Question 1: Mathematical Routines (MR) — 10 points

A (i) For a multistep derivation that includes the equation $\oint \vec{E} \cdot d\vec{A} = \dfrac{q_{\text{enc}}}{\varepsilon_0}$ — **Point A1**

Scoring Note: Vector notation is not required for this point to be earned.

For a correct substitution of the area of an appropriate Gaussian surface with nonzero flux for the region $R_1 < r < R_2$ (e.g., $2\pi r\ell$) — **Point A2**

For a correct expression for the enclosed charge (e.g., $\sigma_1\left(2\pi R_1\ell\right)$) — **Point A3**

Example Response

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$E\left(2\pi r\ell\right) = \frac{\sigma_1\left(2\pi R_1\ell\right)}{\varepsilon_0}$$

$$E = \frac{\sigma_1 R_1}{\varepsilon_0 r}$$

(ii) For substituting the expression for E from part A (i) into $\Delta V = -\int_a^b \vec{E} \cdot d\vec{r}$ — **Point A4**

Scoring Notes:

- Vector notation is not required for this point to be earned.
- The sign of ΔV is not considered for this point to be earned.

For an attempt to solve the integral for $|\Delta V|$ that includes correct limits (e.g., $|\Delta V| = \dfrac{\sigma_1 R_1}{\varepsilon_0}\displaystyle\int_{R_1}^{R_2} \frac{1}{r} dr$) — **Point A5**

Scoring Note: This point may be earned regardless of the order of the limits of integration; for example, $\dfrac{\sigma_1 R_1}{\varepsilon_0}\displaystyle\int_{R_2}^{R_1} \frac{1}{r} dr$.

Example Response

$$|\Delta V| = \left|-\int_a^b \vec{E} \cdot d\vec{r}\right|$$

$$|\Delta V| = \int_{R_1}^{R_2} \frac{\sigma_1 R_1}{r\varepsilon_0} dr = \frac{\sigma_1 R_1}{\varepsilon_0}\int_{R_1}^{R_2} \frac{1}{r} dr$$

$$|\Delta V| = \frac{\sigma_1 R_1}{\varepsilon_0} \ln(r)\Big|_{R_1}^{R_2} = \frac{\sigma_1 R_1}{\varepsilon_0} \ln\left(\frac{R_2}{R_1}\right)$$

(iii) For sketching a graph that is zero for both $0 < r < R_1$ and $r > R_2$ — **Point A6**

For sketching a graph that is decreasing and concave up for $R_1 < r < R_2$ — **Point A7**

Scoring Note: The curve does not have to intersect the vertical dashed lines for this point to be earned.

Example Response

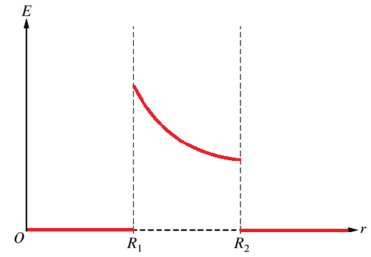


Figure 2

B For a multistep derivation that includes the equation $C = \dfrac{Q}{\Delta V}$ — **Point B1**

For indicating that C with the dielectric material inserted is C without the dielectric material inserted multiplied by κ (e.g., $C = \kappa\dfrac{Q}{\Delta V}$) — **Point B2**

For substitutions of both the total charge Q and the potential difference ΔV that are consistent with part A — **Point B3**

Example Response

Without the dielectric material

$$C = \frac{Q}{\Delta V}$$

$$C = \frac{\sigma_1\left(2\pi L R_1\right)}{\left(\dfrac{\sigma_1 R_1}{\varepsilon_0}\ln\left(\dfrac{R_2}{R_1}\right)\right)}$$

$$C = \frac{2\pi L\varepsilon_0}{\ln\left(\dfrac{R_2}{R_1}\right)}$$

With the dielectric material

$$C = \kappa\frac{Q}{\Delta V}$$

$$C = \frac{2\pi L\kappa\varepsilon_0}{\ln\left(\dfrac{R_2}{R_1}\right)}$$

Question 3: Free-Response Question — 15 points

(a) For a loop rule expression that includes terms for the equivalent resistance $\frac{R}{2}$ and the potential difference across the battery — **1 point**

For an expression that includes charge Q in the term relating the potential difference across the capacitor and includes charge per unit time $\frac{dQ}{dt}$ in the term relating the potential difference across the pair of resistors — **1 point**

Example Response

$$\mathcal{E} - \Delta V_C - \Delta V_{R,eq} = 0$$

$$\mathcal{E} - \frac{Q}{C} - \frac{R}{2}\frac{dQ}{dt} = 0$$

Total for part (a) — 2 points

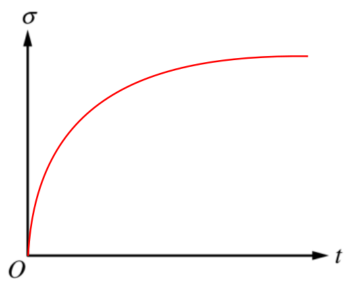
(b) For sketching a concave down and increasing curve on the graph σ as a function of t — **1 point**

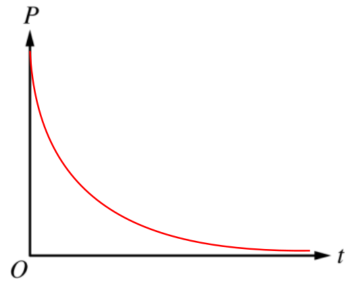
For sketching a curve that is concave up and decreasing on the graph of P as a function of t — **1 point**

For sketching both curves that approach a slope of zero as time increases — **1 point**

Scoring Note: The third point can be earned even if the first two points are not earned.

Example Response





Total for part (b) — 3 points

(c)(i) For a correct justification that could include one of the following: — **1 point**

- An indication that the current is to the right with a justification that includes a statement that indicates that positive charge has accumulated on the top plate of Capacitor 1 and/or negative charge has accumulated on the bottom plate of Capacitor 1 when the switch was closed to Position A
- An indication that the current is to the right with a justification that includes a statement that indicates that the value of the electric potential of the top plate of Capacitor 1 is larger than the electric potential of the bottom plate of Capacitor 1 when the switch was closed to Position A

Example Responses

The current is directed towards the right because the top plate of Capacitor 1 is positively charged, meaning conventional current will flow clockwise.

OR

Toward the right. Current flows from high to low potential so it will flow from the top plate up and right through the switch.

(c)(ii) For indicating that the total charge on the positive plate of Capacitor 2 is $\frac{2}{3}Q_0$ — **1 point**

Scoring Note: This point can be earned without supporting calculations.

Example Response

The potential difference across Capacitor 1 is equal to the potential difference across Capacitor 2. Capacitor 2 has twice the capacitance of Capacitor 1. Therefore, Capacitor 2 stores twice the charge that is stored on Capacitor 1. Due to conservation of charge, Capacitor 2 stores an amount of charge equal to $\frac{2}{3}Q_0$.

(c)(iii) For an indication that the total energy dissipated by the resistors is the difference between an initial electric potential energy stored in one or both capacitors at time $t = t_1$ and a final electric potential energy stored on one or both capacitors after the new steady state conditions have been reached — **1 point**

Example Response

$$E_R = U_C - U_{0C}$$

For indicating that only Capacitor 1 stores nonzero electric potential energy initially and both capacitors store nonzero electric potential energy after the new steady state conditions have been reached, or alternative consistent with part (c)(ii) — **1 point**

Example Response

$$U_{0C} = U_{01}$$
$$U_C = U_1 + U_2$$

For correct substitutions for the charges stored on the capacitors after the new steady state conditions have been reached consistent with part (c)(ii) — **1 point**

Example Response

$$\Delta E_R = U_C - U_{0C}$$
$$\Delta E_R = \left(\frac{1}{2}\left(\frac{1}{C}\right)\left(\frac{Q_0}{3}\right)^2 + \frac{1}{2}\left(\frac{1}{2C}\right)\left(\frac{2Q_0}{3}\right)^2\right) - \frac{1}{2}\frac{Q_0^2}{C}$$

Example Solution

$$\Delta E_R = U_C - U_{0C}$$
$$\Delta E_R = \left(\frac{1}{2}\left(\frac{1}{C}\right)\left(\frac{Q_0}{3}\right)^2 + \frac{1}{2}\left(\frac{1}{2C}\right)\left(\frac{2Q_0}{3}\right)^2\right) - \frac{1}{2}\frac{Q_0^2}{C}$$
$$\Delta E_R = \frac{Q_0^2}{6C} - \frac{1}{2}\frac{Q_0^2}{C}$$
$$\Delta E_R = -\frac{Q_0^2}{3C}$$

Total for part (c) — 5 points

(d) For indicating that the potential difference across each capacitor is the same or that the charge stored on each capacitor in steady state is the same — **1 point**

Example Responses

$$\Delta V_1 = \Delta V_2$$

OR

$$Q_1 = Q_2$$

For recognizing that the capacitance of each capacitor is now the same in the new configuration — **1 point**

Example Response

After steady state conditions are reached, both capacitors have the same potential difference. The new capacitance of Capacitor 2 is equal to the capacitance of Capacitor 1 because the capacitance of a capacitor is inversely related to the distance between the plates of a capacitor. Therefore, since $U_C = \frac{1}{2}C(\Delta V)^2$, $\frac{U_2}{U_1} = 1$.

