

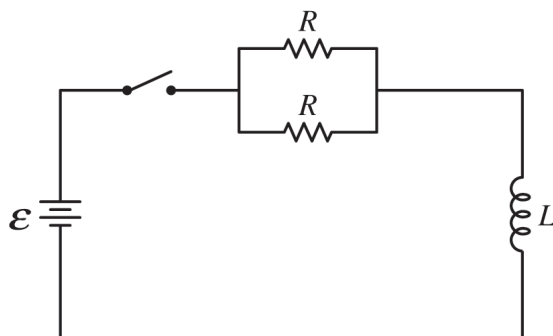
Begin your response to **QUESTION 2** on this page.

Figure 1

2. Students are asked to determine the resistance R of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf $\mathcal{E}$, an inductor of known inductance L , and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that decreases with time to determine R .

(a)

- i. On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that decreases with time.



Voltmeter Symbol

- ii. **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using the measured quantity that decreases with time. Provide enough detail so that another student could replicate the experiment.

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(b)

i. On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.



Figure 2

ii. **Describe** how the information from the graph in part (b)(i) would be used to determine the experimental value for R .

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Continue your response to **QUESTION 2** on this page.

- (c) Starting with an appropriate application of Kirchhoff's loop rule, **derive**, but do NOT solve, a differential equation that can be used to determine the current I in the inductor at time t after the switch is closed. Express your answer in terms of R , $\mathcal{E}$, L , t , and physical constants, as appropriate.

After reaching steady state, the absolute value of the potential difference across the inductor is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across the new inductor is $|\Delta V_2|$.

- (d) **Indicate** whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

_____ $|\Delta V_2| > |\Delta V_1|$ _____ $|\Delta V_2| < |\Delta V_1|$ _____ $|\Delta V_2| = |\Delta V_1|$

Justify your answer.

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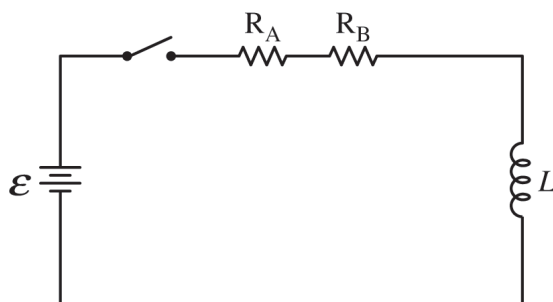
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Figure 1

2. Students are asked to determine the resistance R of identical resistors R_A and R_B . The resistors are connected in series with each other, a battery of known emf $\mathcal{E}$, an inductor of known inductance L , and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that increases with time to determine R .

(a)

- i. On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that increases with time.



Voltmeter Symbol

- ii. **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using a measured quantity that increases with time. Provide enough detail so that another student could replicate the experiment.

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Continue your response to **QUESTION 2** on this page.

- (b)
- i. On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.
- **Label** the quantities graphed on the vertical and horizontal axes.
 - **Sketch** a line or curve that represents the expected trend of the collected data.
 - **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.



Figure 2

- ii. **Describe** how the information from the graph in part (b)(i) would be used to determine the experimental value for R .

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

- (c) Starting with an appropriate application of Kirchhoff's loop rule, **derive**, but do NOT solve, a differential equation that can be used to determine the current I in the inductor at time t after the switch is closed. Express your answer in terms of R , $\mathcal{E}$, L , t , and physical constants, as appropriate.

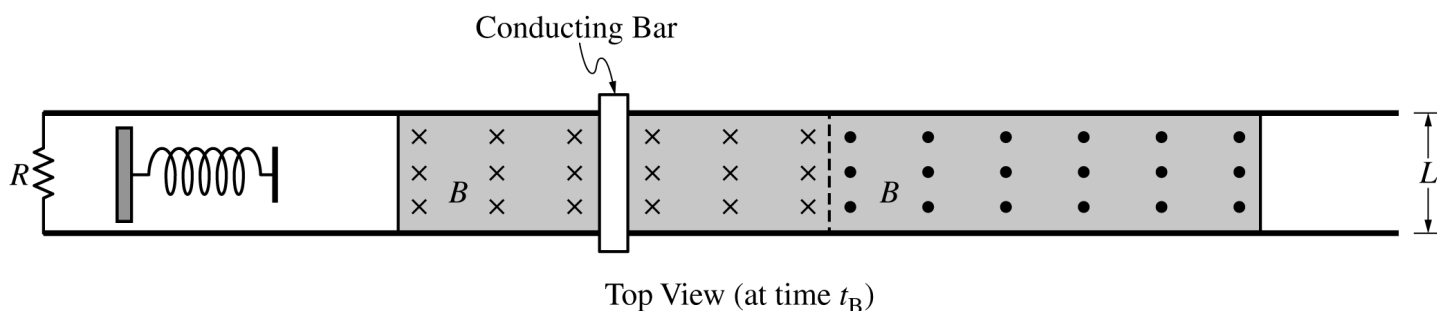
After reaching steady state, the absolute value of the potential difference across R_A is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across R_A is $|\Delta V_2|$.

