Question 1: Mathematical Routines (MR) — 10 points

A	**(i)**	For a multistep derivation that includes the equation $\oint \vec{E}\cdot d\vec{A} = \dfrac{q_{\text{enc}}}{\varepsilon_0}$ **Scoring Note:** Vector notation is not required for this point to be earned.	**Point A1**
		For a correct substitution of the area of an appropriate Gaussian surface with nonzero flux for the region $R_1 < r < R_2$ (e.g., $2\pi r\ell$)	**Point A2**
		For a correct expression for the enclosed charge (e.g., $\sigma_1\left(2\pi R_1\ell\right)$)	**Point A3**
		Example Response $\oint \vec{E}\cdot d\vec{A} = \dfrac{q_{\text{enc}}}{\varepsilon_0}$ $E\left(2\pi r\ell\right) = \dfrac{\sigma_1\left(2\pi R_1\ell\right)}{\varepsilon_0}$ $E = \dfrac{\sigma_1 R_1}{\varepsilon_0 r}$	
	(ii)	For substituting the expression for E from part A (i) into $\Delta V = -\int_a^b \vec{E}\cdot d\vec{r}$ **Scoring Notes:** • Vector notation is not required for this point to be earned. • The sign of ΔV is not considered for this point to be earned.	**Point A4**
		For an attempt to solve the integral for $\lvert\Delta V\rvert$ that includes correct limits (e.g., $\lvert\Delta V\rvert = \dfrac{\sigma_1 R_1}{\varepsilon_0}\int_{R_1}^{R_2}\dfrac{1}{r}dr$) **Scoring Note:** This point may be earned regardless of the order of the limits of integration; for example, $\dfrac{\sigma_1 R_1}{\varepsilon_0}\int_{R_2}^{R_1}\dfrac{1}{r}dr$.	**Point A5**
		Example Response $\lvert\Delta V\rvert = \left\lvert -\int_a^b \vec{E}\cdot d\vec{r}\right\rvert$ $\lvert\Delta V\rvert = \int_{R_1}^{R_2}\dfrac{\sigma_1 R_1}{r\varepsilon_0}dr = \dfrac{\sigma_1 R_1}{\varepsilon_0}\int_{R_1}^{R_2}\dfrac{1}{r}dr$ $\lvert\Delta V\rvert = \dfrac{\sigma_1 R_1}{\varepsilon_0}\ln\left(r\right)\Big\vert_{R_1}^{R_2} = \dfrac{\sigma_1 R_1}{\varepsilon_0}\ln\left(\dfrac{R_2}{R_1}\right)$	
	(iii)	For sketching a graph that is zero for both $0 < r < R_1$ and $r > R_2$	**Point A6**
		For sketching a graph that is decreasing and concave up for $R_1 < r < R_2$ **Scoring Note:** The curve does not have to intersect the vertical dashed lines for this point to be earned.	**Point A7**

Example Response

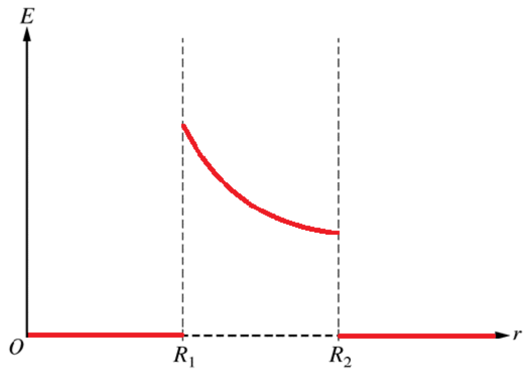


Figure 2

B

For a multistep derivation that includes the equation $C = \dfrac{Q}{\Delta V}$ **Point B1**

For indicating that C with the dielectric material inserted is C without the dielectric material inserted multiplied by κ (e.g., $C = \kappa \dfrac{Q}{\Delta V}$) **Point B2**

For substitutions of both the total charge Q and the potential difference ΔV that are consistent with part A **Point B3**