Total for part (d) — 2 points

(e)(i) For a loop rule that includes the terms for the emf of the battery, the potential difference across the pair of resistors, and the potential difference across Capacitor 1 — **1 point**

Example Response

$$\mathcal{E} - \Delta V_R - \Delta V_C = 0$$

For a correct answer — **1 point**

Example Response

$$I = \frac{Q_0}{RC}$$

Example Solution

$$\mathcal{E} - \Delta V_R - \Delta V_C = 0$$
$$\mathcal{E} - I\left(\frac{R}{2}\right) - \left(\frac{Q_0}{2C}\right) = 0$$
$$\frac{Q_0}{C} - I\left(\frac{R}{2}\right) - \left(\frac{Q_0}{2C}\right) = 0$$
$$\frac{Q_0}{2C} - I\left(\frac{R}{2}\right) = 0$$
$$\frac{Q_0}{2C} = I\left(\frac{R}{2}\right)$$
$$\frac{Q_0}{C} = IR$$
$$I = \frac{Q_0}{RC}$$

(e)(ii) For indicating that the current is zero — **1 point**

Total for part (e) — 3 points

Total for question 3 — 15 points

Question 2: Free-Response Question — 15 points

(a) For a schematic diagram with the capacitor in series with the resistor — **1 point**

OR

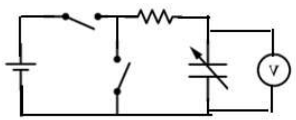
For a schematic diagram with the resistor on the parallel path

For a schematic diagram with the voltmeter in parallel with the capacitor — **1 point**

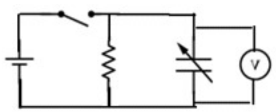
For a schematic diagram that uses a switch to connect the battery to the capacitor — **1 point**

For a schematic diagram that uses a switch that allows the capacitor to discharge through the resistor — **1 point**

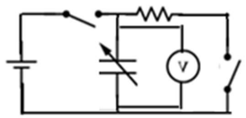
Example Responses



OR



OR



Total for part (a) — **4 points**

(b) For using an appropriate loop equation — **1 point**

Example Response

$V_C - V_R = 0$

For correctly substituting $\frac{q}{C}$ and IR in a loop equation consistent with the first point — **1 point**

Example Response

$\frac{q}{C} = IR$

For using a differential expression consistent with the loop rule in the first point that includes a substitution of $I = -\frac{dq}{dt}$ for the current — **1 point**

Example Response

$$\frac{q}{C} = -\frac{dq}{dt}R$$

$$-\frac{1}{RC}dt = \frac{1}{q}dq$$

$$-\frac{1}{RC}\int_{t=0}^{t=t} dt = \int_{q=q_0}^{q=q} \frac{1}{q}dq$$

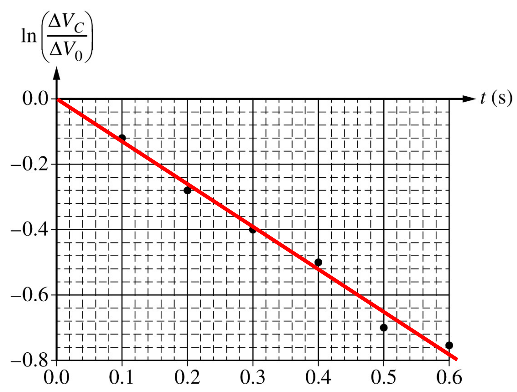
$$\frac{-t}{RC} = \ln\left(\frac{q(t)}{q_0}\right) \therefore \frac{q(t)}{q_0} = e^{-t/RC} \therefore \frac{q(t)}{C} = \frac{q_0}{C}e^{-t/RC} \therefore V(t) = V_0 e^{-t/RC}$$

Scoring Note: The point can be earned regardless of the sign used in the substitution of $\frac{dq}{dt}$ for the current.

Total for part (b) — **3 points**

(c)(i) For drawing an appropriate best-fit line — **1 point**

Example Response



(c)(ii)	For using an appropriate equation to relate the unknown capacitance to the data	**1 point**
	Example Response $V(t) = V_0 e^{-t/RC} \therefore \frac{V(t)}{V_0} = e^{-t/RC} \therefore \ln\left(\frac{V(t)}{V_0}\right) = -\frac{t}{RC}$	
	For correctly calculating the slope using two points from the best-fit line	**1 point**
	Example Response $\text{slope} = \frac{\Delta\left(\ln\left(\frac{V}{V_0}\right)\right)}{\Delta t} = \frac{(-0.70-(-0.40))}{(0.54\text{ s}-0.30\text{ s})} = -1.25\text{ s}^{-1}$ **Scoring Note:** A response may earn this point regardless of the associated unit.	
	For correctly relating the slope to the unknown capacitance	**1 point**
	Example Response $\text{slope} = -\frac{1}{RC} \therefore C = \frac{-1}{\text{slope} \times (R)} = \frac{-1}{\left(-1.25\text{ s}^{-1}\right) \times (150\text{ k}\Omega)} = 5\mu\text{F}$	
	Total for part (c)	**4 points**
(d)	For selecting "Less steep" and an attempt at a relevant justification	**1 point**
	For correctly relating the correct change in capacitance to the slope of the graph	**1 point**
	Example Response *Increasing the area of the plates increases the capacitance of the capacitor; thus, the magnitude of the slope will decrease.* **Scoring Note:** Part (d) is scored consistently with part (c).	
	Total for part (d)	**2 points**
(e)(i)	Correct answer: "No"	
	For selecting "No" AND correctly describing the effects of the internal resistance of the battery on the slope of the graph at $t = 0$	**1 point**
	Example Response *No. The capacitor still discharges only though resistor R, so the slope is the same.*	

(e)(ii)	For selecting "No" AND correctly describing the effects of the internal resistance of the battery on the initial value of the graph	**1 point**
	Example Response *No. The best-fit line does not change, because the internal resistance of the battery does not affect the final potential difference across the charging capacitor.* **Scoring Note:** This point is scored with consistency with the circuit drawn in part (b).	
	Total for part (e)	**2 points**
	Total for question 2	**15 points**

Question 1: Free-Response Question — 15 points

(a) For an indication that the capacitors act like a short circuit immediately after the switch is closed — **1 point**

$R_{EQ} = R_1 = 100\ \Omega$

For using Ohm's law to calculate the current through R_1 — **1 point**

$I = \dfrac{\Delta V}{R} = \dfrac{(10\text{ V})}{(100\ \Omega)}$

$I = 0.10\text{ A}$

Total for part (a) — 2 points

(b) For correctly determining the equivalent resistance during steady-state — **1 point**

$\dfrac{1}{R_P} = \dfrac{1}{R_2} + \dfrac{1}{R_3} = \dfrac{1}{30\ \Omega} + \dfrac{1}{60\ \Omega} \therefore R_P = 20\ \Omega$

