Question 2: Translation Between Representations (TBR) — 12 points

A

For drawing a bar at $\frac{1}{4}T$ — **Point A1**

For drawing a bar at $\frac{1}{4}T$ that has a height of 3 units — **Point A2**

For indicating that $|\mathcal{E}|$ is zero at times $t = 0$ and $\frac{1}{2}T$ — **Point A3**

Example Response

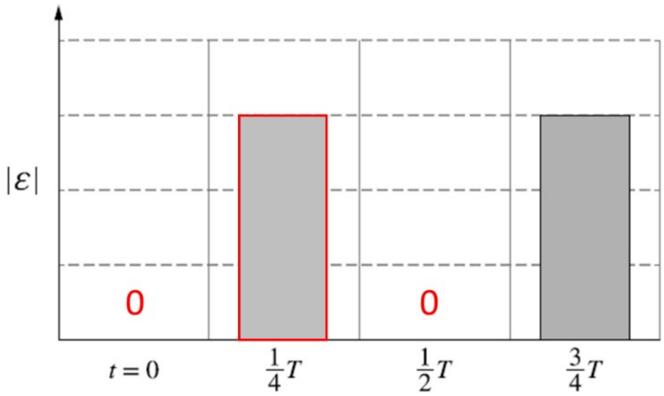


Figure 2

B

For a multistep derivation that includes the equation $\mathcal{E} = -\frac{d\Phi_B}{dt}$ — **Point B1**

Scoring Note: The negative sign does not have to be present in the expression for this point to be earned.

For a correct expression for the induced emf (e.g., $\mathcal{E} = BA\omega \sin(\omega t)$) — **Point B2**

For using Ohm's law to relate current and emf (e.g., $I = \frac{\mathcal{E}}{R}$) — **Point B3**

For a correct expression for the absolute value of the maximum induced current (e.g., $I = \frac{BA\omega}{R}$) — **Point B4**

Example Response

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -\frac{d}{dt}\left[BA\cos(\omega t)\right] = BA\omega\sin(\omega t)$$

$$\mathcal{E}_{max} = BA\omega$$

$$I = \frac{\Delta V}{R} = \frac{\mathcal{E}}{R}$$

$$I_{max} = \frac{\mathcal{E}_{max}}{R} = \frac{BA\omega}{R}$$

C

For a curve that is approximately sinusoidal — **Point C1**

For a curve that shows exactly two cycles — **Point C2**

For a curve that starts at the origin and has equal maximum values and equal minimum values — **Point C3**

Scoring Note: A curve that starts and ends at $P = 0$ and has a single maximum can earn this point.

Example Response

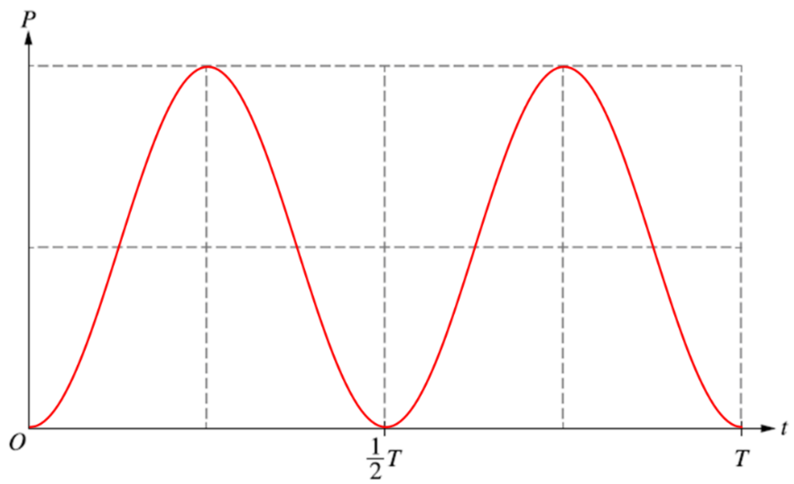


Figure 3

D

For correctly indicating whether the representations are consistent in parts A and C — **Point D1**

For a correct justification that indicates that the maxima and/or minima of the graph in part C align with the bars drawn in part A because P is proportional to $\mathcal{E}^2$ — **Point D2**

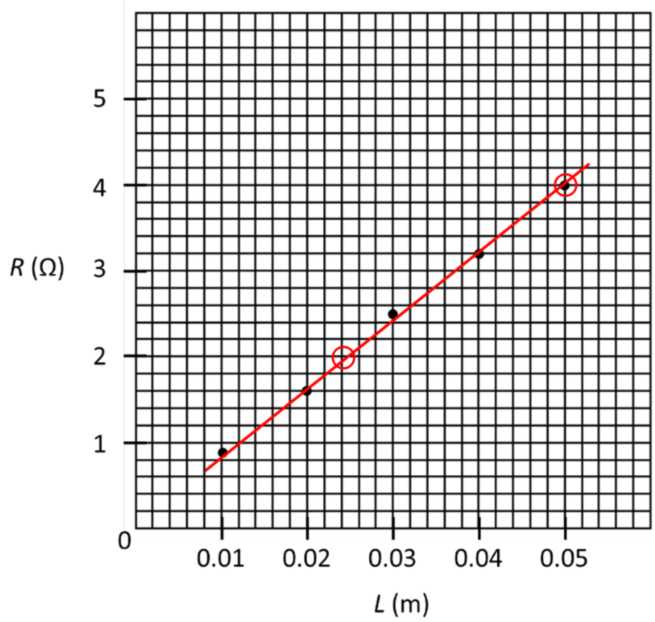
Example Response

Yes, part C is consistent with part A. P is proportional to $\mathcal{E}^2$. *When* $|\mathcal{E}|$ *is maximum, P is maximum.*

Question 3: Experimental Design and Analysis (LAB) — 10 points

A		For a procedure in which the length of and the current in the circuit element are measured, and the cross-sectional area of the circuit element is determined	**Point A1**
		For a procedure that indicates a reasonable method of reducing experimental uncertainty Examples of acceptable responses may include the following: • Making different measurements of the current in the circuit element • Varying the potential difference of the power supply	**Point A2**
		Example Response *Measure the length and diameter of the circuit element. Use the diameter to calculate the area of the circuit element. Measure the current in the circuit element. Repeat the procedure multiple times for different potential difference settings on the variable power supply.*	
B		For indicating quantities that can be graphed appropriately to determine ρ_1, consistent with the procedure from part A, such as potential difference as a function of current **Scoring Notes:** • Responses that include the reciprocals of the preceding example, in addition to other equivalent graphs, also earn this point. • This point may be earned independently of the response in part A.	**Point B1**
		For a correct analysis of the graph, consistent with the first point of part B	**Point B2**
		Example Response *Graph the potential difference across the circuit element as a function of the current in the circuit element. Use the slope of the graph, which is* $\frac{\rho_1 \ell}{\pi r^2}$, *where* ℓ *is the length of the circuit element and r is equal to the radius of the circuit element, to determine* ρ_1.	
C	**(i)**	For indicating appropriate quantities that could be plotted on the graph to determine ρ_2, for example R as a function of L **Scoring Note:** This point may be earned: • if the vertical and horizontal variables are reversed. • for other equivalent graphs.	**Point C1**
	(ii)	For labeling the axes (including units) with a linear scale	**Point C2**
		For correctly plotting data points consistent with **one** of the following: • The quantities indicated in part C (i) • The quantities provided in the table • The axes indicated in the first point of part C (ii)	**Point C3**
	(iii)	For drawing a line or curve that approximates the trend of the plotted data	**Point C4**

Example Response



D	For correctly relating the slope of the best-fit line to ρ_2 (e.g., $\dfrac{\Delta R}{\Delta L} = \text{slope} = \dfrac{\rho_2}{A}$)	**Point D1**
	For a value for ρ_2 that is between $3.5\times10^{-4}\ \Omega\cdot\text{m}$ and $4.5\times10^{-4}\ \Omega\cdot\text{m}$	**Point D2**