Example Response

Without the dielectric material

$$C = \frac{Q}{\Delta V}$$

$$C = \frac{\sigma_1 (2\pi L R_1)}{\left(\dfrac{\sigma_1 R_1}{\varepsilon_0} \ln\left(\dfrac{R_2}{R_1} \right) \right)}$$

$$C = \frac{2\pi L \varepsilon_0}{\ln\left(\dfrac{R_2}{R_1} \right)}$$

With the dielectric material

$$C = \kappa \frac{Q}{\Delta V}$$

$$C = \frac{2\pi L \kappa \varepsilon_0}{\ln\left(\dfrac{R_2}{R_1} \right)}$$

Question 2: Translation Between Representations (TBR) — 12 points

A

For drawing a bar at $\frac{1}{4}T$ — **Point A1**

For drawing a bar at $\frac{1}{4}T$ that has a height of 3 units — **Point A2**

For indicating that $|\mathcal{E}|$ is zero at times $t = 0$ and $\frac{1}{2}T$ — **Point A3**

Example Response

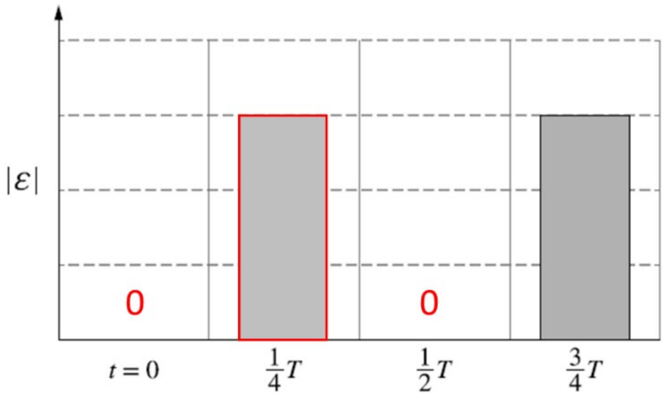


Figure 2

B

For a multistep derivation that includes the equation $\mathcal{E} = -\frac{d\Phi_B}{dt}$ — **Point B1**

Scoring Note: The negative sign does not have to be present in the expression for this point to be earned.

For a correct expression for the induced emf (e.g., $\mathcal{E} = BA\omega \sin(\omega t)$) — **Point B2**

For using Ohm's law to relate current and emf (e.g., $I = \frac{\mathcal{E}}{R}$) — **Point B3**

For a correct expression for the absolute value of the maximum induced current (e.g., $I = \frac{BA\omega}{R}$) — **Point B4**

Example Response

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -\frac{d}{dt}\left[BA\cos(\omega t)\right] = BA\omega\sin(\omega t)$$

$$\mathcal{E}_{max} = BA\omega$$

$$I = \frac{\Delta V}{R} = \frac{\mathcal{E}}{R}$$

$$I_{max} = \frac{\mathcal{E}_{max}}{R} = \frac{BA\omega}{R}$$

C	For a curve that is approximately sinusoidal	**Point C1**
	For a curve that shows exactly two cycles	**Point C2**
	For a curve that starts at the origin and has equal maximum values and equal minimum values **Scoring Note:** A curve that starts and ends at $P = 0$ and has a single maximum can earn this point.	**Point C3**

