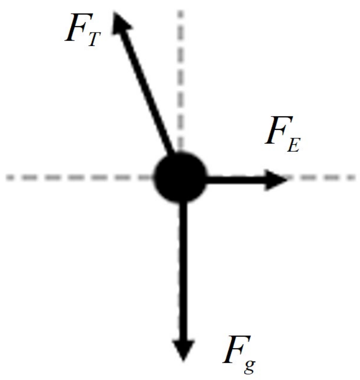
Question 1: Free-Response Question — 15 points

(a)	For correctly drawing and labeling the electrostatic force directed to the right	**1 point**
	For drawing the force of tension up and to the left and the gravitational force in the downward direction	**1 point**

Scoring Note: A maximum of 1 point may be earned if extraneous forces are included.

Example Response



Total for part (a) **2 points**

(b)	For equating the horizontal component of tension to the electrostatic force	**1 point**

Example Response

$F_E = F_T \sin(\theta)$

For equating the vertical component of tension to the gravitational force	**1 point**

Example Response

$F_g = F_{Ty}$

$Mg = F_T \cos\theta$

For an attempt to simultaneously solve the equations	**1 point**

Example Response

$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}\frac{1}{\sin\theta}\cos\theta = Mg$

Example Solution

$\Sigma F_y = 0$

$F_{Ty} = F_g$

$F_T \cos\theta = Mg$

$F_{Tx} = F_E$

$F_T = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}\frac{1}{\sin\theta}$

$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}\frac{1}{\sin\theta}\cos\theta = Mg$

$d^2 = \frac{Qq\cos\theta}{4\pi\varepsilon_0 Mg\sin\theta}$

$d = \sqrt{\frac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}}$

Total for part (b) 3 points

(c)	For applying Coulomb's law to determine tension	**1 point**

Scoring Note: This point may be earned if the student used the vertical component of tension.

Example Response

$$F_E = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2} = F_T\sin(\theta)$$

For correct substitution into an expression for tension consistent with part (b) or a correct expression for tension — **1 point**

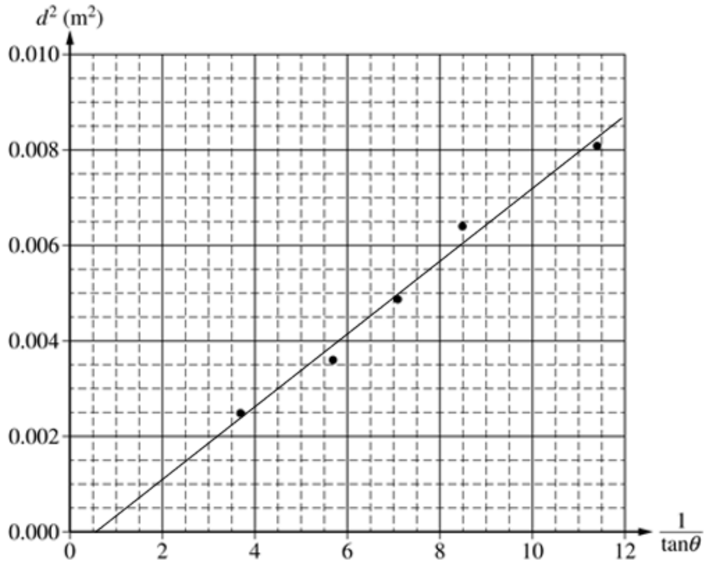
Example Solution

$$\Sigma F = 0$$
$$F_E - F_{Tx} = 0$$
$$F_E = F_{Tx}$$
$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2} = F_T\sin(\theta)$$
$$F_T = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2\sin(\theta)}$$
$$F_T = \frac{1}{4\pi\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\bullet\text{m}^2}\right)}\frac{\left(6.0\times10^{-8}\text{ C}\right)^2}{(0.057\text{ m})^2\sin\left(12^\circ\right)}$$
$$F_T = 0.048\text{ N}$$

Total for part (c) — 2 points

(d)(i)	For a line that approximates the trend of the data	**1 point**

Example Response



(d)(ii) For using two points from the trend line drawn by the student to calculate the slope **1 point**

Scoring Note: Points of data may be used only if points of data are located directly on the line.

Example Response

$$\text{Slope} = \frac{\Delta y}{\Delta x}$$

$$\text{Slope} = \frac{\Delta(d^2)}{\Delta\left(\dfrac{1}{\tan(\theta)}\right)}$$

$$\text{Slope} = \frac{\left(0.0075\text{ m}^2 - 0.001\text{ m}^2\right)}{(10.5 - 2)}$$

$$\text{Slope} = 7.647 \times 10^{-4}\text{ m}^2$$

For correctly relating the slope of the graph to the equation $d = \sqrt{\dfrac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}}$ **1 point**

Example Response

$$d = \sqrt{\frac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}}$$

$$d^2 = \frac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}$$

$$d^2 = \left(\frac{Qq}{4\pi\varepsilon_0 Mg}\right)\frac{1}{\tan\theta}$$

$$\text{slope} = \left(\frac{Qq}{4\pi\varepsilon_0 Mg}\right)$$

For substituting the value of the slope of the graph into the equation $\varepsilon_0 = \dfrac{Qq}{4\pi Mg(\text{slope})}$ to calculate an experimental value of ε_0 **1 point**