- (d) **Indicate** whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

_____ $|\Delta V_2| > |\Delta V_1|$ _____ $|\Delta V_2| < |\Delta V_1|$ _____ $|\Delta V_2| = |\Delta V_1|$

Justify your answer.

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Two horizontal, parallel, conducting rails are separated by distance $L = 0.40$ m. A resistor of resistance $R = 0.30 \, \Omega$ connects the rails. A horizontal ideal spring is located between the rails. The right end of the spring is free to move and the left end is fixed in place. A conducting bar of mass $m = 0.23$ kg is placed on the rails and is in contact with the spring, which is initially compressed. Frictional forces and the resistance of the bar and rails are negligible.

- At time $t = 0$, the bar is released from rest and is pushed to the right by the spring.
- At time t_1 , the bar loses contact with the spring and slides to the right.
- At time t_2 , the bar enters and travels through a uniform magnetic field of magnitude $B = 0.50$ T that is directed into the page, as shown.
- At time t_3 , the bar enters a region where the magnitude of the uniform magnetic field is still $B = 0.50$ T but is directed out of the page.
- At time t_4 , the bar enters a region with no magnetic field.

Consider time t_B such that $t_2 < t_B < t_3$.

- (a) On the following diagram of the bar, draw an arrow indicating the direction of the net force F_{net} exerted on the bar at time t_B . If the net force is zero, write $F_{\text{net}} = 0$.



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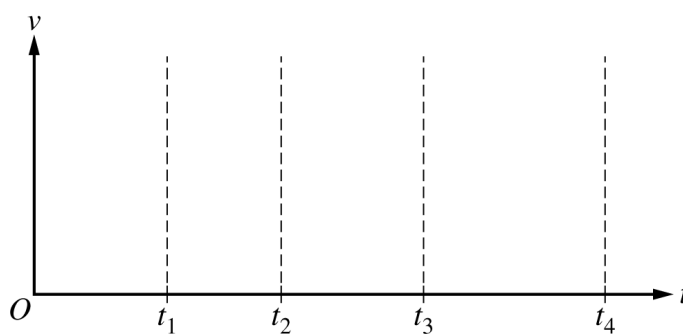
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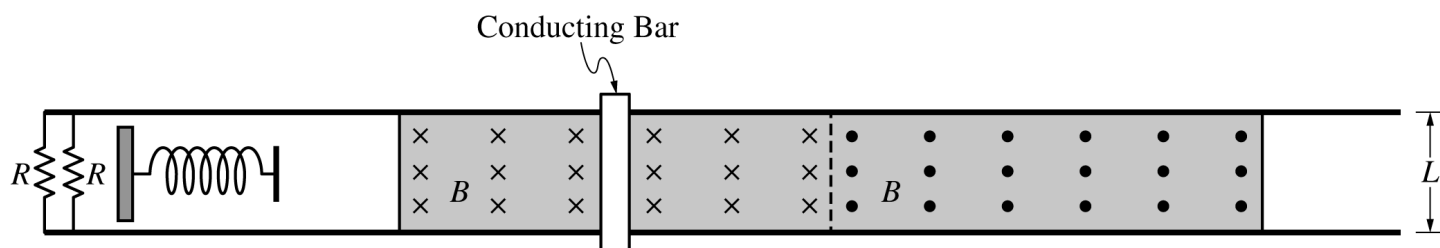
(b) At time t_B , the speed of the bar is $v = 2.5$ m/s.

i. Calculate the magnitude of the current in the bar at time t_B .

ii. Calculate the magnitude of the net force F_{net} exerted on the bar at time t_B .

(c) On the following axes, sketch a graph of the speed v of the bar as a function of time t between $t = 0$ and t_4 .

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Top View (at time t_B)

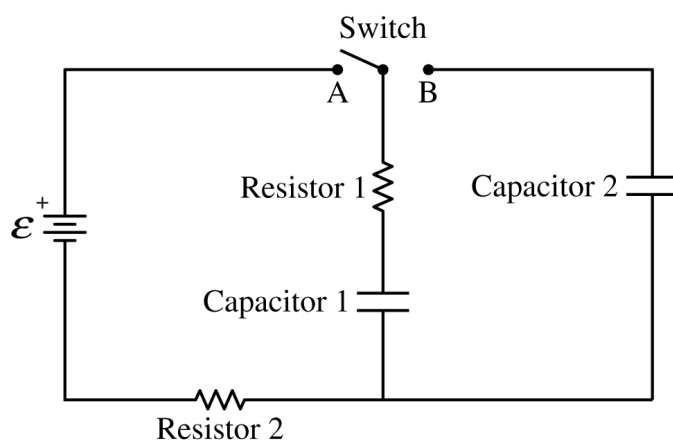
(d) The scenario is repeated but an additional resistor of resistance $R = 0.30 \, \Omega$ is connected, as shown.

i. Determine the total resistance R_{total} of the closed circuit for the new scenario.

ii. In the original scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{original} . In the new scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{new} . Is a_{new} greater than, less than, or equal to a_{original} ? Justify your answer.

