Question 3: Free-Response Question — 15 points

(a)	For a loop rule expression that includes terms for the equivalent resistance $\frac{R}{2}$ and the potential difference across the battery	**1 point**
	For an expression that includes charge Q in the term relating the potential difference across the capacitor and includes charge per unit time $\frac{dQ}{dt}$ in the term relating the potential difference across the pair of resistors	**1 point**

Example Response

$$\mathcal{E} - \Delta V_C - \Delta V_{R,eq} = 0$$

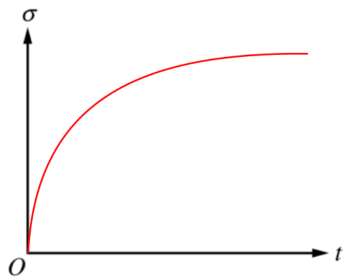
$$\mathcal{E} - \frac{Q}{C} - \frac{R}{2}\frac{dQ}{dt} = 0$$

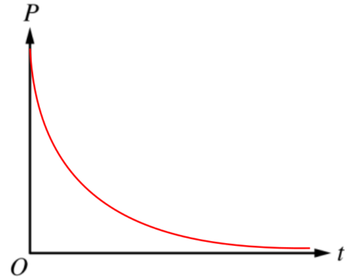
Total for part (a) — 2 points

(b)	For sketching a concave down and increasing curve on the graph σ as a function of t	**1 point**
	For sketching a curve that is concave up and decreasing on the graph of P as a function of t	**1 point**
	For sketching both curves that approach a slope of zero as time increases	**1 point**

Scoring Note: The third point can be earned even if the first two points are not earned.

Example Response





Total for part (b) — 3 points

(c)(i)	For a correct justification that could include one of the following: • An indication that the current is to the right with a justification that includes a statement that indicates that positive charge has accumulated on the top plate of Capacitor 1 and/or negative charge has accumulated on the bottom plate of Capacitor 1 when the switch was closed to Position A • An indication that the current is to the right with a justification that includes a statement that indicates that the value of the electric potential of the top plate of Capacitor 1 is larger than the electric potential of the bottom plate of Capacitor 1 when the switch was closed to Position A	**1 point**

Example Responses

The current is directed towards the right because the top plate of Capacitor 1 is positively charged, meaning conventional current will flow clockwise.

OR

Toward the right. Current flows from high to low potential so it will flow from the top plate up and right through the switch.

(c)(ii)	For indicating that the total charge on the positive plate of Capacitor 2 is $\frac{2}{3}Q_0$	**1 point**

Scoring Note: This point can be earned without supporting calculations.

Example Response

The potential difference across Capacitor 1 is equal to the potential difference across Capacitor 2. Capacitor 2 has twice the capacitance of Capacitor 1. Therefore, Capacitor 2 stores twice the charge that is stored on Capacitor 1. Due to conservation of charge, Capacitor 2 stores an amount of charge equal to $\frac{2}{3}Q_0$.

(c)(iii) For an indication that the total energy dissipated by the resistors is the difference between an initial electric potential energy stored in one or both capacitors at time $t = t_1$ and a final electric potential energy stored on one or both capacitors after the new steady state conditions have been reached — **1 point**

Example Response

$$E_R = U_C - U_{0C}$$

For indicating that only Capacitor 1 stores nonzero electric potential energy initially and both capacitors store nonzero electric potential energy after the new steady state conditions have been reached, or alternative consistent with part (c)(ii) — **1 point**

Example Response

$$U_{0C} = U_{01}$$
$$U_C = U_1 + U_2$$

For correct substitutions for the charges stored on the capacitors after the new steady state conditions have been reached consistent with part (c)(ii) — **1 point**

Example Response

$$\Delta E_R = U_C - U_{0C}$$
$$\Delta E_R = \left(\frac{1}{2}\left(\frac{1}{C}\right)\left(\frac{Q_0}{3}\right)^2 + \frac{1}{2}\left(\frac{1}{2C}\right)\left(\frac{2Q_0}{3}\right)^2\right) - \frac{1}{2}\frac{Q_0^2}{C}$$

Example Solution

$$\Delta E_R = U_C - U_{0C}$$
$$\Delta E_R = \left(\frac{1}{2}\left(\frac{1}{C}\right)\left(\frac{Q_0}{3}\right)^2 + \frac{1}{2}\left(\frac{1}{2C}\right)\left(\frac{2Q_0}{3}\right)^2\right) - \frac{1}{2}\frac{Q_0^2}{C}$$
$$\Delta E_R = \frac{Q_0^2}{6C} - \frac{1}{2}\frac{Q_0^2}{C}$$
$$\Delta E_R = -\frac{Q_0^2}{3C}$$