Example Response

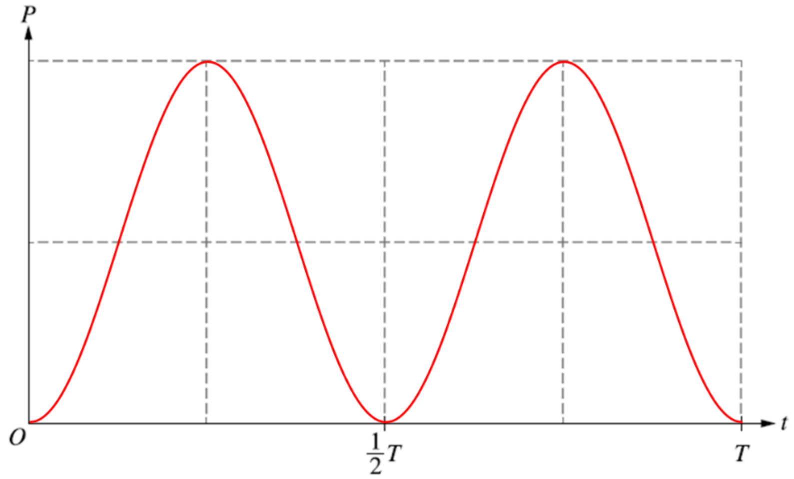


Figure 3

D	For correctly indicating whether the representations are consistent in parts A and C	**Point D1**
	For a correct justification that indicates that the maxima and/or minima of the graph in part C align with the bars drawn in part A because P is proportional to $\mathcal{E}^2$	**Point D2**

Example Response

Yes, part C is consistent with part A. P is proportional to $\mathcal{E}^2$. *When* $|\mathcal{E}|$ *is maximum, P is maximum.*

Question 3: Experimental Design and Analysis (LAB) — 10 points

A

For a procedure in which the length of and the current in the circuit element are measured, and the cross-sectional area of the circuit element is determined — **Point A1**

For a procedure that indicates a reasonable method of reducing experimental uncertainty — **Point A2**

Examples of acceptable responses may include the following:

- Making different measurements of the current in the circuit element
- Varying the potential difference of the power supply

Example Response

Measure the length and diameter of the circuit element. Use the diameter to calculate the area of the circuit element. Measure the current in the circuit element. Repeat the procedure multiple times for different potential difference settings on the variable power supply.

B

For indicating quantities that can be graphed appropriately to determine ρ_1, consistent with the procedure from part A, such as potential difference as a function of current — **Point B1**

Scoring Notes:

- Responses that include the reciprocals of the preceding example, in addition to other equivalent graphs, also earn this point.
- This point may be earned independently of the response in part A.

For a correct analysis of the graph, consistent with the first point of part B — **Point B2**

Example Response

Graph the potential difference across the circuit element as a function of the current in the circuit element. Use the slope of the graph, which is $\frac{\rho_1 \ell}{\pi r^2}$, where ℓ is the length of the circuit element and r is equal to the radius of the circuit element, to determine ρ_1.

C

(i) For indicating appropriate quantities that could be plotted on the graph to determine ρ_2, for example R as a function of L — **Point C1**

Scoring Note: This point may be earned:

- if the vertical and horizontal variables are reversed.
- for other equivalent graphs.

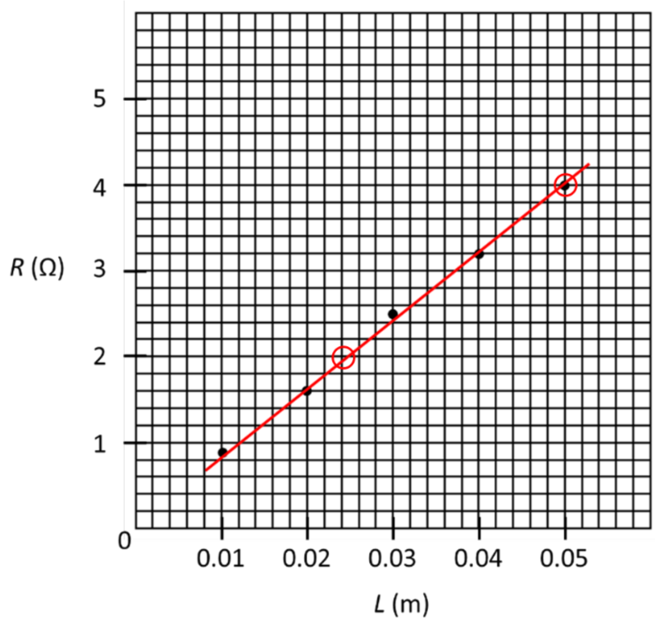
(ii) For labeling the axes (including units) with a linear scale — **Point C2**

For correctly plotting data points consistent with **one** of the following: — **Point C3**

- The quantities indicated in part C (i)
- The quantities provided in the table
- The axes indicated in the first point of part C (ii)