(e) Describe a modification to m , B , or L that will result in a smaller induced potential difference across the original resistor immediately after the bar enters the first uniform magnetic field. Justify your answer.

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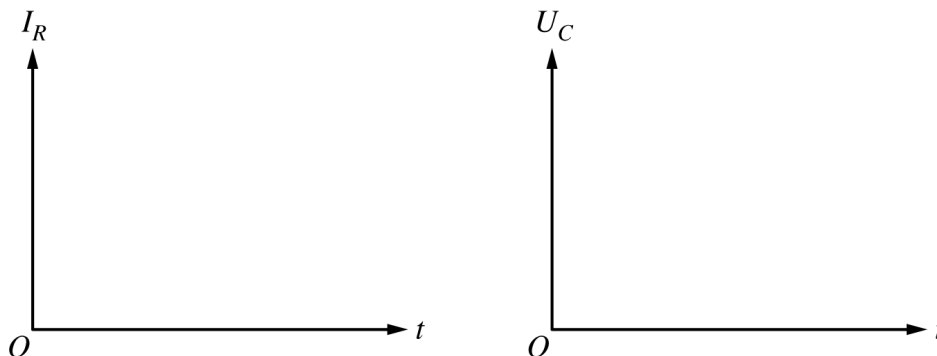
- The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R , capacitors 1 and 2 each with capacitance C , and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

- (a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R , C , Q , t , and fundamental constants, as appropriate.

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- (b) On the axes shown, sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.



A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

- (c) At time t_1 , the switch is closed to Position B.

i. Immediately after t_1 , is the direction of the current in Resistor 1 directed up, directed down, or is there no current? Briefly justify your answer.

ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1 . Express your answer in terms of Q_0 and fundamental constants, as appropriate.

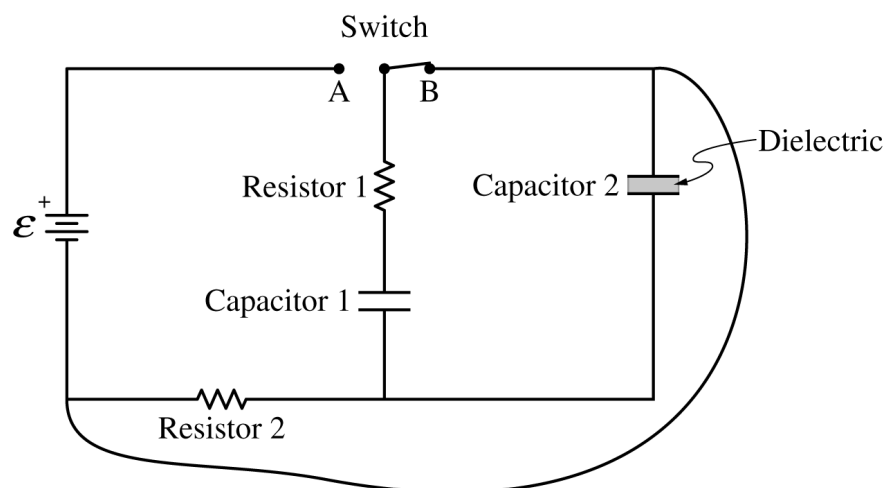
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iii. Derive an expression for the total energy dissipated by Resistor 1 immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C , Q_0 , and fundamental constants, as appropriate.

With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

- (d) Determine the charge on the positive plate of Capacitor 2 a long time after the dielectric has been inserted. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

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With the switch still closed to Position B, a wire of negligible resistance is connected between two corners of the circuit, as shown.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R , C , Q_0 , and fundamental constants, as appropriate.

i. Derive an expression for the current in Resistor 2 immediately after the wire is connected to the circuit.

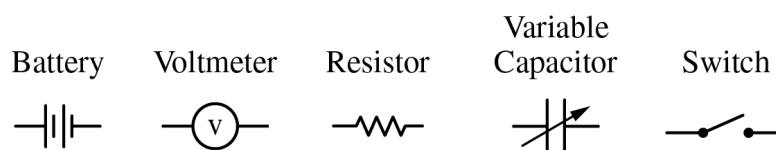
ii. Determine the current in Resistor 2 a long time after the wire is connected to the circuit.

