Question 2: Version J

2. A rotating, circular, conducting loop of area A and resistance R is in an external uniform magnetic field of magnitude B that is directed in the $-z$-direction. At time $t = 0$, the magnetic field is perpendicular to the plane of the loop, as shown in Figure 1. The loop is rotating with constant angular speed ω and period T about the dashed line that is along the diameter of the loop. The value of the magnetic flux through the loop as a function of time t is $\Phi = BA\cos(\omega t)$.

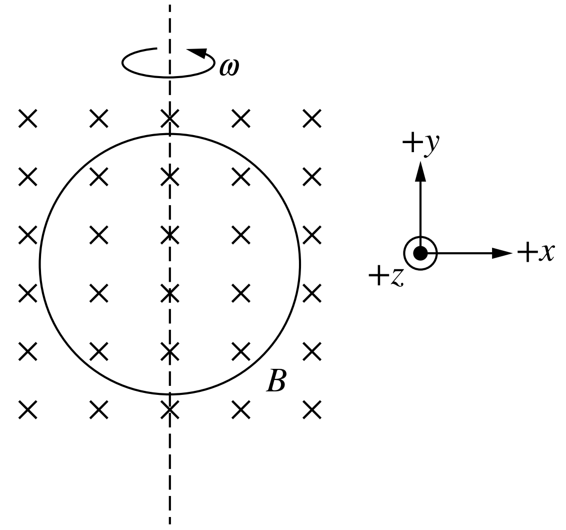


Figure 1

A. The absolute value of the induced emf in the loop is $|\varepsilon|$. The partially completed bar chart in Figure 2 shows a bar that represents $|\varepsilon|$ at $t = \frac{3}{4}T$. In Figure 2, **draw** bars to represent $|\varepsilon|$ at times $t = 0$, $\frac{1}{4}T$, and $\frac{1}{2}T$ relative to $|\varepsilon|$ shown at $\frac{3}{4}T$. If $|\varepsilon| = 0$, **write** a "0" in that column.

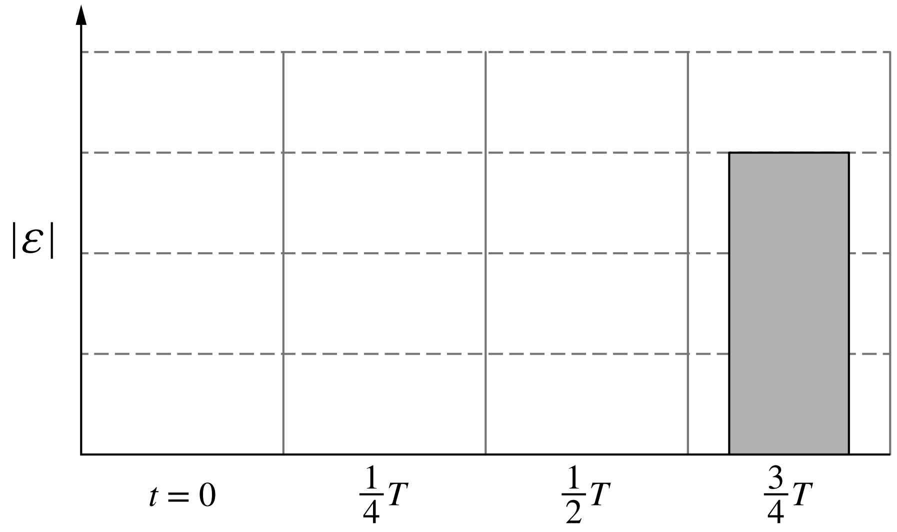


Figure 2

B. Derive an expression for the maximum induced current in the loop in terms of A, R, B, ω, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

C. On the axes shown in Figure 3, **sketch** a graph of the instantaneous power P dissipated by the loop as a function of t during the time interval $0 \leq t \leq T$.