Example Response

$$\text{slope} = \frac{\Delta R}{\Delta L}$$

$$\frac{\Delta R}{\Delta L} = \frac{4.0\ \Omega - 2.0\ \Omega}{0.050\ \text{m} - 0.024\ \text{m}}$$

$$\frac{\Delta R}{\Delta L} = 77\ \frac{\Omega}{\text{m}}$$

$$R = \frac{\rho L}{A}$$

$$R = \left(\frac{\rho_2}{A}\right)L$$

$$\text{slope} = \frac{\rho_2}{A}$$

$$\rho_2 = A(\text{slope})$$

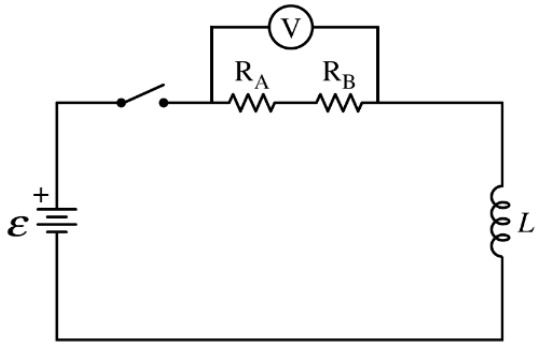
$$\rho_2 = \left(5.0\times10^{-6}\ \text{m}^2\right)\left(77\ \frac{\Omega}{\text{m}}\right)$$

$$\rho_2 \approx 3.9\times10^{-4}\ \Omega\cdot\text{m}$$

Question 2: Free-Response Question — 15 points

(a)(i)	For correctly placing the voltmeter in parallel with Resistor A, Resistor B, or the combination of resistors A and B	**1 point**

Example Response



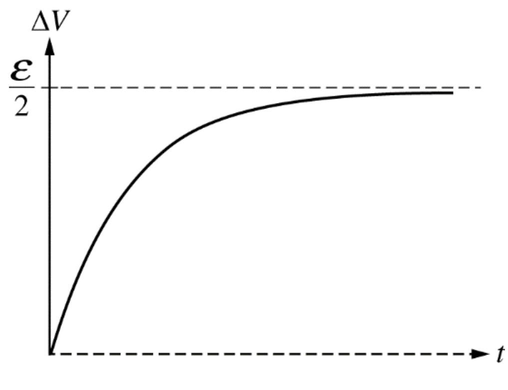
(a)(ii)	For a procedure that indicates that the voltmeter should be used to measure the potential difference for at least one time	**1 point**
	For measuring the potential difference from immediately after the switch is closed to when steady-state conditions have been established or until a time at which the time constant can be determined	**1 point**

Example Response

Close the switch. Using the voltmeter, record the potential difference as a function of time until steady-state conditions are established.

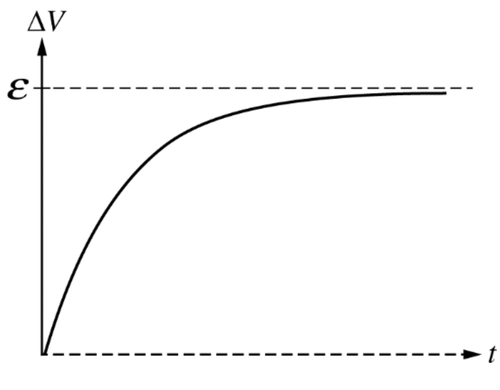
	Total for part (a)	**3 points**
(b)(i)	For correctly labeling potential difference on the vertical axis and time on the horizontal axis	**1 point**
	For a concave-down and increasing curve	**1 point**
	For a curve that starts at the origin and asymptotically approaches a nonzero potential difference value	**1 point**
	For correctly labeling the horizontal asymptote that is consistent with the indication of the connection of the voltmeter in the response in part (a)(i)	**1 point**

Example Responses



One Resistor

OR



Combination of Resistors

(b)(ii) For indicating that a curve fit to the graph is that of an exponential function **1 point**

Alternate Solution

For indicating the time on the graph where the potential difference across Resistor A *or Resistor* B *is approximately* $\frac{0.63\mathcal{E}}{2}$ *or that the potential difference across the combination of resistors* A *and* B *is approximately* $0.63\mathcal{E}$

For indicating that the coefficient in front of the t of the curve-fit equation is equal to $\frac{2R}{L}$ **1 point**

Alternate Solution

For indicating that the time constant is equal to $\frac{L}{2R}$

Example Response

The data in the graph should be fit with an exponential function for the equation $V_R = \mathcal{E}\left(1 - e^{-t\frac{2R}{L}}\right)$. *Because* $\mathcal{E}$ *and* L *are known,* R *can be calculated.*

Alternate Example Response

The potential difference at $0.63\mathcal{E}$ *along the vertical axis corresponds to the time constant along the horizontal axis. Because the time constant is equal to* $\frac{L}{2R}$, *and* L *is known,* R *can be calculated.*

Total for part (b) 6 points

(c)	For a multi-step derivation that begins with an attempt at using Kirchhoff's loop rule	**1 point**
	Example Response $\mathcal{E} - \Delta V_R - \Delta V_L = 0$	
	For indicating that the potential difference across the series combination of resistors is $I(2R)$	**1 point**
	For indicating that the absolute value of the potential difference across the inductor is $L\dfrac{dI}{dt}$	**1 point**
	Example Solution $\mathcal{E} - \Delta V_R - \Delta V_L = 0$ $\mathcal{E} - I(2R) - L\dfrac{dI}{dt} = 0$ $\dfrac{\mathcal{E}}{L} - \dfrac{2IR}{L} = \dfrac{dI}{dt}$ $-\dfrac{R}{L}\left(2I - \dfrac{\mathcal{E}}{R}\right) = \dfrac{dI}{dt}$	
	Total for part (c)	**3 points**
(d)	For selecting $\lvert\Delta V_2\rvert < \lvert\Delta V_1\rvert$ with an attempt at a relevant justification	**1 point**
	For indicating that the total resistance has increased	**1 point**
	For indicating that the current decreases due to the increased resistance **OR** For indicating that the potential difference across the inductor will increase, and, therefore, the potential difference across R_A must decrease	**1 point**
	Example Response $\lvert\Delta V_2\rvert < \lvert\Delta V_1\rvert$*, because the inductor has nonnegligible resistance, and the total resistance of the new circuit increases as compared to the original circuit. Therefore, the current in the new circuit is reduced compared to that of the original circuit when steady-state conditions are established.* **OR** $\lvert\Delta V_2\rvert < \lvert\Delta V_1\rvert$*, because the inductor has nonnegligible resistance, and the total resistance of the new circuit increases as compared to the original circuit. Therefore, the potential difference across the inductor increases, which decreases the potential difference across* R_A *compared to that of the original circuit when steady-state conditions are established.*	
	Total for part (d)	**3 points**
	Total for question 2	**15 points**

Question 3: Free-Response Question — 15 points

(a)	For selecting "Out of the page" and an attempt at a relevant justification	**1 point**
	For a justification that correctly relates how the changing current in the long wire changes the flux through the loop with respect to time	**1 point**
	For indicating the induced current will oppose the change in magnetic flux	**1 point**

Example Response

Because the current in the straight wire is decreasing, the magnetic field, which is originally pointing out of the page, is decreasing. Hence, the induced current produces a field that is directed out of the page to compensate for the decreasing flux.