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STOP

END OF EXAM

The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0 , a single voltmeter, a single resistor of resistance R , a single uncharged variable capacitor set to capacitance C , and one or more switches as needed.



- (a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the potential difference across the capacitor as it discharges through the resistor.

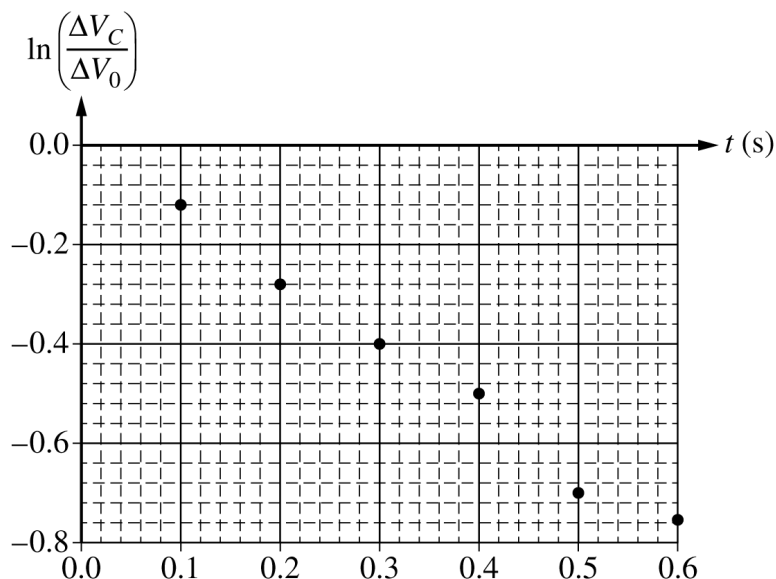
The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

- (b) Show that the potential difference ΔV_C across the capacitor as a function of time t is $\Delta V_C(t) = \Delta V_0 e^{-\frac{t}{RC}}$ as the capacitor discharges.

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- (c) The experiment is performed using a resistor of $R = 150 \text{ k}\Omega$. Data for the potential difference ΔV_C across the capacitor as a function of t are recorded and a plot of $\ln\left(\frac{\Delta V_C}{\Delta V_0}\right)$ as a function of t is created on the graph below.



- Draw the best-fit line for the data.
- Using the best-fit line, calculate a value for the unknown capacitance C .

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Continue your response to **QUESTION 2** on this page.

- (d) The capacitor is adjusted so that the surface area of the plates is increased, and the experiment is repeated. Would the slope of the best-fit line in the second experiment be more steep, less steep, or unchanged compared to the slope of the best-fit line in part (c)?

☐ More steep ☐ Less steep ☐ Unchanged

Briefly justify your answer.

- (e) The ideal battery is then replaced with a non-ideal battery with internal resistance r , and the experiment is repeated.

- i. Would the slope of the graph in this final experiment change compared to the graph in part (c)?

☐ Yes ☐ No

Briefly justify your answer.

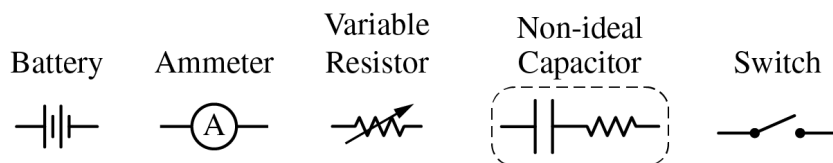
- ii. Would the vertical intercept of the graph in this final experiment change compared to the graph in part (c)?

☐ Yes ☐ No

Briefly justify your answer.

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A non-ideal capacitor has internal resistance that can be modeled as an ideal capacitor in series with a small resistor of resistance r_C . A group of students performs an experiment to determine the internal resistance of a capacitor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0 , a single ammeter, a single variable resistor of resistance R , a single uncharged non-ideal capacitor of capacitance C , and one or more switches as needed.



- (a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the current through the capacitor as it discharges through the resistor.

