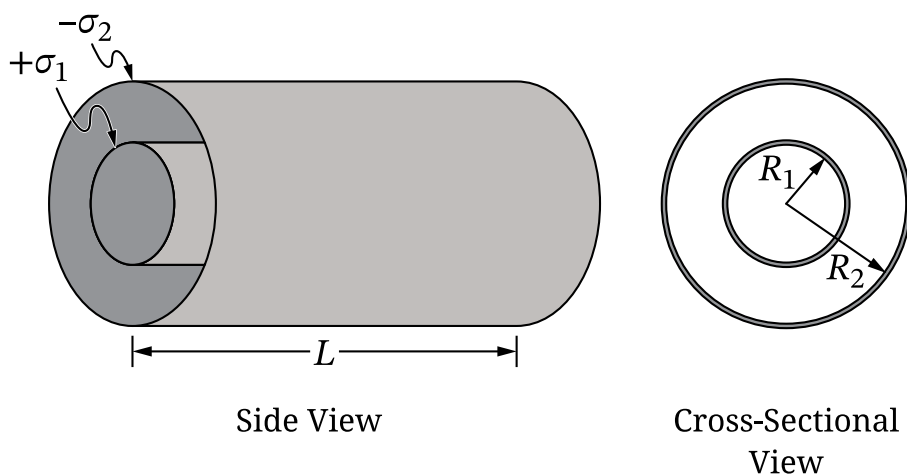


Question 1: Version J

1. An isolated, air-filled, charged capacitor consists of two conducting, coaxial, cylindrical shells that each have length L . The inner shell has radius R_1 and the outer shell has radius R_2 , as shown in Figure 1, where $R_1 < R_2 \ll L$. The surface charge densities (amounts of charge per unit area) of the inner and outer shells are $+\sigma_1$ and $-\sigma_2$, respectively. The absolute values of the total charges on the shells are equal.



Note: Figures not drawn to scale.

Figure 1

A.

- i. Using Gauss's law, **derive** an expression for the magnitude E of the electric field as a function of the radial distance r from the center of the capacitor for the region $R_1 < r < R_2$. Express your answer in terms of R_1 , σ_1 , r , and physical constants, as appropriate.
- ii. **Derive** an expression for the absolute value $|\Delta V|$ of the potential difference between the outer and inner shells in terms of R_1 , R_2 , σ_1 , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

- iii. On the axes shown in Figure 2, **sketch** a graph of E as a function of r from $r = 0$ to a position that is outside the outer shell.

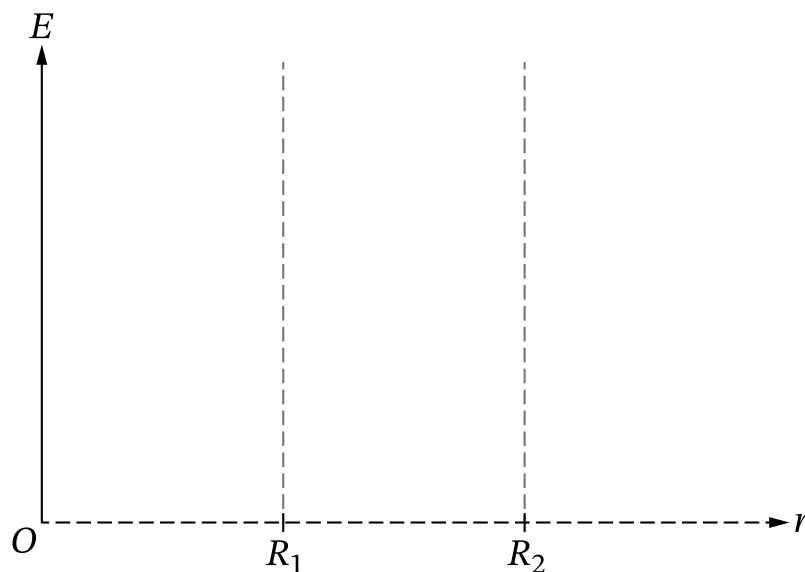
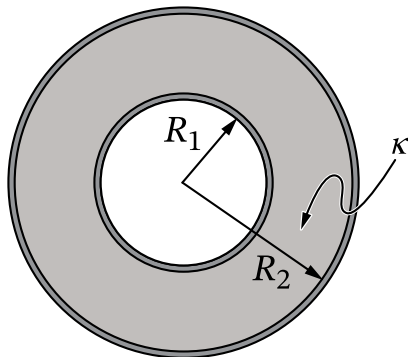


