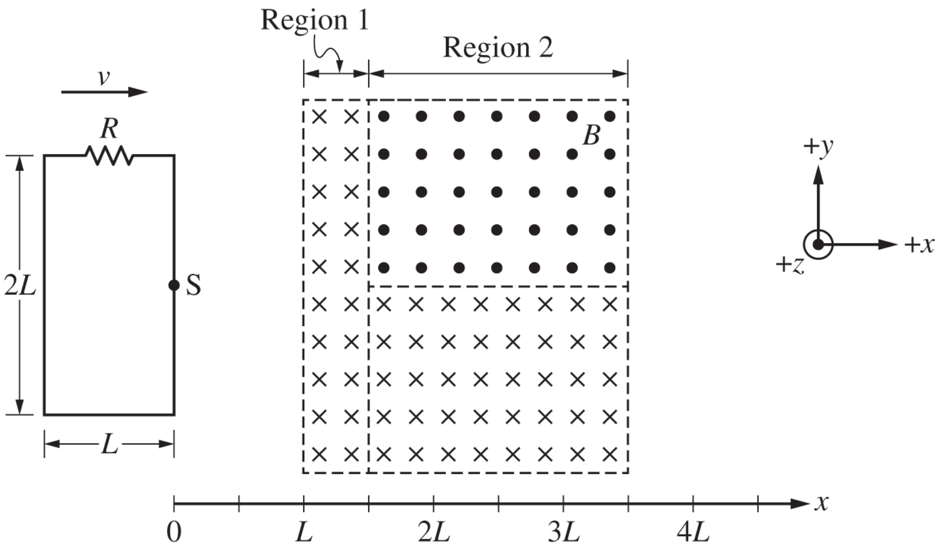



Figure 2

(c) **Indicate** whether $E_{\text{new}}$ is greater than, less than, or equal to $E_{\text{original}}$.

______ $E_{\text{new}} > E_{\text{original}}$ ______ $E_{\text{new}} < E_{\text{original}}$ ______ $E_{\text{new}} = E_{\text{original}}$

Briefly **justify** your answer.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 3** on this page.

The original magnetic fields are modified so that the region $L < x < 3.5L$ contains an external uniform magnetic field of magnitude $B$ that is directed in the $-z$-direction.

A new wire is connected to a resistor of resistance $R$ to form a rigid triangular loop with base length $L$ and height $2L$. An external force is exerted on the loop so that the loop always moves with speed $v$ in the $+x$-direction, as shown in Figure 3. Point S represents the lower-leading corner of the loop.

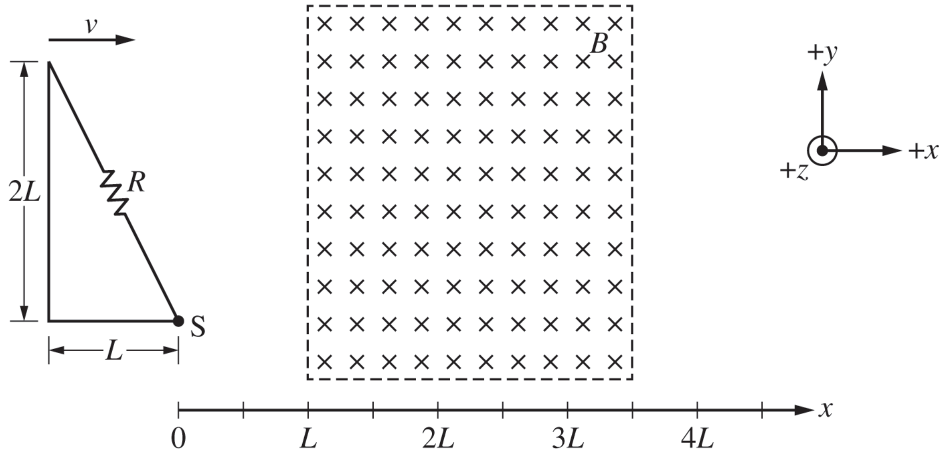
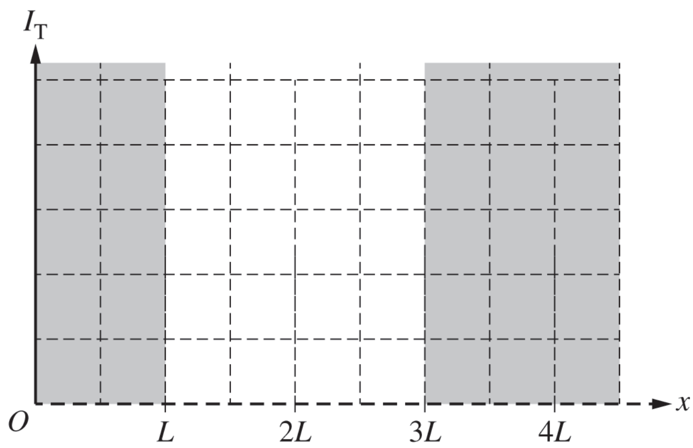



Figure 3

(d) On the following axes, **sketch** a graph of the induced current $I_T$ in the triangular loop as Point S moves from $x = L$ to $x = 3L$.

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**END OF EXAM**

Begin your response to **QUESTION 3** on this page.