$R_{EQ} = R_1 + R_P = 100\ \Omega + 20\ \Omega = 120\ \Omega$

For correctly calculating the current through the battery — **1 point**

$I = \dfrac{\Delta V}{R} = \dfrac{(10\text{ V})}{(120\ \Omega)}$

$I = 0.083\text{ A}$

The potential difference across R_1

$\Delta V = (0.083\text{ A})(100\ \Omega) = 8.3\text{ V}$

Total for part (b) — 2 points

(c) i. For using a correct equation to calculate the potential difference across C_3 — **1 point**

$\Delta V_{C_3} = \Delta V_P = IR_P = (0.083\text{ A})(20\ \Omega) = 1.67\text{ V}$

For using the correct equivalent capacitance of the series combination to calculate the charge stored in capacitor C_2 — **1 point**

$\dfrac{1}{C_S} = \dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{1}{10\ \mu\text{F}} + \dfrac{1}{15\ \mu\text{F}} \therefore C_S = 6.0\ \mu\text{F}$

$Q_2 = Q_S = C_S \Delta V_{C_{12}} = (1.67\text{ V})(6.0\ \mu\text{F}) = 10\ \mu C$

ii. For selecting "Greater than" and attempting a relevant justification — **1 point**

For a correct justification — **1 point**

Example response for part (c)(ii)

The potential difference across the series combination of C_1 and C_2 is the same as the potential difference across C_3; thus, capacitor C_3 has a greater potential difference across its plates than C_2 and has a greater capacitance. Because $Q = C\Delta V$, C_3 stores a greater charge.

Scoring note: A justification showing a mathematical proof earns full credit.

Total for part (c) — 4 points

(d) i. For indicating that the current through R_1 is zero when the switch is opened — **1 point**

$I_1 = 0$

ii. For indicating that the potential difference across R_2 is equal to the potential difference across the capacitors before the switch was opened — **1 point**

$\Delta V_{R_2} = \Delta V_{C_{12}} = 1.67\text{ V}$

For using Ohm's law to calculate the current through R_2 — **1 point**

$I = \dfrac{\Delta V_{R_2}}{R_2} = \dfrac{(1.67\text{ V})}{(30\ \Omega)}$

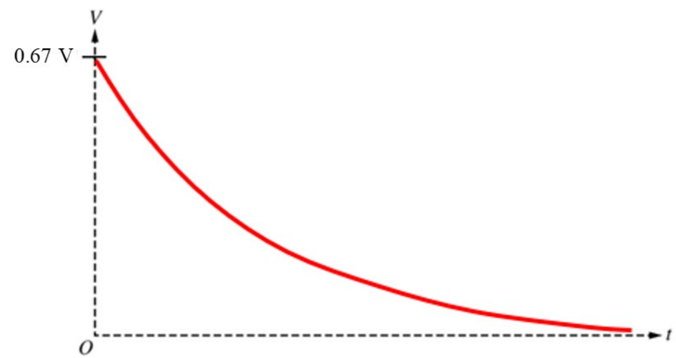
$I = 0.056\text{ A}$

Total for part (d) — 3 points

(e) For a curve starting at a nonzero, labeled maximum value — **1 point**

For a concave-up curve with the horizontal axis as an asymptote — **1 point**

Example response for part (e)



Total for part (e) — 2 points

(f) For selecting "Below" and attempting a relevant justification — **1 point**

For a correct justification — **1 point**

Example response for part (f)

Because the equivalent capacitance of the circuit would decrease, the time constant would also decrease. Thus the capacitors would discharge more rapidly, and the new curve would be below the original curve.

Total for part (f) — 2 points

Total for question 1 — 15 points

2019

AP® CollegeBoard

AP® Physics C: Electricity and Magnetism

Scoring Guidelines Set 2

2019 SCORING GUIDELINES

General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

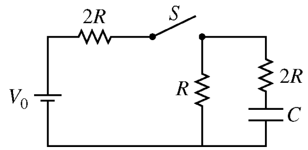
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8\ \text{m}/\text{s}^2$, but the use of $10\ \text{m}/\text{s}^2$ is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 1

15 points



The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(a) LO CNV-7.B.a, SP 5.A, 5.E
2 points

Derive an expression for the steady-state current supplied by the battery.

For using Ohm's law		1 point
$I = \dfrac{V}{R_{eff}} = \dfrac{V_0}{(2R + R)}$		
For correct substitution leading to correct answer		1 point
$I = \dfrac{V_0}{3R}$		

(b) LO CNV-7.B.b, SP 5.A, 5.E
2 points

Derive an expression for the charge on the capacitor.

For using the equation relating stored charge to capacitance		1 point
$q = CV = CV_C = CV_R$		
For determining V_R and substituting into the above equation		1 point
$V_R = IR = \left(\dfrac{V_0}{3R}\right)R = \dfrac{1}{3}V_0$		
$q = CV_R = C\left(\frac{1}{3}V_0\right) = \frac{1}{3}CV_0$		

Question 1 (continued)

(c) LO CNV-7.B.b, SP 5.A, 5.E
2 points

Derive an expression for the energy stored in the capacitor.

For any correct equation for energy stored in a capacitor		1 point
$U = \dfrac{q^2}{2C}$		
Substitute charge and/or voltage from part (b)		1 point
$U = \dfrac{(CV_0/3)^2}{2C} = \dfrac{1}{18}CV_0^2$		

Now the switch is opened at time $t = 0$.

(d) LO CNV-7.D.a, SP 5.A, 5.E
2 points

Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time t after the switch is opened.

For any correct voltage loop equation		1 point
$V_C - V_R - V_{2R} = 0 \therefore V_C = V_R + V_{2R}$		
$\dfrac{q(t)}{C} = I(R + 2R)$		
Note: Any correct loop equation for when the switch is open earns the point		
For substituting $-dq/dt$ or dq/dt for the current, consistent with loop equation		1 point
$\dfrac{q(t)}{C} = -3R\dfrac{dq}{dt}$		

Question 1 (continued)

(e)

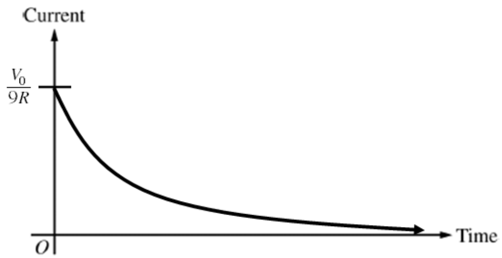
i. LO CNV-7.B.a, SP 5.A, 5.E
2 points

Calculate the current in resistor R immediately after the switch is opened.

For using a voltage consistent with part (b) in $I = \dfrac{V}{R}$		1 point
For substituting the correct resistance ($3R$) into Ohm's law		1 point
$I = \dfrac{V_0/3}{3R} = \dfrac{V_0}{9R}$		

ii. LO CNV-7.E.b, SP 3.C
3 points

On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



For a curve that is concave up throughout graph		1 point
For having the horizontal axis as an asymptote		1 point
For labeling the maximum current consistent with part (e)(i)		1 point

Question 1 (continued)

(f) LO CNV-7.G.a, SP 7.A, 7.C
2 points

Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c)?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

For selecting "Equal to"		1 point
For a correct justification invoking conservation of energy		1 point
Example: After the switch is opened, the capacitor will discharge all of its stored energy and charge. Assuming no energy is lost in the wire, then the only parts of the circuit that will dissipate this energy are the two resistors in series with the capacitor.		
Alternate Solution		*Alternate Points*
For selecting "Less than"		*1 point*
For a correct justification invoking conservation of energy		*1 point*
Example: After the switch is opened, the capacitor will discharge all of its stored energy and charge. Additionally there is energy lost due to resistance in the wire and/or energy loss in the capacitor.		

Question 1 (continued)

Learning Objectives

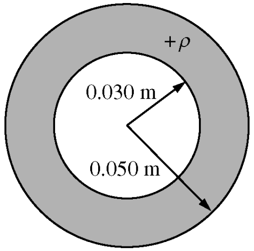
CNV-7.B.a: Calculate the potential difference across a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.
CNV-7.B.b: Calculate the stored charge on a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.
CNV-7.D.a: Derive expressions using calculus to describe the time dependence of the stored charge or potential difference across the capacitor, or the current or potential difference across the resistor in an RC circuit when charging or discharging a capacitor.
CNV-7.E.b: Describe the behavior of the voltage or current behavior over time for a circuit that contains resistors and capacitors in a multi-loop arrangement.
CNV-7.G.a: Describe the energy transfer in charging or discharging a capacitor in an RC circuit.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

Question 2

15 points



A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m}: \ \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m}: \ \rho = b/r \text{, where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

$$r > 0.050 \text{ m}: \ \rho = 0$$

(a) LO CNV-2.D.a, SP 6.B, 6.C
3 points

Calculate the total charge of the sphere.

