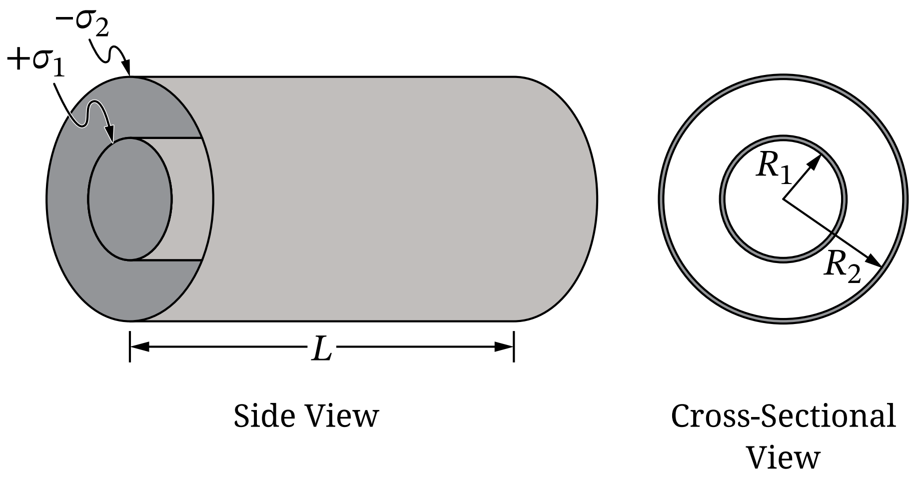
Question 1: Version J

1. An isolated, air-filled, charged capacitor consists of two conducting, coaxial, cylindrical shells that each have length L. The inner shell has radius R_1 and the outer shell has radius R_2, as shown in Figure 1, where $R_1 < R_2 \ll L$. The surface charge densities (amounts of charge per unit area) of the inner and outer shells are $+\sigma_1$ and $-\sigma_2$, respectively. The absolute values of the total charges on the shells are equal.



Note: Figures not drawn to scale.

Figure 1

A.

i. Using Gauss's law, **derive** an expression for the magnitude E of the electric field as a function of the radial distance r from the center of the capacitor for the region $R_1 < r < R_2$. Express your answer in terms of R_1, σ_1, r, and physical constants, as appropriate.

ii. **Derive** an expression for the absolute value $|\Delta V|$ of the potential difference between the outer and inner shells in terms of R_1, R_2, σ_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

iii. On the axes shown in Figure 2, **sketch** a graph of E as a function of r from $r = 0$ to a position that is outside the outer shell.

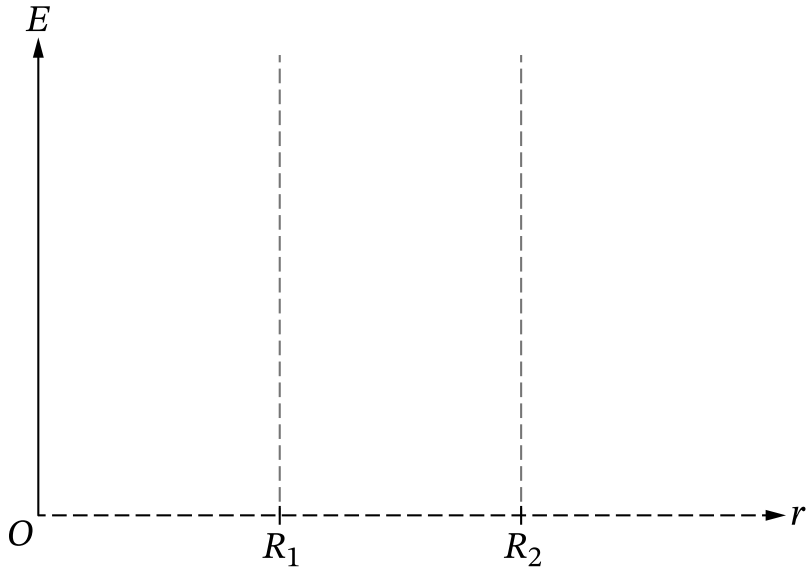
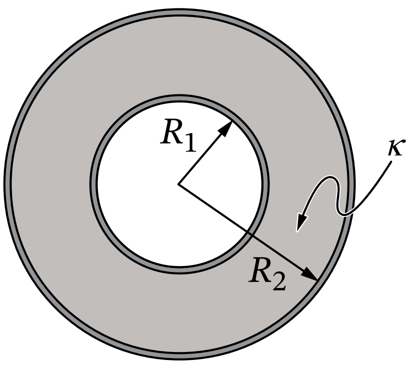


