

The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R , capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

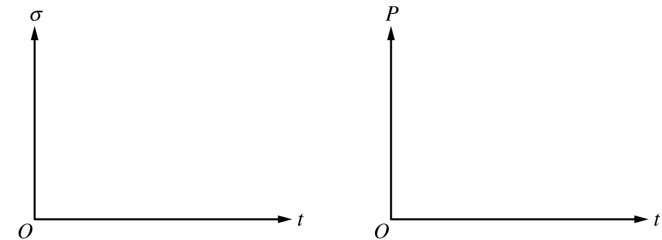
At time $t = 0$, the switch is closed to Position A.

- (a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R , C , Q , t , and fundamental constants, as appropriate.

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Continue your response to **QUESTION 3** on this page.

- (b) On the axes shown, sketch graphs of the surface charge density σ on the positive plate of Capacitor 1 and the total power P dissipated by the resistors as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.



A long time after the switch is closed to Position A, the charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

- (c) At time t_1 , the switch is closed to Position B.
- Immediately after time t_1 , is the direction of the current in the switch directed toward the left, directed toward the right, or is there no current? Briefly justify your answer.
 - Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1 . Express your answer in terms of Q_0 and fundamental constants, as appropriate.

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iii. Derive an expression for the total energy E_R dissipated by resistors 1 and 2 from immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C , Q_0 , and fundamental constants, as appropriate.

With the switch still closed to Position B, the parallel plates of Capacitor 2 are moved so that the separation distance increases by a factor of 2.

(d) Determine the ratio $\frac{U_2}{U_1}$ of the energy U_2 stored in Capacitor 2 to the energy U_1 stored in Capacitor 1 a long time after the plates of Capacitor 2 have been moved. Briefly justify your answer.

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With the capacitors still charged as in part (d), the switch is now closed to Position A.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R , C , Q_0 , and fundamental constants, as appropriate.

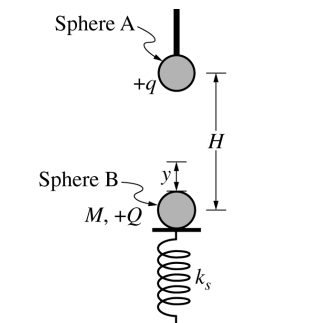
i. Derive an expression for the current I_0 from the battery immediately after the switch is closed to Position A.

ii. Determine the current I_∞ from the battery a long time after the switch is closed to Position A.

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STOP**END OF EXAM****PHYSICS C: ELECTRICITY AND MAGNETISM****SECTION II****Time—45 minutes****3 Questions**

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

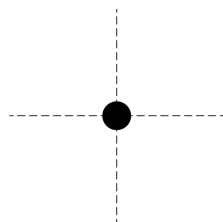
Students perform an experiment to determine the value of vacuum permittivity ϵ_0 . Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M . Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s . Sphere B and the spring are initially at rest.

Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H , the spring has been compressed a distance y , as shown in the figure. The students measure y for different values of H .

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Continue your response to **QUESTION 1** on this page.

- (a) On the following dot that represents Sphere B in the figure on the previous page, draw and label the forces (not components) that are exerted on Sphere B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

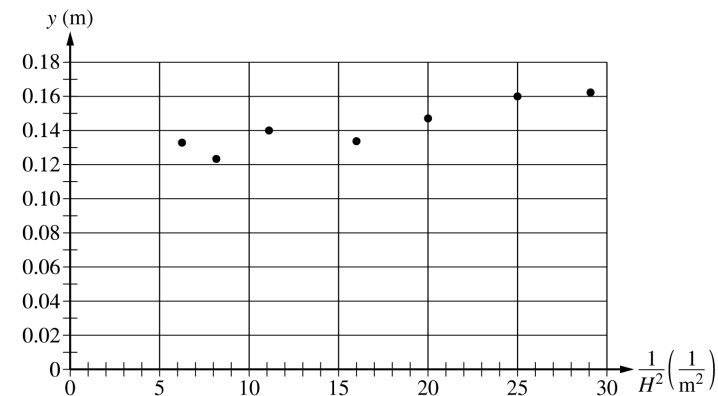


- (b) Derive the relationship between y and H to show that $y = \frac{1}{4\pi\epsilon_0} \frac{Qq}{k_s H^2} + \frac{Mg}{k_s}$.

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Continue your response to **QUESTION 1** on this page.

- (c) The students plot collected data of y as a function of $\frac{1}{H^2}$, as shown in the graph.



- Draw the best-fit line for the data.
- Using the best-fit line, calculate an experimental value for the vacuum permittivity ϵ_0 when $Q = q = 2.00 \times 10^{-6} \text{ C}$ and $k_s = 25 \text{ N/m}$.
- Using the best-fit line, calculate an experimental value for the mass of Sphere B.

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Continue your response to **QUESTION 1** on this page.

(d) The students modify the experiment by replacing nonconducting Sphere B with conducting Sphere C that has the same charge $+Q$ and mass M . Sphere A is brought near Sphere C without touching, compressing the spring. Sphere C comes to rest.