Example Solution

$$\text{slope} = \frac{Qq}{4\pi\varepsilon_0 Mg}$$

$$\varepsilon_0 = \frac{Qq}{4\pi Mg(\text{slope})}$$

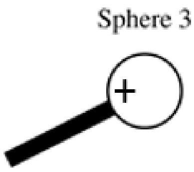
$$\varepsilon_0 = \frac{\left(6.0 \times 10^{-8}\text{ C}\right)^2}{4\pi(0.005\text{ kg})\left(9.8\text{ m/s}^2\right)\left(7.647 \times 10^{-4}\text{ m}^2\right)}$$

$$\varepsilon_0 = 7.6 \times 10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}$$

Total for part (d) 4 points

(e)(i)	For a "+" drawn on the left side of the sphere	**1 point**

Example Response



(e)(ii)	For a statement that indicates correct charge rearrangement on Sphere 3 due to the electric forces from the charges on Sphere 2	**1 point**

Example Response

The negative charges on Sphere 3 move to the right due to the attractive forces from the positive charges on Sphere 2, leaving a net positive charge on the left side of Sphere 3.

(e)(iii)	For selecting "$\theta_2 < \theta_1$" with an attempt at a relevant justification	**1 point**
	For statement that indicates **one** of the following: • The average distance between the repulsive charges is greater. • The electrostatic or repulsive force is less.	**1 point**

Scoring Note: Points 1 and 2 of part (e)(iii) can be earned with an answer that is consistent with the location of the excess positive charges drawn in part (e)(i).

Example Response

Excess charges on Sphere 3 are now free to move, so excess like charges will be concentrated on the far ends of Sphere 3 when the spheres are in static equilibrium. The excess like charges, located on opposite sides of Sphere 3, repel with less force than if the excess charges were located at the centers of Sphere 3. Thus, the downward force due to gravity on Sphere 2 causes the center of Sphere 2 to hang closer to the center of Sphere 3.

Total for part (e)	**4 points**
Total for question 1	**15 points**

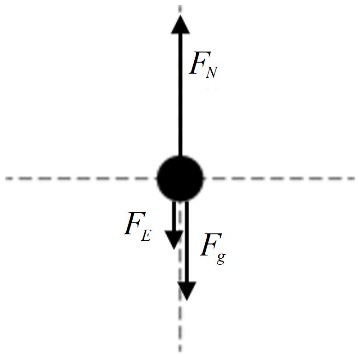
Question 1: Free-Response Question 15 points

(a)	For correctly drawing and labeling the electrostatic force in the downward direction	**1 point**
	For drawing the normal force in the upward direction and a gravitational force in the downward direction	**1 point**

Scoring Note: F_S is an acceptable label to represent the force that the platform exerts on the sphere

Scoring Note: A maximum of 1 point may be earned if extraneous forces are included.

Example Response



Total for part (a) 2 points

(b)	For applying an equation of the forces at equilibrium consistent with the diagram drawn in part (a)	**1 point**

Example Response

$$F_N - F_E - F_g = 0$$

For substituting a $k_s x$ or $k_s y$ that represents the normal force **1 point**

Example Response

$$k_s x - F_E - F_g = 0$$

For substituting a correct expression that represents the electrostatic force **1 point**

Example Response

$$F_E = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{H^2}$$

For substituting a correct expression that represents the gravitational force **1 point**

Example Response

$$F_g = Mg$$

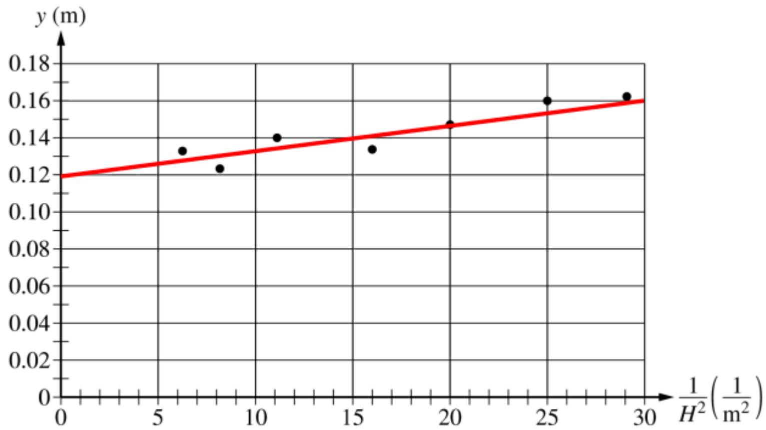
Example Solution

$$F_N - F_E - F_g = 0$$
$$F_N = F_E + F_g$$
$$k_s y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{H^2} + Mg$$
$$y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{k_s H^2} + \frac{Mg}{k_s}$$

Total for part (b) 4 points

(c)(i)	For a line that approximates the trend of the data	**1 point**

Example Response



(c)(ii) For using two points from the trend line drawn by the student to calculate the slope **1 point**

Scoring Note: Points of data may be used only if points of data are located directly on the line.