Figure 2

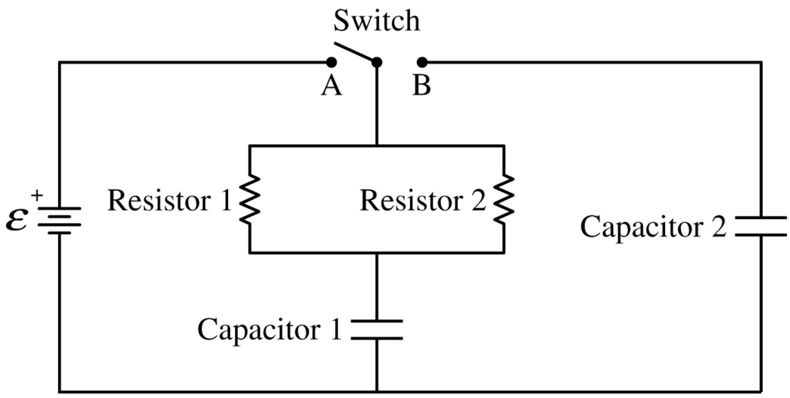
B. A material of dielectric constant κ is inserted into the isolated, charged capacitor such that the material fills the region $R_1 < r < R_2$, as shown in Figure 3.



Cross-Sectional View

Figure 3

Derive an expression for the capacitance C of the capacitor with the material inserted in terms of L, R_1, R_2, κ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.



. The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

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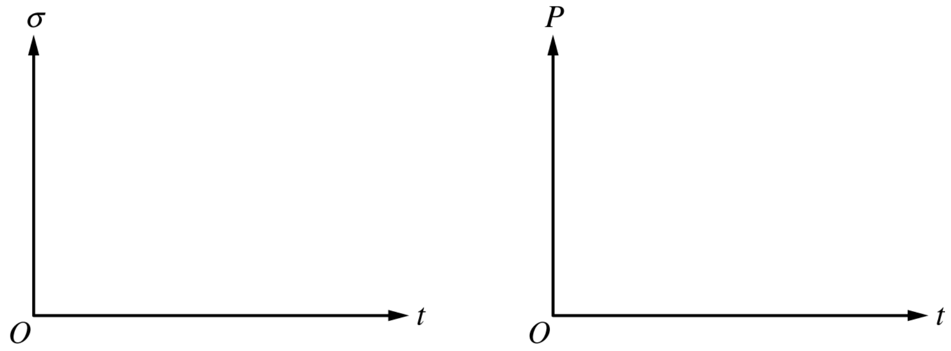
Continue your response to **QUESTION 3** on this page.

(b) On the axes shown, sketch graphs of the surface charge density σ on the positive plate of Capacitor 1 and the total power P dissipated by the resistors as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.



A long time after the switch is closed to Position A, the charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

(c) At time t_1, the switch is closed to Position B.

i. Immediately after time t_1, is the direction of the current in the switch directed toward the left, directed toward the right, or is there no current? Briefly justify your answer.

ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

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iii. Derive an expression for the total energy E_R dissipated by resistors 1 and 2 from immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.

With the switch still closed to Position B, the parallel plates of Capacitor 2 are moved so that the separation distance increases by a factor of 2.

(d) Determine the ratio $\frac{U_2}{U_1}$ of the energy U_2 stored in Capacitor 2 to the energy U_1 stored in Capacitor 1 a long time after the plates of Capacitor 2 have been moved. Briefly justify your answer.

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Continue your response to **QUESTION 3** on this page.

With the capacitors still charged as in part (d), the switch is now closed to Position A.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.

i. Derive an expression for the current I_0 from the battery immediately after the switch is closed to Position A.

ii. Determine the current I_∞ from the battery a long time after the switch is closed to Position A.