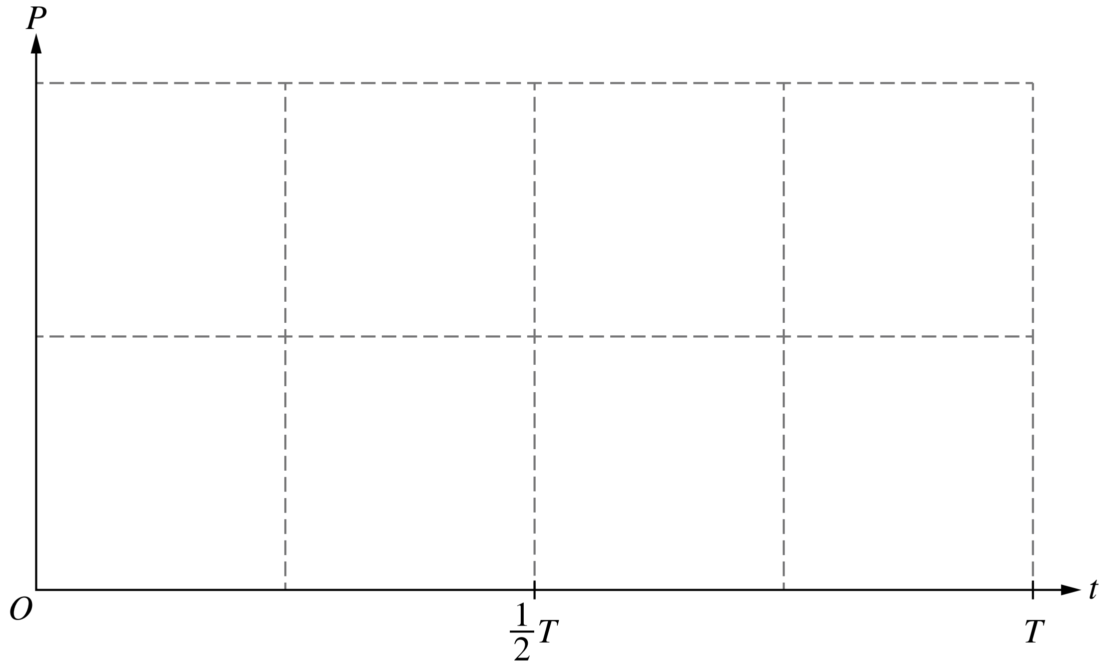


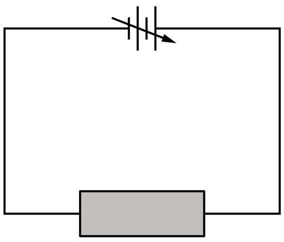
Figure 3

D. Indicate whether the sketch you drew in part C is or is not consistent with the bars that you drew in part A. Briefly **justify** your answer by referencing the functional dependence between P and $|\varepsilon|$.

Question 3

3. In Experiment 1, students are asked to use a graph to determine the resistivity ρ_1 of a circuit element that is connected to a variable power supply, as shown in Figure 1. The circuit element is cylindrical and has uniform resistivity. The students have access to a voltmeter, an ammeter, and a ruler.

Figure 1



Circuit Element

A. Describe a procedure for collecting data that would allow the students to use a graph to determine ρ_1, including any steps necessary to reduce experimental uncertainty.

B. Describe how the collected data could be graphed and how that graph would be analyzed to determine ρ_1.

In Experiment 2, the students are asked to use a graph to determine the resistivity ρ_2 of solid, cylindrical resistors made of the same material but of different lengths L. The cross-sectional area of each resistor is $5.0\times10^{-6}\ \text{m}^2$. The students directly measure the resistance R between the ends of each resistor. Table 1 provides L and R for each resistor.

Table 1

L (m)	R (Ω)
0.010	0.90
0.020	1.6
0.030	2.5
0.040	3.2
0.050	4.0

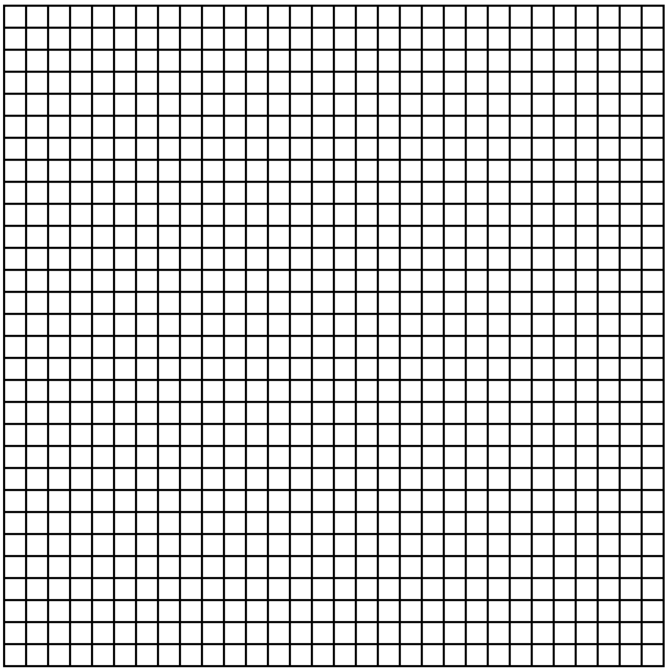
C.

i. **Indicate** two quantities, either measured quantities from Table 1 or additional calculated quantities, that could be graphed to produce a straight line that could be used to determine ρ_2.

Vertical axis: ______ Horizontal axis: ______

ii. On the grid provided, create a graph of the quantities indicated in part C (i).

- Use Table 2 to record the measured or calculated quantities that you will plot.
- Clearly **label** the axes, including units as appropriate.
- **Plot** the points you recorded in Table 2.



iii. **Draw** a best-fit line for the data graphed in part C (ii).

D. Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for ρ_2.

Begin your response to **QUESTION 2** on this page.