For indicating the need to integrate the expression for charge density to determine the total charge on the sphere		1 point
$Q = \int \rho dV$		
For proper substitutions into the integration		1 point
$Q = \int \frac{(1.6 \times 10^{-6})}{r}(4\pi r^2)dr$		
For using the proper limits of integration		1 point
$Q = 4\pi(1.6 \times 10^{-6}) \int_{r=0.03}^{r=0.05} rdr = 4\pi(1.6 \times 10^{-6})\left[\frac{r^2}{2}\right]_{r=0.03}^{r=0.05}$		
$Q = (2\pi)(1.6 \times 10^{-6})(0.05^2 - 0.03^2) = 1.61 \times 10^{-8} \text{ C}$		

Question 2 (continued)

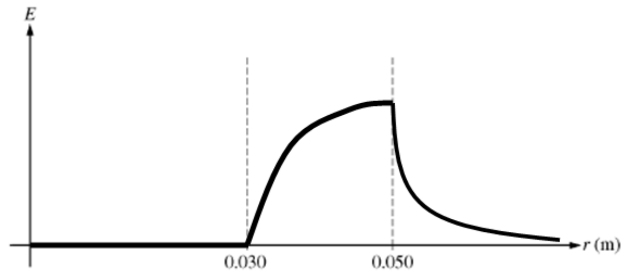
(b) LO CNV-2.D.a, SP 6.C
3 points

Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.

For correctly evaluating the surface integral in Gauss's law		1 point
$\oint E \bullet dA = E(4\pi r^2)$		
For correctly substituting the answer from part (a) and correct radius into above equation		1 point
$\frac{Q_{enc}}{\varepsilon_0} = E(4\pi r^2) \therefore E = \frac{Q_{enc}}{4\pi\varepsilon_0 r^2} = \frac{(1.61 \times 10^{-8} \text{ C})}{(4\pi)(8.85 \times 10^{-12})(0.05)^2}$		
For an answer consistent with part (a) with correct units		1 point
$E = 5.79 \times 10^4 \text{ N/C}$		

(c) LO CNV-2.C, SP 3.C
3 points

On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.



For clearly showing a graph with a value of $E = 0$ for $r < 0.030$ m		1 point
For a continuous graph that starts at zero, is concave down, and increases in value from $r = 0.030$ to $r = 0.050$		1 point
For a continuous graph that decreases asymptotically toward the horizontal axis for $r > 0.050$ m		1 point

Question 2 (continued)

(d) LO CNV-1.G.a, SP 6.B, 6.C
2 points

Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

For substituting the total charge from part (a) into a correct expression for electric potential		1 point
For substituting $r = 0.05$ m into a correct expression for electric potential		1 point
$V_R = \dfrac{Q_{tot}}{4\pi\varepsilon_0 r} = \dfrac{(9\times10^9)(1.61\times10^{-8}\ \text{C})}{(0.05\ \text{m})} = 2900\ \text{V}$		
Alternate Solution		*Alternate Points*
For substituting the total charge from part (a) into an integration for electric potential		*1 point*
$\Delta V_R = V_R - V_\infty = V_R = -\int E dr = -\int \dfrac{Q_{enc}}{4\pi\varepsilon_0 r^2} dr = -\int \dfrac{(9\times10^9)(1.61\times10^{-8}\ \text{C})}{r^2} dr$		
For integrating with correct limits of integration		*1 point*
$V_R = -\displaystyle\int_{r=\infty}^{r=0.05\ \text{m}} \dfrac{145}{r^2} dr = -145\left[-\dfrac{1}{r}\right]_{r=\infty}^{r=0.05\ \text{m}} = 145\left(\dfrac{1}{(0.05\ \text{m})} - \dfrac{1}{\infty}\right) = 2900\ \text{V}$		

A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.

(e)
i. LO ACT-1.D, SP 6.B, 6.C
2 points

Calculate the magnitude of the initial acceleration of the proton.

For using a correct expression of Newton's second law in terms of the electric field		1 point
$F = ma \therefore qE = ma \therefore a = \dfrac{qE}{m}$		
For correctly substituting into equation above		1 point
$a = \dfrac{qE}{m} = \dfrac{(1.6\times10^{-19}\ \text{C})(5.79\times10^4\ \text{N/C})}{1.67\times10^{-27}\ \text{kg}} = 5.55\times10^{12}\ \text{m/s}^2$		

Question 2 (continued)

(e) continued
ii. LO CNV-1.E, SP 6.B, 6.C
2 points

Calculate the speed of the proton after a long time.

For a correct expression of kinetic energy in terms of the electric potential difference		1 point
$-q\Delta V = \dfrac{1}{2}mv^2$		
For correctly substituting into equation above		1 point
$v = \sqrt{\dfrac{2q\Delta V}{m}} = \sqrt{\dfrac{-2(1.6\times10^{-19}\ \text{C})(0 - 2900\ \text{V})}{(1.67\times10^{-27}\ \text{kg})}} = 7.45\times10^5\ \text{m/s}$		

Learning Objectives

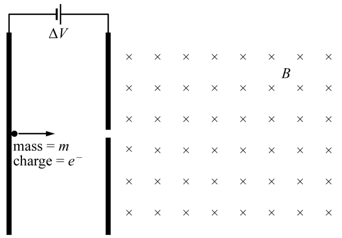
ACT-1.D: Determine the motion of a charged object of specified charge and mass under the influence of an electrostatic force.
CNV-1.E: Calculate the work done or changes in kinetic energy (or changes in speed) of a charged particle when it is moved through some known potential difference.
CNV-1.G.a: Use the general relationship between electric field and electric potential to calculate the relationships between the magnitude of electric field or the potential difference as a function of position.
CNV-2.C: State and use Gauss's law in integral form to derive unknown electric fields for planar, spherical, or cylindrically symmetrical charge distributions.
CNV-2.D.a: Using appropriate mathematics (which may involve calculus), calculate the total charge contained in lines, surfaces, or volumes when given a linear-charge density, a surface-charge density, or a volume-charge density of the charge configuration.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

Question 3

15 points



Two plates are set up with a potential difference V between them. A small sphere of mass m and charge $-e$ is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude B directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of V, m, e, B, and fundamental constants, as appropriate.

(a)

i. LO CHG-1.A.a, SP 7.A
1 point

What is the initial direction of the force on the sphere as it enters the magnetic field?

____ Into the page ____ Out of the page
____ Toward the top of the page ____ Toward the bottom of the page

For selecting "Toward the bottom of the page"		1 point

ii. LO CHG-1.B, SP 7.A
1 point

Describe the path taken by the sphere after it enters the magnetic field.

For describing a circular path for the sphere consistent with the selection from part (a)(i)		1 point
Example: The sphere will move in a circular path toward the bottom of the page.		

Question 3 (continued)

(b) LO CNV-1.E, SP 5.A, 5.E
2 points

Derive an expression for the speed of the sphere as it passes through the small opening.

For using a valid equation relating potential difference to kinetic energy of the sphere		1 point
$\Delta K = -\Delta U \therefore \frac{1}{2}mv^2 = -q\Delta V$		
$\frac{1}{2}mv^2 = eV$		
For the correct answer with supporting work		1 point
$v = \sqrt{\frac{2eV}{m}}$		

(c) LO CHG-1.C, SP 5.A, 5.E
3 points

Derive an expression for the radius of the path taken by the sphere as it moves through the magnetic field.

For an expression relating the magnetic force to the centripetal force		1 point
$\frac{mv^2}{r} = Bqv$		
For substituting the charge of the sphere into the above equation and solving for r		1 point
$\frac{mv}{r} = Be \therefore r = \frac{mv}{Be}$		
For substituting the answer from part (b)		1 point
$r = \frac{m}{Be}\sqrt{\frac{2eV}{m}} = \sqrt{\frac{2mV}{eB^2}}$		

An experiment is performed in which a beam of electrons is accelerated across the space between the plates and passes through the small opening. After passing through the opening, the electrons travel in a semicircular path and strike the right-hand plate. The potential difference between the plates is varied in regular increments, as shown in the table below. For each potential difference, the magnetic field is varied in order to cause the beam to strike the right-hand plate at a distance of 0.020 m from the opening.