Figure 2

- B. A material of dielectric constant κ is inserted into the isolated, charged capacitor such that the material fills the region $R_1 < r < R_2$, as shown in Figure 3.



Cross-Sectional View

Figure 3

Derive an expression for the capacitance C of the capacitor with the material inserted in terms of L , R_1 , R_2 , κ , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Question 2: Version J

2. A rotating, circular, conducting loop of area A and resistance R is in an external uniform magnetic field of magnitude B that is directed in the $-z$ -direction. At time $t = 0$, the magnetic field is perpendicular to the plane of the loop, as shown in Figure 1. The loop is rotating with constant angular speed ω and period T about the dashed line that is along the diameter of the loop. The value of the magnetic flux through the loop as a function of time t is $\Phi = BA \cos(\omega t)$.

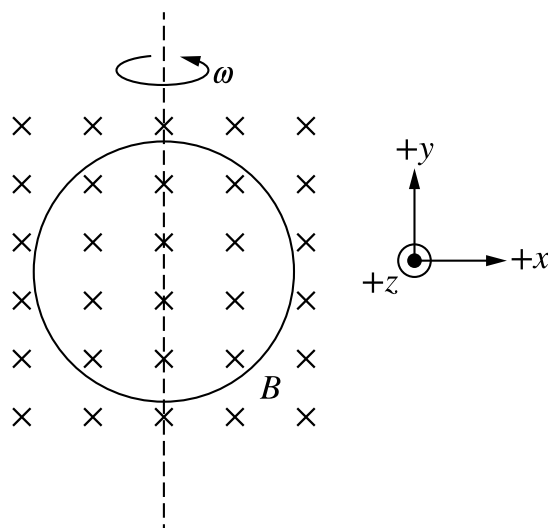


Figure 1

- A. The absolute value of the induced emf in the loop is $|\mathcal{E}|$. The partially completed bar chart in Figure 2 shows a bar that represents $|\mathcal{E}|$ at $t = \frac{3}{4}T$. In Figure 2, **draw** bars to represent $|\mathcal{E}|$ at times $t = 0$, $\frac{1}{4}T$, and $\frac{1}{2}T$ relative to $|\mathcal{E}|$ shown at $\frac{3}{4}T$. If $|\mathcal{E}| = 0$, **write** a “0” in that column.