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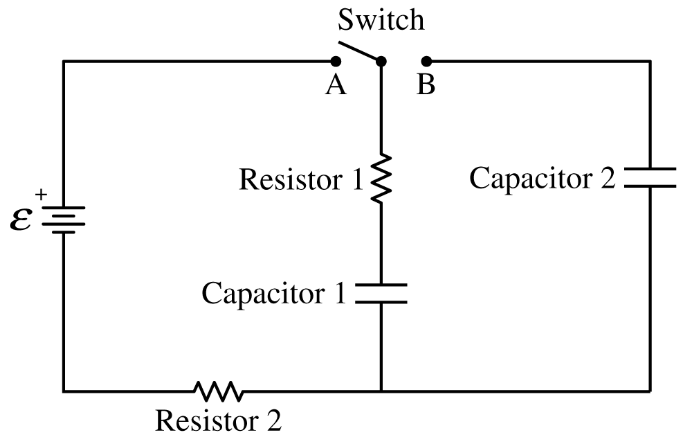
STOP

END OF EXAM



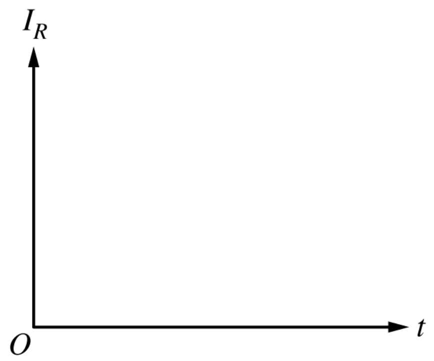
. The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

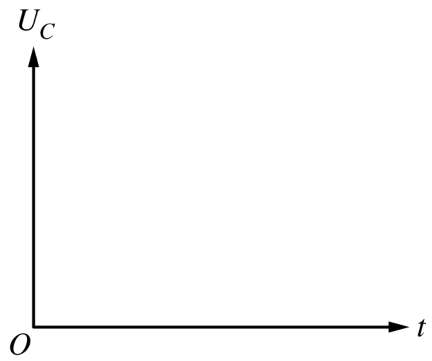
At time $t = 0$, the switch is closed to Position A.

(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

(b) On the axes shown, sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.





A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

(c) At time t_1, the switch is closed to Position B.

i. Immediately after t_1, is the direction of the current in Resistor 1 directed up, directed down, or is there no current? Briefly justify your answer.

ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

iii. Derive an expression for the total energy dissipated by Resistor 1 immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.

With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

(d) Determine the charge on the positive plate of Capacitor 2 a long time after the dielectric has been inserted. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

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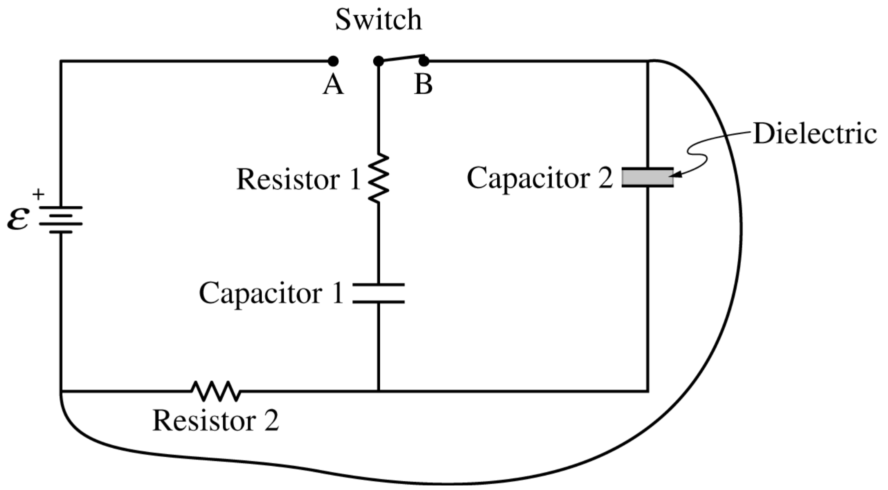
Continue your response to **QUESTION 3** on this page.