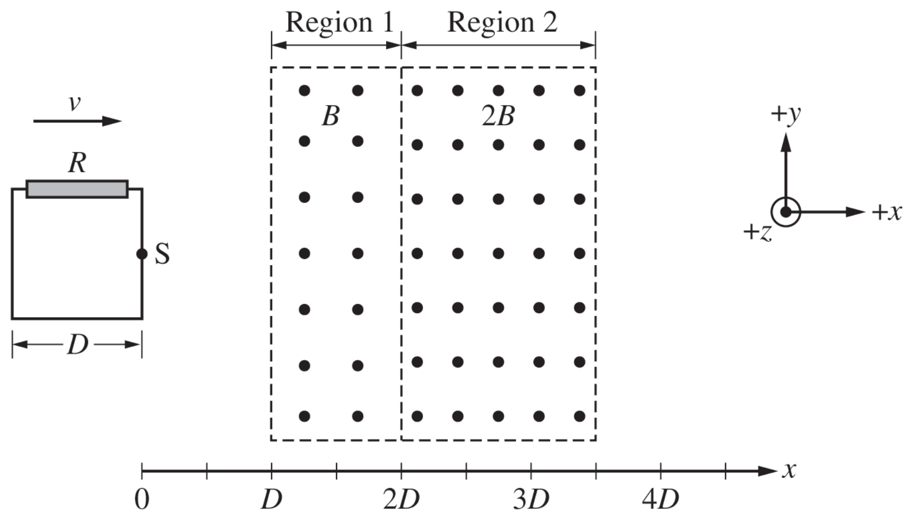


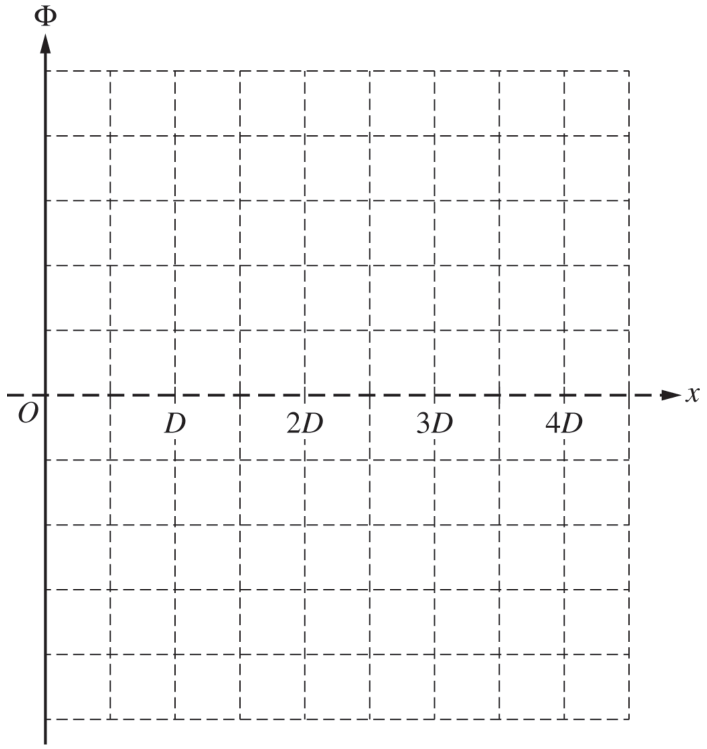

Figure 1

3. A wire is connected to a resistor of resistance $R$ to form a rigid square loop of side length $D$. An external force is exerted on the loop so that the loop always moves with constant speed $v$ in the $+x$ direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude $B$ that is directed in the $+z$-direction. Region 1 has boundaries $x = D$ and $x = 2D$. The loop later enters Region 2 of external uniform magnetic field of magnitude $2B$ that is directed in the $+z$-direction. Region 2 has boundaries $x = 2D$ and $x = 3.5D$. Point S is the midpoint of the leading edge of the loop.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 3** on this page.

(a) On the following axes, **sketch** a graph of the magnetic flux $\Phi$ through the square loop as a function of the position $x$ of Point S from $x = 0$ to $x = 4.5D$. The $+z$-direction indicated in Figure 1 corresponds to $+\Phi$.




(b) Consider the instant when Point S reaches $x = 1.5D$.

i. **Indicate** whether the current $I_S$ that is induced in the square loop when Point S reaches $x = 1.5D$ is clockwise, counterclockwise, or zero.

_____ Clockwise _____ Counterclockwise _____ Zero

Briefly **justify** your answer.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 3** on this page.

ii. **Derive** an expression for $I_S$ when Point S reaches $x = 1.5D$. If $I_S = 0$, indicate how the derived expression shows that $I_S = 0$. Express your answer in terms of $R$, $D$, $v$, $B$, and physical constants, as appropriate.

iii. **Derive** an expression for the power $P$ dissipated by the resistor when Point S reaches $x = 1.5D$. Express your answer in terms of $R$, $D$, $v$, $B$, and physical constants, as appropriate.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 3** on this page.

The total energy dissipated by the resistor in the square loop as Point S moves from $x = 0$ to $x = 4.5D$ is $E_{\text{original}}$.

The vertical boundary between regions 1 and 2 is now shifted to $x = 2.5D$. After the boundary is shifted, the square loop again moves with speed $v$ in the $+x$-direction, as shown in Figure 2. The total energy dissipated by the resistor as Point S moves from $x = 0$ to $x = 4.5D$ is $E_{\text{new}}$.

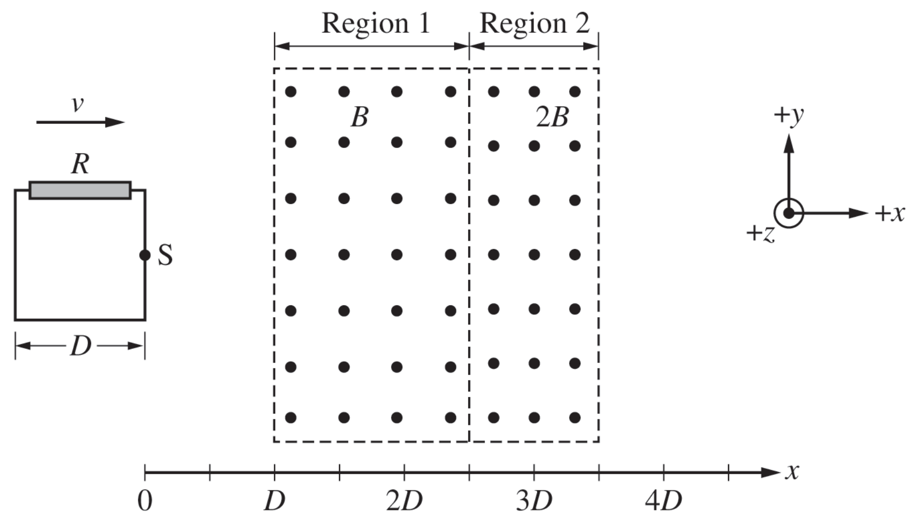



Figure 2

(c) **Indicate** whether $E_{\text{new}}$ is greater than, less than, or equal to $E_{\text{original}}$.

_____ $E_{\text{new}} > E_{\text{original}}$ _____ $E_{\text{new}} < E_{\text{original}}$ _____ $E_{\text{new}} = E_{\text{original}}$

Briefly **justify** your answer.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 3** on this page.

The original magnetic fields are modified so that the region $D < x < 3.5D$ contains an external uniform magnetic field of magnitude $B$ that is directed in the $+z$-direction.

A new wire is connected to a resistor of resistance $R$ to form a rigid triangular loop with base length $D$ and height $D$. An external force is exerted on the loop so that the loop always moves with speed $v$ in the $+x$-direction, as shown in Figure 3. Point S represents the upper-leading corner of the loop.