Example Response

$$\text{Slope} = \frac{\Delta y}{\Delta x}$$

$$\text{Slope} = \frac{\Delta(y)}{\Delta\left(\frac{1}{H^2}\right)}$$

$$\text{Slope} = \frac{(0.15\text{ m} - 0.12\text{ m})}{\left(20\ \frac{1}{\text{m}^2} - 0\ \frac{1}{\text{m}^2}\right)}$$

$$\text{Slope} = 0.0015\text{ m}^3$$

For indicating the relationship between the slope of the graph to ε_0 in the equation **1 point**

$$y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{k_s H^2} + \frac{Mg}{k_s}$$

Example Response

$$y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{k_s H^2} + \frac{Mg}{k_s}$$

$$\text{slope} = \left(\frac{Qq}{4\pi\varepsilon_0 k_s}\right)$$

For correctly substituting the value of the slope of the graph and other given quantities into the equation to calculate an experimental value of ε_0 **1 point**

Scoring Note: A numerical response without units may earn this point.

Example Solution

$$\text{slope} = \frac{Qq}{4\pi\varepsilon_0 k_s}$$

$$\varepsilon_0 = \frac{Qq}{4\pi k_s(\text{slope})}$$

$$\varepsilon_0 = \frac{\left(2\times10^{-6}\text{ C}\right)^2}{4\pi\left(25\frac{\text{N}}{\text{m}}\right)\left(0.0015\text{ m}^3\right)}$$

$$\varepsilon_0 = 8.5\times10^{-12}\ \frac{\text{C}^2}{\text{N}\bullet\text{m}^2}$$

(c)(iii) For identifying that the y-intercept $= \dfrac{Mg}{k_s}$ **1 point**

For correctly substituting the value of the y-intercept of the graph and other given quantities and constants into the equation to calculate an experimental value of M **1 point**

Scoring Note: A numerical response without units may earn this point.

Example Response

y-intercept $= 0.12$ m

$$y\text{-intercept} = \frac{Mg}{k_s}$$

$$M = \frac{(0.12\text{ m})(25\text{ N/m})}{9.8\text{ m/s}^2}$$

$$M = 0.30\text{ kg}$$

Total for part (c) 6 points

(d)(i) For selecting "$y_2 < y_1$" with an attempt at a relevant justification **1 point**

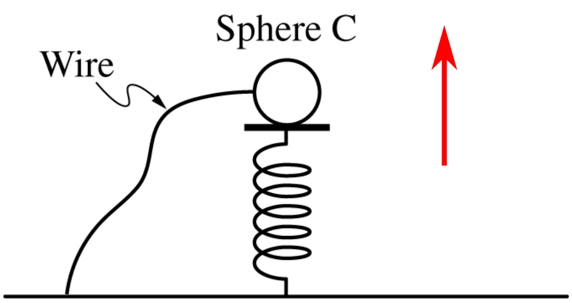
For a justification that indicates correct charge rearrangement in Sphere C due to electrostatic force from Sphere A **1 point**

Example Response

$y_2 < y_1$, *Charges are now free to move within Sphere* C *now that it is a conducting sphere. Electrons on Sphere* C *will be attracted to the excess positive charges on Sphere* A *and move closer to Sphere* A. *This leaves the opposite side of Sphere* C *more positively charged which results in less electric repulsion between the spheres and as a result less compression in the spring.*

(d)(ii) For drawing an upward pointing arrow **1 point**

Example Response



Total for part (d) 3 points

Total for question 1 15 points

Question 1: Free-Response Question — 15 points

(a) For correctly determining the charge on the outer surface of the shell — **1 point**

Example Response

$$q_{net} = q_{inner} + q_{outer}$$
$$q_{outer} = q_{net} - q_{inner}$$
$$q_{outer} = +4Q - (-Q)$$
$$q_{outer} = +5Q$$

Scoring Note: A correct response may earn a point even if no work is shown.

Total for part (a) — 1 point

(b) For using Gauss's law by substituting for either the area or the enclosed charge — **1 point**

Example Response

$$\frac{q_{enc}}{\varepsilon_0} = \oint E \bullet dA = E(2\pi rL)$$

For using a correct expression for q_{enc} as a function of r — **1 point**

Example Response

$$q_{enc} = \rho V = \left(\frac{Q}{\pi R^2 L}\right)\left(\pi r^2 L\right) = Q\frac{r^2}{R^2}$$

For using the correct area — **1 point**

Example Response

$$E = q_{enc}\frac{1}{\varepsilon_0(2\pi rL)} = \left(Q\frac{r^2}{R^2}\right)\frac{1}{\varepsilon_0(2\pi rL)}$$
$$E = Q\frac{r}{2\pi\varepsilon_0 R^2 L}$$

Total for part (b) — 3 points

(c) For recognizing that the E-field varies as the reciprocal of the separation distance from the center of the nonconducting cylinder — **1 point**

Example Response

$$E \propto \frac{1}{r}$$

For the correct value, including units, of electric field at $2R$ — **1 point**

Example Response

$$E_{new} = \frac{E_{old}}{2} = \frac{12\text{ N/C}}{2}$$
$$E_{new} = 6\text{ N/C}$$

Total for part (c) 2 points

(d) For using the equation relating potential difference to the electric field with an attempt at either integration limits or evaluating the integral — **1 point**

Example Response

$$V_f - V_i = -\int_{s_i}^{s_f} E dr$$

$$\Delta V = -\int_{r=R}^{r=3R} E dr$$

For substituting the correct expression for the electric field or an expression consistent with the explicit functional dependence of $E(r)$ from part (c) — **1 point**

Example Response

$$\Delta V = -\int_{r=R}^{r=3R} \left(\frac{Q}{2\pi\varepsilon_0 rL}\right) dr = -\frac{Q}{2\pi\varepsilon_0 L}\int_{r=R}^{r=3R} \frac{1}{r} dr$$