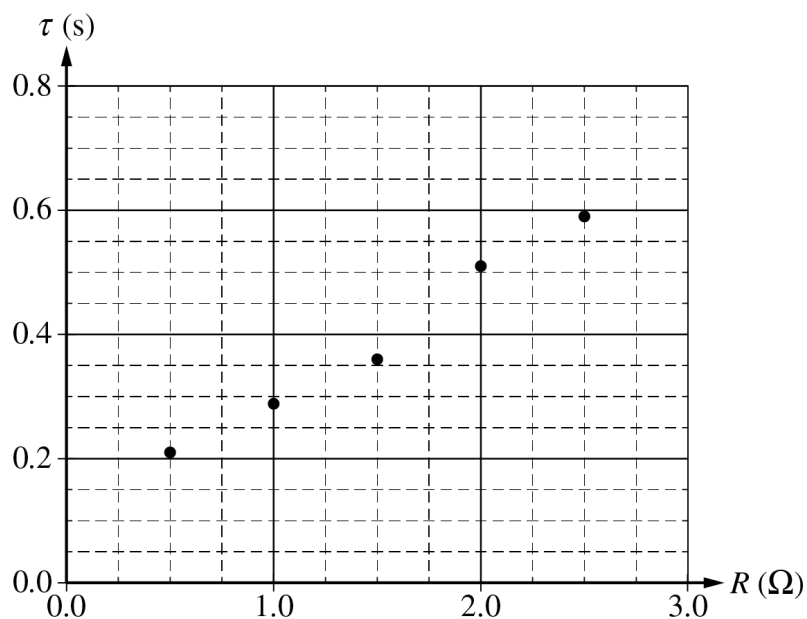
The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

- (b) Show that the current I through the capacitor as a function of time t is $I(t) = I_0 e^{\frac{-t}{(R+r_C)C}}$ as the capacitor discharges.

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- (c) The students determine the time constant τ for the circuit as a function of the resistance R . The students' data are shown in the following graph.



- Draw the best-fit line for the data.
- Using the best-fit line, calculate a value for the internal resistance r_C of the capacitor.

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Continue your response to **QUESTION 2** on this page.

- (d) The ammeter is found to be nonideal. Is the actual value for the internal resistance r_C for the capacitor greater than, less than, or equal to the experimental internal resistance of the capacitor calculated in part (c)?

☐ Greater than ☐ Less than ☐ Equal to

Briefly justify your answer using features of the graph in part (c).

- (e) The values of the variable resistor in the original experiment ranged from $0.5\ \Omega$ to $2.5\ \Omega$. The experiment is repeated with values ranging from $3.0\ \Omega$ to $6.0\ \Omega$. Would the slope of the best-fit line be more steep, be less steep, or remain unchanged compared to the graph in part (c)?

☐ More steep ☐ Less steep ☐ Remain unchanged

Briefly justify your answer.

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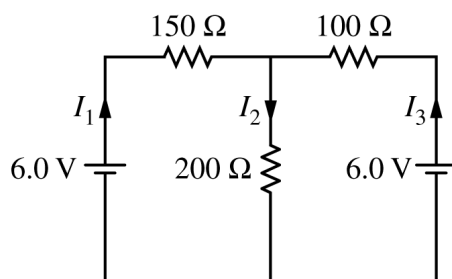


Figure 1

2. The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1 , I_2 , and I_3 in each branch of the circuit are indicated.

- (a)
- Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.
 - Calculate the current in the $200\ \Omega$ resistor.
 - Calculate the power dissipated by the $200\ \Omega$ resistor.

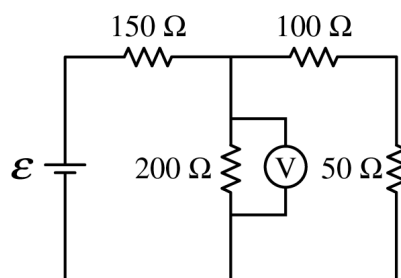


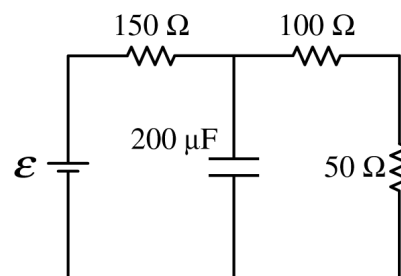
Figure 2

The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

- (b) Calculate the current through the $50\ \Omega$ resistor.
- (c) Calculate the voltage $\mathcal{E}$ of the battery.

(d)

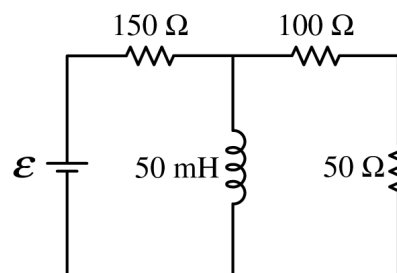
- i. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu\text{F}$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.