Potential difference (V)	60	70	100	110	120	140
Magnetic field $(\mathrm{T} \times 10^{-3})$	2.62	2.78	3.39	3.54	3.78	3.99

Question 3 (continued)

(d) LO CHG-1.C, SP 4.C
1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the mass-to-charge ratio of an electron.

Vertical axis:____________
Horizontal axis:____________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

$$r = \sqrt{\frac{2m_e V}{eB^2}} \therefore B^2 = \frac{2m_e}{er^2}V$$

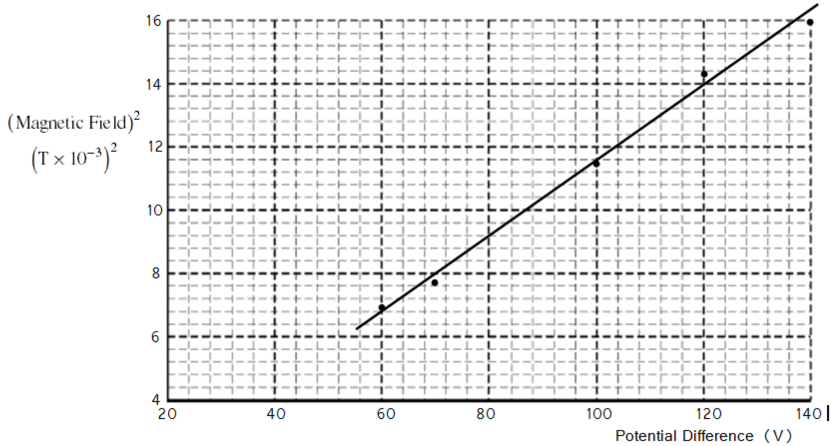
Potential difference (V)	60	70	100	110	120	140
Magnetic field $(\text{T} \times 10^{-3})$	2.62	2.78	3.39	3.54	3.78	3.99
$(\text{Magnetic field})^2$ $(\text{T} \times 10^{-3})^2$	6.86	7.73	11.5	12.5	14.3	15.9

For indicating axis choices with appropriate quantities to produce a straight line		1 point
Note: There are several possible choices.		

Question 3 (continued)

(e) LO CHG-1.C, SP 3.A, 4.C
4 points

On the graph below, plot the relationship determined in part (d). Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



For correctly labeling both axes with quantities and units		1 point
For correctly scaling and labeling the axes so that the points use at least half the grid		1 point
For correctly plotting the data		1 point
For drawing a straight line that represents the data		1 point

(f) LO CHG-1.C, SP 4.D, 5.E
3 points

Using the straight line from part (e), determine the mass-to-charge ratio of an electron.

For correctly calculating the slope from the best-fit straight line and not from the data points unless the points fall on the best-fit line		1 point
slope $= \dfrac{14 - 9.2}{120 - 80} = 0.12 \dfrac{(\text{T} \times 10^{-3})^2}{V} = 1.2 \times 10^{-7} \ \text{T}^2/\text{V}$		
For a correct expression relating the slope to the mass to charge ratio of an electron		1 point
$B^2 = \dfrac{2m_e}{er^2}V \therefore \text{slope} = \dfrac{2m_e}{er^2} \therefore \dfrac{m_e}{e} = \dfrac{r^2 \times \text{slope}}{2}$		
For substituting correct values into the above equation		1 point
$\dfrac{m_e}{e} = \dfrac{(0.01\ \text{m})^2 \times (1.2 \times 10^{-7}\ \text{T}^2/\text{V})}{2} = 6.0 \times 10^{-12}\ \text{kg/C}$		

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Question 3 (continued)

Learning Objectives

CHG-1.A.a: Calculate the magnitude and direction of the magnetic force of interaction between a moving charged particle of specified charge and velocity moving in a region of a uniform magnetic field.
CHG-1.B: Describe the path of different moving charged particles (i.e., of different type of charge or mass) in a uniform magnetic field.
CHG-1.C: Derive an expression for the radius of a circular path for a charged particle of specified characteristics moving in a specified magnetic field.
CNV-1.E: Calculate the work done or changes in kinetic energy (or changes in speed) of a charged particle when it is moved through some known potential difference.

Science Practices

3.A: Select and plot appropriate data.
4.C: Linearize data and/or determine a best-fit line or curve.
4.D: Select relevant features of a graph to describe a physical situation or solve problems.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
7.A: Make a scientific claim.

2018

AP CollegeBoard

AP Physics C: Electricity and Magnetism

Scoring Guidelines

2018 SCORING GUIDELINES

General Notes About 2018 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

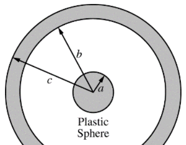
5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

AP® PHYSICS C: ELECTRICITY AND MAGNETISM
2018 SCORING GUIDELINES

Question 1

15 points total **Distribution of points**



Conducting Spherical Shell

A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β , r, a, and physical constants, as appropriate.

(a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

i. 3 points

$r < a$

For using the integral to determine the charge enclosed		1 point
$q_{enc} = \int \rho dV = \int \beta r dV$		
For correctly substituting for dV into the integration to determine the charge enclosed		1 point
$q_{enc} = \int \left(4\pi r'^2\right)(\beta r')dr' = 4\beta\pi \int_{r'=0}^{r'=r} r'^3 dr' = 4\beta\pi \left[\frac{r'^4}{4}\right]_{r'=0}^{r'=r}$		
For a correct answer		1 point
$q_{enc} = \beta\pi r^4$		

ii. 1 points

$a < r < b$

For an answer consistent with (a)(i)		1 point
$q_{enc} = \beta\pi a^4$		

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Question 1 (continued)

Distribution of points

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. 2 points

$r < a$

For correctly substituting the area of a Gaussian sphere into Gauss's law		1 point
$\frac{q_{enc}}{\varepsilon_0} = \oint \vec{E} \cdot d\vec{A}$		
$\frac{q(r)}{\varepsilon_0} = EA_{sphere} = E\left(4\pi r^2\right)$		
For correctly substituting the charge from part (a)(i) into the equation above		1 point
$\frac{\beta\pi r^4}{\varepsilon_0} = E\left(4\pi r^2\right)$		
$E = \frac{\beta r^2}{4\varepsilon_0}$		

ii. 1 point

$a < r < b$

For correctly substituting the charge from part (a)(ii) into Gauss's law to calculate the electric field		1 point
$\frac{\beta\pi a^4}{\varepsilon_0} = E\left(4\pi r^2\right)$		
$E = \frac{\beta a^4}{4\varepsilon_0 r^2}$		

2018 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. 2 points

Determine the charge on the inner surface of the conducting shell.
Justify your answer.

For an answer consistent with part (a)(ii) (must have opposite sign)		1 point
$q_{inner} = -\beta\pi a^4$		
For a correct justification		1 point
$E_{shell} = 0 = \frac{q_{enc}}{\varepsilon_0} \therefore q_{enc} = 0 = q_{sphere} + q_{inner} \therefore q_{inner} = -q_{sphere}$		

ii. 1 point

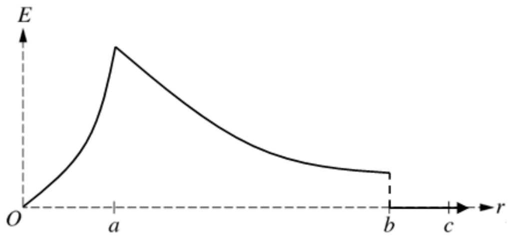
Determine the charge on the outer surface of the conducting shell.

A correct answer		1 point
$q_{outer} = 0$		

(d)

i. 3 points

On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



For concave up graph that begins at the origin and increases to $r = a$		1 point
For a concave up graph that is continuous with the first section and decreases from $r = a$ to nonzero value at $r = b$		1 point
For a graph with a value of $E = 0$ for $r > b$		1 point
Note: Due to scale, discontinuity at point b is not required.		