Total for part (a) — 3 points

(b) For using an appropriate integral equation, with a substitution of an expression for magnetic field, to calculate magnetic flux — **1 point**

Example Response

$$\Phi_B = \int \vec{B} \bullet d\vec{A}$$

For writing a correct equation for the magnetic field as a function of distance from the wire — **1 point**

Example Response

$$B = \frac{\mu_0 I}{2\pi r}$$

For substituting the value of $t = 3\text{ s}$ to find the electric current — **1 point**

Example Response

$$I(t) = C - Dt$$

$$I(3\text{ s}) = 10\text{ A} - (2\text{ A/s})(3\text{ s})$$

$$I(3\text{ s}) = 4\text{ A}$$

For integrating B with correct limits and a correct substitution for dA, to determine the total flux through the loop — **1 point**

Example Response

$$\Phi_B = \int_d^{d+W} \frac{\mu_0 IL}{2\pi r} dr$$

$$\Phi_B = \frac{\mu_0 IL}{2\pi} \ln[r]_d^{d+W}$$

$$\Phi_B = \frac{\mu_0 IL}{2\pi} \ln\left[\frac{d+W}{d}\right]$$

$$\Phi_B = \frac{(4\pi \times 10^{-7}\text{ T}\bullet\text{m/A})(4\text{ A})(0.04\text{ m})}{2\pi} \ln\left[\frac{0.03\text{ m}}{0.01\text{ m}}\right] = 3.52 \times 10^{-8}\text{ T}\bullet\text{m}^2$$

Total for part (b) — 4 points

(c) For using Faraday's law to determine the emf across the light bulb **1 point**

Example Response

$$\varepsilon = \left|\frac{d\Phi}{dt}\right|$$

$$\varepsilon = \frac{d}{dt}\left(\frac{\mu_0 (C - Dt)L}{2\pi}\ln\left[\frac{d+W}{d}\right]\right)$$

$$\varepsilon = \left(\frac{\mu_0 DL}{2\pi}\ln\left[\frac{d+W}{d}\right]\right)$$

For using Ohm's law to find the current in the light bulb consistent with the emf determined from the previous point **1 point**

Example Response

$$I = \frac{\varepsilon}{R}$$

$$I = \left(\frac{\mu_0 DL}{2\pi R}\ln\left[\frac{d+W}{d}\right]\right)$$

For correct substitutions of the values of $\frac{dI}{dt}$ and R **1 point**

Example Response

$$I = \frac{(4\pi \times 10^{-7}\ \text{T}\bullet\text{m/A})(2.0\ \text{A/s})(0.04\ \text{m})}{(2\pi \bullet 10\Omega)}\ln\left[\frac{0.03\ \text{m}}{0.01\ \text{m}}\right]$$

$$I = 1.8 \times 10^{-9}\ \text{A}$$

Total for part (c) 3 points

(d) For selecting: "The current in the long wire changes at a faster rate than expected." **1 point**

For correctly justifying the selection **1 point**

Scoring Note: A response cannot earn this point if the incorrect selection is chosen.

Example Response

If the current in the wire changes at a faster rate, there will be a greater rate of change of magnetic flux. So the induced emf and current will be higher.

Total for part (d) 2 points

(e)	For selecting "$I_2 < I_1$" with an attempt at a relevant justification	**1 point**
	For indicating the total flux in the loop is less in the new orientation	**1 point**
	For correctly relating the rate of change of the flux to the total flux inside the loop	**1 point**

Example Response

With the new orientation, some parts of the rectangle are further away from the straight wire, which means that the magnetic flux through the rectangle will be less. The rate of change of the flux has the same dependence on distance and will also decrease, resulting in a smaller current.

Total for part (e) 3 points

Total for question 3 15 points

Question 3: Free-Response Question — 15 points

(a)	For selecting "Counterclockwise" and an attempt at a relevant justification	**1 point**
	For a justification that correctly relates how the changing current in the solenoid changes the flux through the loop with respect to time	**1 point**
	For a justification that has a correct relationship between the change in magnetic flux through the loop of wire and how the current in the loop changes to oppose the change in magnetic flux	**1 point**

Example Response

In the end view, the magnetic field due to the solenoid is directed into the page. The current in the solenoid is increasing; thus, the magnetic flux is increasing. According to Lenz's law, because the magnetic flux is increasing and into the page, the current in the loop must create a magnetic field directed out of the page; thus, the current in the loop must be counterclockwise the counter the change in magnetic flux.

Total for part (a) 3 points

(b)	For using an appropriate equation to calculate emf in the loop	**1 point**

Example Response

$$\varepsilon = \left|\frac{d\Phi}{dt}\right| = \frac{d(BA)}{dt} = A\frac{dB}{dt}$$

For correctly substituting the magnetic field for a solenoid into above equation — **1 point**

Example Response

$$\varepsilon = A\frac{d(\mu_0 nI)}{dt} = \mu_0 nA\frac{dI}{dt}$$

For using Ohm's law to calculate the current in the loop — **1 point**

Example Response

$$I_{\text{loop}} = \frac{\varepsilon}{R} = \frac{\mu_0 nA}{R}\frac{dI}{dt}$$

For correctly substituting into the previous equation for the induced current in the loop — **1 point**

Example Response

$$I_{\text{loop}} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\bullet\text{m/A}\right)\left(\dfrac{500\ \text{turns}}{0.25\ \text{m}}\right)\left((\pi)(0.10\ \text{m})^2\right)}{(3\ \Omega)}(5.0\ \text{A/s}) = 1.3 \times 10^{-4}\ \text{A}$$

Total for part (b) 4 points

(c)	For using an appropriate expression for the power for a constant current through a resistor **Example Response** $P = I_{\text{loop}}{}^2 R$	**1 point**
	For a correct substitution consistent with (b) into a correct expression for electrical power **Example Response** $P = I_{\text{loop}}{}^2 R = \left(\frac{\mu_0 nA}{R}\frac{dI}{dt}\right)^2 R = \left(1.3\times10^{-4}\text{ A}\right)^2 (3.0\ \Omega) = 5.2\times10^{-8}\text{ W}$	**1 point**
	For correctly substituting the power dissipated by the loop of wire into a correct energy equation **Example Response** $P = \frac{dE}{dt} = \frac{\Delta E}{\Delta t} \therefore \Delta E = P\Delta t$ $\Delta E = \left(5.2\times10^{-8} W\right)(2.0\text{ s}) = 1.0\times10^{-7}\text{ J}$	**1 point**
	Total for part (c)	**3 points**
(d)	For selecting "The plane of the loop is not perpendicular to the axis of the solenoid."	**1 point**
	For correctly justifying the selection **Scoring Note:** A response cannot earn this point if the incorrect selection is chosen.	**1 point**
	Example Response *If the plane of the loop is not perpendicular to the axis of the solenoid it is not perpendicular to the magnetic field. Therefore, the magnetic flux and emf will be less, so the current will be less.*	
	Total for part (d)	**2 points**
(e)	For selecting "$\frac{I_2}{I_1} = 2$" with an attempt at a relevant justification **Scoring Note:** The response can earn this point if selecting "$\frac{I_2}{I_1} > 2$" and only referencing the increase in flux through the loop in the justification.	**1 point**
	For a statement that indicates that the resistance of the second loop is double the resistance of the original loop	**1 point**
	For a statement that indicates that the new emf is four times greater than the original emf due to the increased flux through the loop resulting from the quadrupled area	**1 point**

Example Response

The magnetic flux quadruples. This is because of the increase in area of the loop, which quadruples the emf. The resistance of the loop doubles because the length of the wire doubles. Therefore, $\frac{I_2}{I_1} = 2$.

$$\varepsilon = \mu_0 n \left(A_{loop}\right) = \mu_0 n \left(\pi r^2\right) \frac{dI}{dt}$$

The radius of the loop doubles. Therefore, the area of the loop quadruples.

$$R = \rho \frac{2\pi r}{A_{wire}}$$

The radius of the loop doubles. Therefore, the resistance of the loop doubles.

$$\frac{I_2 = \frac{\varepsilon}{R} = \frac{\mu_0 n(4) A}{(2) R} \frac{dI}{dt}}{I_1 = \frac{\varepsilon}{R} = \frac{\mu_0 n A}{R} \frac{dI}{dt}} = 2$$

Total for part (e) 3 points

Total for question 3 15 points

2019

AP® CollegeBoard

AP® Physics C: Electricity and Magnetism

Scoring Guidelines Set 1

2019 SCORING GUIDELINES

General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

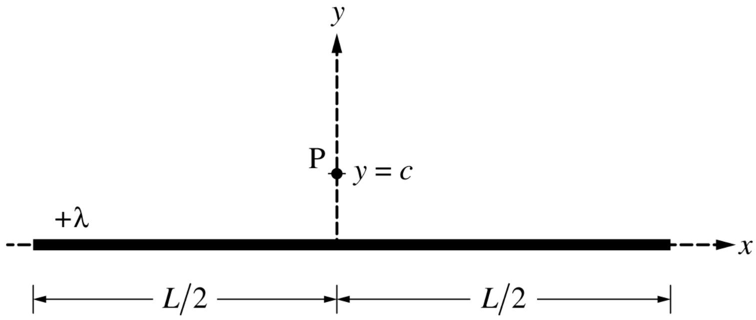
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 1

15 points



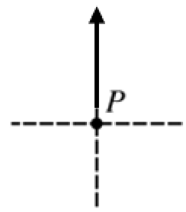
Note: Figure not drawn to scale.