For correctly integrating the electric field expression that was substituted in the previous point with the correct limits of of integration — **1 point**

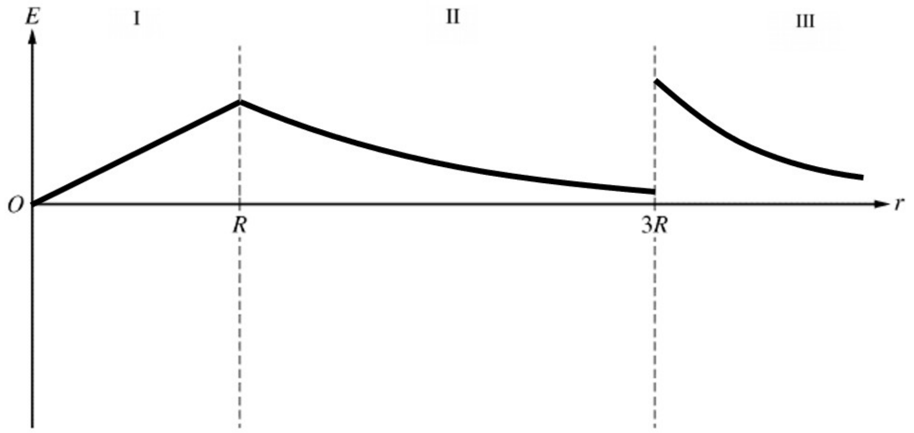
Example Response

$$\Delta V = -\frac{Q}{2\pi\varepsilon_0 L}\left[\ln(r)\right]_{r=R}^{r=3R} = -\frac{Q}{2\pi\varepsilon_0 L}\left[\ln(3R) - \ln(R)\right]$$
$$\Delta V = -\frac{Q}{2\pi\varepsilon_0 L}\ln(3)$$

Total for part (d) 3 points

(e)(i)	For indicating E is positive and the magnitude increases linearly from 0 to R	**1 point**
	For a positive, decreasing concave up graph for $r > R$	**1 point**
	For a discontinuous increase in magnitude at $r = 3R$	**1 point**

Example Response

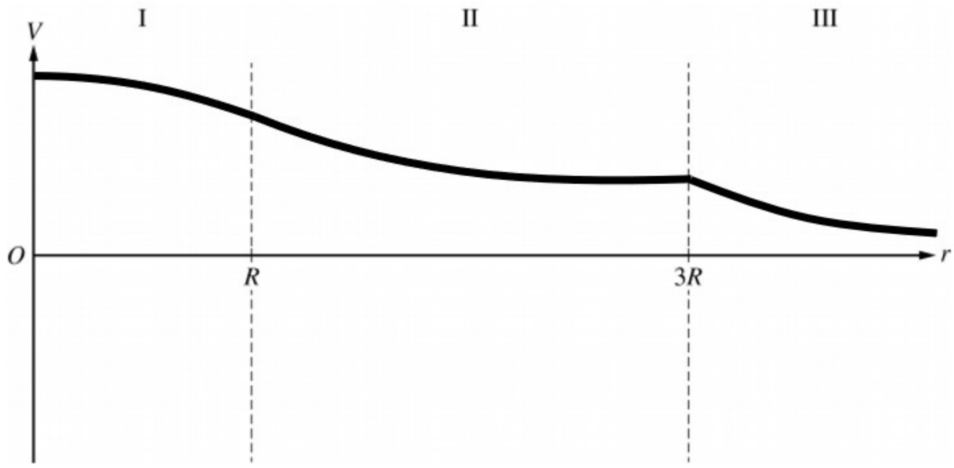


(e)(ii)	For a graph that is concave down and always decreasing as r increases within Region I	**1 point**
	For a graph that is concave up and always decreasing as r increases within each of the Regions II and III	**1 point**
	For a continuous graph across all three regions	**1 point**

Scoring Notes:

- The intercept of the curve on the vertical axis is irrelevant. The intercept of the curve on the horizontal axis is irrelevant. The curve can, hence, cross the horizontal axis at any location or even be entirely on the negative side of the horizontal axis as long as the other criteria are met.
- The response can earn a maximum of 2 points if all three segments are reflected across the horizontal axis.

Example Response



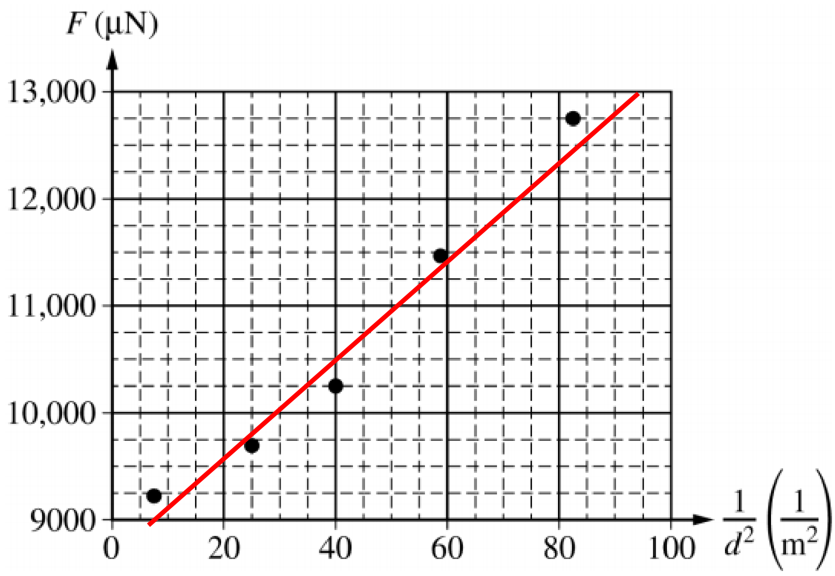
Total for part (e) 6 points

Total for question 1 15 points

Question 2: Free-Response Question — 15 points

(a) i. For drawing an appropriate best-fit line — **1 point**

Example response for part (a)(i)