2018 SCORING GUIDELINES

Question 1 (continued)

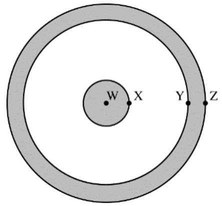
Distribution of points

(d) (continued)

ii. 2 points

The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

__1__ W   __2__ X   __3__ Y   __3__ Z

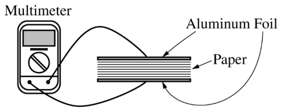


For any ranking that has Y and Z at the same potential		1 point
For ranking the electric potentials: W > X > Y & Z (because Y = Z is the first point)		1 point

AP PHYSICS C: ELECTRICITY AND MAGNETISM
2018 SCORING GUIDELINES

Question 2

15 points total **Distribution of points**



An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \text{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	*d* (m)	*C* (F)		
500	0.045	6.5×10^{-11}		
400	0.036	7.4×10^{-11}		
300	0.027	8.9×10^{-11}		
200	0.018	11.9×10^{-11}		
100	0.010	21.0×10^{-11}		

(a) 1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: ____________

Horizontal axis: ____________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

For indicating variables that will create a straight line whose slope can be used to determine the dielectric constant of the paper		1 point
Example: Vertical axis: C *Horizontal axis:* $\dfrac{1}{d}$		
Note: Student earns full credit if the axes are reversed or if they use another acceptable combination.		

AP Physics C: **Electricity and Magnetism: 2018**

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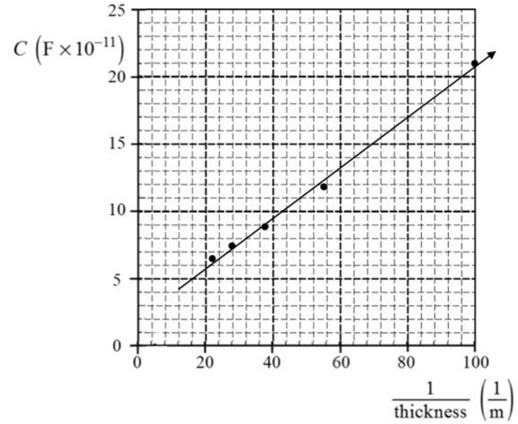
2018 SCORING GUIDELINES

Question 2 (continued)

Distribution of points

(b) 4 points

Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



For a correct scale that uses more than half the grid		1 point
For correctly labeling the axes including units		1 point
For correctly plotting the data		1 point
For drawing a straight line consistent with the plotted data		1 point

(c) 3 points

Using the straight line, calculate a dielectric constant for the paper.

For correctly calculating the slope from the best-fit line and not the data points unless the points fall on the best-fit line		1 point
$\text{slope} = \dfrac{\Delta y}{\Delta x} = \dfrac{(17-8)\left(\text{F} \times 10^{-11}\right)}{(80-32)(1/\text{m})} = 0.19\ \text{F•m}$ (Linear regression $= 0.187\ \text{F•m}$)		
For correctly relating the slope to the dielectric constant		1 point
$\text{slope} = \kappa \varepsilon_0 A$		
$\kappa = \dfrac{\text{slope}}{\varepsilon_0 A} = \dfrac{(0.19\ \text{F•m})}{\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N•m}^2\right)\left(0.060\ \text{m}^2\right)}$		
For a correct answer		1 point
$\kappa = 3.58$ (Linear regression $= 3.52$)		

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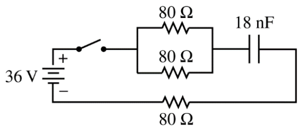
Question 2 (continued)

Distribution of points



The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical $80\ \Omega$ resistors, and an open switch, as shown in the figure above.

(d) 3 points

Calculate the current in the battery immediately after the switch is closed.

For calculating the equivalent resistance for the parallel resistors		1 point
$\dfrac{1}{R_P} = \dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{1}{80\ \Omega} + \dfrac{1}{80\ \Omega} = \dfrac{2}{80\ \Omega} = \dfrac{1}{40\ \Omega}$		
$R_P = 40\ \Omega$		
For using Ohm's law with the potential difference across the capacitor equal to zero		1 point
$I = \dfrac{V}{R} = \dfrac{V}{R_P + R_3} = \dfrac{(36\ \text{V})}{R_P + R_3}$		
For substitution of values for resistance including the value for combined resistance above		1 point
$I = \dfrac{V}{R_P + R_3} = \dfrac{(36\ \text{V})}{(40\ \Omega + 80\ \Omega)} = 0.30\ \text{A}$		

(e) 2 points

Determine the time constant for this circuit.

For using the equation for the time constant with the equivalent resistance from above		1 point
$\tau = R_{eq}C = (120\ \Omega)(18\ \text{nF})$		
For an answer with units consistent with part (d)		1 point
$t = 2.16\ \mu\text{s}$		

AP® PHYSICS C: ELECTRICITY AND MAGNETISM
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Question 2 (continued)

Distribution of points

(f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. 1 point

Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

____ Larger than measured   ____ Smaller than measured

Explain experimentally what could account for this.

Select "Larger than measured"		
For an appropriate explanation		1 point
Example: The battery is not ideal and has internal resistance, so the actual resistance for the circuit is larger than the measured resistance.		

ii. 1 points

Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

____ Larger than measured   ____ Smaller than measured

Explain experimentally what could account for this.

Select "Larger than measured"		
For an appropriate explanation		1 point
Example: Some of the sheets of paper may be thinner than expected, so the actual capacitance for the circuit is larger than the measured capacitance.		

AP PHYSICS C: ELECTRICITY AND MAGNETISM
2018 SCORING GUIDELINES

Question 3

15 points total **Distribution of points**

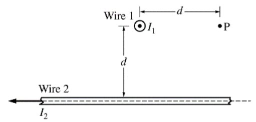


Figure 1. Side view

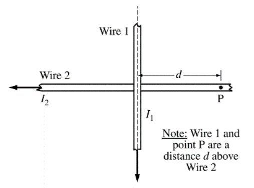


Figure 2. Top view

The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance *d* above wire 2. Point P is a horizontal distance *d* from wire 1 and a distance *d* directly above wire 2. Express your answers to parts (a) and (b) in terms of I, *d*, and physical constants, as appropriate.

(a) 2 points

Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

For attempting to use Ampere's law to calculate the magnetic field at point *P*		1 point
$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$		
$B(2\pi d) = \mu_0 I$		
For a correct answer		1 point
$B = \dfrac{\mu_0 I}{2\pi d}$		

(b) 2 points

Derive an expression for the magnitude of the net magnetic field at point P.

For indicating $B_2 = 2B_1$		1 point
$B_1 = \dfrac{\mu_0 I}{2\pi d}$ & $B_2 = \dfrac{\mu_0(2I)}{2\pi d} = \dfrac{\mu_0 I}{\pi d}$		
For an indication that the magnitude of the magnetic field is the vector sum of B_1 and B_2		1 point
$B_{net} = \sqrt{B_1^2 + B_2^2} = \sqrt{B^2 + B^2} = \sqrt{\left(\dfrac{\mu_0 I}{2\pi d}\right)^2 + \left(\dfrac{\mu_0(2I)}{2\pi d}\right)^2} = \dfrac{\sqrt{5}\mu_0 I}{2\pi d}$		

AP PHYSICS C: ELECTRICITY AND MAGNETISM
2018 SCORING GUIDELINES

Question 3 (continued)

Distribution of points

(c) 2 points

Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

For correctly relating the angle to the individual magnetic fields		1 point
$\theta = \tan^{-1}\left(\dfrac{B_1}{B_2}\right)$		
For correctly substituting B_1 and B_2 into the equation		1 point
$\theta = \tan^{-1}\left(\dfrac{B_1}{B_2}\right) = \tan^{-1}\left(\dfrac{\left(\dfrac{\mu_0 I}{2\pi d}\right)}{\left(\dfrac{\mu_0(2I)}{2\pi d}\right)}\right) == \tan^{-1}\left(\dfrac{1}{2}\right) = 26.6°$		

(d) 2 points

Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2? Assume gravitational effects are negligible.

