Week 25

AP Physics C: Electricity and Magnetism

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

AP Physics C: Electricity and Magnetism Name: ____________ Score: ______

1. Magnetic flux through a flat loop is:
   - ☐ $BA\cos\theta$
   - ☐ $BA\sin\theta$
   - ☐ B/A
   - ☐ $B+A$
2. Faraday's law says the induced EMF depends on:
   - ☐ the flux itself
   - ☐ the rate of change of flux
   - ☐ the wire's resistance
   - ☐ the coil's colour
3. The induced current in a coil always flows so that its magnetic field:
   - ☐ helps the change in flux
   - ☐ opposes the change in flux
   - ☐ points north always
   - ☐ does nothing
4. The self-induced EMF of an inductor is:
   - ☐ $-L\,dI/dt$
   - ☐ $-L\,I$
   - ☐ $-I/L$
   - ☐ $-L/t$
5. The time constant of an LR circuit is:
   - ☐ RC
   - ☐ L/R
   - ☐ R/L
   - ☐ LR
6. A 2 T field passes straight through ($\theta = 0$) a 0.3 m^2 loop. Find the flux (in Wb).

   ______________ Wb

7. A coil in a strong but steady magnetic field has zero induced EMF.

☐ True ☐ False

8. A 4 H inductor's current changes at 2 A/s. Find the induced EMF magnitude (in V).

________________ V

9. An LR circuit has $L = 10$ H and $R = 5\ \Omega$. Find τ (in s).

________________ s

10. An LC circuit has $L = 4$ H and $C = 1$ F. Find ω (in rad/s).

________________ rad/s

AP Physics C: Electricity and Magnetism

1. $BA\cos\theta$

   $\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta$.

2. **the rate of change of flux**

   $\varepsilon = -N\,d\Phi_B/dt$ — the rate of change of flux.

3. **opposes the change in flux**

   Lenz's law: the induced field always opposes the change.

4. $-L\,dI/dt$

   $\varepsilon = -L\,dI/dt$ — proportional to the rate of current change.

5. L/R

   For an LR circuit $\tau = L/R$.

6. **0.6 Wb**

   $\Phi_B = BA\cos 0 = 2\times 0.3\times 1 = 0.6$ Wb.

7. **True**

   No change in flux means no induced EMF, however strong the field.

8. **8 V**

   $|\varepsilon| = L\,dI/dt = 4\times 2 = 8$ V.

9. **2 s**

   $\tau = L/R = 10/5 = 2$ s.

10. **0.5 rad/s**

    $\omega = 1/\sqrt{LC} = 1/\sqrt{4} = 0.5$ rad/s.

13.1 Magnetic Flux

Name: ______________ Class: __________ Date: ____________ **Total: 10 marks**

KEY WORDS magnetic flux 磁通量 • emf 电动势

Objective

Build the skills to answer exam questions on **magnetic flux**.

You must be able to:

- define **magnetic flux 磁通量** through a surface as $\Phi_B = \int \vec{B} \cdot d\vec{A}$
- relate flux to the perpendicular component of $\vec{B}$ and the area
- calculate the flux through a flat loop at different orientations
- see how changing the field, area, or angle changes the flux

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **Magnetic flux**

$\Phi_B = \int \vec{B} \cdot d\vec{A}$. For a uniform field through a flat loop this is $\Phi_B = BA\cos\theta$, where θ is the angle between $\vec{B}$ and the loop's normal.

■ **When flux is largest**

Flux is greatest when the loop **faces** the field ($\theta = 0$) and zero when the field lies **in** the plane of the loop.

■ **Central to induction**

A **changing** flux is what induces an emf — so flux links magnetic fields to circuits.

2 Practice

2.1 Find the flux through a 0.10 m^2 loop placed face-on in a 0.30 T field. [2]

2.2 State three ways to change the flux through a loop. [2]

2.3 State what a changing magnetic flux produces. [1]

3 Exam-style questions

3.1 Magnetic flux is [1]

- **A** $\int \vec{B} \cdot d\vec{A}$
- **B** $\int \vec{E} \cdot d\vec{A}$
- **C** BIL
- **D** qvB

3.2 The flux through a loop is greatest when its plane is [1]

- **A** parallel to the field
- **B** perpendicular to the field
- **C** at 45° to the field
- **D** irrelevant to the field

3.3 A 0.050 m^2 coil sits in a 0.40 T field.

(a) Find the flux when the coil faces the field. [2]

(b) Find the flux when the field lies in the plane of the coil. [1]

Solutions

2.1 $\Phi_B = BA = 0.30 \times 0.10 = 0.030$ Wb.