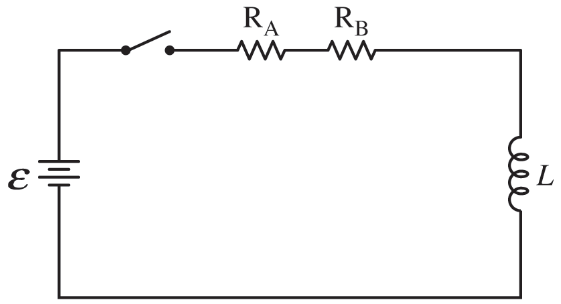


Figure 1

2. Students are asked to determine the resistance R of identical resistors R_A and R_B. The resistors are connected in series with each other, a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that increases with time to determine R.

(a)

i. On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that increases with time.

Voltmeter Symbol

ii. **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using a measured quantity that increases with time. Provide enough detail so that another student could replicate the experiment.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b)

i. On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.

Figure 2

ii. **Describe** how the information from the graph in part (b)(i) would be used to determine the experimental value for R.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(c) Starting with an appropriate application of Kirchhoff's loop rule, **derive,** but do NOT solve, a differential equation that can be used to determine the current I in the inductor at time t after the switch is closed. Express your answer in terms of R, $\mathcal{E}$, L, t, and physical constants, as appropriate.

After reaching steady state, the absolute value of the potential difference across R_A is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across R_A is $|\Delta V_2|$.

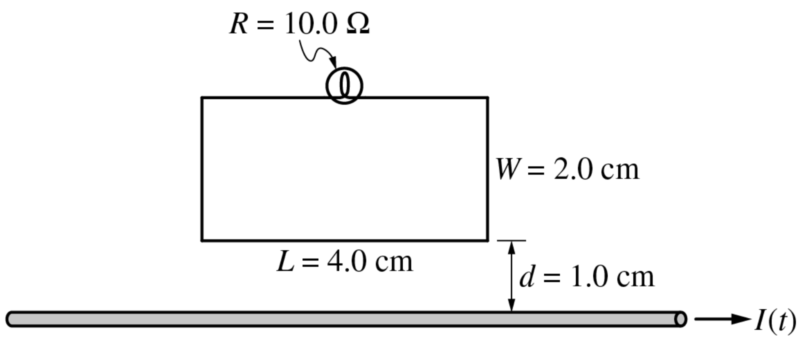
(d) **Indicate** whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

____ $|\Delta V_2| > |\Delta V_1|$ ____ $|\Delta V_2| < |\Delta V_1|$ ____ $|\Delta V_2| = |\Delta V_1|$

Justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 3** on this page.



A lightbulb of resistance $R = 10.0\ \Omega$ is connected to a rectangular loop of wire of negligible resistance near a very long current-carrying wire. The rectangular loop has a length $L = 4.0$ cm and a width $W = 2.0$ cm and is positioned so one of the longer sides of the loop is a distance $d = 1.0$ cm above and parallel to the long wire, as shown. The current in the long wire is initially flowing to the right and is given by $I(t) = C - Dt$, where $C = 10.0$ A and $D = 2.0$ A/s. At time $t = 5.0$ s, the current in the long wire is instantaneously zero as the current changes direction.

(a) What is the direction, if any, of the magnetic field produced by the induced current in the rectangular loop as the current in the long wire changes direction?

___ Into the page   ___ Out of the page   ___ No direction, because the field is zero

Justify your answer.

(b) Calculate the magnetic flux through the <u>loop</u> due to only the long wire at time $t = 3.0$ s.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(c) Calculate the current through the lightbulb at time $t = 3.0\,\text{s}$.

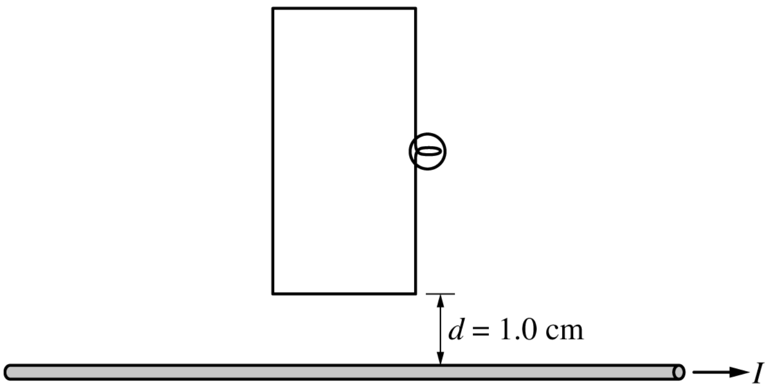
(d) A group of students attempts to experimentally verify whether the current through the lightbulb is consistent with the current calculation from part (c). The current in the rectangular loop is measured to be greater than the current calculated in part (c). Which of the following could explain this discrepancy? Select one answer.

___ The students did not account for Earth's magnetic field.
___ The rectangular loop is tilted and is not in the same plane as the wire.
___ The resistance of the lightbulb is greater than the recorded value.
___ The long side of the rectangular loop is shorter than the recorded value.
___ The current in the long wire changes at a faster rate than expected.