____ Wire 1 will not move.

____ Wire 1 will move upward as viewed in Figure 1.

____ Wire 1 will move downward as viewed in Figure 1.

____ Wire 1 will rotate clockwise as viewed in Figure 2.

____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

For stating there is no translational motion since the magnetic forces on the wire cancel		1 point
For stating there is a net torque which causes rotation		1 point
Example: In Figure 2, the top portion of wire 1 will be in a magnetic field into the page from wire 2 and, thus, will experience a force to the right. The bottom portion of wire 1 will be in a magnetic field out of the page from wire 2 and, thus, will experience a force to the left. So the net force will be zero, but there will be a net clockwise torque, so the wire will rotate clockwise.		

AP PHYSICS C: ELECTRICITY AND MAGNETISM
2018 SCORING GUIDELINES

Question 3 (continued)

Distribution of points

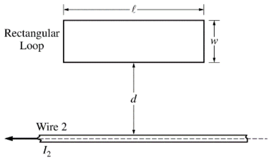


Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1-kt)$, where k is a positive constant with units of s^{-1}.

(e) 1 point

Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

__X__ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

For selecting $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$		1 point

AP PHYSICS C: ELECTRICITY AND MAGNETISM
2018 SCORING GUIDELINES

Question 3 (continued)

Distribution of points

(f) 3 points

Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \frac{\mu_0 I_0 \ell(1-kt)}{\pi} \ln\left(\frac{d+w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

For attempting to take the time derivative of the magnetic flux to calculate the emf		1 point
$\varepsilon = \left\lvert -\frac{d\Phi}{dt} \right\rvert = \frac{\mu_0 I_0 \ell}{\pi} \ln\left(\frac{d+w}{d}\right) \left\lvert \frac{d}{dt}[(1-kt)] \right\rvert$		
$\varepsilon = \frac{\mu_0 I_0 k \ell}{\pi} \ln\left(\frac{d+w}{d}\right)$		
For dividing the emf by the resistance to calculate the current		1 point
$I = \frac{\varepsilon}{R} = \frac{\frac{\mu_0 I_0 k \ell}{\pi} \ln\left(\frac{d+w}{d}\right)}{R}$		
For a correct answer		1 point
$I = \frac{\mu_0 I_0 k \ell}{\pi R} \ln\left(\frac{d+w}{d}\right)$		

(g) 3 points

What is the direction of the current, if any, induced in the loop as seen in Figure 3?

____ Clockwise ____ Counterclockwise

____ Undefined because there is no current induced in the loop

Justify your answer.

For selecting "Clockwise" with an attempt at a relevant justification		1 point
For indicating that the flux inside the loop will decrease		1 point
For using Lenz's law to relate the decrease in the flux to the clockwise current		1 point
Example: Because the current in the wire is decreasing, the flux in the loop will decrease. According to Lenz's law, the induced current should create a magnetic field to oppose this decrease. Thus the induced magnetic field must be into the page, and the current in the loop must be clockwise.		

2017

AP Physics C: Electricity and Magnetism
Scoring Guidelines

AP® PHYSICS
2017 SCORING GUIDELINES

General Notes About 2017 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth one point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections—Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based and AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8\ \mathrm{m/s^2}$, but use of $10\ \mathrm{m/s^2}$ is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

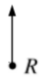
2017 SCORING GUIDELINES

Question

15 points total | **Distribution of ints**

(a)

i. 1 point



For correctly drawing a single vector pointing upward at point *R* — 1 point

ii. 1 point

For correctly drawing a single vector pointing downward at point *S* — 1 point

(b)

i. 2 points

For using Gauss's law to calculate the electric flux — 1 point

$$\Phi = \frac{Q}{\varepsilon_0} = \frac{\rho V}{\varepsilon_0}$$

For a correct answer — 1 point

$$\Phi = \frac{\rho_0 A h}{\varepsilon_0}$$

ii. 3 points

For correctly applying Gauss's law to the slab — 1 point

$$\frac{Q}{\varepsilon_0} = \oint \vec{E} \bullet d\vec{A} = EA_{top} + EA_{bottom}$$

For substituting the flux from part (b)(i) into equation above and for including the contributions of the top and bottom parts of the Gaussian surface — 1 point

$$\frac{\rho_0 A h}{\varepsilon_0} = 2EA$$

For correctly relating h to $2z$ — 1 point

$$\frac{\rho_0 A (2z)}{\varepsilon_0} = 2EA$$

$$E = \frac{\rho_0 z}{\varepsilon_0}$$

2017 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

(c)

i. 1 point

Selecting "$z_0 < 0$"

For a correct justification — 1 point

Example: The electric field due to the plates alone is directed toward the top of the page. Therefore, the net electric field can only be zero where the electric field of the slab is directed toward the bottom of the page. So the net electric field can only be zero when $z_0 < 0$.

ii. 3 points

For setting the net electric field between the plates equal to zero and relating it to the sum of the electric fields of the plates and the slab — 1 point

$$E_{total} = 0 = E_{plate} + E_{slab}$$

For using the correct formula for the electric field between the plates regardless of sign — 1 point

$$\frac{\sigma}{\varepsilon_0} = -\frac{\rho_0 z_0}{\varepsilon_0}$$

$$z_0 = -\frac{\sigma}{\rho_0}$$

Substitute into equation above

$$z_0 = -\frac{\left(2.0 \times 10^{-6}\ \mathrm{C/m^2}\right)}{\left(1.0 \times 10^{-3}\ \mathrm{C/m^3}\right)}$$

For an answer that is consistent with the formula provided by the student above (must include units) — 1 point

$$z_0 = -2.0 \times 10^{-3}\ \mathrm{m}$$

2017 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

(d) 4 points

For using the sum of the electric potential of the slab and the plates as the total potential difference between the center and top of the slab — 1 point

$\Delta V = \Delta V_{slab} + \Delta V_{plate}$

$\Delta V = \int E_{total}\, dr = \int \left(E_{slab} + E_{plate}\right) dr$

$\Delta V = \int E_{slab} dr + \int E_{plate}\, dr$

For integrating with proper limits or constants of integration — 1 point

$$\Delta V = \int_{z=0}^{z=0.005} \frac{\rho_0 z}{\varepsilon_0} dz + \int_{z=0}^{z=0.005} \frac{\sigma}{\varepsilon_0} dz$$

For correctly integrating each term regardless of whether they contain the correct expressions — 1 point

$$\Delta V = \frac{\rho_0}{\varepsilon_0}\left[\frac{z^2}{2}\right]_{z=0}^{z=0.005} + \frac{\sigma}{\varepsilon_0}[z]_{z=0}^{z=0.005} = \frac{\rho_0}{2\varepsilon_0}\left(0.005^2 - 0\right) + \frac{\sigma}{\varepsilon_0}(0.005 - 0)$$

$$\Delta V = \frac{\rho_0 z^2}{2\varepsilon_0} + \frac{\sigma z}{\varepsilon_0}$$

$$\Delta V = \frac{\left(1.0 \times 10^{-3}\ \text{C/m}^3\right)\left(5.0 \times 10^{-3}\ \text{m}\right)^2}{(2)\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\bullet\text{m}^2\right)} + \frac{\left(2.0 \times 10^{-6}\ \text{C/m}^2\right)\left(5.0 \times 10^{-3}\ \text{m}\right)}{\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\bullet\text{m}^2\right)}$$

For a correct answer — 1 point

$\Delta V = 2540\ \text{V}$

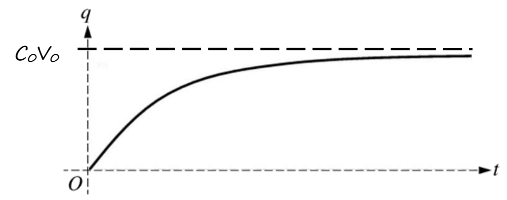
2017 SCORING GUIDELINES

Question

15 points total

Distribution of points

(a) 3 points

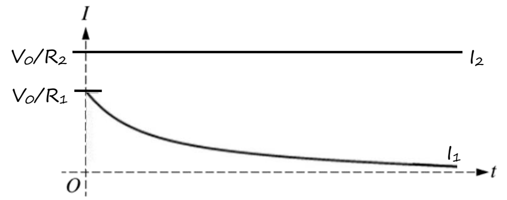


For a concave down curve starting at the origin — 1 point

For a horizontal asymptote — 1 point

For properly labeling the horizontal asymptote — 1 point

(b) 4 points



For showing that I_2 is horizontal with the correct vertical intercept — 1 point

For showing that I_1 is concave up and asymptotic to I = 0 — 1 point

For correctly labeling the vertical intercept of the I_1 graph — 1 point

For drawing the I_2 graph always above the I_1 graph — 1 point

(c) 2 points

Select "$\Delta t_C < \Delta t_D$"

For indicating that the equivalent resistance during discharging is greater than during charging — 1 point

For a statement relating the greater resistance to the greater time constant, or to a smaller current — 1 point

Example: Because the resistance as the capacitor discharges is greater than when it charges, the time constant is larger for discharging. Therefore, the time to charge to 50% of its maximum charge is less than the time to discharge to 50% of its maximum charge.