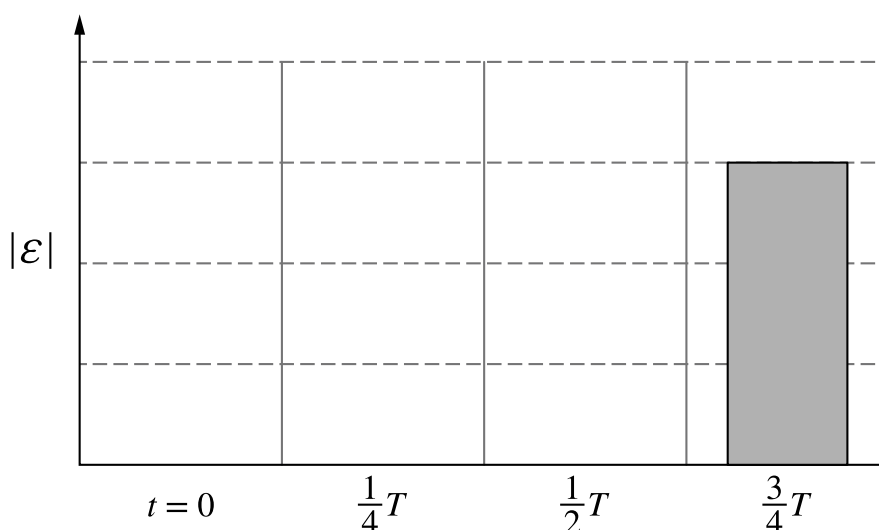


Figure 2

- B. Derive** an expression for the maximum induced current in the loop in terms of A , R , B , ω , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.
- C.** On the axes shown in Figure 3, **sketch** a graph of the instantaneous power P dissipated by the loop as a function of t during the time interval $0 \leq t \leq T$.

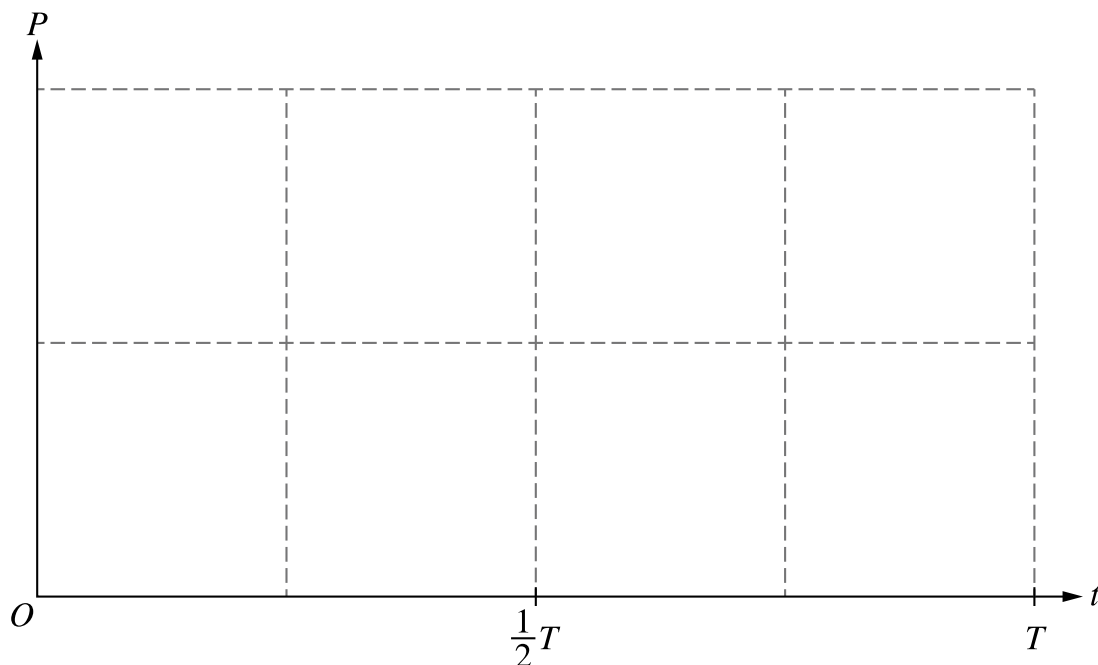
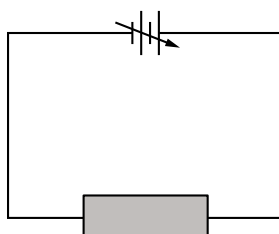


Figure 3

- D. Indicate** whether the sketch you drew in part C is or is not consistent with the bars that you drew in part A. Briefly **justify** your answer by referencing the functional dependence between P and $|\mathcal{E}|$.

Question 3

3. In Experiment 1, students are asked to use a graph to determine the resistivity ρ_1 of a circuit element that is connected to a variable power supply, as shown in Figure 1. The circuit element is cylindrical and has uniform resistivity. The students have access to a voltmeter, an ammeter, and a ruler.

Figure 1

Circuit Element

- A. **Describe** a procedure for collecting data that would allow the students to use a graph to determine ρ_1 , including any steps necessary to reduce experimental uncertainty.
- B. **Describe** how the collected data could be graphed and how that graph would be analyzed to determine ρ_1 .