(iii) For drawing a line or curve that approximates the trend of the plotted data — **Point C4**

Example Response



D	For correctly relating the slope of the best-fit line to ρ_2 (e.g., $\frac{\Delta R}{\Delta L} = \text{slope} = \frac{\rho_2}{A}$)	**Point D1**
	For a value for ρ_2 that is between $3.5 \times 10^{-4}\ \Omega \cdot \text{m}$ and $4.5 \times 10^{-4}\ \Omega \cdot \text{m}$	**Point D2**

Example Response

$$\text{slope} = \frac{\Delta R}{\Delta L}$$

$$\frac{\Delta R}{\Delta L} = \frac{4.0\ \Omega - 2.0\ \Omega}{0.050\ \text{m} - 0.024\ \text{m}}$$

$$\frac{\Delta R}{\Delta L} = 77\ \frac{\Omega}{\text{m}}$$

$$R = \frac{\rho L}{A}$$

$$R = \left(\frac{\rho_2}{A}\right)L$$

$$\text{slope} = \frac{\rho_2}{A}$$

$$\rho_2 = A(\text{slope})$$

$$\rho_2 = \left(5.0 \times 10^{-6}\ \text{m}^2\right)\left(77\ \frac{\Omega}{\text{m}}\right)$$

$$\rho_2 \approx 3.9 \times 10^{-4}\ \Omega \cdot \text{m}$$

Question 4: Qualitative Quantitative Translation (QQT) — 8 points

A	For indicating that $F_2 > F_1$	**Point A1**
	For correctly relating the magnitude of the magnetic force to the magnitude of the magnetic field	**Point A2**
	For indicating that the magnitude of the magnetic field at the location of Sphere 2 is greater than the magnitude of the magnetic field at the location of Sphere 1 by referring to either: • The relative distance from the spheres to Wires S and T • The direction of the magnetic fields from Wires S and T at the locations of the spheres	**Point A3**
	Example Response *The magnitude of the magnetic force exerted on a sphere is directly proportional to Q, v, and B. Because Q and v are the same for both spheres, the difference in the magnitudes of the magnetic forces is due to the difference in the magnitudes of the magnetic fields at the locations of the spheres. The magnitude of the magnetic field is greater at the location of Sphere 2 than at the location of Sphere 1 because the fields from the wires are in the same direction. Therefore,* $F_2 > F_1$.	
B	For a multistep derivation that includes the equation $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$ or $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\left(d\vec{\ell} \times \hat{r}\right)}{r^2}$ **Scoring Note:** Vector notation is not required for this point to be earned. A multistep derivation that includes $B = \frac{\mu_0 I}{2\pi d}$ can earn this point.	**Point B1**
	For a correct expression for the magnitude of the magnetic field due to the current in one wire at the location of Sphere 2	**Point B2**
	For indicating that the magnitude of the total field is twice that due to one wire	**Point B3**
	Example Response *Determine the magnitude B of the magnetic field due to one wire.* $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$ $B(2\pi d) = \mu_0 I$ $B = \frac{\mu_0 I}{2\pi d}$ *Determine* B_{tot}. $B_{\text{tot}} = \frac{\mu_0 I}{2\pi d} + \frac{\mu_0 I}{2\pi d}$ $B_{\text{tot}} = \frac{\mu_0 I}{\pi d}$	

C	For indicating $F_{\text{new}} = F_2$	**Point C1**
	For indicating $B_{\text{tot}} = \frac{3\mu_0 I}{2\pi d} - \frac{\mu_0 I}{2\pi d}$ OR For indicating that the net magnitude of the magnetic field at the location of Sphere 2 will remain the same even though the field from Wire T will change	**Point C2**
	Example Response *The magnetic field at the location of Sphere 2 due to the current in Wire T is in the opposite direction to the magnetic field due to the current in Wire S. The current in Wire T is increased by a factor of 3. Therefore,* $B_{\text{tot}} = \frac{3\mu_0 I}{2\pi d} - \frac{\mu_0 I}{2\pi d} = \frac{\mu_0 I}{\pi d}$ *in this scenario. Thus,* $F_{\text{new}} = F_2$.	