A very long, thin, nonconducting cylinder of length L is centered on the y-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y-axis at $y = c$, where $L >> c$.

(a)

i. LO CNV-3.B.a, SP 7.A
1 point

On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the long cylinder. The arrow should start on and point away from the dot.



For drawing an arrow at point P that points upward		1 point

ii. LO CNV-2.F, SP 7.C
1 point

Describe the shape and location of a Gaussian surface that can be used to determine the electric field at point P due to the long cylinder.

For describing a Gaussian surface that could be used to determine the electric field at point P		1 point
Example: Drawing a cylinder that is coaxial with the thin cylinder and whose surface contains point P can be used to determine the electric field at point P.		
Note: Credit is earned if the student draws the correct surface on the figure.		

Question 1 (continued)

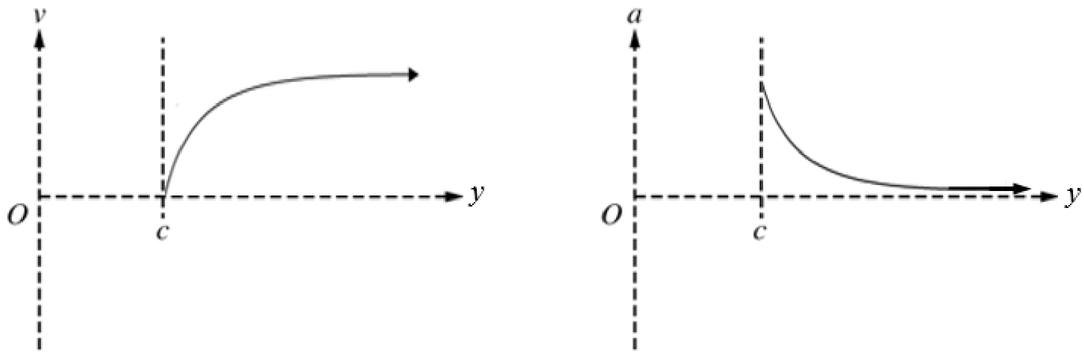
(a) continued
iii. LO CNV-2.C, SP 5.A, 5.E
3 points

Use your Gaussian surface to derive an expression for the magnitude of the electric field at point P. Express your answer in terms of λ, c, L, and physical constants, as appropriate.

For using Gauss's law to determine the electric field at point *P*		1 point
$\frac{q_{enc}}{\varepsilon_0} = \int E \bullet dA \therefore \frac{Q}{\varepsilon_0} = EA$		
For correctly substituting for the charge into the equation above		1 point
For correctly substituting for the area or into the equation above		1 point
$\frac{\lambda L}{\varepsilon_0} = E(2\pi cL) \therefore E = \frac{\lambda}{2\pi\varepsilon_0 c} = \frac{2k\lambda}{c}$		

(b) LO ACT-1.D, SP 3.C
2 points

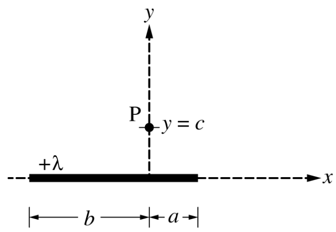
A proton is released from rest at point P. On the axes below, sketch the velocity v as a function of <u>position</u> y and the acceleration a as a function of <u>position</u> y for the proton.



For a concave down graph for v as a function of position x that does not start at the origin		1 point
For a concave up graph for a as a function of position x that has an asymptote at the horizontal axis		1 point

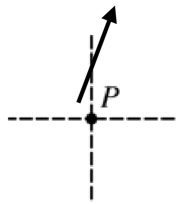
Question 1 (continued)

The original cylinder is now replaced with a much shorter thin, nonconducting cylinder with the same uniform linear charge density $+\lambda$, as shown in the figure below. The length of the cylinder to the right of the y-axis is a, and the length of the cylinder to the left of the y-axis is b, where $a < b$.



(c) LO CNV-3.B.a, SP 7.A
2 points

On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the shorter cylinder. The arrow should start on and point away from the dot.



For drawing an arrow at point P that points to the right		1 point
For drawing an arrow at point P that points up and to the right		1 point

(d)
i. and ii. LO CNV-2.F, SP 7.A
1 point

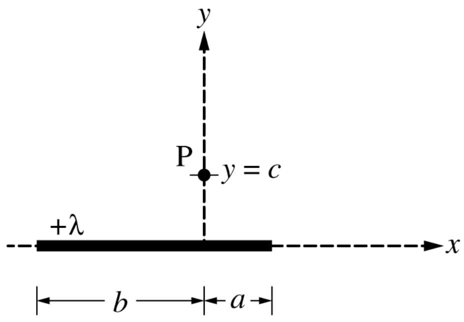
Is there a single Gaussian surface that can be used with Gauss's law to derive an expression for the electric field at point P?

____ Yes ____ No

If your answer to part (d)(i) is yes, explain how you can use Gauss's law to derive an expression for the field at point P. If your answer to part (d)(i) is no, explain why Gauss's law cannot be applied to derive an expression for the electric field in this case.

For selecting "No" with a valid explanation		1 point
Claim: Select "No." Evidence: The length of the cylinder is not much greater than the distance from the cylinder to point P and the charge distribution is asymmetric. Reasoning: Therefore, cannot use the approximation of the constant magnitude of electric field over a cylindrical surface.		

Question 1 (continued)



<u>Note</u>: This figure is shown again for reference.

A student in class argues that using the integral shown below might be a useful approach for determining the electric field at point P.

$$E = \int \frac{1}{4\pi\varepsilon_0} \frac{1}{r^2}\, dq$$

The student uses this approach and writes the following two integrals for the magnitude of the horizontal and vertical components of the electric field at point P.

$$\text{Horizontal component: } \left|E_x\right| = \frac{\lambda}{4\pi\varepsilon_0} \int_{-b}^{a} \frac{x}{\left(c^2 + x^2\right)^{3/2}}\, dx$$

$$\text{Vertical component: } \left|E_y\right| = \frac{\lambda}{4\pi\varepsilon_0} \int_{-b}^{a} \frac{y}{\left(c^2 + x^2\right)}\, dy$$

(e)

i. LO CNV-3.A, SP 7.A
1 point

One of the two expressions above is not correct. Which expression is not correct?

____ Horizontal component ____ Vertical component

For correctly selecting “Vertical component”		1 point

Question 1 (continued)

(e) continued
ii. LO CNV-3.A, SP 7.D
4 points

Identify two mistakes in the incorrect expression, and explain how to correct the mistakes.

For indicating the integral is not along the length of the cylinder		1 point
For an appropriate correction		1 point
Claim: Change *dy* to *dx*. Evidence: The integral is not along the cylinder. Reasoning: This change will make the integral valid.		
For indicating the power on the denominator term for the vertical component is incorrect		1 point
For an appropriate correction		1 point
Claim: The power on the term in the denominator should be $3/2$. Evidence: The units of the integrand are not valid. Reasoning: This change will make the integrand valid.		