Briefly justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.



(e) Later, the same rectangular loop with lightbulb is rotated such that a short side of the loop is 1.0 cm above and parallel to the long current-carrying wire, as shown. The current in the wire is again initially flowing from left to right and given by $I(t) = C - Dt$, where $C = 10.0$ A and $D = 2.0$ A/s. The current through the lightbulb in the loop's new orientation at time $t = 3.0$ s is I_2. Which of the following correctly relates the current I_2 to I_1, the current through the lightbulb in part (c)?

____ $I_2 < I_1$ ____ $I_2 = I_1$ ____ $I_2 > I_1$

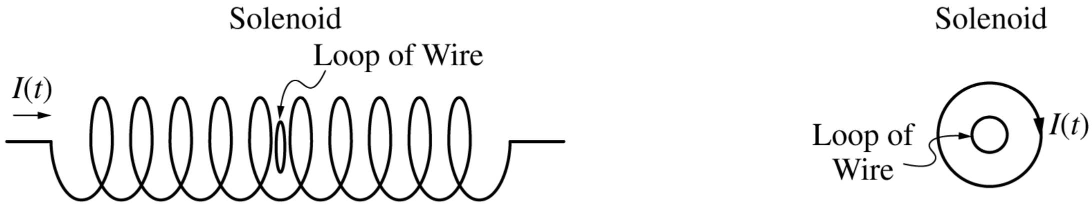
Justify your answer.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

Begin your response to **QUESTION 3** on this page.



Side View End View

Note: Figures not drawn to scale.

. A single loop of wire with resistance $3.0\,\Omega$ and radius 0.10 m is placed inside a solenoid, with the normal to the loop parallel to the axis of the solenoid. The solenoid has 500 turns, is 0.25 m long, and is connected to a power supply that is not shown. At time $t = 0$, the power supply is turned on, and the current I in the solenoid as a function of t is given by the equation $I(t) = \beta t$, where $\beta = 5.0$ A/s. The direction of the current in the solenoid is clockwise, as shown in the end view.

(a) At time $t = 2.0$ s, is the induced current in the loop, as seen from the end view shown, clockwise, counterclockwise, or zero?

___ Clockwise ___ Counterclockwise ___ Zero

Justify your answer.

(b) Calculate the current in the loop of wire at time $t = 2.0$ s.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(c) Calculate the total energy dissipated by the loop of wire from time $t = 0$ to time $t = 2.0\text{ s}$.

(d) A group of students attempts to verify experimentally the calculation of the current from part (b). The current in the inner circular loop at time $t = 2.0\text{ s}$ is measured to be less than the current calculated in part (b). Which of the following could explain this discrepancy? Select one answer.

___ The experiment did not account for Earth's magnetic field.
___ The plane of the loop is not perpendicular to the axis of the solenoid.
___ The center of the loop is not on the axis of the solenoid.
___ The resistance of the loop is less than the given value.
___ The radius of the loop is actually larger than 0.10 m.

Justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(e) The power supply is now turned off. The original loop of wire is then replaced with a second loop made from wire that has the same thickness and is made from the same material as the original loop of wire. The second loop has radius 0.20 m, is placed in the same orientation as the original loop, and fits completely inside the solenoid. The power supply is turned on, and the current I in the solenoid as a function of t is again given by the equation $I(t) = \beta t$, where $\beta = 5.0$ A/s. Which of the following expressions correctly indicates the ratio $\frac{I_2}{I_1}$ where I_1 represents the current induced in the original loop of wire in part (b) and I_2 represents the current induced in the second loop of wire?

___ $\frac{I_2}{I_1} = 1$ ___ $1 < \frac{I_2}{I_1} < 2$ ___ $\frac{I_2}{I_1} = 2$ ___ $\frac{I_2}{I_1} > 2$

Justify your answer.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

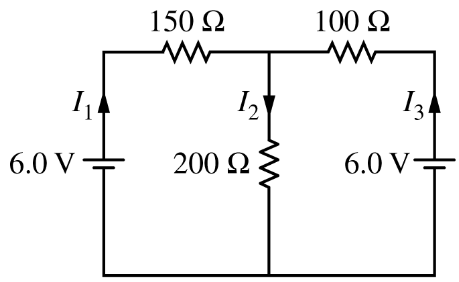


Figure 1

2. The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(a)

i. Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

ii. Calculate the current in the $200\ \Omega$ resistor.

iii. Calculate the power dissipated by the $200\ \Omega$ resistor.