In Experiment 2, the students are asked to use a graph to determine the resistivity ρ_2 of solid, cylindrical resistors made of the same material but of different lengths L . The cross-sectional area of each resistor is $5.0 \times 10^{-6} \text{ m}^2$. The students directly measure the resistance R between the ends of each resistor. Table 1 provides L and R for each resistor.

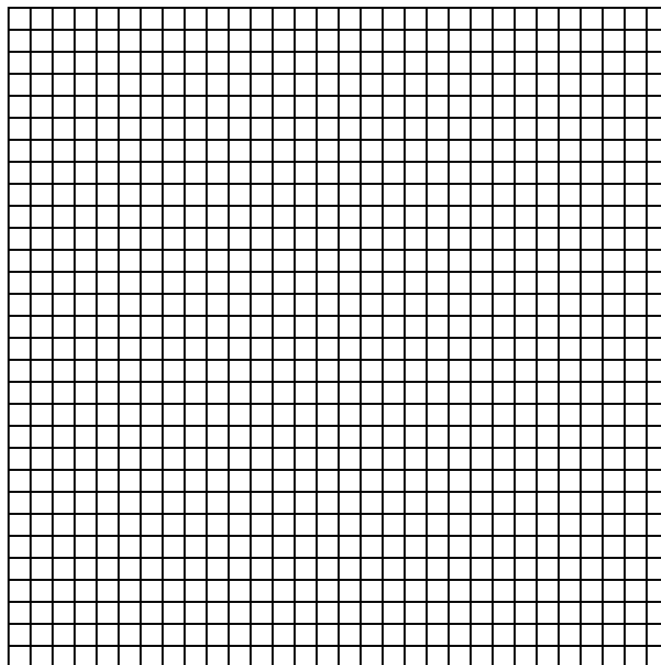
Table 1

L (m)	R (Ω)
0.010	0.90
0.020	1.6
0.030	2.5
0.040	3.2
0.050	4.0

- C.
- i. **Indicate** two quantities, either measured quantities from Table 1 or additional calculated quantities, that could be graphed to produce a straight line that could be used to determine ρ_2 .

Vertical axis: _____ Horizontal axis: _____

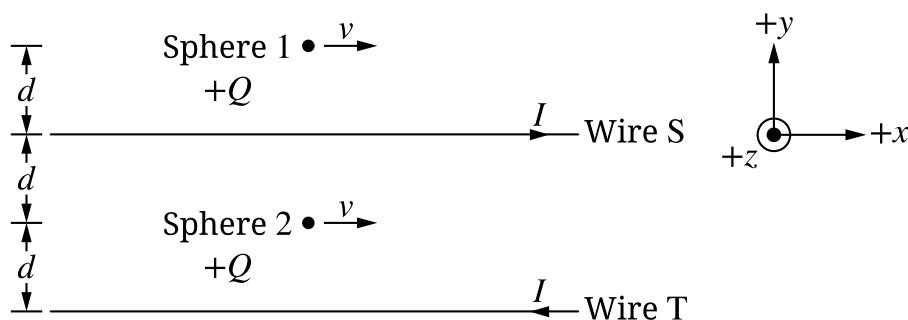
- ii. On the grid provided, create a graph of the quantities indicated in part C (i).
- Use Table 2 to record the measured or calculated quantities that you will plot.
 - Clearly **label** the axes, including units as appropriate.
 - **Plot** the points you recorded in Table 2.



- iii. **Draw** a best-fit line for the data graphed in part C (ii).
- D.** Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for ρ_2 .

Question 4

4. Long, parallel wires S and T are a distance $2d$ apart. Both wires carry equal currents I , but the currents are in opposite directions. Both wires are parallel to the x -axis. At the instant shown in Figure 1, Sphere 1 is a distance d above Wire S, Sphere 2 is a distance d below Wire S, and both spheres are moving with speed v in the $+x$ -direction. Each sphere has positive charge $+Q$. Gravitational effects are negligible.