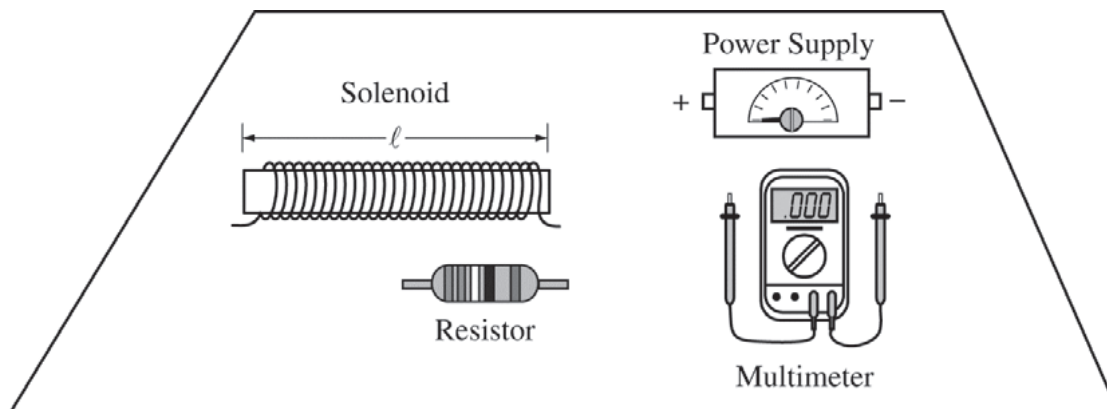


- ii. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal $50\ \text{mH}$ inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b) ?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.





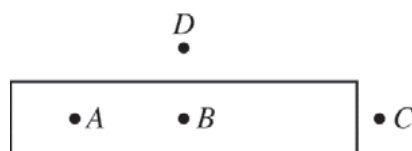
3. When studying Ampere's law, students collect data on the magnetic field of two different solenoids in order to determine the magnetic permeability of free space μ_0 . The solenoids are created by wrapping wire around a hollow plastic tube. The solenoids of length ℓ with N turns of wire will be connected in series to a power supply and resistor. A multimeter will be used as an ammeter to measure the magnitude of the current I through the solenoids. The main components for the setup with one of the solenoids are shown in the figure above.

(a)

- On the figure above, draw wire connections between the solenoid, power supply, resistor, and multimeter that will complete the circuit and allow students to measure the magnitude of the current through the solenoid.
- Using the connections you made in part (a)i above, what will be the direction of the magnetic field inside the solenoid?

☐ Toward the top of the page	☐ To the left	☐ Out of the page
☐ Toward the bottom of the page	☐ To the right	☐ Into the page

The rectangle shown below represents the solenoid (the loops of wire are not shown). Points A , B , and C are along the central axis of the solenoid with point B at the middle of the solenoid. Point D is directly above point B .

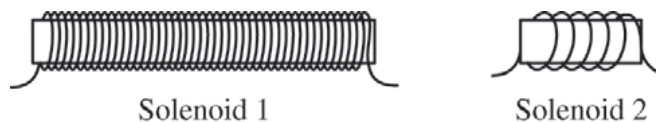


- From the choices below, select the point where you would place a magnetic field probe (a probe that can measure the magnitude of the magnetic field) to best measure the strength of the magnetic field of the solenoid in order to determine the magnetic permeability of free space μ_0 .

☐ A ☐ B ☐ C ☐ D

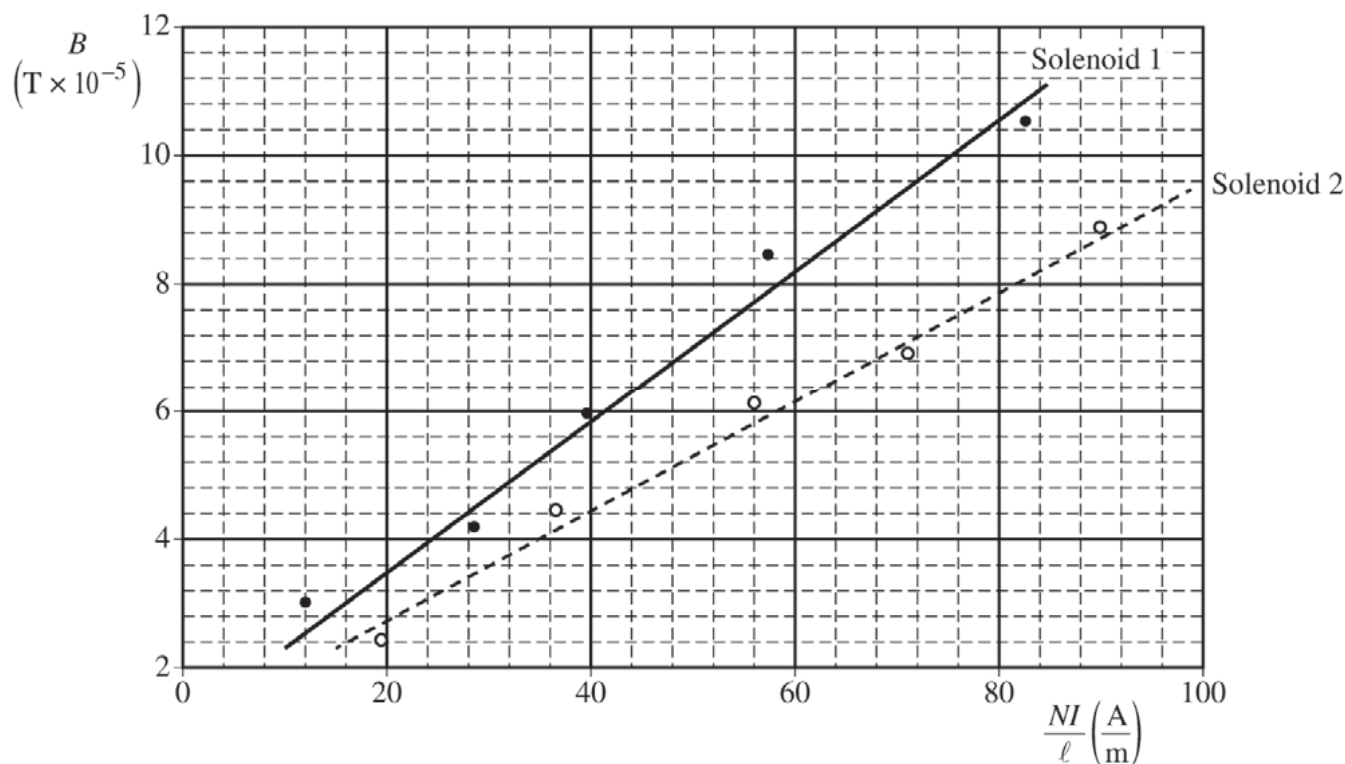
Justify your answer based on the model for a simple solenoid.