Question 1: Free-Response Question — 15 points

(a) For using a correct equation for electric flux — **1 point**

Example Response

$$\Phi_E = \frac{Q}{\varepsilon_0}$$

For the correct numerical answer — **1 point**

Scoring Note: This point can be earned if a negative sign is included in the final answer or if units are missing and/or the units are incorrect.

Example Response

$$|\Phi_E| = 226 \frac{\mathrm{N \cdot m^2}}{\mathrm{C}}$$

Example Solution

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$\Phi_E = \frac{Q}{\varepsilon_0}$$

$$\Phi_E = \frac{-2.0 \times 10^{-9}\ \mathrm{C}}{8.85 \times 10^{-12} \frac{\mathrm{C^2}}{\mathrm{N \cdot m^2}}}$$

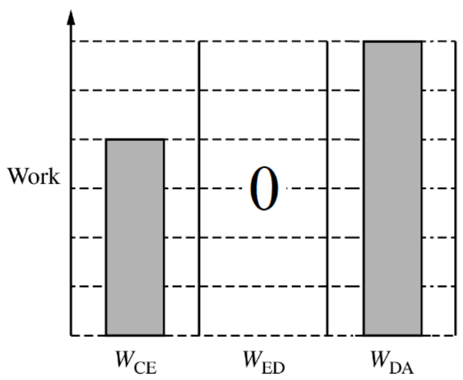
$$|\Phi_E| = 226 \frac{\mathrm{N \cdot m^2}}{\mathrm{C}}$$

Total for part (a) — 2 points

(b)(i) For indicating that $W_{\mathrm{ED}} = 0$ — **1 point**

For drawing a bar representing W_{DA} that has a height of six units — **1 point**

Example Response



(b)(ii) For using an equation that relates the electric field to potential difference **1 point**

Scoring Note: This point can be earned if the response begins with a correct relationship between electric field and potential difference in which numerical values are already substituted.

Example Responses

$E_x = -\frac{dV}{dx}$ **OR** $|E_x| = \left|-\frac{dV}{dx}\right|$ **OR** $|E_x| = \left|-\frac{\Delta V}{\Delta x}\right|$ **OR** $\Delta V = -\int \vec{E} \cdot d\vec{r}$

For correct substitutions of values of electric potential and the distance between equipotential lines that can be used to calculate the approximate magnitude of the electric field at Position B **1 point**

Example Response

$|E_x| = \left|-\frac{20.0 \text{ V} - 0.0 \text{ V}}{0.65 \text{ m}}\right|$

Example Solution

$$E_x = -\frac{dV}{dx}$$
$$|E_x| = \left|-\frac{dV}{dx}\right|$$
$$|E_x| = \left|-\frac{\Delta V}{\Delta x}\right|$$
$$|E_x| = \left|-\frac{20.0 \text{ V} - 0.0 \text{ V}}{0.65 \text{ m}}\right|$$
$$|E_x| = 31 \frac{\text{V}}{\text{m}}$$

Total for part (b) 4 points

(c) For selecting only $+y$ with an attempt at a relevant justification **1 point**

For indicating that the direction of the electric field vector is perpendicular to a line that is tangent to the equipotential line at Position D **1 point**

For indicating one of the following: **1 point**

- The test charge moves from a higher electric potential to a lower electric potential.
- The test charge and the sphere have charges of opposite sign.
- The test charge moves in the direction of the electric field, which is directed upward.