Figure 1

- A. F_1 is the magnitude of the magnetic force exerted on Sphere 1 due to the currents in wires S and T. F_2 is the magnitude of the magnetic force exerted on Sphere 2 due to the currents in wires S and T.

Indicate whether F_2 is greater than, less than, or equal to F_1 by writing one of the following.

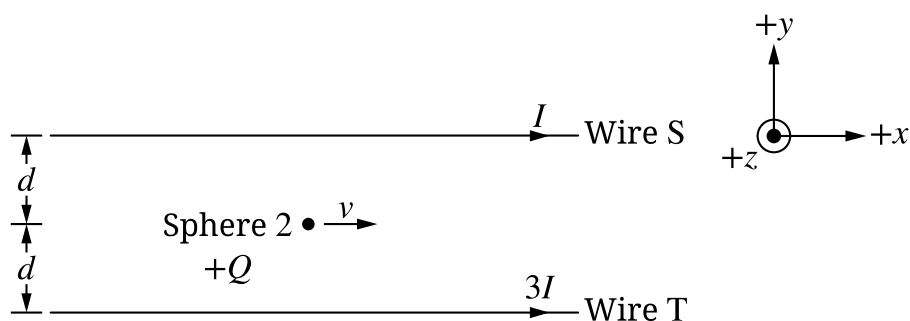
- $F_2 > F_1$
- $F_2 < F_1$
- $F_2 = F_1$

Justify your answer.

- B. **Derive** an expression for the magnitude B_{tot} of the magnetic field at the location of Sphere 2 due to the currents in wires S and T in terms of d , I , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

- C. Later, Wire T carries current $3I$ in the $+x$ -direction. At the instant shown in Figure 2, Sphere 2 is a distance d below Wire S and is moving with speed v in the $+x$ -direction. F_{new} is the new magnitude of the magnetic force exerted on Sphere 2 due to the currents in wires S and T.

Figure 2



Indicate whether F_{new} is greater than, less than, or equal to F_2 by writing one of the following.

- $F_{\text{new}} > F_2$
- $F_{\text{new}} < F_2$
- $F_{\text{new}} = F_2$

Briefly **justify** your answer by referencing your derivation in part B.

STOP
END OF EXAM

Begin your response to **QUESTION 1** on this page.**PHYSICS C: ELECTRICITY AND MAGNETISM****SECTION II****Time—45 minutes****3 Questions**