Learning Objectives

ACT-1.D: Determine the motion of a charged object of specified charge and mass under the influence of an electrostatic force.
CNV-2.C: State and use Gauss's law in integral form to derive unknown electric fields for planar, spherical, or cylindrically symmetrical charge distributions.
CNV-2.F: Describe the general features of an unknown charge distribution, given other features of the system.
CNV-3.A: Derive expressions for the electric field of specified charge distributions using integration and the principle of superposition. Examples of such charge distributions include a uniformly charged wire, a thin ring of charge (along the axis of the ring), and a semicircular or part of a semicircular arc.
CNV-3.B.a: Identify and qualitatively describe situations in which the direction and magnitude of the electric field can be deduced from symmetry considerations and understanding the general behavior of certain charge distributions.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.
7.D: Provide reasoning to justify a claim using physical principles or laws.

Question 2

15 points

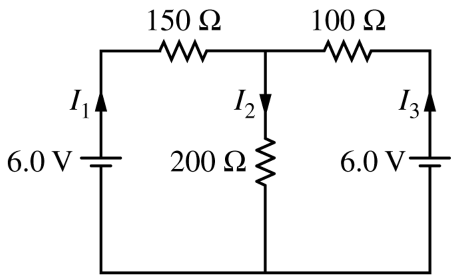


Figure 1

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(a)

i. LO CNV-6.F.b, SP 6.A
3 points

Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

For an equation representing the sum of the currents at one of the junctions		1 point
$I_1 - I_2 + I_3 = 0$		
For an equation representing the sum of the potential differences around one of the loops		1 point
For an equation representing the sum of the potential differences around a loop different from the above loop		1 point
$6 - 150I_1 - 200I_2 = 0$		
$6 - 100I_3 - 200I_2 = 0$		
$6 - 150I_1 + 100I_3 - 6 = 0$		
Note: Full credit is earned for two correct loop equations using loop currents.		

ii. LO CNV-6.F.b, SP 6.C
2 points

Calculate the current in the $200\ \Omega$ resistor.

For combining the equations from part (a)(i)		1 point
$I_1 - I_2 + I_3 = 0$ $-I_1 - 1.33I_2 = -.04 \therefore -4.33I_2 = -0.10$ $-I_3 - 2I_2 = -.06$		
Note: Credit is earned if students indicate they used a calculator function to solve the system of equations.		
For a correct answer with correct units		1 point
$-4.33I_2 = -0.10 \therefore I_2 = 0.023\text{ A}$		

Question 2 (continued)

(a) continued

iii. LO CNV-5.A.a, SP 6.C
1 point

Calculate the power dissipated by the $200\ \Omega$ resistor.

For using a correct equation to calculate the power in the $200\ \Omega$ resistor		1 point
$P = I^2R = (0.023\text{ A})^2(200\ \Omega)\ \therefore\ P = 0.107\text{ W}$		

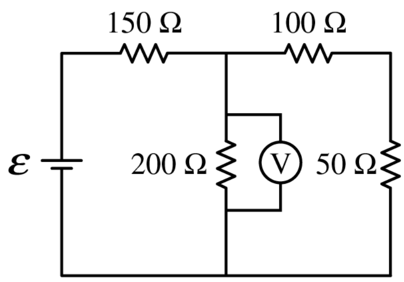


Figure 2

The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

(b) LO CNV-6.C.a, SP 6.B, 6.C
2 points

Calculate the current through the $50\ \Omega$ resistor.

For correctly calculating the equivalent resistance of the branch with the $50\ \Omega$ resistor		1 point
$R = 100\ \Omega + 50\ \Omega = 150\ \Omega$		
For using the correct potential difference in Ohm's law to calculate the current in the $50\ \Omega$ resistor		1 point
$I = \dfrac{V}{R} = \dfrac{(4.4\text{ V})}{(100\ \Omega + 50\ \Omega)}\ \therefore\ I = 0.029\text{ A}$		

Question 2 (continued)

(c) LO CNV-6.C.a, SP 6.A, 6.C
3 points

Calculate the voltage ε of the battery.

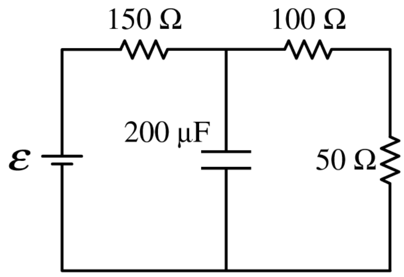
For using a correct equation to determine the current through the $150\ \Omega$ resistor		1 point
For correctly substituting the current from part (b) as I_3		1 point
$I_1 = I_2 + I_3 = \dfrac{(4.4\text{ V})}{(200\ \Omega)} + 0.029\text{ A} = 0.051\text{ A}$		
For using a correct equation to determine the emf of the battery		1 point
$\varepsilon = I_1R_1 + V_2 = (0.051\text{ A})(150\ \Omega) + 4.4\text{ V} = 12.1\text{ V}$		
Alternate Third Point		
For calculating the equivalent resistance of the circuit and substituting this resistance into a correct equation to determine the emf of the battery		*1 point*
$R_T = 150\ \Omega + \dfrac{1}{\dfrac{1}{200\ \Omega} + \dfrac{1}{150\ \Omega}} = 236\ \Omega$		
$\varepsilon = I_1R_T = (0.051\text{ A})(236\ \Omega) = 12.0\text{ V}$		
Alternate Solution		*Alternate Points*
For using a correct equation to determine the equivalent resistance of the parallel resistors		*1 point*
$\dfrac{1}{R_P} = \dfrac{1}{200\ \Omega} + \dfrac{1}{150\ \Omega}\ \therefore\ R_P = 86\ \Omega$		
For correctly substituting the given potential difference and the calculated equivalent resistance to determine the total current of the circuit		*1 point*
$I_T = \dfrac{(4.4\text{ V})}{(86\ \Omega)} = 0.051\text{ A}$		
For calculating the equivalent resistance of the circuit and substituting into a correct equation to determine the emf of the battery		*1 point*
$R_T = 150\ \Omega + 86\ \Omega = 236\ \Omega$		
$\varepsilon = I_1R_T = (0.051\text{ A})(236\ \Omega) = 12.0\text{ V}$		

Question 2 (continued)

(d)

i. LO CNV-7.B.a, SP 6.A, 6.C
2 points

The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu\text{F}$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.

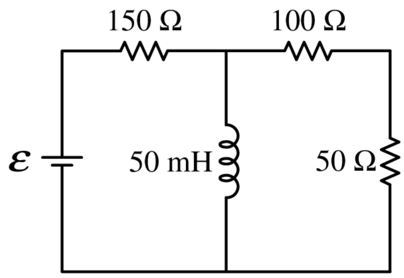


For substituting the voltage consistent with part (c) into Ohm's law		1 point
For correctly calculating the equivalent resistance of the circuit		1 point
$I = \dfrac{\mathcal{E}}{R_{tot}} = \dfrac{(12.1\text{ V})}{(150\ \Omega + 100\ \Omega + 50\ \Omega)} = 40.3\text{ mA}$		

ii. LO CNV-10.C.a, SP 7.A, 7.C
2 points

The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b)?

____ Greater than ____ Less than ____ Equal to



Justify your answer.

For correctly selecting "Less than" with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because steady state is reached, the inductor will act as a short circuit. So all the current will pass through the inductor and no current will pass through the $50\ \Omega$ resistor.		

Question 2 (continued)

Learning Objectives

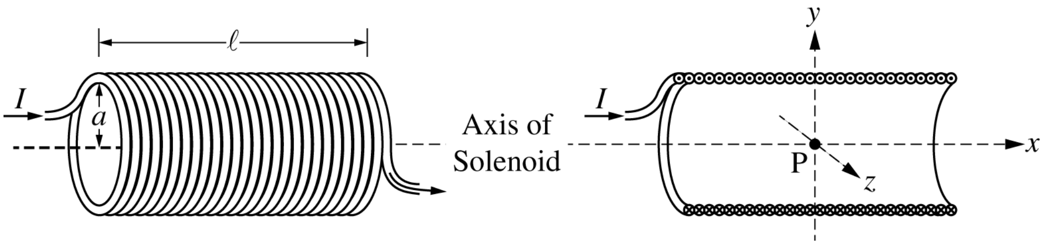
CNV-5.A.a: Derive expressions that relate current, voltage, and resistance to the rate at which heat is produced in a resistor.
CNV-6.C.a: Calculate voltage, current, and power dissipation for any resistor in a circuit containing a network of known resistors with a single battery or energy source.
CNV-6.F.b: Set up simultaneous equations to calculate at least two unknowns (currents or resistance values) in a multi-loop circuit.
CNV-7.B.a: Calculate the potential difference across a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.
CNV-10.C.a: Calculate initial transient currents and final steady-state currents through any part of a series or parallel circuit containing an inductor and one or more resistors.

