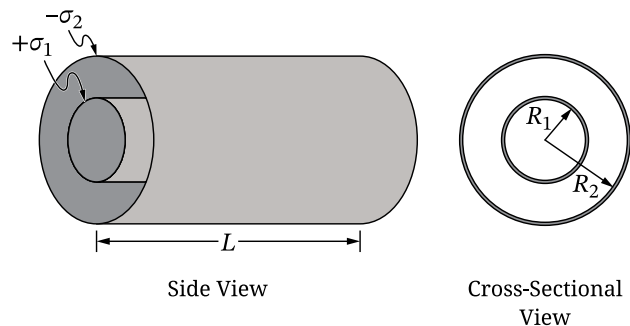


Question 1: Version J

1. An isolated, air-filled, charged capacitor consists of two conducting, coaxial, cylindrical shells that each have length L . The inner shell has radius R_1 and the outer shell has radius R_2 , as shown in Figure 1, where $R_1 < R_2 \ll L$. The surface charge densities (amounts of charge per unit area) of the inner and outer shells are $+\sigma_1$ and $-\sigma_2$, respectively. The absolute values of the total charges on the shells are equal.



Note: Figures not drawn to scale.

Figure 1

A.

- Using Gauss's law, **derive** an expression for the magnitude E of the electric field as a function of the radial distance r from the center of the capacitor for the region $R_1 < r < R_2$. Express your answer in terms of R_1 , σ_1 , r , and physical constants, as appropriate.
- Derive** an expression for the absolute value $|\Delta V|$ of the potential difference between the outer and inner shells in terms of R_1 , R_2 , σ_1 , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

- On the axes shown in Figure 2, **sketch** a graph of E as a function of r from $r = 0$ to a position that is outside the outer shell.

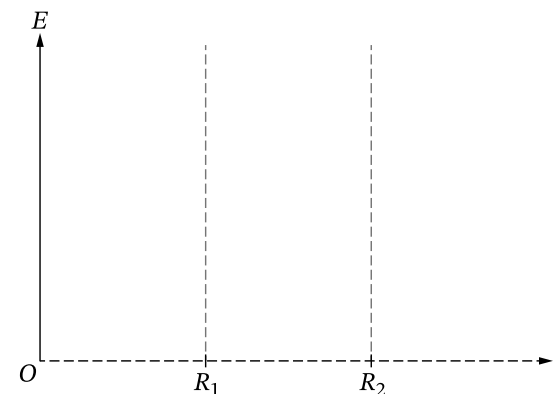
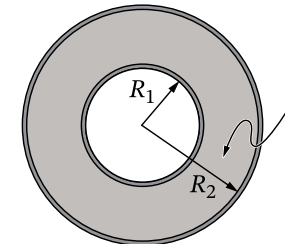


Figure 2

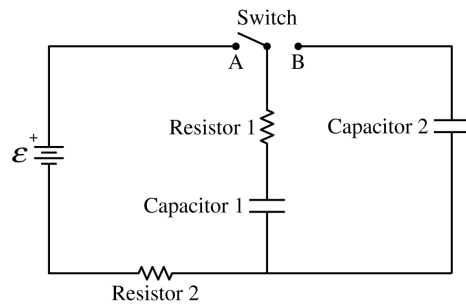
- B. A material of dielectric constant κ is inserted into the isolated, charged capacitor such that the material fills the region $R_1 < r < R_2$, as shown in Figure 3.



Cross-Sectional View

Figure 3

Derive an expression for the capacitance C of the capacitor with the material inserted in terms of L , R_1 , R_2 , κ , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.



The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R , capacitors 1 and 2 each with capacitance C , and a switch. The switch is initially open and both capacitors are uncharged.

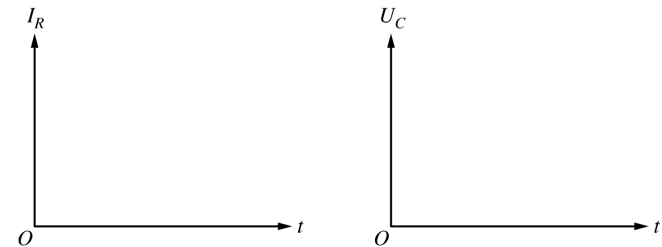
At time $t = 0$, the switch is closed to Position A.

- (a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R , C , Q , t , and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to Question 3 on this page.

- (b) On the axes shown, sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.



A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

- (c) At time t_1 , the switch is closed to Position B.
- Immediately after t_1 , is the direction of the current in Resistor 1 directed up, directed down, or is there no current? Briefly justify your answer.
 - Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1 . Express your answer in terms of Q_0 and fundamental constants, as appropriate.