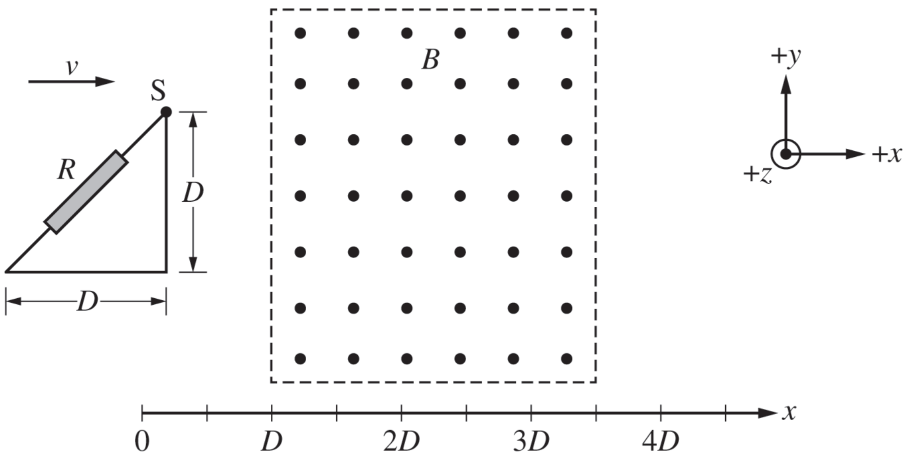
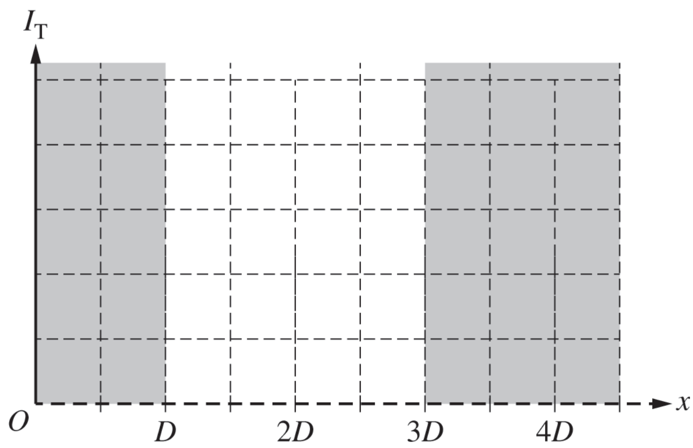



Figure 3

(d) On the following axes, **sketch** a graph of the induced current $I_T$ in the loop as Point S moves from $x = D$ to $x = 3D$.



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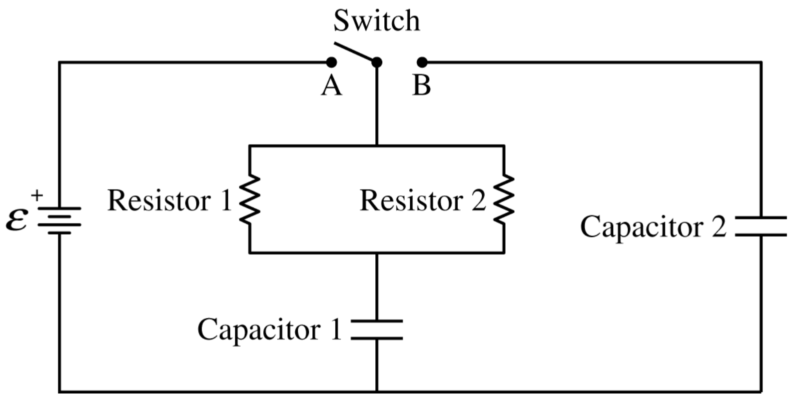


. The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance $R$, capacitors 1 and 2 with capacitances $C$ and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

(a) Write, but do NOT solve, a differential equation that can be used to determine the charge $Q$ on the positive plate of Capacitor 1 as a function of time $t$ after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, $R$, $C$, $Q$, $t$, and fundamental constants, as appropriate.

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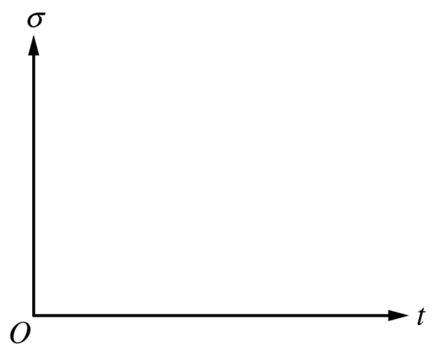
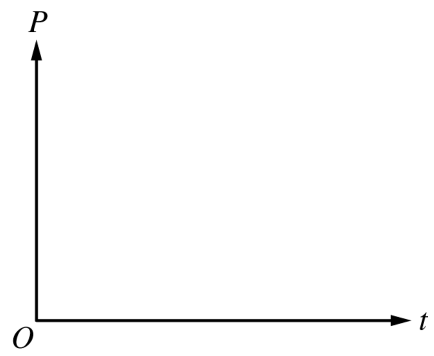
Continue your response to **QUESTION 3** on this page.

(b) On the axes shown, sketch graphs of the surface charge density $\sigma$ on the positive plate of Capacitor 1 and the total power $P$ dissipated by the resistors as functions of time $t$ from time $t = 0$ until steady-state conditions are nearly reached.




A long time after the switch is closed to Position A, the charge on the positive plate of Capacitor 1 is $Q_0$ and Capacitor 2 is uncharged.

(c) At time $t_1$, the switch is closed to Position B.

i. Immediately after time $t_1$, is the direction of the current in the switch directed toward the left, directed toward the right, or is there no current? Briefly justify your answer.

ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after $t_1$. Express your answer in terms of $Q_0$ and fundamental constants, as appropriate.

**GO ON TO THE NEXT PAGE.**

iii. Derive an expression for the total energy $E_R$ dissipated by resistors 1 and 2 from immediately after time $t_1$ until new steady-state conditions have been reached. Express your answer in terms of $C$, $Q_0$, and fundamental constants, as appropriate.

With the switch still closed to Position B, the parallel plates of Capacitor 2 are moved so that the separation distance increases by a factor of 2.

(d) Determine the ratio $\frac{U_2}{U_1}$ of the energy $U_2$ stored in Capacitor 2 to the energy $U_1$ stored in Capacitor 1 a long time after the plates of Capacitor 2 have been moved. Briefly justify your answer.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 3** on this page.

With the capacitors still charged as in part (d), the switch is now closed to Position A.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of $R$, $C$, $Q_0$, and fundamental constants, as appropriate.

i. Derive an expression for the current $I_0$ from the battery immediately after the switch is closed to Position A.

ii. Determine the current $I_\infty$ from the battery a long time after the switch is closed to Position A.