With the switch still closed to Position B, a wire of negligible resistance is connected between two corners of the circuit, as shown.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.

i. Derive an expression for the current in Resistor 2 immediately after the wire is connected to the circuit.

ii. Determine the current in Resistor 2 a long time after the wire is connected to the circuit.

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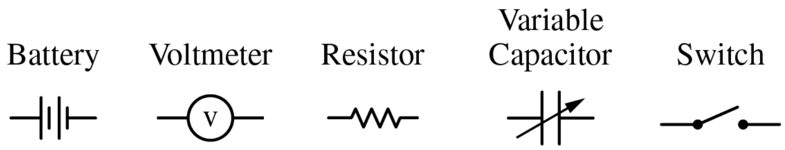
STOP

END OF EXAM

. The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.



(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the potential difference across the capacitor as it discharges through the resistor.

The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

(b) Show that the potential difference ΔV_C across the capacitor as a function of time t is $\Delta V_C(t) = \Delta V_0 e^{-\frac{t}{RC}}$ as the capacitor discharges.

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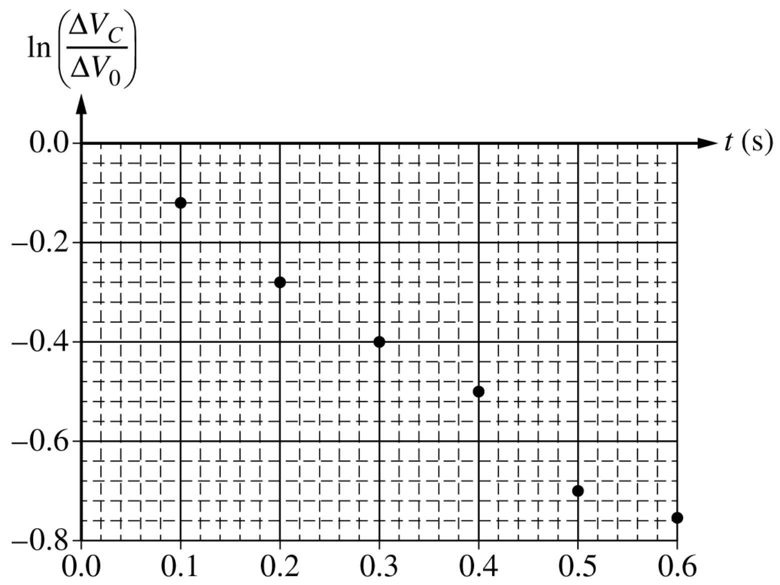
Continue your response to **QUESTION 2** on this page.

(c) The experiment is performed using a resistor of $R = 150\,\text{k}\Omega$. Data for the potential difference ΔV_C across the capacitor as a function of t are recorded and a plot of $\ln\left(\frac{\Delta V_C}{\Delta V_0}\right)$ as a function of t is created on the graph below.



i. Draw the best-fit line for the data.

ii. Using the best-fit line, calculate a value for the unknown capacitance C.

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Continue your response to **QUESTION 2** on this page.

(d) The capacitor is adjusted so that the surface area of the plates is increased, and the experiment is repeated. Would the slope of the best-fit line in the second experiment be more steep, less steep, or unchanged compared to the slope of the best-fit line in part (c)?

___ More steep ___ Less steep ___ Unchanged

Briefly justify your answer.

(e) The ideal battery is then replaced with a non-ideal battery with internal resistance r, and the experiment is repeated.

i. Would the slope of the graph in this final experiment change compared to the graph in part (c)?

___ Yes ___ No

Briefly justify your answer.

ii. Would the vertical intercept of the graph in this final experiment change compared to the graph in part (c)?

___ Yes ___ No

Briefly justify your answer.

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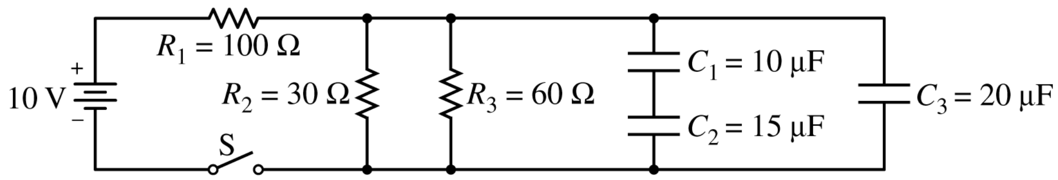
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. The circuit shown above is composed of an ideal 10 V battery, three resistors and three capacitors with the values shown, and an open switch S. The capacitors are initially uncharged. Switch S is now closed.