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

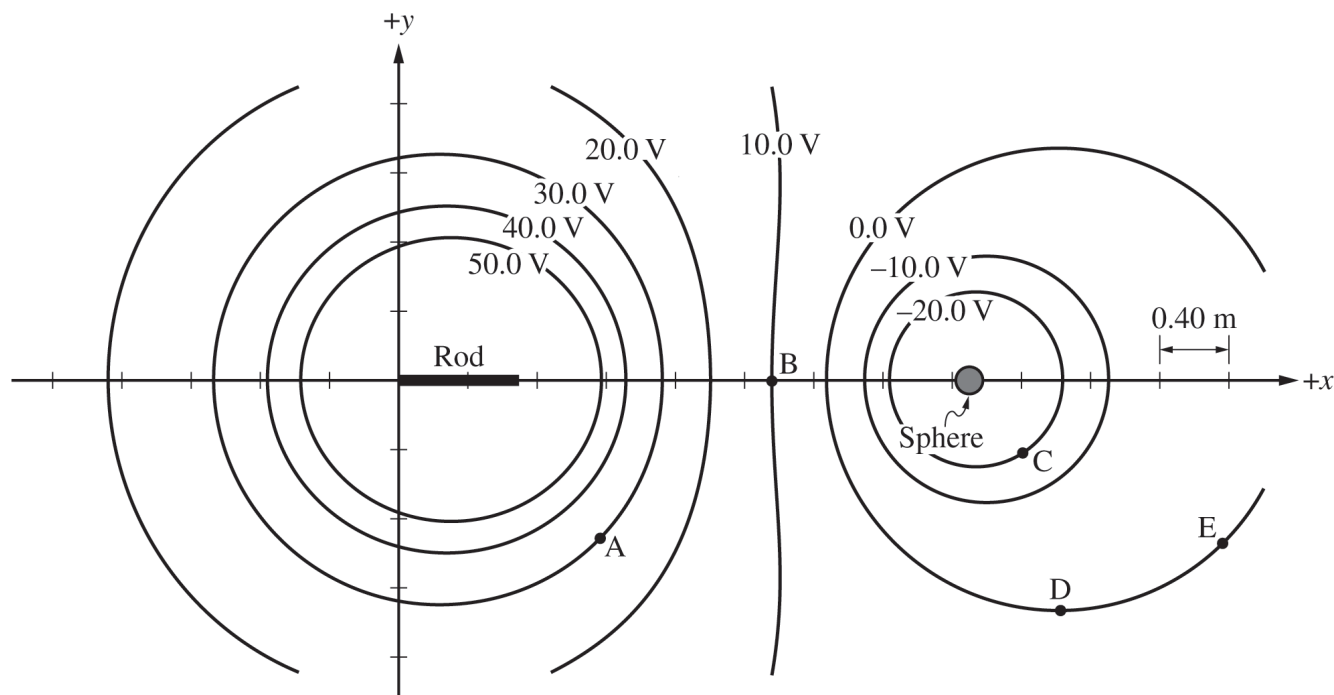


Figure 1

1. A nonconducting rod of uniform positive linear charge density is near a sphere with charge -2.0 nC. The rod and sphere are held at rest on the x -axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x -axis and the y -axis are 0.40 m apart.
- (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the -20.0 V equipotential line.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of $C \rightarrow E \rightarrow D \rightarrow A$. The test charge is momentarily held at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE} . If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a “0” in the corresponding columns, as appropriate.

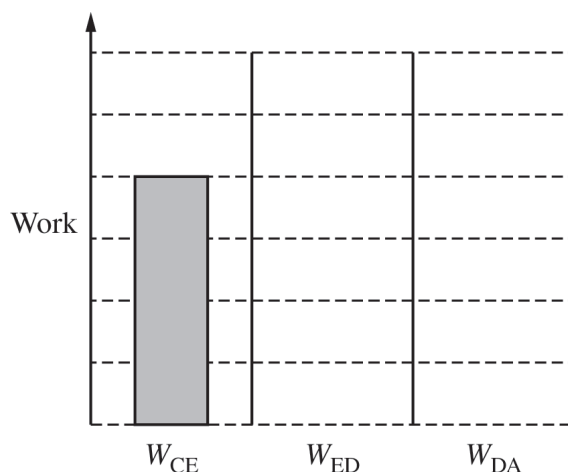


Figure 2

ii. **Calculate** the approximate magnitude of the x -component of the electric field at Position B.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position D. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

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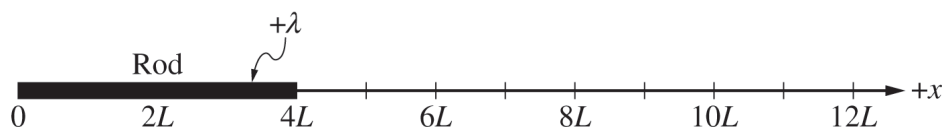
Continue your response to **QUESTION 1** on this page.

Figure 3

The sphere and the test charge are removed. The rod has length $4L$ and uniform positive linear charge density $+\lambda$. The rod is held at rest on the x -axis in the orientation shown in Figure 3. Position P (not shown) is located on the x -axis a distance x_P from the origin, where $x_P > 4L$.

(d) The electric potential V_P at x_P is $V_P = k\lambda \ln\left(\frac{x_P}{x_P - 4L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

- ii. On Figure 4, **sketch** a graph of the x -component E_x of the electric field from the rod as a function of x in the region $4L < x < 12L$.