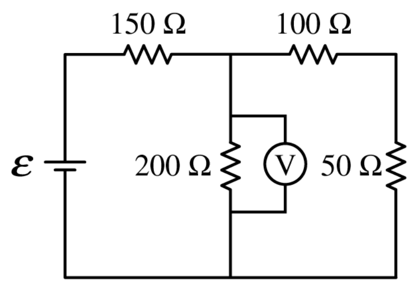


Figure 2

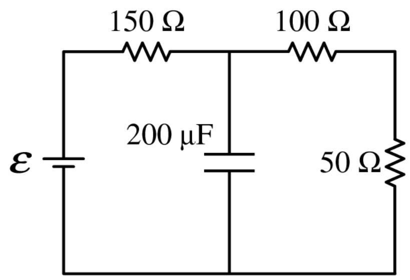
The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

(b) Calculate the current through the $50\ \Omega$ resistor.

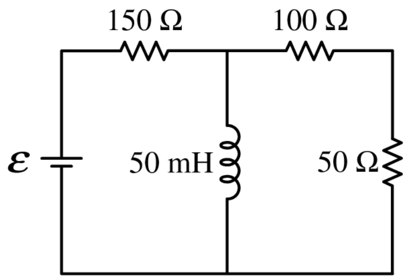
(c) Calculate the voltage $\mathcal{E}$ of the battery.

(d)

i. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu\text{F}$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.

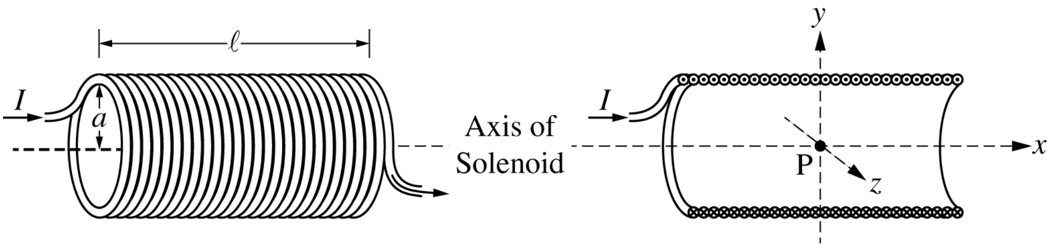


ii. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b) ?



____ Greater than ____ Less than ____ Equal to

Justify your answer.



Solenoid Cutaway View

Note: Figures not drawn to scale.

3. A solenoid is used to generate a magnetic field. The solenoid has an inner radius a, length ℓ, and N total turns of wire. A power supply, not shown, is connected to the solenoid and generates current I, as shown in the figure on the left above. The x-axis runs along the axis of the solenoid. Point P is in the middle of the solenoid at the origin of the xyz-coordinate system, as shown in the cutaway view on the right above. Assume $\ell >> a$.

(a) Select the correct direction of the magnetic field at point P.

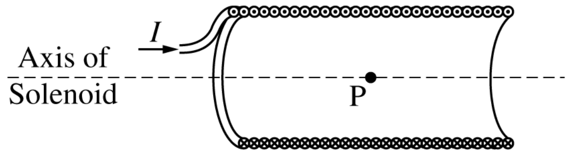
____ $+x$-direction ____ $+y$-direction ____ $+z$-direction

____ $-x$-direction ____ $-y$-direction ____ $-z$-direction

Justify your selection.

(b)

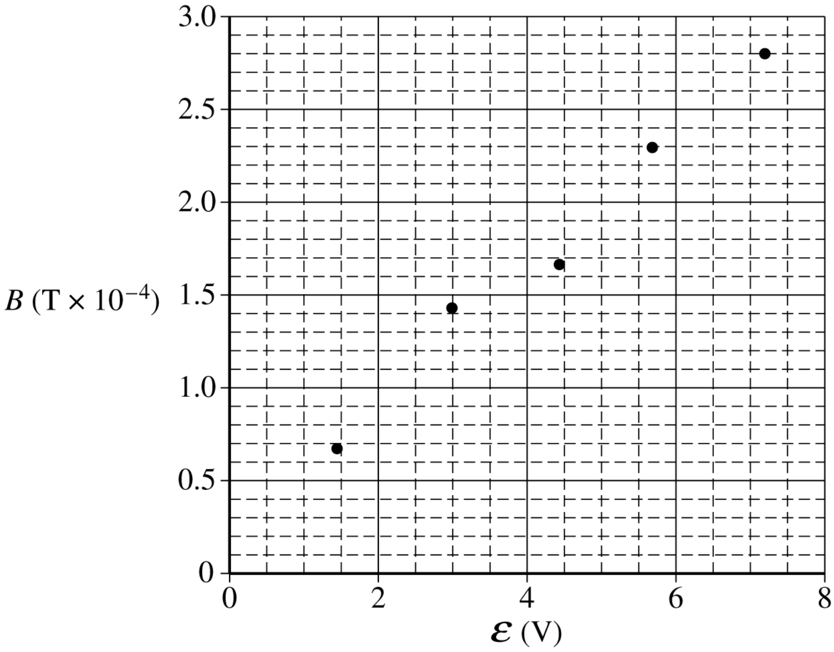
i. On the cutaway view below, clearly draw an Amperian loop that can be used to determine the magnetic field at point P at the center of the solenoid.