Total for part (c) — 5 points

AP Physics C: **Electricity and Magnetism: 2023 Set 1**

(d) For indicating that the potential difference across each capacitor is the same or that the charge stored on each capacitor in steady state is the same — **1 point**

Example Responses

$$\Delta V_1 = \Delta V_2$$

OR

$$Q_1 = Q_2$$

For recognizing that the capacitance of each capacitor is now the same in the new configuration — **1 point**

Example Response

After steady state conditions are reached, both capacitors have the same potential difference. The new capacitance of Capacitor 2 *is equal to the capacitance of Capacitor* 1 *because the capacitance of a capacitor is inversely related to the distance between the plates of a capacitor. Therefore, since* $U_C = \frac{1}{2}C(\Delta V)^2$, $\frac{U_2}{U_1} = 1$.

Total for part (d) — 2 points

(e)(i) For a loop rule that includes the terms for the emf of the battery, the potential difference across the pair of resistors, and the potential difference across Capacitor 1 — **1 point**

Example Response

$$\mathcal{E} - \Delta V_R - \Delta V_C = 0$$

For a correct answer — **1 point**

Example Response

$$I = \frac{Q_0}{RC}$$

Example Solution

$$\mathcal{E} - \Delta V_R - \Delta V_C = 0$$
$$\mathcal{E} - I\left(\frac{R}{2}\right) - \left(\frac{Q_0}{2C}\right) = 0$$
$$\frac{Q_0}{C} - I\left(\frac{R}{2}\right) - \left(\frac{Q_0}{2C}\right) = 0$$
$$\frac{Q_0}{2C} - I\left(\frac{R}{2}\right) = 0$$
$$\frac{Q_0}{2C} = I\left(\frac{R}{2}\right)$$
$$\frac{Q_0}{C} = IR$$
$$I = \frac{Q_0}{RC}$$

(e)(ii) For indicating that the current is zero — **1 point**

Total for part (e) — 3 points

Total for question 3 — 15 points

Question 1: Free-Response Question — 15 points

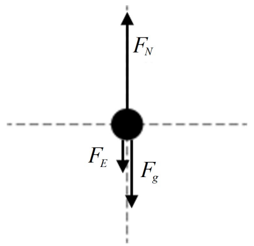
(a) For correctly drawing and labeling the electrostatic force in the downward direction — **1 point**

For drawing the normal force in the upward direction and a gravitational force in the downward direction — **1 point**

Scoring Note: F_S is an acceptable label to represent the force that the platform exerts on the sphere

Scoring Note: A maximum of 1 point may be earned if extraneous forces are included.

Example Response



Total for part (a) — 2 points

(b) For applying an equation of the forces at equilibrium consistent with the diagram drawn in part (a) — **1 point**

Example Response

$$F_N - F_E - F_g = 0$$

For substituting a $k_s x$ or $k_s y$ that represents the normal force — **1 point**

Example Response

$$k_s x - F_E - F_g = 0$$

For substituting a correct expression that represents the electrostatic force — **1 point**

Example Response

$$F_E = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{H^2}$$

For substituting a correct expression that represents the gravitational force — **1 point**

Example Response

$$F_g = Mg$$

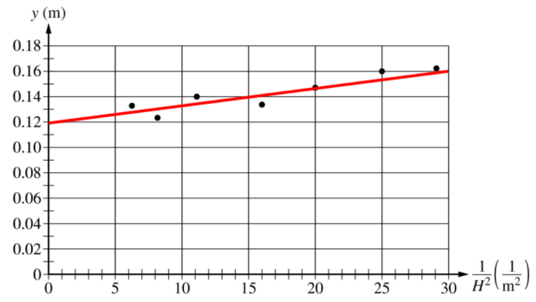
Example Solution

$$F_N - F_E - F_g = 0$$
$$F_N = F_E + F_g$$
$$k_s y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{H^2} + Mg$$
$$y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{k_s H^2} + \frac{Mg}{k_s}$$

Total for part (b) — 4 points

(c)(i) For a line that approximates the trend of the data — **1 point**

Example Response



(c)(ii) For using two points from the trend line drawn by the student to calculate the slope **1 point**

Scoring Note: Points of data may be used only if points of data are located directly on the line.