ii. For calculating the slope using two points from the best-fit line — **1 point**

$$slope = m = \frac{\Delta y}{\Delta x} = \frac{(12{,}000 - 10{,}500)\ \mu\text{N}}{(72 - 40)\ \text{m}^{-2}} = 4.7 \times 10^{-5}\ \text{N}\bullet\text{m}^2$$

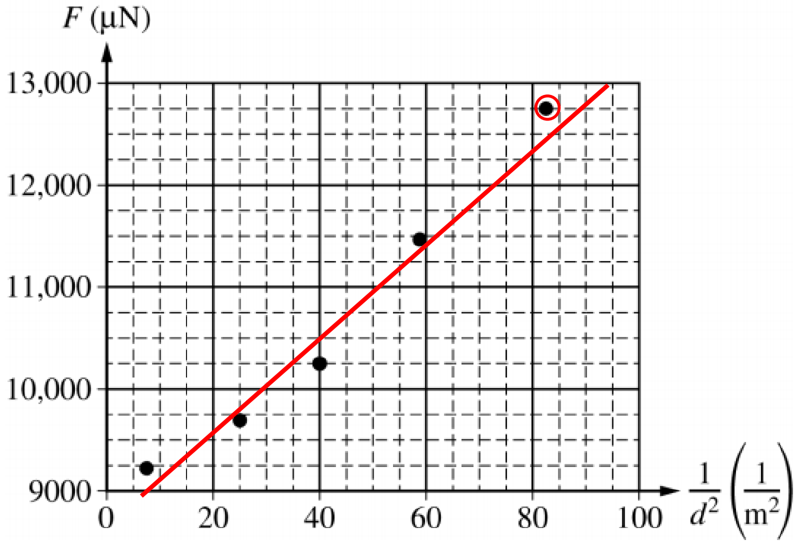
For correctly relating the slope to the charge — **1 point**

$$F = \frac{kQ^2}{r^2}\ \therefore\ \text{slope} = kQ^2$$

For correctly calculating the charge consistent with the slope from the best-fit line — **1 point**

$$Q = \sqrt{\frac{\text{slope}}{k}} = \sqrt{\frac{\left(4.7 \times 10^{-5}\ \text{N}\bullet\text{m}^2\right)}{\left(9 \times 10^{9}\ \text{N}\bullet\text{m}^2/\text{C}^2\right)}} = 7.2 \times 10^{-8}\ \text{C}$$

(a)	**iii.**	For drawing a circle around the appropriate data point	**1 point**



	iv.	For a correct answer	**1 point**

From the graph, $\dfrac{1}{d^2} = 82.5\ \text{m}^{-2}$

$d = 0.11\ \text{m}$

	v.	For indicating that the y-intercept should be the weight of the conducting sphere and insulating rod	**1 point**
		For a correct justification	**1 point**

Example responses for part (a)(v).

When the spheres are infinitely far apart, there is no electric force between them. The only force recorded by the balance must be the weight of the lower sphere and insulating rod.

OR

The scale is zeroed before the insulating rod, and lower sphere are placed on it, so the nonzero y-intercept is their weight.

Total for part (a) **8 points**

(b)	For selecting "Yes" and providing a correct justification	**1 point**

Example response for part (b)

Because the later force measurements are lower than the expected values on the best-fit line, then a decrease in charge could explain the discrepancy seen in the data.

(c) i. For correctly indicating the position of the excess positive charges **1 point**

For correctly indicating that the net charge is positive **1 point**

Example response for part (c)(i)

ii. For a correct justification **1 point**

Example response for part (c)(ii)

On a conductor, excess charges are free to move and will repel each other. The excess charges in the sphere will rearrange themselves until they are as far apart from each other as possible; thus, the repelling force decreases.

Total for part (c) 3 points

(d) i. For appropriately indicating the position of the excess charges on both spheres **1 point**

ii. For selecting "Less than" and attempting a relevant justification **1 point**

For a correct justification **1 point**

Example response for part (d)

Because the opposite charged spheres will attract each other, this will decrease the upward normal force exerted by the scale on the lower sphere; thus, the reading on the scale will be less than when the spheres had the same charge.