Cutaway View

ii. Use Ampere's law to derive an expression for the magnetic field strength at point P. Express your answer in terms of I, ℓ, N, a, and physical constants, as appropriate.

Some physics students conduct an experiment to determine the resistance R_S of a solenoid with radius $a = 0.015$ m, total turns $N = 100$, and total length $\ell = 0.40$ m. The students connect the solenoid to a variable power supply. A magnetic field sensor is used to measure the magnetic field strength along the central axis at the center of the solenoid. The plot of the magnetic field strength B as a function of the emf $\mathcal{E}$ of the power supply is shown below.



(c)

i. On the graph above, draw a best-fit line for the data.

ii. Use the straight line to determine the resistance R_S of the solenoid used in the experiment.

(d) One of the students notes that the horizontal component of the magnetic field of Earth is 2.5×10^{-5} T.

i. Is there evidence from the graph that the horizontal orientation of the solenoid affects the measured values for B ?

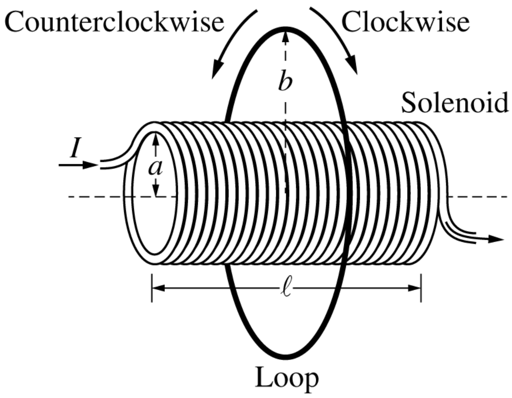
____ Yes ____ No

Justify your answer.

ii. Would the horizontal orientation of the solenoid affect the calculated value for R_S ?

____ Yes ____ No

Justify your answer.



A thin conducting loop of radius b and resistance R_L is placed concentric with the solenoid, as shown above. The current in the solenoid is decreased from I to zero over time Δt.

(e)

i. Is the direction of the induced current in the loop clockwise or counterclockwise during the time period that the current in the solenoid is decreasing?

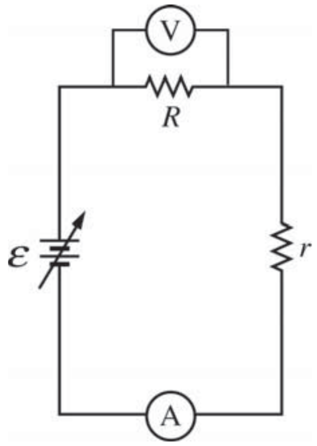
____ Clockwise ____ Counterclockwise

Justify your answer.

ii. Derive an equation for the average induced current i_{IND} in the loop during the time period that the current in the solenoid is decreasing. Express your answer in terms of I, ℓ, N, a, b, R_L, R_S, Δt, and physical constants, as appropriate.

STOP

END OF EXAM



E&M.2.

The circuit shown above consists of a source of variable emf $\mathcal{E}$, an ideal ammeter A, an ideal voltmeter V, a resistor of resistance *R*, and a sample of wire with resistance *r*.

(a) How does the current through the wire sample compare with the current through the resistor *R* ?

_____ It is greater through *R*.   _____ It is greater through the sample.

_____ It is the same through both.   _____ It depends on the resistance of the sample.

Justify your answer.

(b) How does the potential difference across the wire sample compare with the potential difference across the resistor *R* ?

_____ It is greater across *R*.   _____ It is greater across the sample.

_____ It is the same across both.   _____ It depends on the resistance of the sample.

Justify your answer.

With the sample of wire in place, the emf of the source is set to a given value. The current through and potential difference across the resistor *R* are measured. This is repeated for several values of emf, and the data are recorded in the table below.

$\mathcal{E}$ (V)	V_R (V)	I_R (A)		
0.250	0.179	0.162		
0.500	0.335	0.327		
0.750	0.520	0.490		
1.000	0.670	0.687		

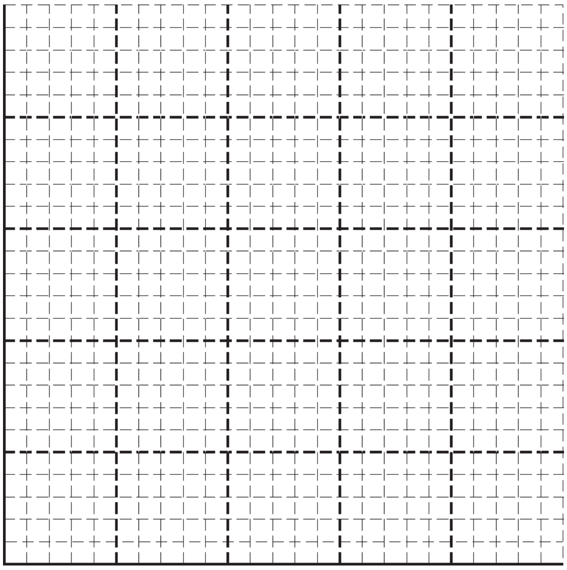
(c) Indicate below which quantities should be graphed to yield a straight line that could be used to calculate a numerical value for the resistance of the wire sample.