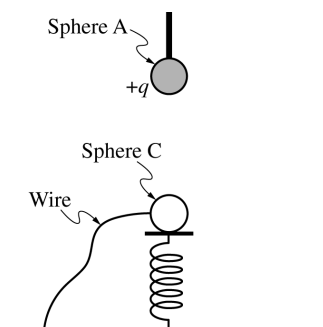
i. In the original experiment, when the centers of spheres A and B are a vertical distance H_1 apart, the spring is compressed a distance y_1 . In the modified experiment, when the centers of spheres A and C are a vertical distance H_1 apart, the spring is compressed a distance y_2 .

Is y_2 greater than, less than, or equal to y_1 ?

_____ $y_2 > y_1$ _____ $y_2 < y_1$ _____ $y_2 = y_1$

Justify your answer.

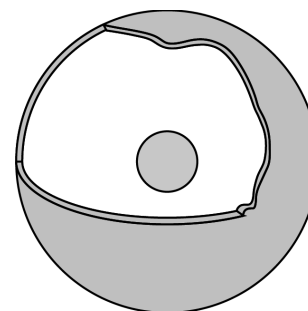
ii. Sphere C is then grounded with a wire. On the following figure, draw an arrow indicating the direction that the platform will move immediately after being grounded. If the platform remains stationary, write “does not move.”



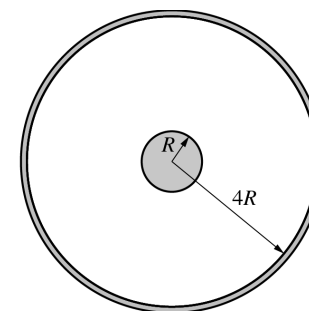
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Begin your response to **QUESTION 1** on this page.**PHYSICS C: ELECTRICITY AND MAGNETISM****SECTION II****Time—45 minutes****3 Questions**

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Cutout View



Cross-Section View

Note: Figures not drawn to scale.

A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.

(a) Determine the charge on the outer surface of the shell.

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Continue your response to **QUESTION 1** on this page.

- (b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the sphere for $r < R$. Express your answer in terms of Q , R , r , and physical constants, as appropriate.

- (c) The magnitude of the electric field at $r = R$ is 8 N/C . Calculate the value of the electric field at $r = 2R$.

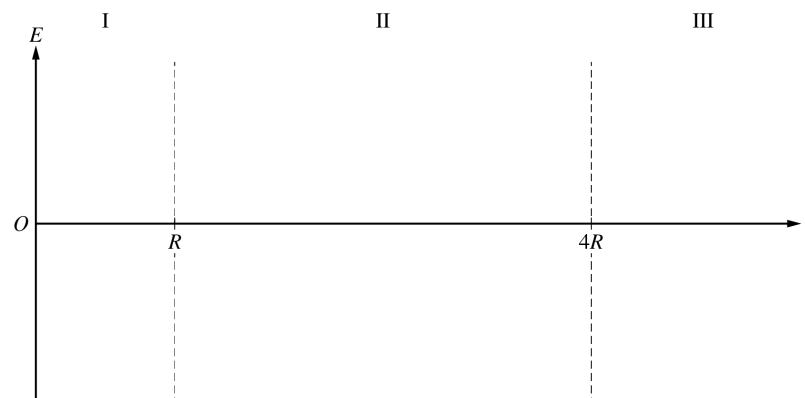
- (d) Derive an expression for the absolute value of the potential difference between the outer surface of the sphere and the inner surface of the shell. Express your answer in terms of Q , R , and physical constants, as appropriate.

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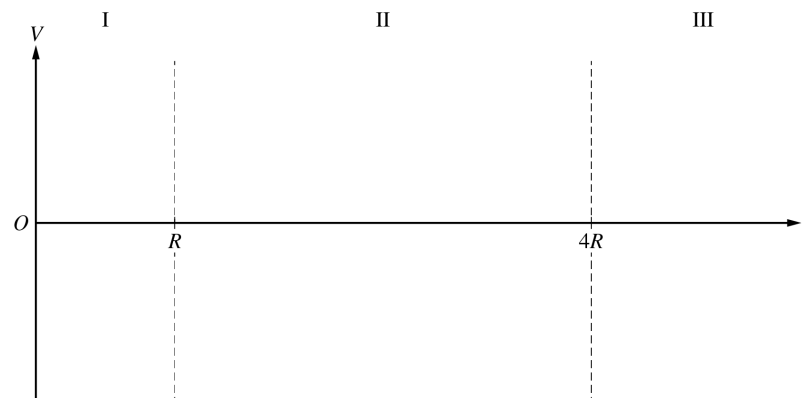
Continue your response to **QUESTION 1** on this page.

(e)

- i. On the following axes that include regions I, II, and III, sketch a graph of the electric field E as a function of the distance r from the center of the sphere.



- ii. On the following axes that include regions I, II, and III, sketch a graph of the electric potential V as a function of the distance r from the center of the sphere.



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