Example Response

$+y$. The direction of an electric field vector is perpendicular to an equipotential line. Because the test charge has a positive charge, the test charge would move from a position of higher electric potential to a position of lower electric potential when an electric force is exerted on the test charge. Therefore, at Position D*, the electric force is upward because that is the direction that is perpendicular to the equipotential line and in the direction of decreasing electric potential.*

Total for part (c) 3 points

(d)(i) For using an appropriate equation for determining the electric potential from a line of uniform charge **1 point**

Example Responses

$$V = \frac{1}{4\pi\varepsilon_0}\sum\frac{q_i}{r_i} \quad \textbf{OR} \quad V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} \quad \textbf{OR} \quad \Delta V = -\int\vec{E}\cdot d\vec{r}$$

For a correct determination of r, the distance between Point P and a point on the line of uniform charge **1 point**

Example Responses

$$V_\mathrm{P} = k\sum\frac{Q}{x_\mathrm{P} - x} \quad \textbf{OR} \quad V_\mathrm{P} = k\int\left(\frac{1}{x_\mathrm{P} - x}\right)dq$$

For a correct integral with λdx substituted for dq **1 point**

Example Response

$$V_\mathrm{P} = k\lambda\int\left(\frac{1}{x_\mathrm{P} - x}\right)dx$$

For the correct limits of integration **1 point**

Example Response

$$V_\mathrm{P} = k\lambda\int_0^{4L}\left(\frac{1}{x_\mathrm{P} - x}\right)dx$$

Example Solutions

$$V = \frac{1}{4\pi\varepsilon_0}\sum\frac{q_i}{r_i}$$

$$V_P = k\int\left(\frac{1}{x_P - x}\right)dq$$

$$dq = \lambda\, dx$$

$$V_P = k\lambda\int\left(\frac{1}{x_P - x}\right)dx$$

$$V_P = k\lambda\int_0^{4L}\left(\frac{1}{x_P - x}\right)dx$$

$$V_P = -k\lambda\ln(x_P - x)\,\Big|_0^{4L} = k\lambda\ln(x_P - x)\,\Big|_{4L}^{0}$$

$$V_P = k\lambda\ln\left(\frac{x_P}{x_P - 4L}\right)$$

OR

$$\Delta V = -\int\vec{E}\cdot d\vec{r}$$

$$E(x) = \int\frac{k}{r^2}\,dq$$

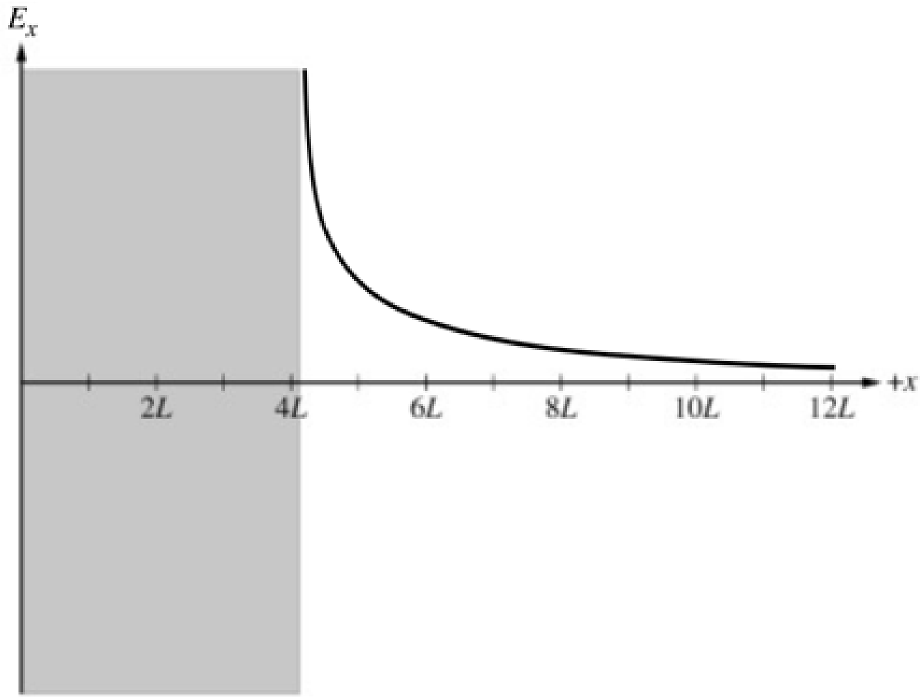
$$E(x) = \int_0^{4L}\frac{k\lambda}{(x - x')^2}\,dx' = k\lambda\left(\frac{1}{x - 4L} - \frac{1}{x}\right)$$

$$V_P = -\int_\infty^{x_P}E(x)\,dx = -k\lambda\int_\infty^{x_P}\left(\frac{1}{x - 4L} - \frac{1}{x}\right)dx$$

$$V_P = k\lambda\ln\left(\frac{x_P}{x_P - 4L}\right)$$

(d)(ii)	For sketching a curve or line that continually approaches the horizontal axis as position increases	**1 point**
	For sketching a concave up curve that is always positive	**1 point**