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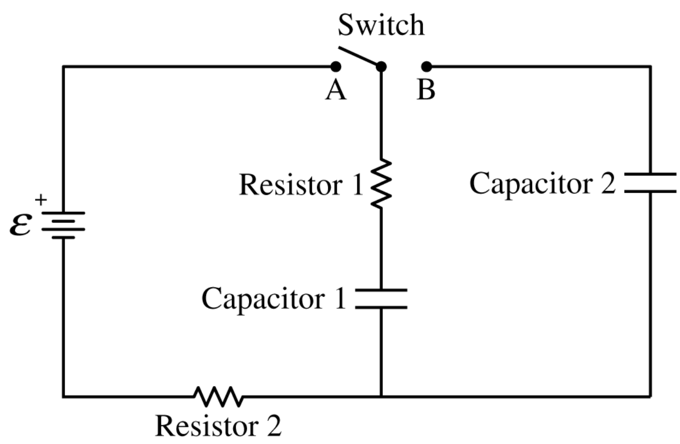


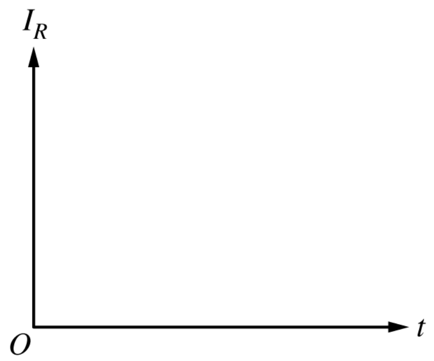

. The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance $R$, capacitors 1 and 2 each with capacitance $C$, and a switch. The switch is initially open and both capacitors are uncharged.

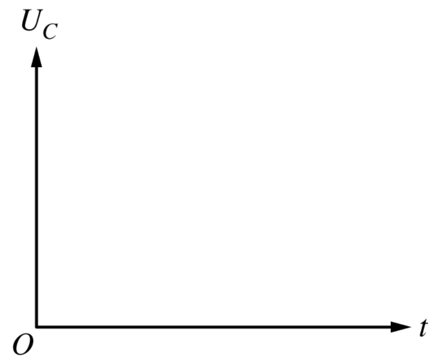
At time $t = 0$, the switch is closed to Position A.

(a) Write, but do NOT solve, a differential equation that can be used to determine the charge $Q$ on the positive plate of Capacitor 1 as a function of time $t$ after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, $R$, $C$, $Q$, $t$, and fundamental constants, as appropriate.

**GO ON TO THE NEXT PAGE.**

(b) On the axes shown, sketch graphs of the current $I_R$ in Resistor 1 and the energy $U_C$ stored in Capacitor 1 as functions of time $t$ from time $t = 0$ until steady-state conditions are nearly reached.




A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is $Q_0$ and Capacitor 2 is uncharged.

(c) At time $t_1$, the switch is closed to Position B.

i. Immediately after $t_1$, is the direction of the current in Resistor 1 directed up, directed down, or is there no current? Briefly justify your answer.

ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after $t_1$. Express your answer in terms of $Q_0$ and fundamental constants, as appropriate.

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Continue your response to **QUESTION 3** on this page.

iii. Derive an expression for the total energy dissipated by Resistor 1 immediately after time $t_1$ until new steady-state conditions have been reached. Express your answer in terms of $C$, $Q_0$, and fundamental constants, as appropriate.

With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

(d) Determine the charge on the positive plate of Capacitor 2 a long time after the dielectric has been inserted. Express your answer in terms of $Q_0$ and fundamental constants, as appropriate.

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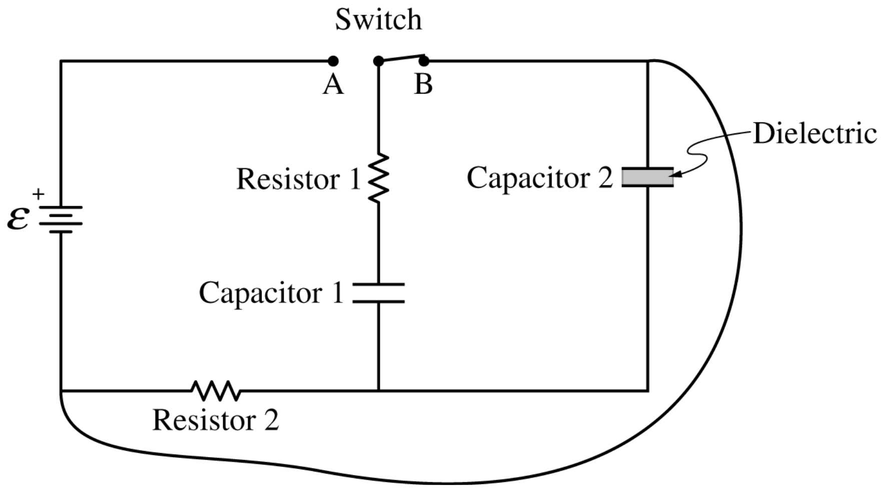
Continue your response to **QUESTION 3** on this page.




With the switch still closed to Position B, a wire of negligible resistance is connected between two corners of the circuit, as shown.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of $R$, $C$, $Q_0$, and fundamental constants, as appropriate.

i. Derive an expression for the current in Resistor 2 immediately after the wire is connected to the circuit.

ii. Determine the current in Resistor 2 a long time after the wire is connected to the circuit.

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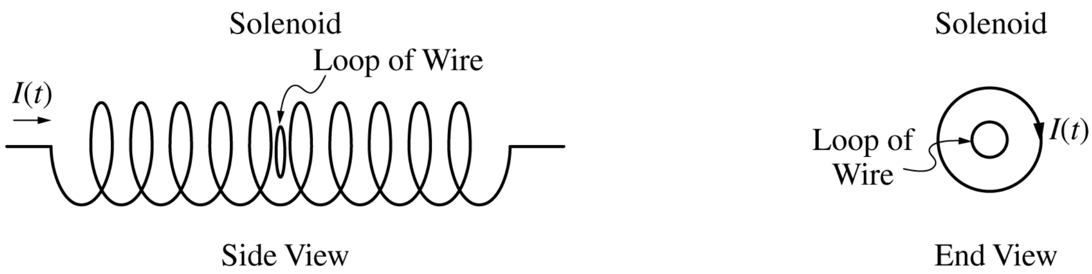


<u>Note:</u> Figures not drawn to scale.