2017 SCORING GUIDELINES

Question 2 (continued)

Distribution of points

(d)

i. 2 points

For using a correct statement of Ohm's law or loop rule with the switch open — 1 point

$V = IR$

$I = \dfrac{V_C}{R_{tot}} = \dfrac{V_0}{R_1 + R_2}$

Substitute correct values into equation above

$I = \dfrac{(1.5\text{ V})}{(100\ \Omega + 150\ \Omega)}$

For a correct answer — 1 point

$I = 6.0\text{ mA}$ with units, or 0.006 without units

ii. 1 point

For a correct justification — 1 point

Example: Once the capacitor begins to discharge, the charge stored in the capacitor will decrease and the potential difference across the capacitor decreases. Therefore, the current through resistor R_2 decreases.

(e)

i. 1 point

For correctly substituting into an equation for the energy stored in a capacitor, including units for numerical values — 1 point

$U = \dfrac{1}{2}CV^2 = \dfrac{1}{2}(80\ \mu\text{F})(1.5\text{ V})^2 = 90\ \mu\text{J}$

ii. 2 points

For setting up a valid equation or argument to calculate the energy dissipated by resistor R_1 — 1 point

$E_{dis} = Pt = I^2Rt \therefore E_{dis} \sim R \therefore E_{dis,1} = E_{dis,tot}\dfrac{R_1}{(R_1 + R_2)}$

For correctly substituting into the equation above — 1 point

$E_{dis,1} = (90\ \mu\text{J})\dfrac{(150\ \Omega)}{(150\ \Omega + 100\ \Omega)} = 54\ \mu\text{J}$

Alternate Solution on next page.

2017 SCORING GUIDELINES

Question 2 (continued)

Distribution of points

(e)(ii) continued

Alternate Solution — *Alternate Points*

Derive an equation for the current through resistor R_1 as a function of time

$I(t) = I_{MAX}e^{-t/\tau} = \dfrac{V_{MAX}}{(R_1 + R_2)}e^{-t/(R_1+R_2)C} = \dfrac{(1.5\text{ V})}{(250\ \Omega)}e^{-t/(250\ \Omega)(80\ \mu\text{F})}$

$I(t) = (6.0\text{ mA})e^{-t/(0.02)}$

For using a correct formula with integral calculus to calculate the energy dissipated by resistor R_1 — 1 point

$E = \int Pdt = \int I^2Rdt = \int I(t)^2 R_1dt = \int\left((6.0\text{ mA})e^{-t/(0.02)}\right)^2(150\ \Omega)dt$

For integrating the above equation with correct limits or constants of integration — 1 point

$E = (0.0054)\int_{t=0}^{t=\infty}\left(e^{-t/(0.02)}\right)^2 dt = (0.0054)\int_{t=0}^{t=\infty}e^{-100t}dt = (0.0054)\left[\dfrac{e^{-100t}}{(-100)}\right]_{t=0}^{t=\infty}$

$E = \left(-5.4\times10^{-5}\right)\left(e^{-\infty} - e^0\right) = 54\ \mu\text{J}$

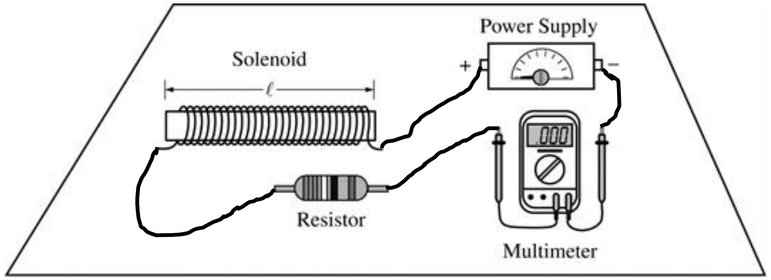
2017 SCORING GUIDELINES

Question

15 points total — **Distribution of points**

(a)

i. 2 points



For connections that create a circuit with the power supply, solenoid, and resistor in series with each other — 1 point

For connections that create a circuit with the multimeter in series with the inductor/resistor combination — 1 point

ii. 1 point

For selecting the answer consistent with the answer in part (a)(i) — 1 point

iii. 3 points

For selecting point *B* — 1 point

For any indication that the magnetic field outside the solenoid is less than inside the solenoid — 1 point

For any indication that the closer you are to the center of the solenoid, the better the approximation of an ideal solenoid due to the lack of edge effects — 1 point

Example: The equation for the magnetic field for a solenoid is for an ideal solenoid. This does not work outside the solenoid (this eliminates *C* and *D*). In addition, as you approach the edge of the solenoid, the ideal approximation is not as accurate (this eliminates *A*). Thus, point *B* gives the best approximation of an ideal solenoid.

2017 SCORING GUIDELINES

Question 3 (continued)

Distribution of points

(b) 2 points

For any indication that Solenoid 1 has more turns per unit length — 1 point

For any indication that Solenoid 1 is longer and therefore more like an ideal solenoid — 1 point

Examples:

The equation for the magnetic field of a solenoid $(B = \mu_0 nI)$ is based on an ideal solenoid in which you can ignore edge effects. Because Solenoid 1 has more turns per unit length, it is a better approximation of an ideal solenoid and thus its slope will give a better value for the permeability of free space.

The slope for Solenoid 1 is closer to the accepted value than the slope for Solenoid 2; therefore, the graph for Solenoid 1 is better.

(c)

i. 3 points

For correctly calculating the slope for the solenoid chosen in part (b) using the best-fit line and not the data points — 1 point

$$\text{slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(10-4)\left(\mathrm{T}\times 10^{-5}\right)}{(75-24)(\mathrm{A/m})} = 1.17\times 10^{-6}\ \mathrm{T{\bullet}m/A}$$

For correctly relating the magnetic permittivity of free space to the slope — 1 point

$\mu_0 = \text{slope}$

For correct units — 1 point

$\mu_0 = 1.17\times 10^{-6}\ \mathrm{T{\bullet}m/A}$

For Solenoid 2

$$\text{slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(8-4)\left(\mathrm{T}\times 10^{-5}\right)}{(82-35)(\mathrm{A/m})} = 8.51\times 10^{-7}\ \mathrm{T{\bullet}m/A}$$

ii. 1 point

For using a correct equation for % error including the substitution of the accepted and the measured values — 1 point

$$\%\ \text{error} = \frac{|\text{acc} - \text{exp}|}{\text{acc}}\times 100\% = \frac{\left|\left(1.26\times 10^{-6}\right) - \left(1.18\times 10^{-6}\right)\right|}{\left(1.26\times 10^{-6}\right)}\times 100\% = 6.35\%$$

For Solenoid 2

$$\%\ \text{error} = \frac{|\text{acc} - \text{exp}|}{\text{acc}}\times 100\% = \frac{\left|\left(1.26\times 10^{-6}\right) - \left(8.51\times 10^{-7}\right)\right|}{\left(1.26\times 10^{-6}\right)}\times 100\% = 32.4\%$$

2017 SCORING GUIDELINES

Question 3 (continued)

	Distribution of points
(d)	
i. 1 point	
For a correct explanation involving any extraneous *B* fields	1 point
Example: A component of Earth's magnetic field is in the direction of the inductor's magnetic field.	
ii. 2 points	
For stating that either the solenoid or the circuit has near zero resistance	1 point
For stating that as a result of near zero resistance the current becomes very high	1 point
Example: The inductor has near zero resistance. Therefore, without the resistor in series with it, the inductor will draw a near infinite current that the multimeter cannot handle.	