Example Response

$$\text{Slope} = \frac{\Delta y}{\Delta x}$$

$$\text{Slope} = \frac{\Delta(y)}{\Delta\left(\frac{1}{H^2}\right)}$$

$$\text{Slope} = \frac{(0.15\ \text{m} - 0.12\ \text{m})}{\left(20\ \frac{1}{\text{m}^2} - 0\ \frac{1}{\text{m}^2}\right)}$$

$$\text{Slope} = 0.0015\ \text{m}^3$$

For indicating the relationship between the slope of the graph to ε_0 in the equation **1 point**

$$y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{k_s H^2} + \frac{Mg}{k_s}$$

Example Response

$$y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{k_s H^2} + \frac{Mg}{k_s}$$

$$\text{slope} = \left(\frac{Qq}{4\pi\varepsilon_0 k_s}\right)$$

For correctly substituting the value of the slope of the graph and other given quantities into the equation to calculate an experimental value of ε_0 **1 point**

Scoring Note: A numerical response without units may earn this point.

Example Solution

$$\text{slope} = \frac{Qq}{4\pi\varepsilon_0 k_s}$$

$$\varepsilon_0 = \frac{Qq}{4\pi k_s(\text{slope})}$$

$$\varepsilon_0 = \frac{\left(2\times10^{-6}\ \text{C}\right)^2}{4\pi\left(25\frac{\text{N}}{\text{m}}\right)\left(0.0015\ \text{m}^3\right)}$$

$$\varepsilon_0 = 8.5\times10^{-12}\ \frac{\text{C}^2}{\text{N}\bullet\text{m}^2}$$

(c)(iii) For identifying that the y-intercept $= \frac{Mg}{k_s}$ **1 point**

For correctly substituting the value of the y-intercept of the graph and other given quantities and constants into the equation to calculate an experimental value of M **1 point**

Scoring Note: A numerical response without units may earn this point.

Example Response

$$y\text{-intercept} = 0.12\ \text{m}$$

$$y\text{-intercept} = \frac{Mg}{k_s}$$

$$M = \frac{(0.12\ \text{m})(25\ \text{N/m})}{9.8\ \text{m/s}^2}$$

$$M = 0.30\ \text{kg}$$

Total for part (c) 6 points

(d)(i) For selecting "$y_2 < y_1$" with an attempt at a relevant justification **1 point**

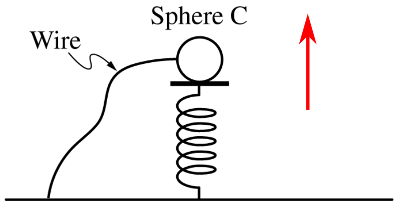
For a justification that indicates correct charge rearrangement in Sphere C due to electrostatic force from Sphere A **1 point**

Example Response

$y_2 < y_1$, Charges are now free to move within Sphere C now that it is a conducting sphere. Electrons on Sphere C will be attracted to the excess positive charges on Sphere A and move closer to Sphere A. This leaves the opposite side of Sphere C more positively charged which results in less electric repulsion between the spheres and as a result less compression in the spring.

(d)(ii) For drawing an upward pointing arrow **1 point**

Example Response



Total for part (d) 3 points

Total for question 1 15 points

Question 1: Free-Response Question — 15 points

(a) For correctly determining the charge on the outer surface of the shell — **1 point**

Example Response

$$q_{net} = q_{inner} + q_{outer}$$
$$q_{outer} = q_{net} - q_{inner}$$
$$q_{outer} = +3Q - (+Q)$$
$$q_{outer} = +2Q$$

Scoring Note: A correct response may earn a point even if no work is shown.

Total for part (a) — 1 points

(b) For using Gauss's law by substituting for either the area or the enclosed charge — **1 point**

Example Response

$$\frac{q_{enc}}{\varepsilon_0} = \oint E \bullet dA = E\left(4\pi r^2\right)$$

For using a correct expression for q_{enc} as a function of r — **1 point**

Example Response

$$q_{enc} = \rho V = \left(\frac{-Q}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) = -Q\frac{r^3}{R^3}$$

Scoring Note: The response may earn this point regardless of the sign of q_{enc}.