Science Practices

6.A: Extract quantities from narratives or mathematical relationships to solve problems.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

Question 3

15 points



Solenoid

Cutaway View

Note: Figures not drawn to scale.

A solenoid is used to generate a magnetic field. The solenoid has an inner radius a, length ℓ, and N total turns of wire. A power supply, not shown, is connected to the solenoid and generates current I, as shown in the figure on the left above. The x-axis runs along the axis of the solenoid. Point P is in the middle of the solenoid at the origin of the xyz-coordinate system, as shown in the cutaway view on the right above. Assume $\ell >> a$.

(a) LO CNV-8.E.a, SP 7.A, 7.C
2 points

Select the correct direction of the magnetic field at point P.

____ $+x$-direction ____ $+y$-direction ____ $+z$-direction
____ $-x$-direction ____ $-y$-direction ____ $-z$-direction

Justify your selection.

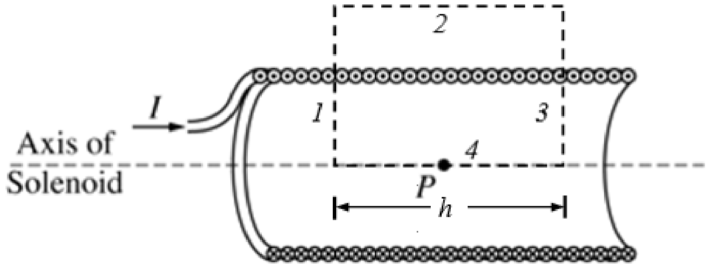
For choosing the "$+x$-direction" and providing a justification		1 point
For a correct justification		1 point
Example: Using the right-hand rule for current on the left side of the solenoid, the fingers curl into the loop, so the magnetic field points to the right, or in the $+x$-direction.		
Example: Using the right-hand rule for solenoids, when the fingers curl around the solenoid in the direction of the current, the thumb points to the right, therefore the magnetic field is to the right, or in the $+x$-direction.		

Question 3 (continued)

(b)

i. LO CNV-8.C.c, SP 3.D
1 point

On the cutaway view below, clearly draw an Amperian loop that can be used to determine the magnetic field at point P at the center of the solenoid.



For drawing a rectangle with one side along the central axis of the solenoid and another side outside the solenoid and whose edges do not extend beyond the solenoid		1 point

ii. LO CNV-8.C.c, SP 5.A, 5.E
2 points

Use Ampere's law to derive an expression for the magnetic field strength at point P. Express your answer in terms of I, ℓ, N, a, and physical constants, as appropriate.

For using Ampere's law to calculate the magnetic field along the axis of the solenoid		1 point
$\int B \bullet d\ell = \mu_0 I_{enc} \therefore \left(\int B \bullet d\ell\right)_1 + \left(\int B \bullet d\ell\right)_2 + \left(\int B \bullet d\ell\right)_3 + \left(\int B \bullet d\ell\right)_4 = \mu_0 I_{enc}$		
$\left(\int B \bullet d\ell\right)_1 + 0 + \left(-\left(\int B \bullet d\ell\right)_1\right) + Bh = \mu_0 \frac{N}{\ell} hI$		
$Bh = \mu_0 \frac{N}{\ell} hI$		
For a correct answer		1 point
$B = \frac{\mu_0 NI}{\ell}$		

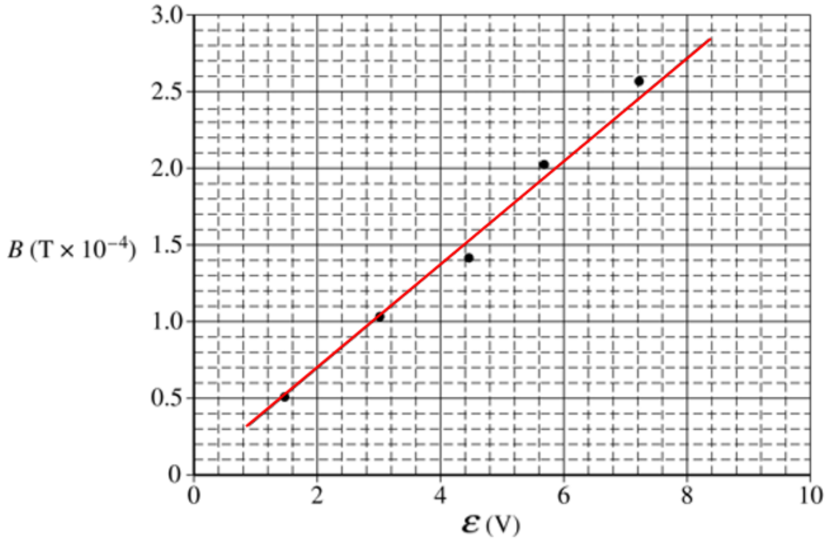
Some physics students conduct an experiment to determine the resistance R_S of a solenoid with radius $a = 0.015$ m, total turns $N = 100$, and total length $\ell = 0.40$ m. The students connect the solenoid to a variable power supply. A magnetic field sensor is used to measure the magnetic field strength along the central axis at the center of the solenoid. The plot of the magnetic field strength B as a function of the emf $\mathcal{E}$ of the power supply is shown below.

Question 3 (continued)

(c) LO CNV-8.C.c, SP 4.C

i. 1 point

On the graph above, draw a best-fit line for the data.



For drawing a best-fit line with at least one point above and one point below the line		1 point

ii. LO CNV-8.C.c, SP 6.B, 6.C

2 points

Use the straight line to determine the resistance R_S of the solenoid used in the experiment.

For calculating the slope using the best-fit line and not the data points unless they fall on the best-fit line		1 point
$slope = \dfrac{\Delta y}{\Delta x} = \dfrac{(2.5-0.9)\left(\times 10^{-4}\ \text{T}\right)}{(6.4-2.0)\ \text{V}} = 0.36\times 10^{-4} = 3.6\times 10^{-5}\ \ \text{T/V}$		
For correctly giving the expression that relates the slope to the resistance of the solenoid		1 point
$B = \dfrac{\mu_0 NI}{\ell} = \dfrac{\mu_0 N\mathcal{E}}{\ell R}\ \therefore\ slope = \dfrac{\mu_0 N}{\ell R_S}\ \therefore\ R_S = \dfrac{\mu_0 N}{\ell \times slope}$ $R_S = \dfrac{\left(4\pi\times 10^{-7}\ \ (\text{T}\cdot\text{m})/\text{A}\right)(100\ \text{turns})}{(0.40\ \text{m})\left(3.6\times 10^{-5}\ \ \text{T/V}\right)} = 8.7\ \Omega$		

One of the students notes that the horizontal component of the magnetic field of Earth is 2.5×10^{-5} T.

Question 3 (continued)

(d)

i. LO CNV-8.E.a, SP 2.D
1 point

Is there evidence from the graph that the horizontal orientation of the solenoid affects the measured values for B?

____ Yes ____ No

Justify your answer.

If the line on the graph does not go through the origin, select "Yes"		
For a correct justification		1 point
Example: The horizontal component of Earth's magnetic field will add or subtract from the magnetic field of the solenoid depending on the orientation of the solenoid.		
Alternate Solution		*Alternate Points*
If the line on the graph does pass through the origin, select "No"		
For a correct justification		*1 point*
Example: Based on the graph, the line passes through the origin, so the magnetic field is zero when the emf of the power supply is zero, therefore Earth's magnetic field is not affecting the values of B.		

ii. LO CNV-8.E.a, SP 2.E
1 point

Would the horizontal orientation of the solenoid affect the calculated value for R_S ?