Horizontal axis: ____________

Vertical axis: ____________

You may use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given.

(d) On the grid below, plot the straight line data points from part (c). Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(e) Use your straight line to calculate the value of the resistance of the wire sample.

(f) The wire sample has a length of 3.00 m and a radius of 1.00×10^{-3} m . Calculate the resistivity of the material from which the wire sample is made.

(g)

i. Suppose the ammeter used to collect these data was not ideal. Would the actual value of the resistance of the wire sample be greater than, less than, or equal to that calculated in part (e) ?

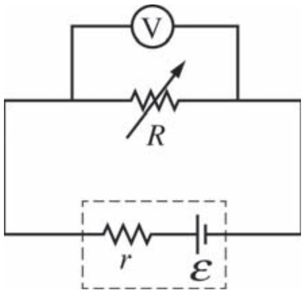
____ Greater than ____ Less than ____ Equal to

Justify your answer.

ii. If the ideal voltmeter is replaced by a voltmeter that is not ideal and the experiment is repeated, would the readings of the ideal ammeter be greater than, less than, or equal to those in the data chart before part (c) ?

____ Greater than ____ Less than ____ Equal to

Justify your answer.



E&M.2.

A student performs an experiment to determine the emf $\mathcal{E}$ and internal resistance r of a given battery. The student connects the battery in series to a variable resistance R, with a voltmeter across the variable resistor, as shown in the figure above, and measures the voltmeter reading V as a function of the resistance R. The data are shown in the table below.

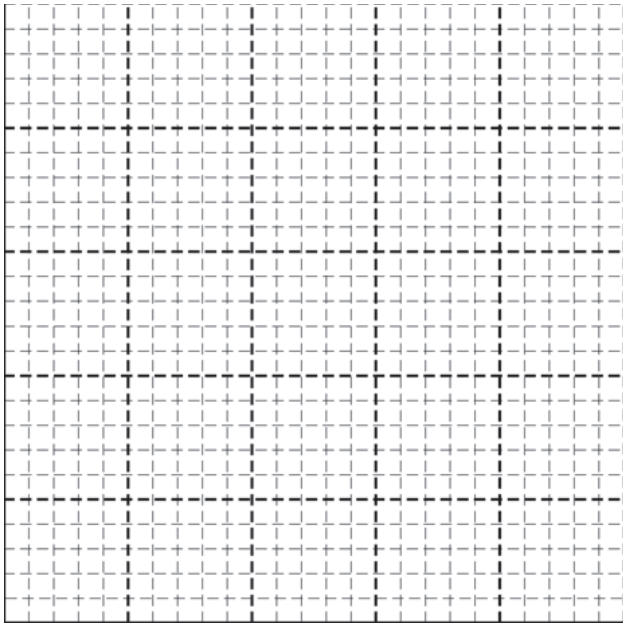
Trial #	Resistance (Ω)	Voltage (V)	$1/R$ (1/Ω)	$1/V$ (1/V)
1	0.50	5.6	2.00	0.179
2	1.0	7.4	1.00	0.135
3	2.0	9.4	0.50	0.106
4	3.0	10.6	0.33	0.094
5	5.0	10.9	0.20	0.092
6	10	11.4	0.10	0.088

(a)

i. Derive an expression for the measured voltage V. Express your answer in terms of R, $\mathcal{E}$, r, and physical constants, as appropriate.

ii. Rewrite your expression from part (a)-i to express $1/V$ as a function of $1/R$.

(b) On the grid below, plot data points for the graph of $1/V$ as a function of $1/R$. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.



(c) Use the straight line from part (b) to obtain values for the following.

i. $\mathcal{E}$

ii. r

(d) Using the results of the experiment, calculate the maximum current that the battery can provide.

(e) A voltmeter is to be used to determine the emf of the battery after removing the battery from the circuit. Two voltmeters are available to take this measurement—one with low internal resistance and one with high internal resistance. Indicate which voltmeter will provide the most accurate measurement.

____ The voltmeter with low resistance will provide the most accurate measurement.

____ The voltmeter with high resistance will provide the most accurate measurement.

____ The two voltmeters will provide equal accuracy.

Justify your answer.