(a) Calculate the current through R_1 immediately after switch S is closed.

Switch S has been closed for a long time, and the circuit has reached a steady state.

(b) Calculate the potential difference across R_1.

(c)
i. Calculate the charge stored on the positive plate of capacitor C_2.

ii. Is the charge stored on capacitor C_3 greater than, less than, or equal to the charge stored on capacitor C_2 ?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

GO ON TO THE NEXT PAGE.

Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.

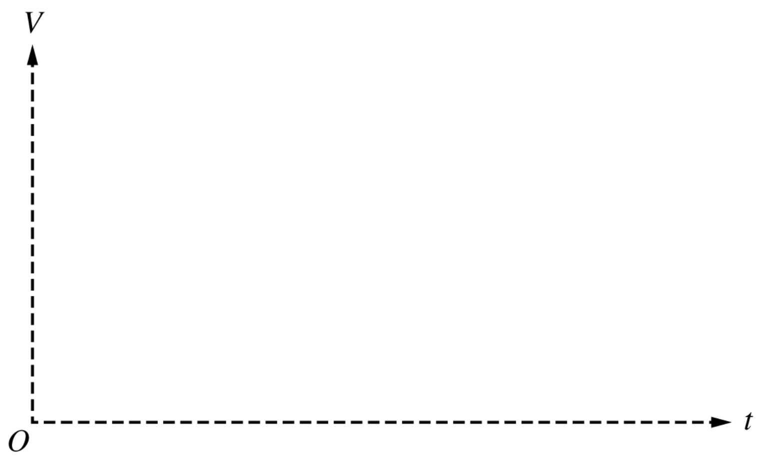
Switch S is then opened.

(d)

i. Determine the current through R_1 immediately after the switch is opened.

ii. Calculate the current through R_2 immediately after the switch is opened.

(e) On the axes below, sketch a graph of the potential difference V across capacitor C_2 as a function of time t if switch S is opened at time $t = 0$. Label the maximum value.



Capacitor C_3 is replaced by two $10\,\mu\text{F}$ capacitors connected in series, switch S is closed, and the circuit reaches equilibrium. Switch S is then opened at time $t = 0$.

(f) For $t > 0$, would the sketch of a graph of the new voltage across C_2 as a function of time be above, below, or the same as the sketch for part (e) ?

_____ Above   _____ Below   _____ The same

Justify your answer.

GO ON TO THE NEXT PAGE.

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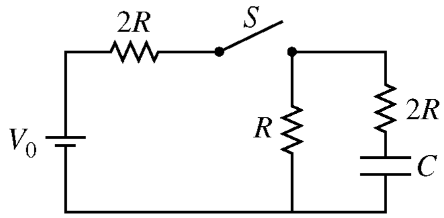


1. The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

   (a) Derive an expression for the steady-state current supplied by the battery.

   (b) Derive an expression for the charge on the capacitor.

   (c) Derive an expression for the energy stored in the capacitor.

   Now the switch is opened at time $t = 0$.

   (d) Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time t after the switch is opened.

(e)

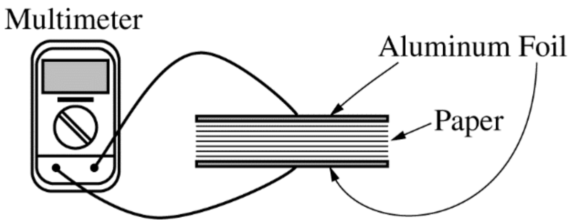
i. Calculate the current in resistor R immediately after the switch is opened.

ii. On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.

(f) Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c) ?

____ Greater than ____ Less than ____ Equal to

Justify your answer.