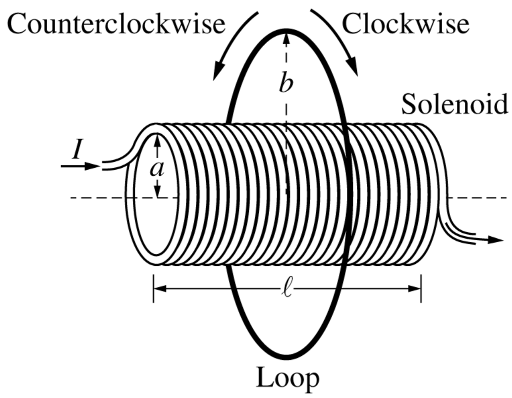
____ Yes ____ No

Justify your answer.

Select "No"		
For a correct justification		1 point
Example: The horizontal component of Earth's magnetic field will not affect the change in magnetic field as the emf is changed. Therefore, the value for the resistance of the solenoid will not change.		

2019 SCORING GUIDELINES

Question 3 (continued)



A thin conducting loop of radius b and resistance R_L is placed concentric with the solenoid, as shown above. The current in the solenoid is decreased from I to zero over time Δt.

(e)

i. LO FIE-6.A.b, SP 7.A, 7.C
2 points

Is the direction of the induced current in the loop clockwise or counterclockwise during the time period that the current in the solenoid is decreasing?

____ Clockwise ____ Counterclockwise

Justify your answer.

Select "Clockwise"		
For a justification indicating that the magnetic field inside the solenoid, and therefore the loop, will decrease		1 point
For a justification using Lenz's law to relate the change in magnetic field to the direction of the current		1 point
Example: As the current in the solenoid decreases, the magnetic field inside the solenoid decreases. As the solenoid's magnetic field decreases, the induced current in the loop will create a magnetic field to oppose this change. Because the solenoid's magnetic field is toward the right and decreasing, the magnetic field due to the current in the loop must be toward the right. Therefore, the current in the loop must be clockwise.		

Question 3 (continued)

(e) continued

ii. LO FIE-6.A.d, SP 5.A, 5.E
3 points

Derive an equation for the average induced current i_{IND} in the loop during the time period that the current in the solenoid is decreasing. Express your answer in terms of I, ℓ, N, a, b, R_L, R_S, Δt, and physical constants, as appropriate.

For using Faraday's law to calculate the emf in the loop		1 point
$\varepsilon = \left\lvert \frac{d\Phi}{dt} \right\rvert = \frac{d(BA)}{dt} = A\frac{\Delta B}{\Delta t}$		
For using Ohm's law to calculate the current in the loop		1 point
$I_{\text{IND}} = \frac{V}{R} = \frac{A\Delta B/\Delta t}{R_L} = \frac{A\Delta B}{R_L \Delta t}$		
For using the correct radius for the area in the equation above		1 point
$I_{\text{IND}} = \frac{\pi a^2 \mu_0 N \Delta I/\ell}{R_L \Delta t} = \frac{\pi a^2 \mu_0 N I}{R_L \ell \Delta t}$		

Learning Objectives

CNV-8.C.c: Derive the expression for the magnetic field of an ideal solenoid (length dimension is much larger than the radius of the solenoid) using Ampère's law.
CNV-8.E.a: Describe the direction of a magnetic field at a point in space due to various combinations of conductors, wires, cylindrical conductors, or loops.
FIE-6.A.b: Describe the direction of an induced current in a conductive loop that is placed in a changing magnetic field.
FIE-6.A.d: Calculate the magnitude and direction of induced EMF and induced current in a conductive loop (or conductive bar) when the magnitude of either the field or area of loop is changing at a constant rate.

Science Practices

2.D: Make observations or collect data from representations of laboratory setups or results.
2.E: Identify or describe potential sources of experimental error.
3.D: Create appropriate diagrams to represent physical situations.
4.C: Linearize data and/or determine a best-fit line or curve.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

AP® Physics C: Electricity and Magnetism 2016 Scoring Guidelines

SCORING GUIDELINES

Question

15 points total | **Distribution of points**

3 points

For indicating that the total potential is the sum of the potential from individual point charges — 1 point

ample using point B: $V_B = 0 = V_1 + V_2$

orrectly substituting into the above equation (signs are ignored at this step) — 1 point

$$-V_1 = V_2$$

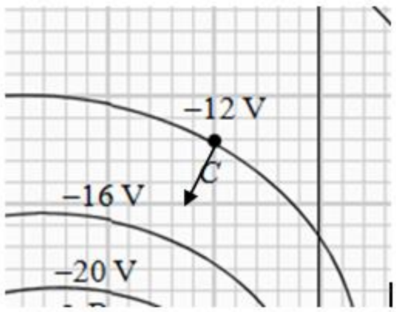
$$-\frac{kq_1}{r_1} = \frac{kq_2}{r_2}$$

$$-\frac{q_1}{(5 \times 0.5m)} = \frac{(2.0\text{ nC})}{(2 \times 0.5\text{ m})}$$

For a correct answer with correct sign and units — 1 point

$$q_1 = -5.0\text{ nC}$$

2 points



For drawing a vector perpendicular to the equipotential line for *C* — 1 point

For drawing a vector in the direction of the -16 V line — 1 point

2 points

For using the equation relating the electric field to potential difference — 1 point

$$E = -\frac{dV}{dx}$$

$$|E| \simeq \frac{\Delta V}{\Delta x}$$

For substituting values from the figure — 1 point

$$E = \frac{(-20\text{ V} - (-24\text{ V}))}{(2 \times 0.1\text{ m})}$$

$$E = 20\text{ N/C}$$

;CORING GUIDELINES

Question 1 (continued)

Distribution of points

2 points

For using a correct equation for the electric flux — 1 point

$$\Phi_E = \frac{q_{enc}}{\varepsilon_0}$$

$$\Phi_E = \frac{(2.0\text{ nC})}{\left(8.85 \times 10^{-12}\ \text{C}^2/\left(\text{N}\bullet\text{m}^2\right)\right)}$$

For a correct answer with units — 1 point

$$\Phi_E = 226\ \left(\text{N}\bullet\text{m}^2\right)/\text{C}$$

(

2 points

.sing an equation that relates the work done to the change in potential energy — 1 point

$$W = -q\Delta V = -(1.6 \times 10^{-19}\ \text{C})(-4\ \text{V} - (4\ \text{V}))$$

For a correct answer including sign and units — 1 point

$$W = 1.28 \times 10^{-18}\ \text{J} = 8.0\ \text{eV}$$

2 points

ndicating that the kinetic energy of the proton at E will be equal to the work done — 1 point

$$W = \Delta K = \frac{1}{2}mv^2$$

orrectly substituting the answer from part (e) i. into the above equation — 1 point

$$\left(1.28 \times 10^{-18}\ \text{J}\right) = \left(\frac{1}{2}\right)\left(1.67 \times 10^{-27}\ \text{kg}\right)v^2$$

$$v = 3.92 \times 10^4\ \text{m/s}$$

) 2 points

;orrectly selecting Left — 1 point

For a correct justification — 1 point

Example: Electrons accelerate in the direction perpendicular to equipotential surfaces and toward higher potential. Therefore, at point B, an electron would accelerate toward the left.

No points are earned if the wrong answer is selected.

;CORING GUIDELINES

Question

15 points total **Distribution of points**

2 points

For selecting "It is the same through both." 1 point

For a correct justification 1 point

Example: The sample of wire and the resistor are in series with each other and therefore will have the same current.

Note: Because the voltmeter is ideal, it will have no effect on the result.

No points are earned if the wrong answer is selected.

2 points

For selecting "It will depend on the resistance of the sample." 1 point

For a correct justification including argument using Ohm's law, voltage drop across resistors, etc. 1 point

Example: Because the sample of wire has resistance, it cannot be known whether the wire of the resistor has the greatest resistance and which has the higher potential difference across it. Therefore, it will depend on the resistance of the wire sample.