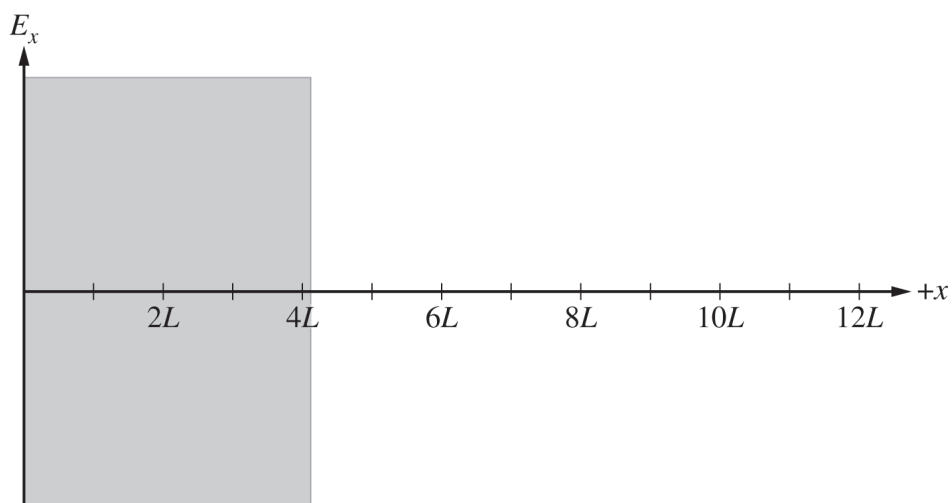
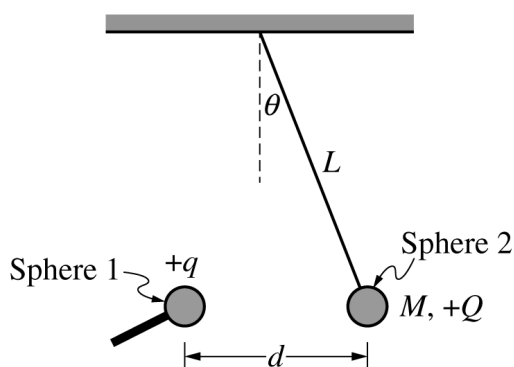


Figure 4

GO ON TO THE NEXT PAGE.

PHYSICS C: ELECTRICITY AND MAGNETISM**SECTION II****Time—45 minutes****3 Questions**

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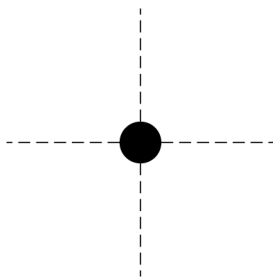


Students perform an experiment to determine the value of vacuum permittivity ϵ_0 . Sphere 1 is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere 2 is nonconducting with charge $+Q$ and has mass M . Sphere 2 is hung from a string of negligible mass and length L . Sphere 1 is brought near, without touching, Sphere 2, as shown. Equilibrium is established when the centers of the two spheres have the same vertical position, are a horizontal distance d apart, and the string is at an angle θ from the vertical.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

- (a) On the following dot that represents Sphere 2 at the position shown in the previous figure, draw and label the forces (not components) that act on Sphere 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