Example Response



Total for part (d) **6 points**

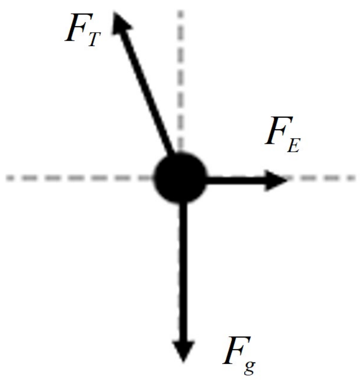
Total for question 1 **15 points**

Question 1: Free-Response Question — 15 points

(a)	For correctly drawing and labeling the electrostatic force directed to the right	**1 point**
	For drawing the force of tension up and to the left and the gravitational force in the downward direction	**1 point**

Scoring Note: A maximum of 1 point may be earned if extraneous forces are included.

Example Response



Total for part (a) **2 points**

(b)	For equating the horizontal component of tension to the electrostatic force	**1 point**

Example Response

$$F_E = F_T \sin(\theta)$$

For equating the vertical component of tension to the gravitational force	**1 point**

Example Response

$$F_g = F_{Ty}$$
$$Mg = F_T \cos\theta$$

For an attempt to simultaneously solve the equations	**1 point**

Example Response

$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}\frac{1}{\sin\theta}\cos\theta = Mg$$

Example Solution

$$\Sigma F_y = 0$$
$$F_{Ty} = F_g$$
$$F_T \cos\theta = Mg$$
$$F_{Tx} = F_E$$
$$F_T = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}\frac{1}{\sin\theta}$$
$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}\frac{1}{\sin\theta}\cos\theta = Mg$$
$$d^2 = \frac{Qq\cos\theta}{4\pi\varepsilon_0 Mg\sin\theta}$$
$$d = \sqrt{\frac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}}$$

Total for part (b) 3 points

(c) For applying Coulomb's law to determine tension **1 point**

Scoring Note: This point may be earned if the student used the vertical component of tension.

Example Response

$$F_E = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2} = F_T \sin(\theta)$$

For correct substitution into an expression for tension consistent with part (b) or a correct expression for tension **1 point**