No points are earned if the wrong answer is selected.

(1 point

ıdicating the correct quantities for each axis 1 point

Horizontal axis: I_R

Vertical axis: $V_r = \mathcal{E} - V_R$

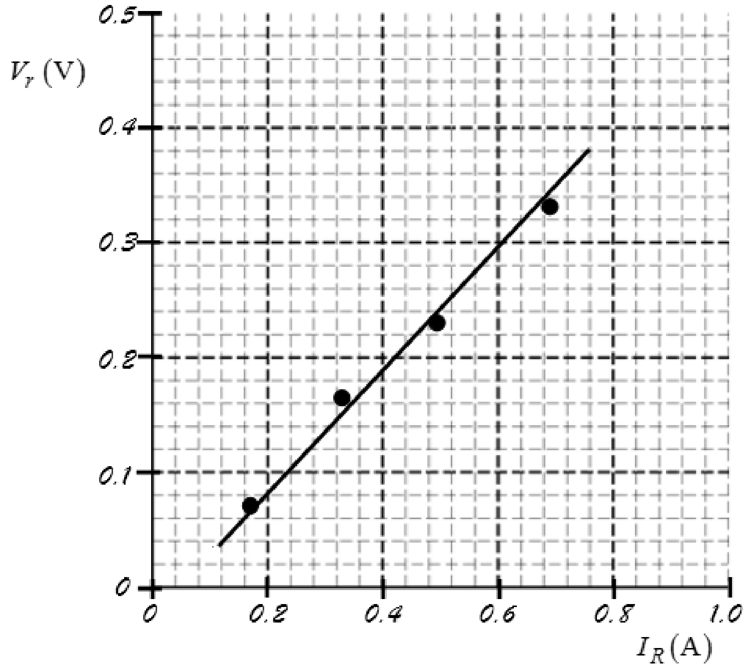
Note: Credit is received if the axes are reversed.

SCORING GUIDELINES

Question 2 (continued)

Distribution of points

3 points



For a correct scale that uses more than half the grid and for correctly labeling the axes including units — 1 point

For correctly plotting given data — 1 point

For drawing a straight line consistent with the given data — 1 point

(2 points

For correctly calculating slope using the best-fit straight line and not data points — 1 point

$$\text{slope} = \frac{(V_2 - V_1)}{(I_2 - I_1)} = \frac{(0.30 - 0.10)\ \text{V}}{(0.60 - 0.22)\ \text{A}} = 0.526\ \text{V/A}$$

Note: Linear regression gives $\text{slope} = 0.485\ \text{V/A}$. (Student must indicate that the value comes from a linear regression from the calculator, and the equation of the line must be present.)

For correctly relating r to the slope — 1 point

$V_r = I_R r$, therefore the slope equals r

$r = 0.526\ \Omega$

Note: Linear regression gives $r = 0.485\ \Omega$.

SCORING GUIDELINES

Question 2 (continued)

Distribution of points

) 1 point

For using the equation relating resistance to resistivity with the correct or consistent substitutions — 1 point

$$R = \frac{\rho L}{A} \quad \text{so} \quad \rho = \frac{AR}{L}$$

$$\rho = \frac{(\pi)\left(1.00 \times 10^{-3}\ \text{m}\right)^2 (0.526\ \Omega)}{(3.00\ \text{m})}$$

$$\rho = 5.51 \times 10^{-7}\ \Omega \bullet \text{m}$$

Note: Linear regression gives $\rho = 5.08 \times 10^{-7}\ \Omega \bullet \text{m}$.

2 points

electing "Less than" with an attempt at a justification — 1 point

For a correct justification — 1 point

Example: The resistance calculated from the graph is the sum of the sample resistance plus the ammeter resistance because it is not ideal. The actual resistance is the calculated resistance minus the ammeter resistance, and therefore less than the calculated resistance.

oints are earned if the wrong answer is selected.

2 points

For selecting "Greater than" with an attempt at a justification — 1 point

For a correct justification — 1 point

Example: If the voltmeter is not ideal, that would add an additional resistor in parallel. A parallel resistor reduces the total resistance of the circuit. This would lead to an increase in current and a higher reading on the ammeter.

No points are earned if the wrong answer is selected.

;CORING GUIDELINES

Question

15 points total **Distribution of points**

1 point

For selecting "Left" 1 point

2 points

For selecting "Less than" 1 point
For a correct justification 1 point
Example: Because as the bar falls the flux at point C is increasing, the emf generated must create a magnetic field to oppose this change. Therefore, it will create a magnetic field to decrease the flux and thus decrease the magnetic field.
No points are earned if the wrong answer is selected.

2 points

For selecting "Greater than" 1 point
For a correct justification 1 point
Example: The field at point C, which is above the bar, is less than the original magnetic field, and point D is on the other side of the bar. Therefore, the direction of the magnetic field from the bar at point D is the opposite of the direction at point C, so the net magnetic field at D when the bar is falling must be greater than the original magnetic field.
No points are earned if the wrong answer is selected.

(4 points

For correctly applying Newton's second law to the motion of the bar 1 point

$F_{net} = Mg - F_M = Mg - BIL$

$I = \mathcal{E}/R$

For attempting to use Faraday's law to obtain an expression for the emf in the bar 1 point

$\mathcal{E} = d\Phi/dt = BL(dx/dt)$

For correctly using the expression for emf to obtain an expression for the current 1 point

$$I = \frac{BL(dx/dt)}{R} = \frac{BLv}{R}$$

$$Ma = Mg - B\frac{BLv}{R}L$$

$$a = g - \frac{B^2L^2v}{MR}$$

vriting the acceleration as dv/dt 1 point

$$\frac{dv}{dt} = g - \frac{B^2L^2v}{MR}$$

;CORING GUIDELINES

Question 3 (continued)

Distribution of points

2 points

For setting the net force at terminal velocity equal to zero — 1 point

$$0 = Mg - \frac{B^2L^2v_T}{R}$$

$$\frac{B^2L^2v_T}{R} = Mg$$

ın answer consistent with part (c) — 1 point

$$v_T = \frac{MgR}{B^2L^2}$$

(1 point

For a correct or consistent substitution in an appropriate power equation — 1 point

$P = I^2R$ or $V = IR$

$$\frac{BLv_T}{R} = I$$

$$P = \frac{B^2L^2{v_T}^2}{R}$$

Alternate Solution — *Alternate points*

For a correct or consistent substitution in an appropriate power equation — *1 point*

$$P = I^2R$$

$$Mg = BIL$$

$$I = \frac{Mg}{BL}$$

$$P = \frac{M^2g^2R}{B^2L^2}$$

ʖCORING GUIDELINES

Question 3 (continued)

Distribution of points

) 3 points

Using the equation from part (c)

$$\frac{dv}{dt} = g - \frac{B^2L^2v}{MR}$$

$$\frac{dv}{dt} = \frac{-B^2L^2}{MR}\left(v - \frac{MRg}{B^2L^2}\right)$$

For attempting separation of variables — 1 point

$$\frac{dv}{v - \frac{MRg}{B^2L^2}} = \frac{-B^2L^2}{MR}dt$$

For attempting to integrate with the correct limits or the correct constant of integration — 1 point

$$\int_{v'=0}^{v'=v(t)} \frac{1}{\left(v' - \frac{MRg}{B^2L^2}\right)}\, dv' = \int_{t'=0}^{t'=t} \frac{-B^2L^2}{MR} dt'$$

$$\ln\left[v' - \frac{MRg}{B^2L^2}\right]_{v'=0}^{v'=v(t)} = \frac{-B^2L^2}{MR}t$$

$$\ln\left(\frac{v(t) - \frac{MRg}{B^2L^2}}{\frac{MRg}{B^2L^2}}\right) = \frac{-B^2L^2}{MR}t$$

For a correct answer — 1 point

$$v(t) = \frac{MRg}{B^2L^2}\left(1 - e^{-\frac{B^2L^2}{MR}t}\right)$$

ng a trial solution in the differential equation and verifying its correctness is also acceptable.