Total for part (d) 3 points

Total for question 2 15 points

2018

AP® CollegeBoard

AP Physics C: Electricity and Magnetism

Scoring Guidelines

2018 SCORING GUIDELINES

General Notes About 2018 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

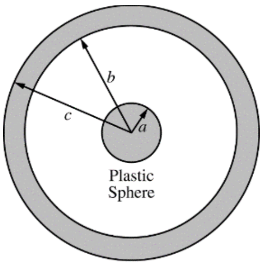
5. The scoring guidelines typically show numerical results using the value $g = 9.8\ \text{m/s}^2$, but the use of $10\ \text{m/s}^2$ is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

AP PHYSICS C: ELECTRICITY AND MAGNETISM
2018 SCORING GUIDELINES

Question 1

15 points total — **Distribution of points**



Conducting Spherical Shell

A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

(a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

i. 3 points

$r < a$

For using the integral to determine the charge enclosed		1 point
$q_{enc} = \int \rho dV = \int \beta r dV$		
For correctly substituting for dV into the integration to determine the charge enclosed		1 point
$q_{enc} = \int \left(4\pi r'^2\right)(\beta r')dr' = 4\beta\pi \int_{r'=0}^{r'=r} r'^3 dr' = 4\beta\pi \left[\frac{r'^4}{4}\right]_{r'=0}^{r'=r}$		
For a correct answer		1 point
$q_{enc} = \beta\pi r^4$		

ii. 1 points

$a < r < b$

For an answer consistent with (a)(i)		1 point
$q_{enc} = \beta\pi a^4$		

AP® PHYSICS C: ELECTRICITY AND MAGNETISM
2018 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. 2 points

$r < a$

For correctly substituting the area of a Gaussian sphere into Gauss's law		1 point
$\frac{q_{enc}}{\varepsilon_0} = \oint \vec{E} \bullet d\vec{A}$		
$\frac{q(r)}{\varepsilon_0} = EA_{sphere} = E\left(4\pi r^2\right)$		
For correctly substituting the charge from part (a)(i) into the equation above		1 point
$\frac{\beta\pi r^4}{\varepsilon_0} = E\left(4\pi r^2\right)$		
$E = \frac{\beta r^2}{4\varepsilon_0}$		

ii. 1 point

$a < r < b$

For correctly substituting the charge from part (a)(ii) into Gauss's law to calculate the electric field		1 point
$\frac{\beta\pi a^4}{\varepsilon_0} = E\left(4\pi r^2\right)$		
$E = \frac{\beta a^4}{4\varepsilon_0 r^2}$		

AP PHYSICS C: ELECTRICITY AND MAGNETISM
2018 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. 2 points

Determine the charge on the inner surface of the conducting shell.
Justify your answer.

For an answer consistent with part (a)(ii) (must have opposite sign)		1 point
$q_{inner} = -\beta\pi a^4$		
For a correct justification		1 point
$E_{shell} = 0 = \frac{q_{enc}}{\varepsilon_0} \therefore q_{enc} = 0 = q_{sphere} + q_{inner} \therefore q_{inner} = -q_{sphere}$		

ii. 1 point

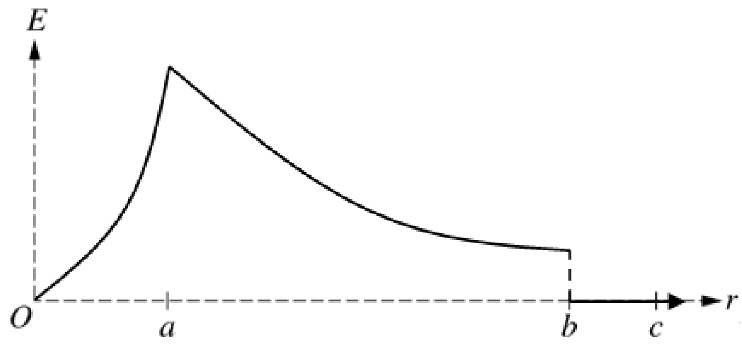
Determine the charge on the outer surface of the conducting shell.

A correct answer		1 point
$q_{outer} = 0$		

(d)

i. 3 points

On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



For concave up graph that begins at the origin and increases to $r = a$		1 point
For a concave up graph that is continuous with the first section and decreases from $r = a$ to nonzero value at $r = b$		1 point
For a graph with a value of $E = 0$ for $r > b$		1 point
Note: Due to scale, discontinuity at point b is not required.		

AP® PHYSICS C: ELECTRICITY AND MAGNETISM
2018 SCORING GUIDELINES

Question 1 (continued)

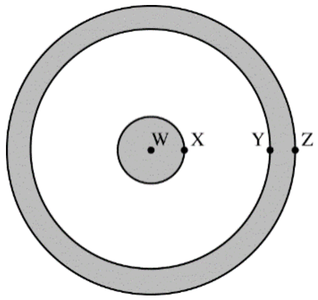
Distribution of points

(d) (continued)

ii. 2 points

The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

__1__ W   __2__ X   __3__ Y   __3__ Z



For any ranking that has Y and Z at the same potential		1 point
For ranking the electric potentials: W > X > Y & Z (because Y = Z is the first point)		1 point

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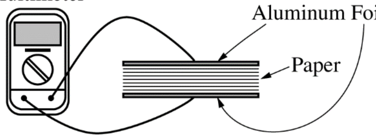
Question 2

15 points total | **Distribution of points**



An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \text{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)		
500	0.045	6.5×10^{-11}		
400	0.036	7.4×10^{-11}		
300	0.027	8.9×10^{-11}		
200	0.018	11.9×10^{-11}		
100	0.010	21.0×10^{-11}		

(a) 1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: ____________

Horizontal axis: ____________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

For indicating variables that will create a straight line whose slope can be used to determine the dielectric constant of the paper		1 point
Example: Vertical axis: C *Horizontal axis:* $\frac{1}{d}$		
Note: Student earns full credit if the axes are reversed or if they use another acceptable combination.		