2.2 change the field strength B, the area A, or the angle θ.

2.3 an induced emf.

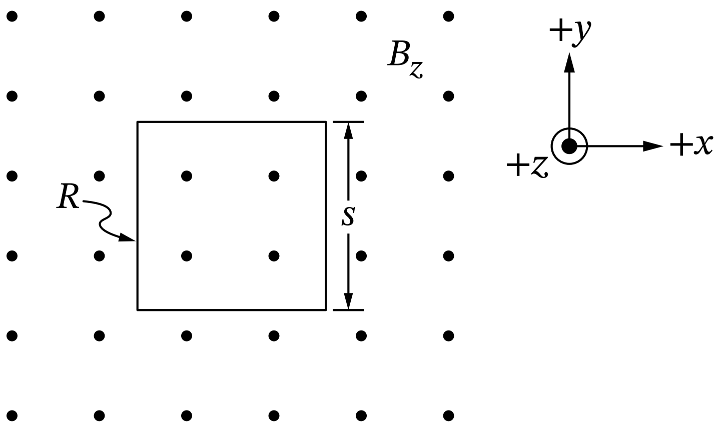
3.1 **A**.

3.2 **B** — the plane perpendicular to the field means the loop faces it.

3.3 (a) $\Phi_B = BA = 0.40 \times 0.050 = 0.020$ Wb. (b) zero ($\theta = 90°$).

Question 4

4. In Scenario 1, a square conducting loop with side length s has total resistance R. The loop is held fixed in the xy-plane in an external, uniform, time-varying magnetic field that is initially directed in the $+z$-direction, as shown. The z-component of the magnetic field changes as a function of time t as described by $B_z = B_0 \cos\left(2\pi \frac{t}{t_1}\right)$, where B_0 and t_1 are positive constants.



Part A

Indicate whether the induced current I in the loop is clockwise, counterclockwise, or zero during the time interval $0 < t < \frac{t_1}{4}$ by writing one of the following.

- I is clockwise.
- I is counterclockwise.
- I is zero.

Justify your answer without only manipulating equations.

Part B

Derive an expression for I as a function of t in terms of s, R, B_0, t_1, t, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

In Scenario 2, a new square conducting loop with side length $2s$ and total resistance $2R$ is placed in the same external, uniform, time-varying magnetic field as in Scenario 1.

Indicate whether the maximum induced current I_2 in Scenario 2 is greater than, less than, or equal to the maximum induced current I_1 in Scenario 1 by writing one of the following.

- $I_2 > I_1$
- $I_2 < I_1$
- $I_2 = I_1$

Briefly **justify** your answer by referencing the expression you derived in part B.

STOP

END OF EXAM

Begin your response to **QUESTION 3** on this page.