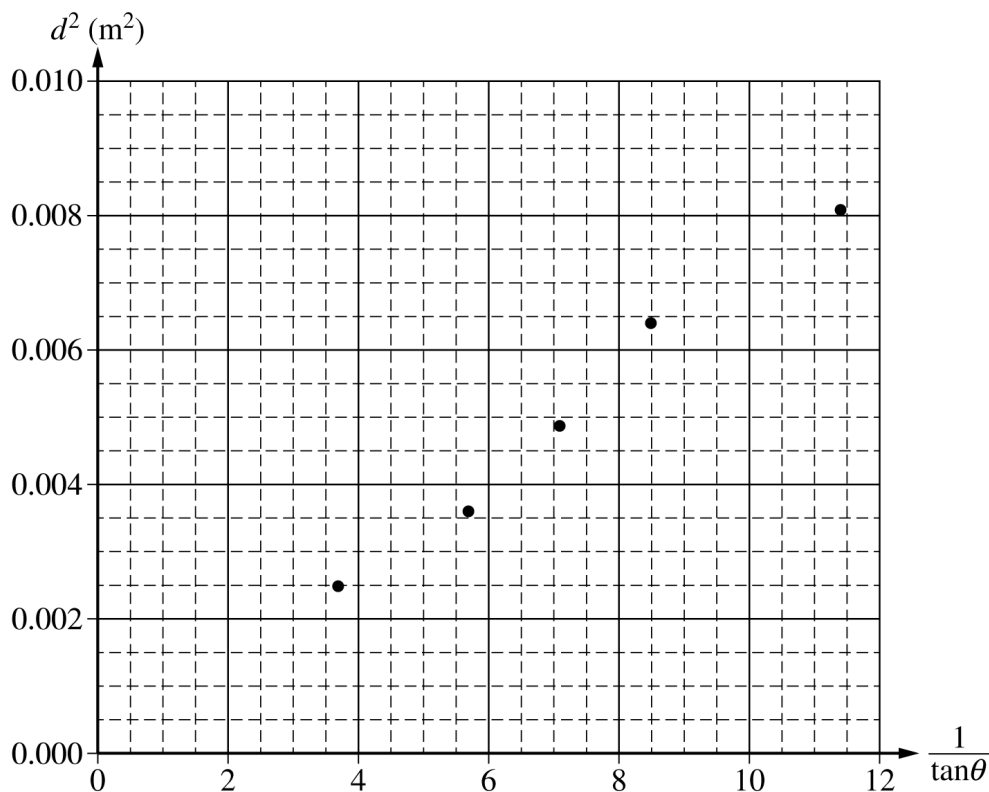


- (b) Derive the relationship between the distance d and the angle θ to show that $d = \sqrt{\frac{Qq}{4\pi\epsilon_0 Mg \tan\theta}}$.

- (c) These values are collected in one trial: $Q = q = 6.0 \times 10^{-8} \text{ C}$, $\theta = 12^\circ$, and $d = 0.057 \text{ m}$. Calculate the expected force of tension exerted on Sphere 2 by the string.

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- (d) The students vary d and measure θ after equilibrium is reached. The students use the collected data to plot the following graph of d^2 vs. $\frac{1}{\tan\theta}$.

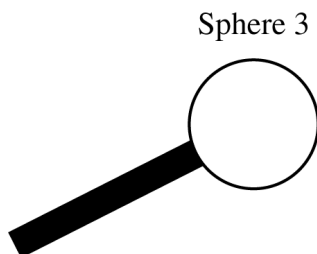


- Draw the best-fit line for the data.
- Using the best-fit line, calculate an experimental value for the vacuum permittivity ϵ_0 when $M = 0.0050$ kg and $Q = q = 6.0 \times 10^{-8}$ C.

GO ON TO THE NEXT PAGE.

(e) The students modify the experiment by replacing Sphere 1 with a conducting Sphere 3 that has the same size and charge $+q$. The experiment is repeated.

- i. The circle in the following figure represents Sphere 3 when spheres 2 and 3 are at equilibrium. On the circle, draw a single “+” sign to represent the location of highest concentration of the excess positive charges.



- ii. Briefly explain your reasoning for the sketch drawn in part (e)(i).

iii. In the original experiment, when the centers of the two spheres are a horizontal distance d_1 apart, the string makes an angle θ_1 from the vertical. In the modified experiment, when the centers of the two spheres are a horizontal distance d_1 apart, the string makes an angle θ_2 from the vertical.

Is θ_2 greater than, less than, or equal to θ_1 ?

_____ $\theta_2 > \theta_1$ _____ $\theta_2 < \theta_1$ _____ $\theta_2 = \theta_1$

Briefly justify your answer.

GO ON TO THE NEXT PAGE.