. A single loop of wire with resistance $3.0\,\Omega$ and radius $0.10$ m is placed inside a solenoid, with the normal to the loop parallel to the axis of the solenoid. The solenoid has 500 turns, is 0.25 m long, and is connected to a power supply that is not shown. At time $t = 0$, the power supply is turned on, and the current $I$ in the solenoid as a function of $t$ is given by the equation $I(t) = \beta t$, where $\beta = 5.0$ A/s. The direction of the current in the solenoid is clockwise, as shown in the end view.

(a) At time $t = 2.0$ s, is the induced current in the loop, as seen from the end view shown, clockwise, counterclockwise, or zero?

___ Clockwise ___ Counterclockwise ___ Zero

Justify your answer.

(b) Calculate the current in the <u>loop of wire</u> at time $t = 2.0$ s.

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Continue your response to **QUESTION 3** on this page.

(c) Calculate the total energy dissipated by the loop of wire from time $t = 0$ to time $t = 2.0\text{ s}$.

(d) A group of students attempts to verify experimentally the calculation of the current from part (b). The current in the inner circular loop at time $t = 2.0\text{ s}$ is measured to be less than the current calculated in part (b). Which of the following could explain this discrepancy? Select one answer.

___ The experiment did not account for Earth's magnetic field.
___ The plane of the loop is not perpendicular to the axis of the solenoid.
___ The center of the loop is not on the axis of the solenoid.
___ The resistance of the loop is less than the given value.
___ The radius of the loop is actually larger than $0.10\text{ m}$.

Justify your answer.

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Continue your response to **QUESTION 3** on this page.

(e) The power supply is now turned off. The original loop of wire is then replaced with a second loop made from wire that has the same thickness and is made from the same material as the original loop of wire. The second loop has radius 0.20 m, is placed in the same orientation as the original loop, and fits completely inside the solenoid. The power supply is turned on, and the current $I$ in the solenoid as a function of $t$ is again given by the equation $I(t) = \beta t$, where $\beta = 5.0$ A/s. Which of the following expressions correctly indicates the ratio $\frac{I_2}{I_1}$ where $I_1$ represents the current induced in the original loop of wire in part (b) and $I_2$ represents the current induced in the second loop of wire?

___ $\frac{I_2}{I_1} = 1$ ___ $1 < \frac{I_2}{I_1} < 2$ ___ $\frac{I_2}{I_1} = 2$ ___ $\frac{I_2}{I_1} > 2$

Justify your answer.

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**END OF EXAM**

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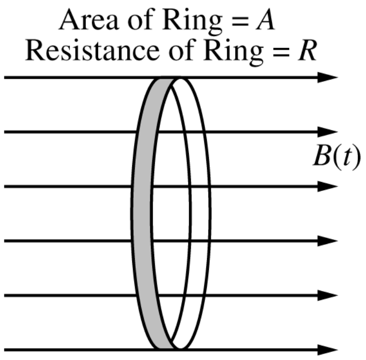


3. A thin, conducting ring of area $A$ and resistance $R$ is aligned in a uniform magnetic field directed to the right and perpendicular to the plane of the ring, as shown. At time $t = 0$, the magnitude of the magnetic field is $B_0$. At $t = 1\text{ s}$, the magnitude of the magnetic field begins to decrease according to the equation $B(t) = \dfrac{\beta}{t}$, where $\beta$ has units of $\text{T}\cdot\text{s}$.

(a) Derive an equation for the magnitude of the induced current $I$ in the ring as a function of $t$ for $t > 1\text{ s}$. Express your answer in terms of $\beta$, $A$, $R$, $t$, and physical constants, as appropriate.

Assume $A = 0.50\text{ m}^2$, $R = 2.0\,\Omega$, and $\beta = 0.50\text{ T}\cdot\text{s}$.

(b) Calculate the electrical energy dissipated in the ring from $t = 1\text{ s}$ to $t = 2\text{ s}$.

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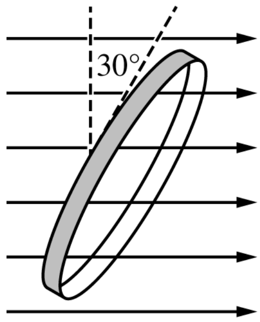


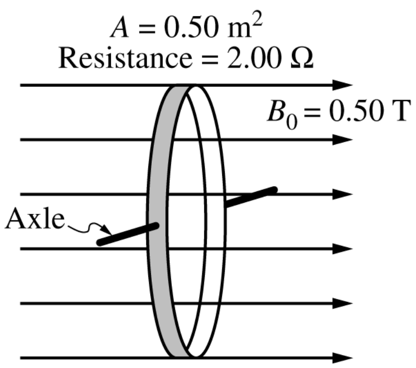

The ring is then rotated so that the plane of the ring is aligned at a $30°$ angle to the magnetic field, as shown.

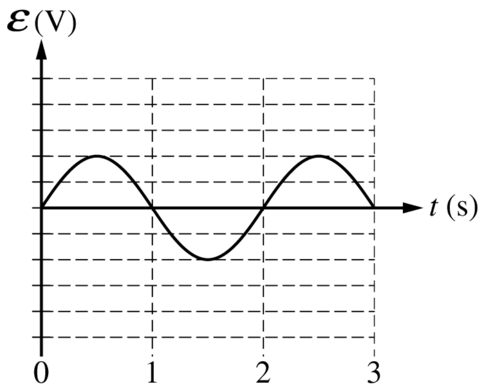
The magnitude of the magnetic field is reset to a magnitude of $B_0$ at a new time $t = 0$ and again begins to decrease at $t = 1\text{ s}$ according to the equation $B(t) = \dfrac{\beta}{t}$, where $\beta$ has units of T·s.

(c) Will the amount of energy dissipated in the ring from $t = 1\text{ s}$ to $t = 2\text{ s}$ be greater than, less than, or equal to the energy dissipated in part (b) ?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.




The ring is now mounted on an axle that is perpendicular to the magnetic field. The magnitude of the magnetic field is now held at a constant $B_0 = 0.50\text{ T}$, as shown. The ring rotates about the axle, and the emf $\varepsilon$ induced in the ring as a function of time $t$ is shown on the graph.

(d) Calculate the angular speed $\omega$ of the rotating ring in rad/s.

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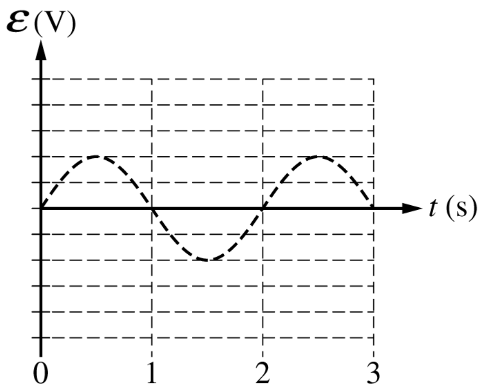
(e) Calculate the magnitude of the maximum emf $\varepsilon_{MAX}$ induced in the ring.