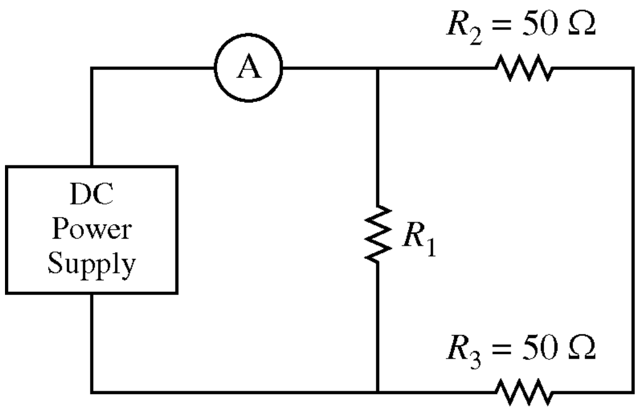
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

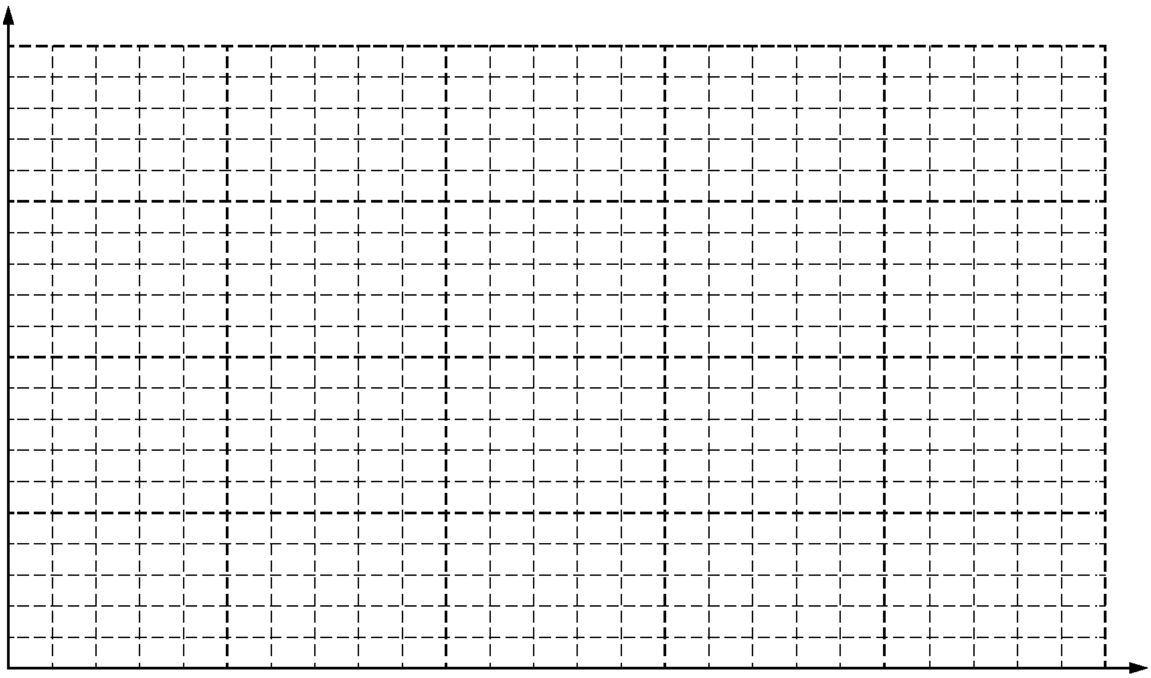


E&M. 1.

Physics students are analyzing the circuit above. A variable DC power supply is connected to an ammeter and three resistors. The resistances of two of the resistors are known to be $R_2 = R_3 = 50\ \Omega$, but the resistance of the third resistor is unknown. The students collect data on the potential difference across the power supply and the current measured by the ammeter, as follows.

Potential Difference (V)	2	4	6	8	10
Current (mA)	40	55	97	138	155

(a) On the grid below, plot the data points for the current as a function of the potential difference. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



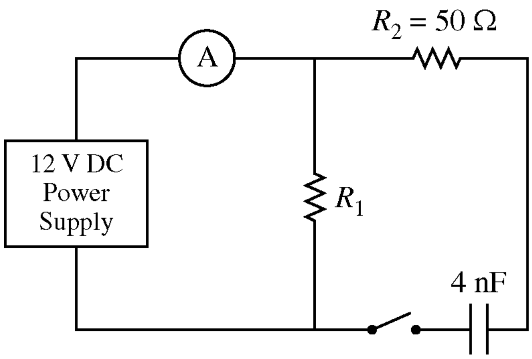
(b) Using the straight line from part (a), calculate the total resistance of the three-resistor combination.

(c) Calculate the value of R_1.

The power supply is now fixed at 12 V.

(d) Calculate the current through R_2.

(e) Resistor 3 is now removed and replaced by an open switch in series with an uncharged 4 nF capacitor, as shown below. The power supply is still fixed at 12 V.



i. Calculate the current in R_2 immediately after the switch is closed.

ii. A long time after the switch is closed, will the magnitude of the current in R_2 be greater than, less than, or equal to the current through R_2 found in part (d) ?

__ Greater than __ Less than __ Equal to

Justify your answer.

(f) The 4 nF capacitor is replaced with an uncharged 10 nF capacitor. Will the magnitude of the current in R_2 immediately after the switch is closed be greater than, less than, or equal to the current in part (e)i ?

__ Greater than __ Less than __ Equal to

Justify your answer.