The figures below show two different solenoids that will be connected in the circuit above. Solenoid 1 has a length $\ell = 25$ cm with $N = 100$ turns. Solenoid 2 has a length $\ell = 5.0$ cm with $N = 5$ turns.



Note: Figures not drawn to scale.

A graph of the magnitude of the magnetic field B as a function of NI/ℓ is shown below. The best-fit lines for the data are shown as a solid line for solenoid 1 and as a dashed line for solenoid 2.



- (b) Which solenoid's best-fit line would give the best results for determining a value for the magnetic permeability of free space μ_0 ?

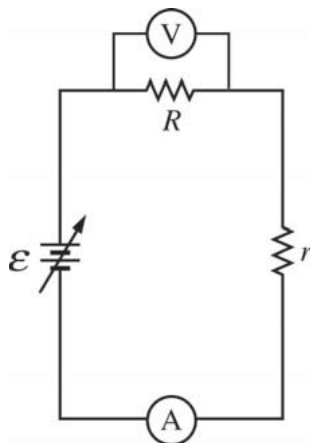
_____ Solenoid 1 _____ Solenoid 2

Justify your answer.

- (c)
- Use the slope of the best-fit line for the solenoid chosen in part (b) to calculate the magnetic permeability of free space μ_0 .
 - Calculate the percent error for the experimental value of the magnetic permeability of free space μ_0 determined in part (c)i.

- (d)
- What is a reasonable physical explanation for a best-fit line that does not pass through the origin?
 - Suppose a student connects the solenoid in a closed circuit similar to the circuit in part (a)i but without the resistor. The student notices the multimeter stops functioning after the power supply is turned on. Explain what causes the failure of the multimeter.

STOP
END OF EXAM



E&M.2.

The circuit shown above consists of a source of variable emf $\mathcal{E}$, an ideal ammeter A, an ideal voltmeter V, a resistor of resistance R , and a sample of wire with resistance r .

- (a) How does the current through the wire sample compare with the current through the resistor R ?

_____ It is greater through R . _____ It is greater through the sample.
 _____ It is the same through both. _____ It depends on the resistance of the sample.

Justify your answer.

- (b) How does the potential difference across the wire sample compare with the potential difference across the resistor R ?

_____ It is greater across R . _____ It is greater across the sample.
 _____ It is the same across both. _____ It depends on the resistance of the sample.

Justify your answer.

With the sample of wire in place, the emf of the source is set to a given value. The current through and potential difference across the resistor R are measured. This is repeated for several values of emf, and the data are recorded in the table below.

$\mathcal{E}$ (V)	V_R (V)	I_R (A)		
0.250	0.179	0.162		
0.500	0.335	0.327		
0.750	0.520	0.490		
1.000	0.670	0.687		

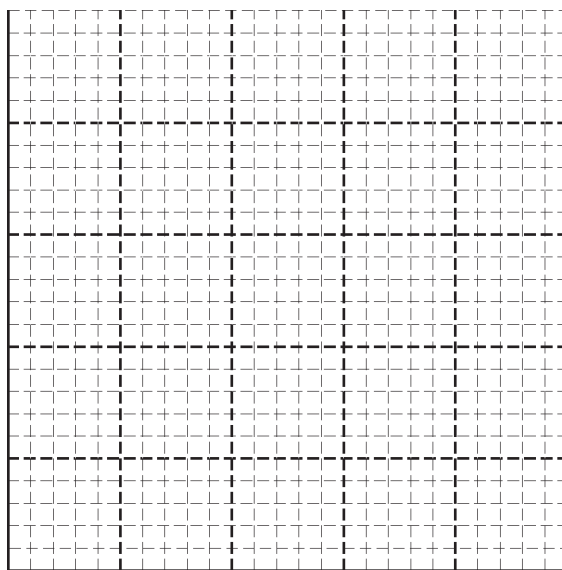
- (c) Indicate below which quantities should be graphed to yield a straight line that could be used to calculate a numerical value for the resistance of the wire sample.