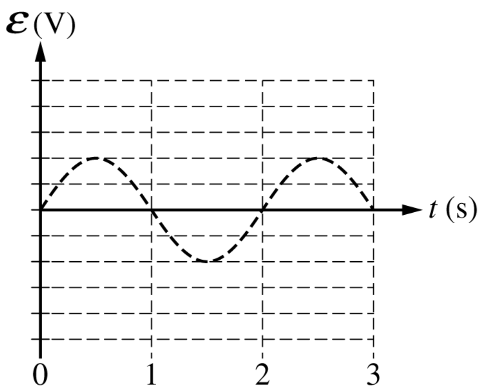
The ring now begins to rotate at an angular speed $2\omega$.

(f) On the graph below, draw a curve to indicate the new induced emf $\varepsilon$ in the ring. The dashed curve shows the emf induced under the original conditions.




Justify your sketch, specifically identifying and addressing any similarities or differences between the sketch and the original graph.

PRACTICE GRAPH - Use the graph below to practice your sketch for part (f). Any work shown on the graph below will NOT be graded.




**GO ON TO THE NEXT PAGE.**

**STOP**

**END OF EXAM**

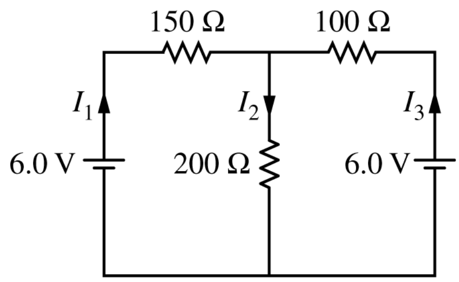



Figure 1

2. The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents $I_1$, $I_2$, and $I_3$ in each branch of the circuit are indicated.

(a)

i. Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

ii. Calculate the current in the $200\ \Omega$ resistor.

iii. Calculate the power dissipated by the $200\ \Omega$ resistor.

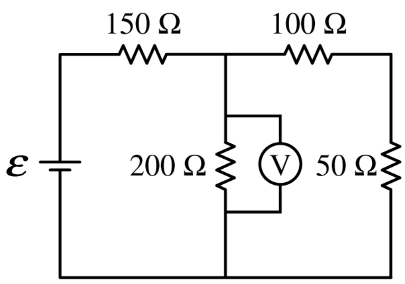



Figure 2

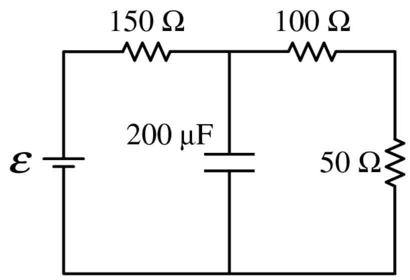
The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

(b) Calculate the current through the $50\ \Omega$ resistor.

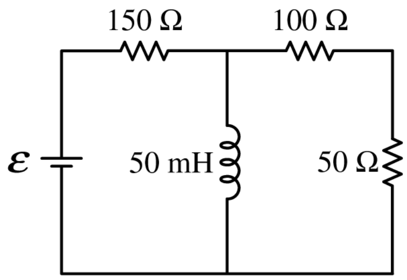
(c) Calculate the voltage $\mathcal{E}$ of the battery.

(d)

i. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu F$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.

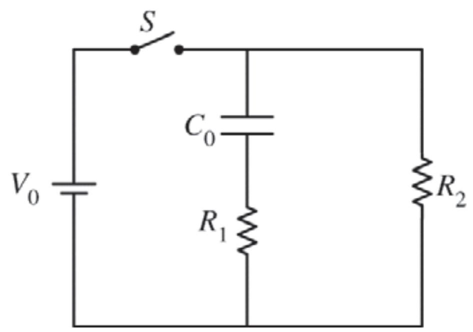



ii. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b) ?



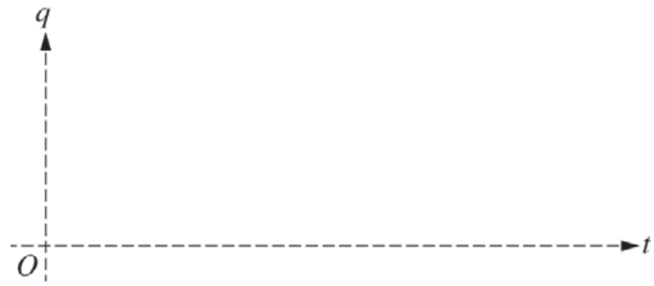

____ Greater than ____ Less than ____ Equal to

Justify your answer.

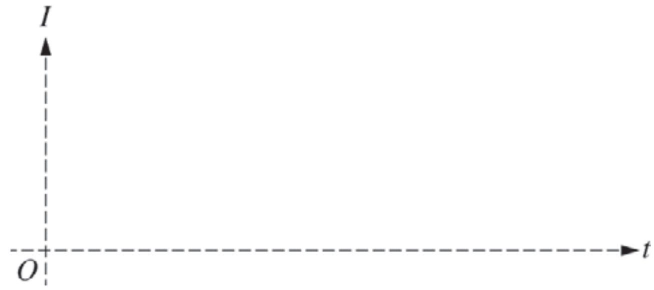



2. In the circuit above, an ideal battery of voltage $V_0$ is connected to a capacitor with capacitance $C_0$ and resistors with resistances $R_1$ and $R_2$, with $R_1 > R_2$. The switch $S$ is open, and the capacitor is initially uncharged.

(a) The switch is closed at time $t = 0$. On the axes below, sketch the charge $q$ on the capacitor as a function of time $t$. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.




(b) On the axes below, sketch the current $I$ through each resistor as a function of time $t$. Clearly label the two curves as $I_1$ and $I_2$, the currents through resistors $R_1$ and $R_2$, respectively. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.




The circuit is constructed using an ideal 1.5 V battery, an 80 $\mu$F capacitor, and resistors $R_1 = 150\ \Omega$ and $R_2 = 100\ \Omega$. The switch is closed, allowing the capacitor to fully charge. The switch is then opened, allowing the capacitor to discharge.

(c) The time it takes to charge the capacitor to 50% of its maximum charge is $\Delta t_C$. The time it takes for the capacitor to discharge to 50% of its maximum charge is $\Delta t_D$. Which of the following correctly relates the two time intervals?