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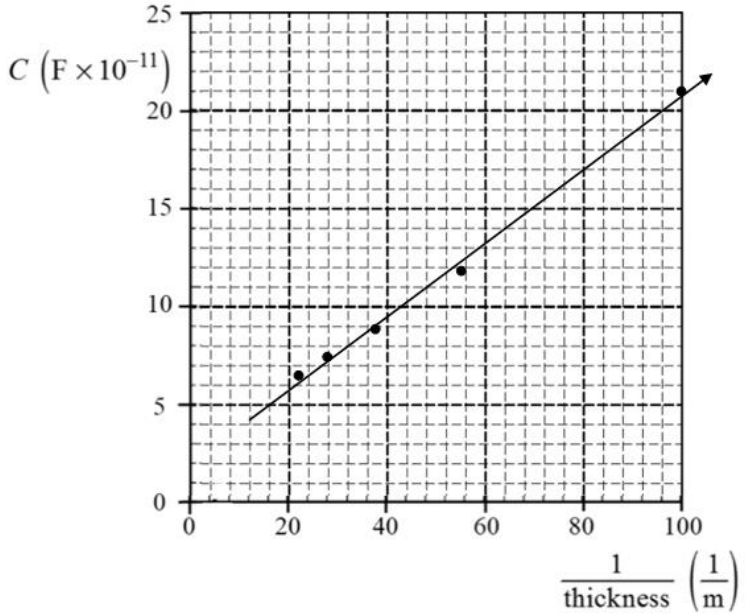
Question 2 (continued)

Distribution of points

(b) 4 points

Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



For a correct scale that uses more than half the grid		1 point
For correctly labeling the axes including units		1 point
For correctly plotting the data		1 point
For drawing a straight line consistent with the plotted data		1 point

(c) 3 points

Using the straight line, calculate a dielectric constant for the paper.

For correctly calculating the slope from the best-fit line and not the data points unless the points fall on the best-fit line		1 point
$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{(17-8)(\mathrm{F}\times10^{-11})}{(80-32)(1/\mathrm{m})} = 0.19\ \mathrm{F{\bullet}m}$ (Linear regression $= 0.187\ \mathrm{F{\bullet}m}$)		
For correctly relating the slope to the dielectric constant		1 point
$\text{slope} = \kappa\varepsilon_0 A$		
$\kappa = \frac{\text{slope}}{\varepsilon_0 A} = \frac{(0.19\ \mathrm{F{\bullet}m})}{(8.85\times10^{-12}\ \mathrm{C^2/N{\bullet}m^2})(0.060\ \mathrm{m^2})}$		
For a correct answer		1 point
$\kappa = 3.58$ (Linear regression $= 3.52$)		

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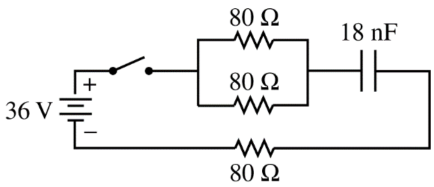
Question 2 (continued)

Distribution of points



The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical $80\ \Omega$ resistors, and an open switch, as shown in the figure above.

(d) 3 points

Calculate the current in the battery immediately after the switch is closed.

For calculating the equivalent resistance for the parallel resistors		1 point
$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{80\ \Omega} + \frac{1}{80\ \Omega} = \frac{2}{80\ \Omega} = \frac{1}{40\ \Omega}$		
$R_P = 40\ \Omega$		
For using Ohm's law with the potential difference across the capacitor equal to zero		1 point
$I = \frac{V}{R} = \frac{V}{R_P + R_3} = \frac{(36\ \text{V})}{R_P + R_3}$		
For substitution of values for resistance including the value for combined resistance above		1 point
$I = \frac{V}{R_P + R_3} = \frac{(36\ \text{V})}{(40\ \Omega + 80\ \Omega)} = 0.30\ \text{A}$		

(e) 2 points

Determine the time constant for this circuit.

For using the equation for the time constant with the equivalent resistance from above		1 point
$\tau = R_{eq}C = (120\ \Omega)(18\ \text{nF})$		
For an answer with units consistent with part (d)		1 point
$t = 2.16\ \mu\text{s}$		

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Question 2 (continued)

Distribution of points

(f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. 1 point

Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

____ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

Select "Larger than measured"		
For an appropriate explanation		1 point
Example: The battery is not ideal and has internal resistance, so the actual resistance for the circuit is larger than the measured resistance.		

ii. 1 points

Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

____ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

Select "Larger than measured"		
For an appropriate explanation		1 point
Example: Some of the sheets of paper may be thinner than expected, so the actual capacitance for the circuit is larger than the measured capacitance.		

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Question 3

15 points total **Distribution of points**

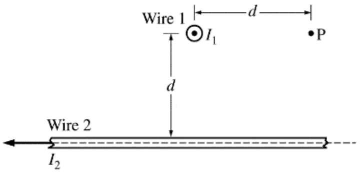

Figure 1. Side view

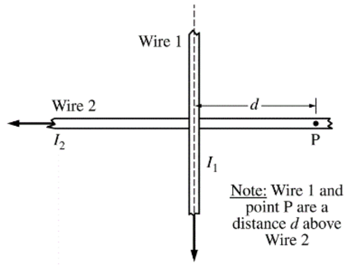

Figure 2. Top view

The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance *d* above wire 2. Point P is a horizontal distance *d* from wire 1 and a distance *d* directly above wire 2. Express your answers to parts (a) and (b) in terms of I, *d*, and physical constants, as appropriate.