2. An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060 \text{ m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)		
500	0.045	6.5×10^{-11}		
400	0.036	7.4×10^{-11}		
300	0.027	8.9×10^{-11}		
200	0.018	11.9×10^{-11}		
100	0.010	21.0×10^{-11}		

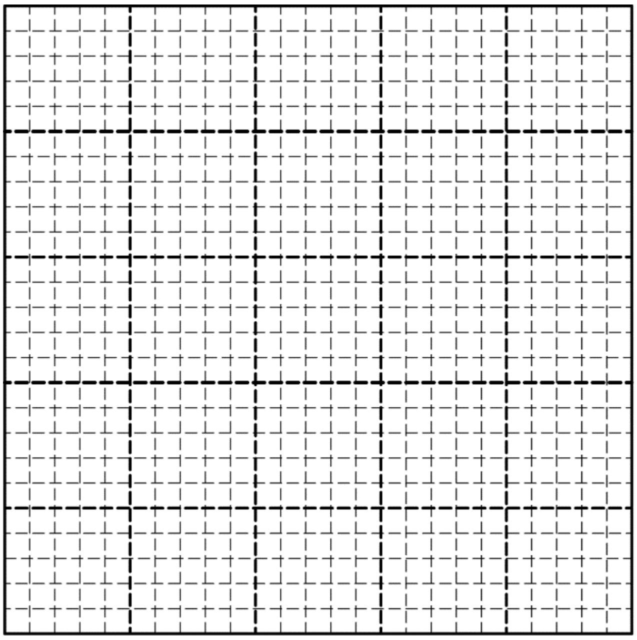
(a) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: ____________

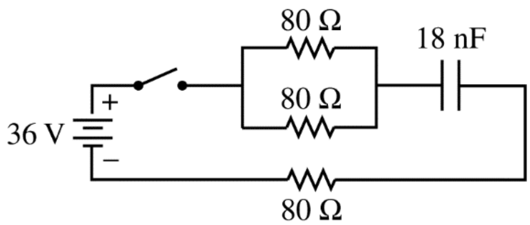
Horizontal axis: ____________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

(b) Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(c) Using the straight line, calculate a dielectric constant for the paper.



The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical $80\ \Omega$ resistors, and an open switch, as shown in the figure above.

(d) Calculate the current in the battery immediately after the switch is closed.

(e) Determine the time constant for this circuit.

(f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

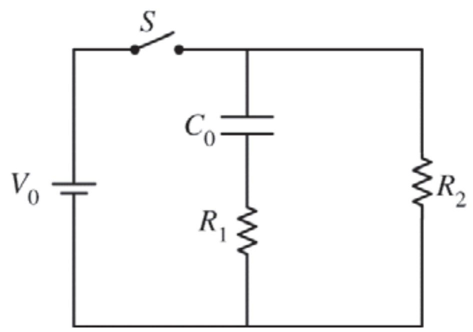
____ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

ii. Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

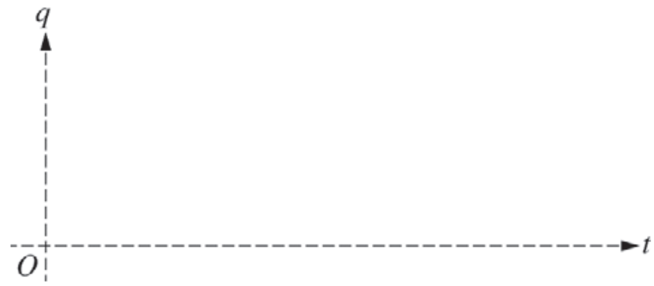
____ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

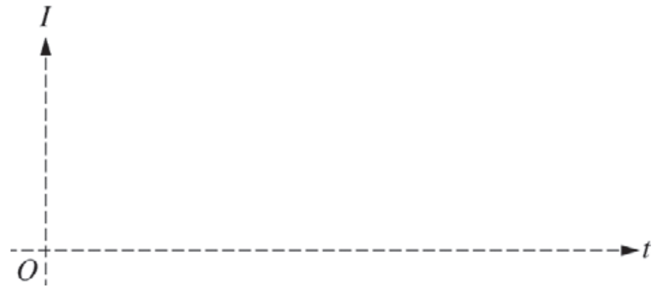


2. In the circuit above, an ideal battery of voltage V_0 is connected to a capacitor with capacitance C_0 and resistors with resistances R_1 and R_2, with $R_1 > R_2$. The switch *S* is open, and the capacitor is initially uncharged.