____ $\Delta t_C > \Delta t_D$ ____ $\Delta t_C = \Delta t_D$ ____ $\Delta t_C < \Delta t_D$

Justify your answer.

(d)

i. Calculate the current through resistor $R_2$ immediately after the switch is opened.

ii. Is the current through resistor $R_2$ increasing, decreasing, or constant immediately after the switch is opened?

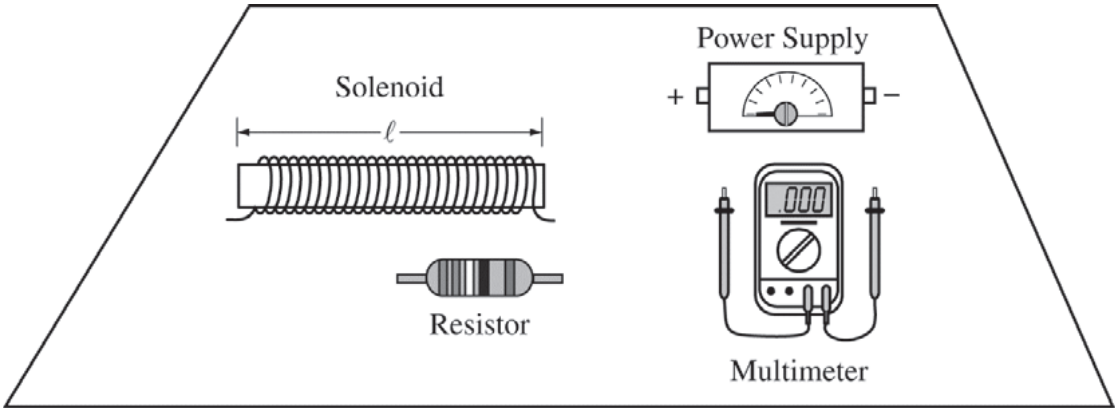
____ Increasing ____ Decreasing ____ Constant

Justify your answer.

(e)

i. Calculate the energy stored in the capacitor immediately after the switch is opened.

ii. Calculate the energy dissipated by resistor $R_1$ as the capacitor completely discharges.




3. When studying Ampere's law, students collect data on the magnetic field of two different solenoids in order to determine the magnetic permeability of free space $\mu_0$. The solenoids are created by wrapping wire around a hollow plastic tube. The solenoids of length $\ell$ with $N$ turns of wire will be connected in series to a power supply and resistor. A multimeter will be used as an ammeter to measure the magnitude of the current $I$ through the solenoids. The main components for the setup with one of the solenoids are shown in the figure above.

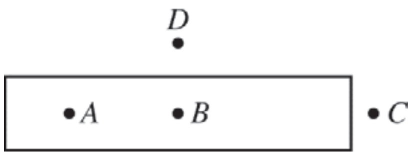
(a)

i. On the figure above, draw wire connections between the solenoid, power supply, resistor, and multimeter that will complete the circuit and allow students to measure the magnitude of the current through the solenoid.

ii. Using the connections you made in part (a)i above, what will be the direction of the magnetic field inside the solenoid?

| | | |
|---|---|---|
| ____ Toward the top of the page | _____ To the left | ____ Out of the page |
| ____ Toward the bottom of the page | _____ To the right | ____ Into the page |

The rectangle shown below represents the solenoid (the loops of wire are not shown). Points $A$, $B$, and $C$ are along the central axis of the solenoid with point $B$ at the middle of the solenoid. Point $D$ is directly above point $B$.

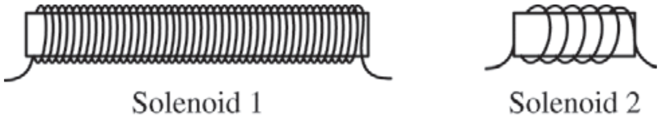



iii. From the choices below, select the point where you would place a magnetic field probe (a probe that can measure the magnitude of the magnetic field) to best measure the strength of the magnetic field of the solenoid in order to determine the magnetic permeability of free space $\mu_0$.

____ $A$ ____ $B$ ____ $C$ ____ $D$

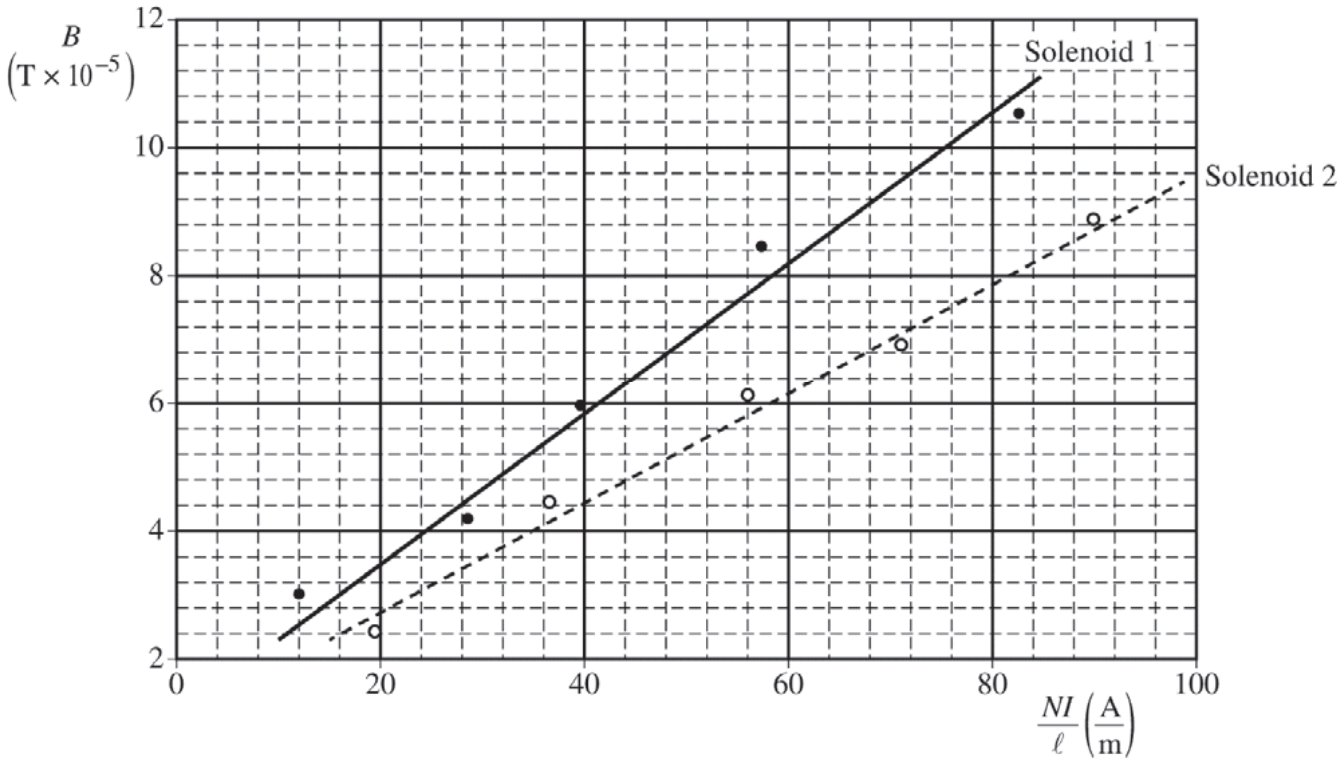
Justify your answer based on the model for a simple solenoid.