For using the correct area — **1 point**

Example Response

$$E = q_{enc}\frac{1}{\varepsilon_0\left(4\pi r^2\right)} = \left(-Q\frac{r^3}{R^3}\right)\frac{1}{\varepsilon_0\left(4\pi r^2\right)}$$
$$E = -Q\frac{r}{4\pi\varepsilon_0 R^3}$$

Scoring Note: The response may earn this point regardless of the sign of E.

Total for part (b) — 3 points

(c) For recognizing the electric field varies as an inverse square of the separation distance from the center of the nonconducting sphere — **1 point**

Example Response

$$E \propto \frac{1}{r^2}$$

For the correct magnitude, including units, of the electric field at $2R$ — **1 point**

Example Response

$$E_{new} = \frac{E_{old}}{4} = \frac{8\text{ N/C}}{4}$$
$$E_{new} = 2\text{ N/C}$$

Total for part (c) — 2 points

(d) For using the equation relating potential difference to the electric field with an attempt at either integration limits or evaluating the integral — **1 point**

Example Response

$$V_f - V_i = -\int_{s_i}^{s_f} E\,dr$$
$$\Delta V = -\int_{r=R}^{r=4R} E\,dr$$

For substituting the correct expression for the electric field or an expression for E that is consistent with the explicit functional dependence of $E(r)$ from part (c) — **1 point**

Example Response

$$\Delta V = -\int_{r=R}^{r=4R} -\frac{Q}{4\pi\varepsilon_0 r^2}dr$$
$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\int_{r=R}^{r=4R}\frac{1}{r^2}dr$$

	For correctly integrating the electric field expression that was substituted in the previous point with correct limits of integration	**1 point**

Example Response

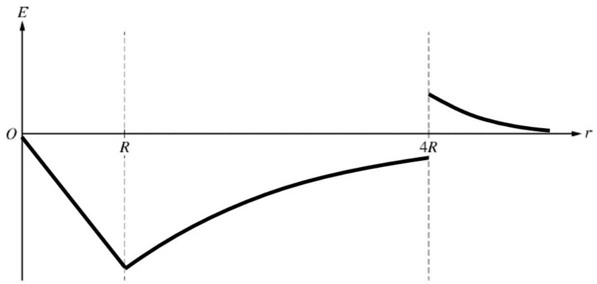
$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{r=R}^{r=4R} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{r=4R}^{r=R} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{4R}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{4-1}{4R}\right)$$

$$\Delta V = \frac{3Q}{16\pi\varepsilon_0 R}$$

Alternate Solution

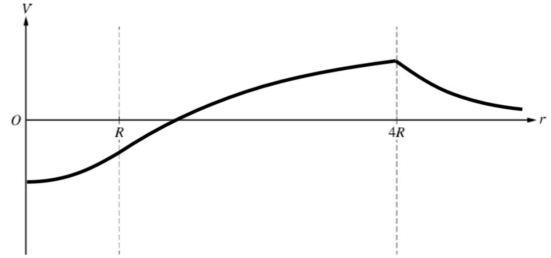
	For using a difference in potentials treating the inner sphere as a point charge by symmetry $\Delta V = \frac{Q}{4\pi\varepsilon_0 r_f} - \frac{Q}{4\pi\varepsilon_0 r_i}$	**1 Point**
	For using the correct charge on both of the terms; Q *not* Q *and* $3Q$	**1 Point**
	For using the correct values for r $\Delta V = \frac{Q}{4\pi\varepsilon_0 R} - \frac{Q}{4\pi\varepsilon_0 4R}$ $\Delta V = \frac{3Q}{16\pi\varepsilon_0 R}$	**1 Point**
	Total for part (d)	**3 points**
(e)(i)	For indicating that E is negative and the magnitude of E increases linearly from 0 to R	**1 point**
	For a curve that is continuous at R and asymptotically approaches zero in the region $R < r < 4R$	**1 point**
	For a positive, concave-up, and decreasing curve for $r > 4R$	**1 point**

Example Response



(e)(ii)	For a potential curve that is always increasing from 0 to $4R$	**1 point**
	For a potential curve that is decreasing for $r > 4R$	**1 point**
	For a continuous graph across all three regions	**1 point**

Example Response



Scoring Note: The intercept of the curve on the vertical axis is irrelevant. The intercept of the curve on the horizontal axis is irrelevant. The curve can, hence, cross the horizontal axis at any location or even be entirely on the negative side of the horizontal axis as long as the other criteria are met.

Total for part (e)	**6 points**
Total for question 1	**15 points**