(a) 2 points

Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

For attempting to use Ampere's law to calculate the magnetic field at point *P*		1 point
$\oint \vec{B} \bullet d\vec{\ell} = \mu_0 I$		
$B(2\pi d) = \mu_0 I$		
For a correct answer		1 point
$B = \dfrac{\mu_0 I}{2\pi d}$		

(b) 2 points

Derive an expression for the magnitude of the net magnetic field at point P.

For indicating $B_2 = 2B_1$		1 point
$B_1 = \dfrac{\mu_0 I}{2\pi d}$ & $B_2 = \dfrac{\mu_0 (2I)}{2\pi d} = \dfrac{\mu_0 I}{\pi d}$		
For an indication that the magnitude of the magnetic field is the vector sum of B_1 and B_2		1 point
$B_{net} = \sqrt{B_1^2 + B_2^2} = \sqrt{B^2 + B^2} = \sqrt{\left(\dfrac{\mu_0 I}{2\pi d}\right)^2 + \left(\dfrac{\mu_0 (2I)}{2\pi d}\right)^2} = \dfrac{\sqrt{5}\mu_0 I}{2\pi d}$		

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Question 3 (continued)

Distribution of points

(c) 2 points

Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

For correctly relating the angle to the individual magnetic fields		1 point
$\theta = \tan^{-1}\left(\dfrac{B_1}{B_2}\right)$		
For correctly substituting B_1 and B_2 into the equation		1 point
$\theta = \tan^{-1}\left(\dfrac{B_1}{B_2}\right) = \tan^{-1}\left(\dfrac{\left(\dfrac{\mu_0 I}{2\pi d}\right)}{\left(\dfrac{\mu_0 (2I)}{2\pi d}\right)}\right) == \tan^{-1}\left(\dfrac{1}{2}\right) = 26.6°$		

(d) 2 points

Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2? Assume gravitational effects are negligible.

____ Wire 1 will not move.

____ Wire 1 will move upward as viewed in Figure 1.

____ Wire 1 will move downward as viewed in Figure 1.

____ Wire 1 will rotate clockwise as viewed in Figure 2.

____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

For stating there is no translational motion since the magnetic forces on the wire cancel		1 point
For stating there is a net torque which causes rotation		1 point
Example: In Figure 2, the top portion of wire 1 will be in a magnetic field into the page from wire 2 and, thus, will experience a force to the right. The bottom portion of wire 1 will be in a magnetic field out of the page from wire 2 and, thus, will experience a force to the left. So the net force will be zero, but there will be a net clockwise torque, so the wire will rotate clockwise.		

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Question 3 (continued)

Distribution of points

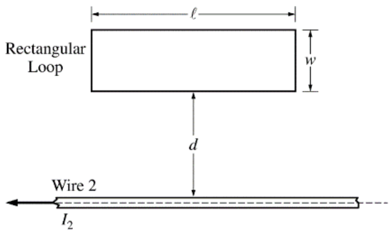

Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width *w*, and resistance *R*. The loop is placed a distance *d* from wire 2, as shown. The loop, wire, and distance *d* are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time *t* according to the equation $I_2 = 2I_0(1 - kt)$, where *k* is a positive constant with units of s^{-1}.

(e) 1 point

Of the following, select the integration that will give an expression for the flux Φ as a function of time *t*.

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

__X__ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

For selecting $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$		1 point

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Question 3 (continued)

Distribution of points

(f) 3 points

Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \dfrac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\dfrac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

For attempting to take the time derivative of the magnetic flux to calculate the emf		1 point
$\varepsilon = \left\lvert -\dfrac{d\Phi}{dt} \right\rvert = \dfrac{\mu_0 I_0 \ell}{\pi} \ln\left(\dfrac{d + w}{d}\right) \left\lvert \dfrac{d}{dt}[(1 - kt)] \right\rvert$		
$\varepsilon = \dfrac{\mu_0 I_0 k \ell}{\pi} \ln\left(\dfrac{d + w}{d}\right)$		
For dividing the emf by the resistance to calculate the current		1 point
$I = \dfrac{\varepsilon}{R} = \dfrac{\dfrac{\mu_0 I_0 k \ell}{\pi} \ln\left(\dfrac{d + w}{d}\right)}{R}$		
For a correct answer		1 point
$I = \dfrac{\mu_0 I_0 k \ell}{\pi R} \ln\left(\dfrac{d + w}{d}\right)$		

(g) 3 points

What is the direction of the current, if any, induced in the loop as seen in Figure 3?

____ Clockwise ____ Counterclockwise

____ Undefined because there is no current induced in the loop

Justify your answer.

For selecting "Clockwise" with an attempt at a relevant justification		1 point
For indicating that the flux inside the loop will decrease		1 point
For using Lenz's law to relate the decrease in the flux to the clockwise current		1 point
Example: Because the current in the wire is decreasing, the flux in the loop will decrease. According to Lenz's law, the induced current should create a magnetic field to oppose this decrease. Thus the induced magnetic field must be into the page, and the current in the loop must be clockwise.		