The figures below show two different solenoids that will be connected in the circuit above. Solenoid 1 has a length $\ell = 25$ cm with $N = 100$ turns. Solenoid 2 has a length $\ell = 5.0$ cm with $N = 5$ turns.




Note: Figures not drawn to scale.

A graph of the magnitude of the magnetic field $B$ as a function of $NI/\ell$ is shown below. The best-fit lines for the data are shown as a solid line for solenoid 1 and as a dashed line for solenoid 2.




(b) Which solenoid's best-fit line would give the best results for determining a value for the magnetic permeability of free space $\mu_0$?

____ Solenoid 1 ____ Solenoid 2

Justify your answer.

(c)

i. Use the slope of the best-fit line for the solenoid chosen in part (b) to calculate the magnetic permeability of free space $\mu_0$.

ii. Calculate the percent error for the experimental value of the magnetic permeability of free space $\mu_0$ determined in part (c)i.

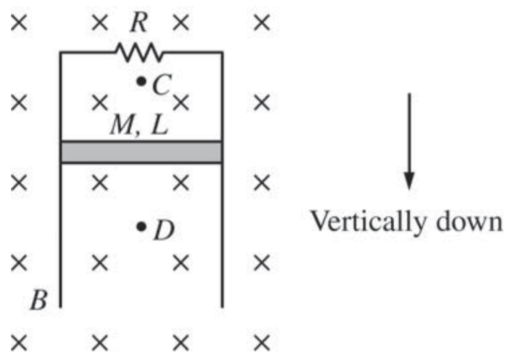
(d)

i. What is a reasonable physical explanation for a best-fit line that does not pass through the origin?

ii. Suppose a student connects the solenoid in a closed circuit similar to the circuit in part (a)i but without the resistor. The student notices the multimeter stops functioning after the power supply is turned on. Explain what causes the failure of the multimeter.

**STOP**

**END OF EXAM**




E&M.3.

A conducting bar of mass $M$, length $L$, and negligible resistance is connected to two long vertical conducting rails of negligible resistance. The two rails are connected by a resistor of resistance $R$ at the top. The entire apparatus is located in a magnetic field of magnitude $B$ directed into the page, as shown in the figure above. The bar is released from rest and slides without friction down the rails.

(a) What is the direction of the current in the resistor?

____ Left ____ Right

(b)

i. Is the magnitude of the net magnetic field above the bar at point $C$ greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

ii. While the bar is above point D, is the magnitude of the net magnetic field at point $D$ greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Express your answers to parts (c) and (d) in terms of $M$, $L$, $R$, $B$, and physical constants, as appropriate.

(c) Write, but do NOT solve, a differential equation that could be used to determine the velocity of the falling bar as a function of time $t$.

(d) Determine an expression for the terminal velocity $v_T$ of the bar.

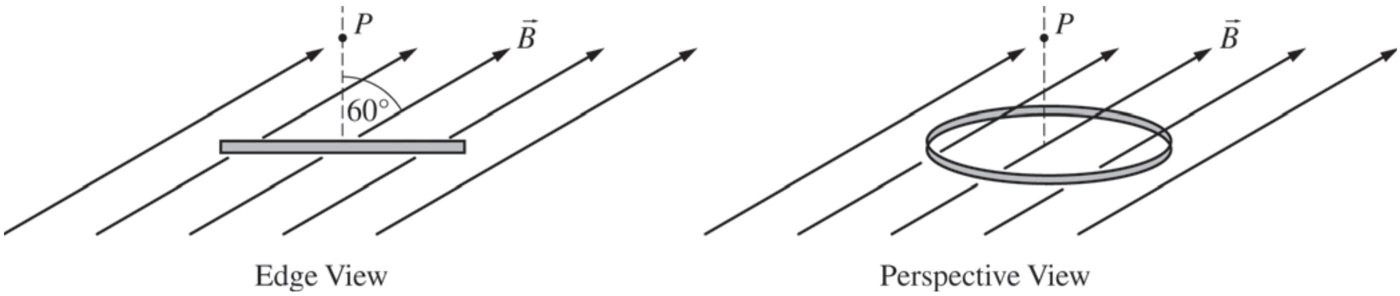
Express your answers to parts (e) and (f) in terms of $v_T$, $M$, $L$, $R$, $B$, and physical constants, as appropriate.

(e) Derive an expression for the power dissipated in the resistor when the bar is falling at terminal velocity.

(f) Using your differential equation from part (c), derive an expression for the speed of the falling bar $v(t)$ as a function of time $t$.

**STOP**

**END OF EXAM**




E&M. 3.

A circular wire loop with radius 0.10 m and resistance 50 Ω is held in place horizontally in a magnetic field $\vec{B}$ directed upward at an angle of 60° with the vertical, as shown in the figure above. The magnetic field in the direction shown is given as a function of time $t$ by $B(t) = a(1 - bt)$, where $a = 4.0$ T and $b = 0.20\ s^{-1}$.

(a) Derive an expression for the magnetic flux through the loop as a function of time $t$.

(b) Calculate the numerical value of the induced emf in the loop.

(c)

i. Calculate the numerical value of the induced current in the loop.

ii. What is the direction of the induced current in the loop as viewed from point $P$ ?

____ Clockwise ____ Counterclockwise

Justify your answer.

(d) Assuming the loop stays in its current position, calculate the energy dissipated in the loop in 4.0 seconds.

(e) Indicate whether the net magnetic force and net magnetic torque on the loop are zero or nonzero while the loop is in the magnetic field.

Net magnetic force: ____ Zero ____ Nonzero

Net magnetic torque: ____ Zero ____ Nonzero

Justify both of your answers.

**STOP**

**END OF EXAM**