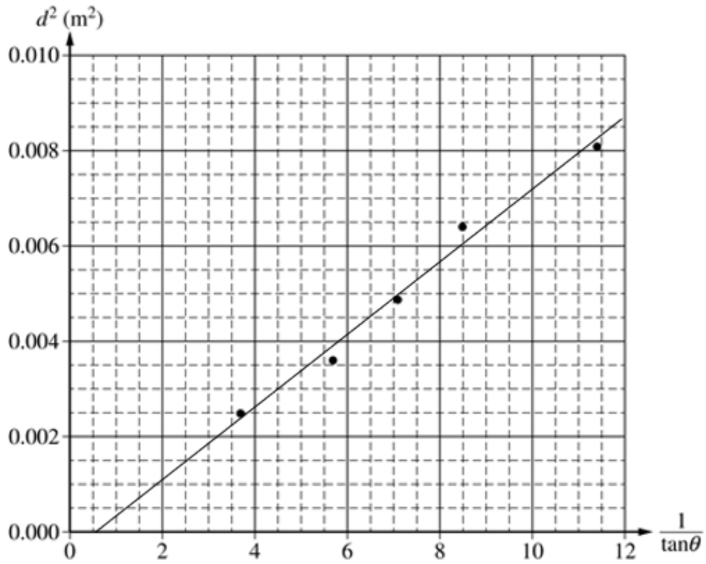
Example Solution

$$\Sigma F = 0$$
$$F_E - F_{Tx} = 0$$
$$F_E = F_{Tx}$$
$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2} = F_T \sin(\theta)$$
$$F_T = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2 \sin(\theta)}$$
$$F_T = \frac{1}{4\pi\left(8.85\times10^{-12}\ \dfrac{\mathrm{C}^2}{\mathrm{N{\bullet}m}^2}\right)}\frac{\left(6.0\times10^{-8}\ \mathrm{C}\right)^2}{(0.057\ \mathrm{m})^2\sin\left(12^\circ\right)}$$
$$F_T = 0.048\ \mathrm{N}$$

Total for part (c) 2 points

(d)(i) For a line that approximates the trend of the data **1 point**

Example Response



(d)(ii) For using two points from the trend line drawn by the student to calculate the slope **1 point**

Scoring Note: Points of data may be used only if points of data are located directly on the line.

Example Response

$$\text{Slope} = \frac{\Delta y}{\Delta x}$$

$$\text{Slope} = \frac{\Delta(d^2)}{\Delta\left(\dfrac{1}{\tan(\theta)}\right)}$$

$$\text{Slope} = \frac{\left(0.0075\ \text{m}^2 - 0.001\ \text{m}^2\right)}{(10.5 - 2)}$$

$$\text{Slope} = 7.647 \times 10^{-4}\ \text{m}^2$$

For correctly relating the slope of the graph to the equation $d = \sqrt{\dfrac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}}$ **1 point**

Example Response

$$d = \sqrt{\frac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}}$$

$$d^2 = \frac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}$$

$$d^2 = \left(\frac{Qq}{4\pi\varepsilon_0 Mg}\right)\frac{1}{\tan\theta}$$

$$\text{slope} = \left(\frac{Qq}{4\pi\varepsilon_0 Mg}\right)$$

For substituting the value of the slope of the graph into the equation $\varepsilon_0 = \dfrac{Qq}{4\pi Mg(\text{slope})}$ to calculate an experimental value of ε_0 **1 point**

Example Solution

$$\text{slope} = \frac{Qq}{4\pi\varepsilon_0 Mg}$$

$$\varepsilon_0 = \frac{Qq}{4\pi Mg(\text{slope})}$$

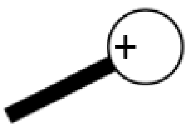
$$\varepsilon_0 = \frac{\left(6.0 \times 10^{-8}\ \text{C}\right)^2}{4\pi(0.005\ \text{kg})\left(9.8\ \text{m/s}^2\right)\left(7.647 \times 10^{-4}\ \text{m}^2\right)}$$

$$\varepsilon_0 = 7.6 \times 10^{-12}\ \frac{\text{C}^2}{\text{N}\bullet\text{m}^2}$$

Total for part (d) 4 points

(e)(i)	For a "+" drawn on the left side of the sphere	**1 point**

Example Response



(e)(ii)	For a statement that indicates correct charge rearrangement on Sphere 3 due to the electric forces from the charges on Sphere 2	**1 point**

Example Response

The negative charges on Sphere 3 move to the right due to the attractive forces from the positive charges on Sphere 2, leaving a net positive charge on the left side of Sphere 3.

(e)(iii)	For selecting "$\theta_2 < \theta_1$" with an attempt at a relevant justification	**1 point**
	For statement that indicates **one** of the following: • The average distance between the repulsive charges is greater. • The electrostatic or repulsive force is less.	**1 point**

Scoring Note: Points 1 and 2 of part (e)(iii) can be earned with an answer that is consistent with the location of the excess positive charges drawn in part (e)(i).

Example Response

Excess charges on Sphere 3 are now free to move, so excess like charges will be concentrated on the far ends of Sphere 3 when the spheres are in static equilibrium. The excess like charges, located on opposite sides of Sphere 3, repel with less force than if the excess charges were located at the centers of Sphere 3. Thus, the downward force due to gravity on Sphere 2 causes the center of Sphere 2 to hang closer to the center of Sphere 3.

Total for part (e) 4 points

Total for question 1 15 points