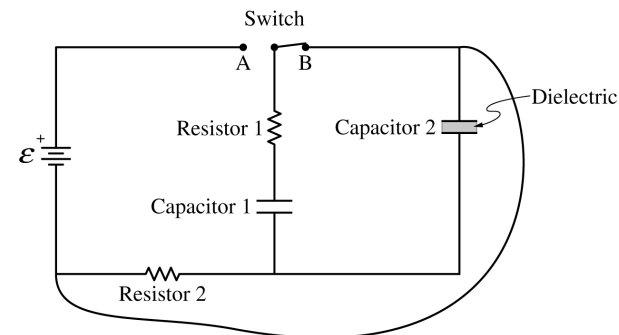
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iii. Derive an expression for the total energy dissipated by Resistor 1 immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C , Q_0 , and fundamental constants, as appropriate.

With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

(d) Determine the charge on the positive plate of Capacitor 2 a long time after the dielectric has been inserted.
Express your answer in terms of Q_0 and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.



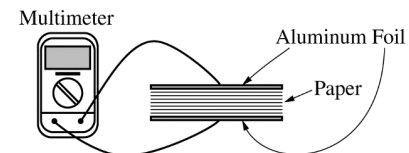
With the switch still closed to Position B, a wire of negligible resistance is connected between two corners of the circuit, as shown.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R , C , Q_0 , and fundamental constants, as appropriate.

i. Derive an expression for the current in Resistor 2 immediately after the wire is connected to the circuit.

ii. Determine the current in Resistor 2 a long time after the wire is connected to the circuit.

GO ON TO THE NEXT PAGE.

STOP**END OF EXAM**

2. An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060 \text{ m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)		
500	0.045	6.5×10^{-11}		
400	0.036	7.4×10^{-11}		
300	0.027	8.9×10^{-11}		
200	0.018	11.9×10^{-11}		
100	0.010	21.0×10^{-11}		

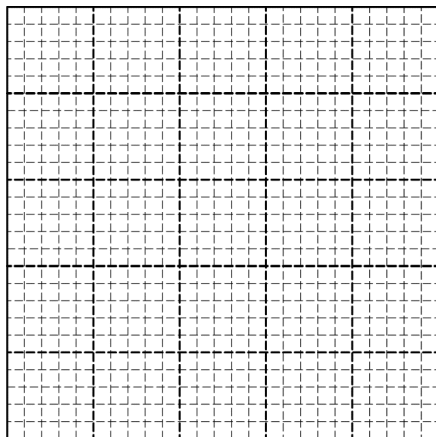
- (a) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: _____

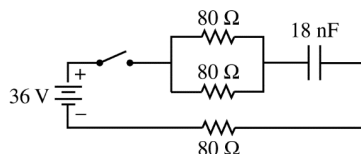
Horizontal axis: _____

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

- (b) Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



- (c) Using the straight line, calculate a dielectric constant for the paper.



The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical 80 Ω resistors, and an open switch, as shown in the figure above.

- (d) Calculate the current in the battery immediately after the switch is closed.
- (e) Determine the time constant for this circuit.
- (f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.
- Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?
 ____ Larger than measured ____ Smaller than measured
 Explain experimentally what could account for this.
 - Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?
 ____ Larger than measured ____ Smaller than measured
 Explain experimentally what could account for this.

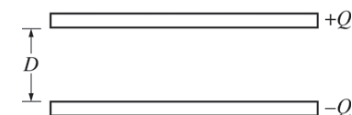
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

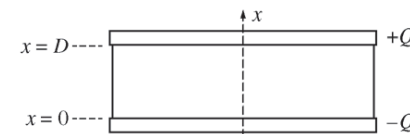
Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



E&M.1.

A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D . The plates of the capacitor are each given a charge of magnitude Q , as shown in the figure above. Ignore edge effects.

- (a)
- On the figure above, draw an arrow to indicate the direction of the electric field between the plates.
 - On the figure above, draw an appropriate Gaussian surface that will be used to derive an expression for the magnitude of the electric field E between the plates.
 - Using Gauss's law and the Gaussian surface from part (a)-ii, derive an expression for the magnitude of the electric field E between the plates. Express your answer in terms of A , D , Q , and physical constants, as appropriate.



The space between the plates is now filled with a dielectric material that is engineered so that its dielectric constant varies with the distance from the bottom plate to the top plate, defined by the x -axis indicated in the diagram above. As a result, the electric field between the plates is given by $\vec{E} = -\frac{Q}{\epsilon_0 \kappa_0 e^{-x/D} A} \hat{i}$, where κ_0 is a positive constant. Express all algebraic answers to the remaining parts in terms of A , D , Q , κ_0 , x , and physical constants, as appropriate.

- (b) Determine an expression for the dielectric constant κ as a function of x .

Name: _____

(c)

- i. Write, but do NOT solve, an equation that could be used to determine the potential difference V between the plates of the capacitor.
- ii. Using the equation from part (c)-i, derive an expression for the potential difference $V_D - V_0$, where V_D is the potential of the top plate and V_0 is the potential of the bottom plate.

(d) Determine the capacitance of the capacitor.

(e) The energy stored in the capacitor that has a varying dielectric is U_V . A second capacitor that has a constant dielectric of value κ_0 is also given a charge Q . The energy stored in the second capacitor is U_C . How do the values of U_V and U_C compare?

_____ $U_V < U_C$ _____ $U_V > U_C$ _____ $U_V = U_C$

Justify your answer.