(a) The switch is closed at time $t = 0$. On the axes below, sketch the charge q on the capacitor as a function of time t. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



(b) On the axes below, sketch the current I through each resistor as a function of time t. Clearly label the two curves as I_1 and I_2, the currents through resistors R_1 and R_2, respectively. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



The circuit is constructed using an ideal 1.5 V battery, an 80 μF capacitor, and resistors $R_1 = 150\ \Omega$ and $R_2 = 100\ \Omega$. The switch is closed, allowing the capacitor to fully charge. The switch is then opened, allowing the capacitor to discharge.

(c) The time it takes to charge the capacitor to 50% of its maximum charge is Δt_C. The time it takes for the capacitor to discharge to 50% of its maximum charge is Δt_D. Which of the following correctly relates the two time intervals?

____ $\Delta t_C > \Delta t_D$ ____ $\Delta t_C = \Delta t_D$ ____ $\Delta t_C < \Delta t_D$

Justify your answer.

(d)

i. Calculate the current through resistor R_2 immediately after the switch is opened.

ii. Is the current through resistor R_2 increasing, decreasing, or constant immediately after the switch is opened?

____ Increasing ____ Decreasing ____ Constant

Justify your answer.

(e)

i. Calculate the energy stored in the capacitor immediately after the switch is opened.

ii. Calculate the energy dissipated by resistor R_1 as the capacitor completely discharges.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

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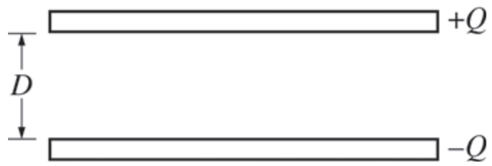


E&M.1.

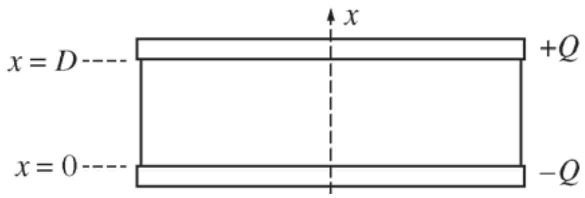
A parallel-plate capacitor is constructed of two parallel metal plates, each with area *A* and separated by a distance *D*. The plates of the capacitor are each given a charge of magnitude *Q*, as shown in the figure above. Ignore edge effects.

(a)

i. On the figure above, draw an arrow to indicate the direction of the electric field between the plates.

ii. On the figure above, draw an appropriate Gaussian surface that will be used to derive an expression for the magnitude of the electric field *E* between the plates.

iii. Using Gauss's law and the Gaussian surface from part (a)-ii, derive an expression for the magnitude of the electric field *E* between the plates. Express your answer in terms of *A*, *D*, *Q*, and physical constants, as appropriate.



The space between the plates is now filled with a dielectric material that is engineered so that its dielectric constant varies with the distance from the bottom plate to the top plate, defined by the x-axis indicated in the diagram above. As a result, the electric field between the plates is given by $\vec{E} = -\dfrac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\hat{i}$, where κ_0 is a positive constant. Express all algebraic answers to the remaining parts in terms of *A, D*, *Q*, κ_0, *x*, and physical constants, as appropriate.

(b) Determine an expression for the dielectric constant κ as a function of *x*.

(c)

i. Write, but do NOT solve, an equation that could be used to determine the potential difference V between the plates of the capacitor.

ii. Using the equation from part (c)-i, derive an expression for the potential difference $V_D - V_0$, where V_D is the potential of the top plate and V_0 is the potential of the bottom plate.

(d) Determine the capacitance of the capacitor.

(e) The energy stored in the capacitor that has a varying dielectric is U_V. A second capacitor that has a constant dielectric of value κ_0 is also given a charge Q. The energy stored in the second capacitor is U_C. How do the values of U_V and U_C compare?

____ $U_V < U_C$ ____ $U_V > U_C$ ____ $U_V = U_C$

Justify your answer.