Horizontal axis: _____

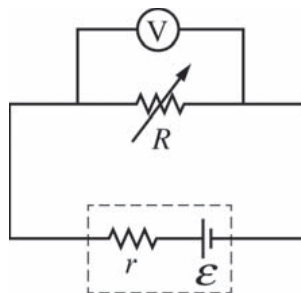
Vertical axis: _____

You may use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given.

- (d) On the grid below, plot the straight line data points from part (c). Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



- (e) Use your straight line to calculate the value of the resistance of the wire sample.
- (f) The wire sample has a length of 3.00 m and a radius of 1.00×10^{-3} m . Calculate the resistivity of the material from which the wire sample is made.
- (g)
- Suppose the ammeter used to collect these data was not ideal. Would the actual value of the resistance of the wire sample be greater than, less than, or equal to that calculated in part (e) ?
____ Greater than ____ Less than ____ Equal to
Justify your answer.
 - If the ideal voltmeter is replaced by a voltmeter that is not ideal and the experiment is repeated, would the readings of the ideal ammeter be greater than, less than, or equal to those in the data chart before part (c) ?
____ Greater than ____ Less than ____ Equal to
Justify your answer.



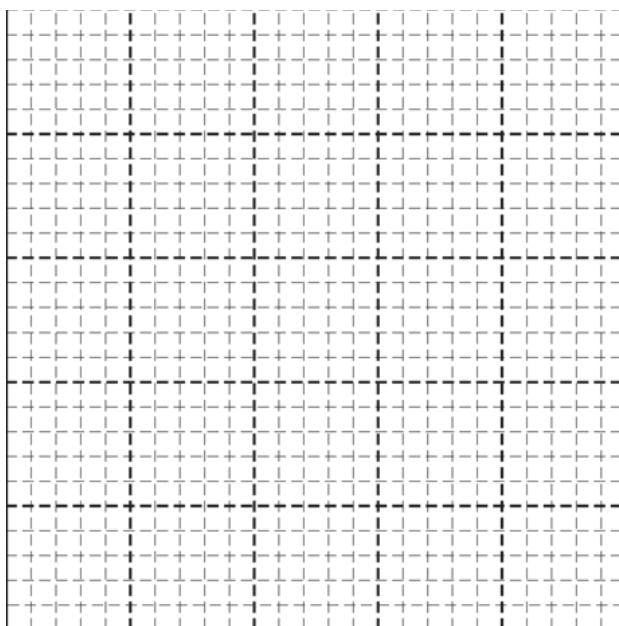
E&M.2.

A student performs an experiment to determine the emf $\mathcal{E}$ and internal resistance r of a given battery. The student connects the battery in series to a variable resistance R , with a voltmeter across the variable resistor, as shown in the figure above, and measures the voltmeter reading V as a function of the resistance R . The data are shown in the table below.

Trial #	Resistance (Ω)	Voltage (V)	$1/R$ ($1/\Omega$)	$1/V$ ($1/V$)
1	0.50	5.6	2.00	0.179
2	1.0	7.4	1.00	0.135
3	2.0	9.4	0.50	0.106
4	3.0	10.6	0.33	0.094
5	5.0	10.9	0.20	0.092
6	10	11.4	0.10	0.088

- (a)
- Derive an expression for the measured voltage V . Express your answer in terms of R , $\mathcal{E}$, r , and physical constants, as appropriate.
 - Rewrite your expression from part (a)-i to express $1/V$ as a function of $1/R$.

- (b) On the grid below, plot data points for the graph of $1/V$ as a function of $1/R$. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.



- (c) Use the straight line from part (b) to obtain values for the following.
- $\mathcal{E}$
 - r
- (d) Using the results of the experiment, calculate the maximum current that the battery can provide.
- (e) A voltmeter is to be used to determine the emf of the battery after removing the battery from the circuit. Two voltmeters are available to take this measurement—one with low internal resistance and one with high internal resistance. Indicate which voltmeter will provide the most accurate measurement.
- ___ The voltmeter with low resistance will provide the most accurate measurement.
- ___ The voltmeter with high resistance will provide the most accurate measurement.
- ___ The two voltmeters will provide equal accuracy.

Justify your answer.