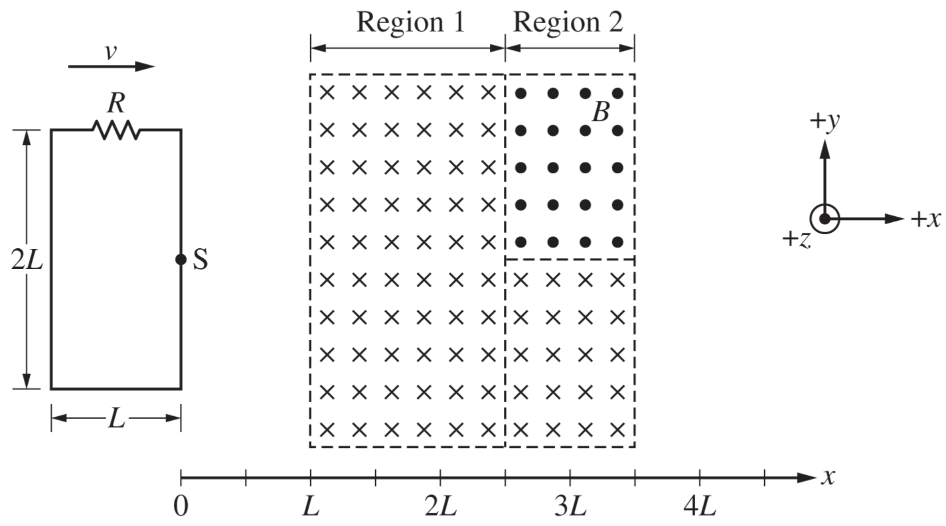


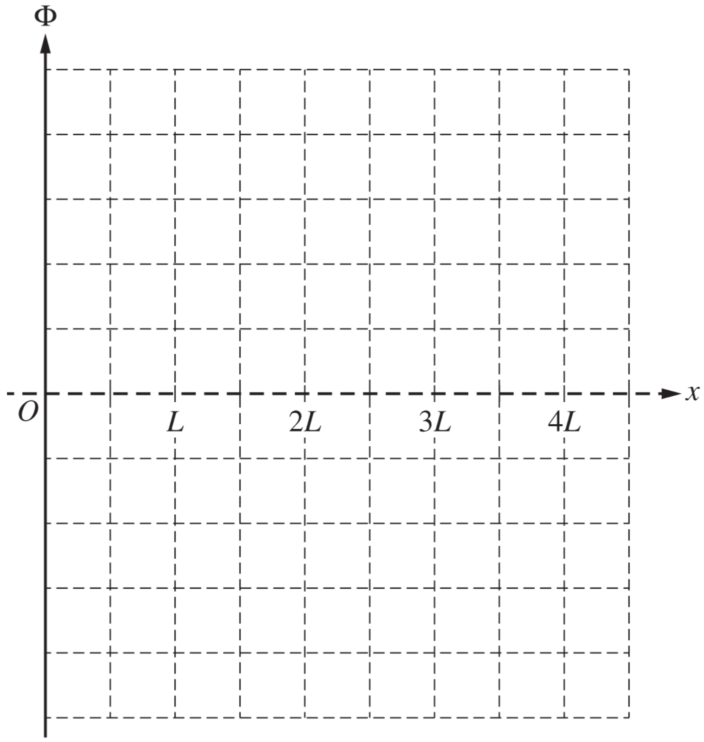
Figure 1

3. A wire is connected to a resistor of resistance R to form a rigid rectangular loop of width L and height $2L$. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$-direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $-z$-direction. Region 1 has boundaries $x = L$ and $x = 2.5L$. The loop later enters Region 2 with two external, uniform magnetic fields, each of magnitude B, that are parallel but are directed in opposite z-directions. Region 2 has boundaries $x = 2.5L$ and $x = 3.5L$. Point S is the midpoint of the leading edge of the loop and is aligned with the horizontal boundary in Region 2 that separates the two magnetic fields.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(a) On the following axes, **sketch** a graph of the magnetic flux Φ through the rectangular loop as a function of the position x of Point S from $x = 0$ to $x = 4.5L$. The $+z$-direction indicated in Figure 1 corresponds to $+\Phi$.



(b) Consider the instant when Point S reaches $x = 1.5L$.

i. **Indicate** whether the current I_R that is induced in the rectangular loop when Point S reaches $x = 1.5L$ is clockwise, counterclockwise, or zero.

_____ Clockwise _____ Counterclockwise _____ Zero

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

ii. **Derive** an expression for I_R when Point S reaches $x = 1.5L$. If $I_R = 0$, indicate how the derived expression shows that $I_R = 0$. Express your answer in terms of R, L, v, B, and physical constants, as appropriate.

iii. **Derive** an expression for the power P dissipated by the resistor when Point S reaches $x = 1.5L$. Express your answer in terms of R, L, v, B, and physical constants, as appropriate.

Continue your response to **QUESTION 3** on this page.

The total energy dissipated by the resistor in the rectangular loop as Point S moves from $x = 0$ to $x = 4.5L$ is E_{original}.

The vertical boundary between regions 1 and 2 is now shifted to $x = 1.5L$. After the boundary is shifted, the rectangular loop again moves with speed v in the $+x$-direction, as shown in Figure 2. The total energy dissipated by the resistor as Point S moves from $x = 0$ to $x = 4.5L$ is E_{new}.

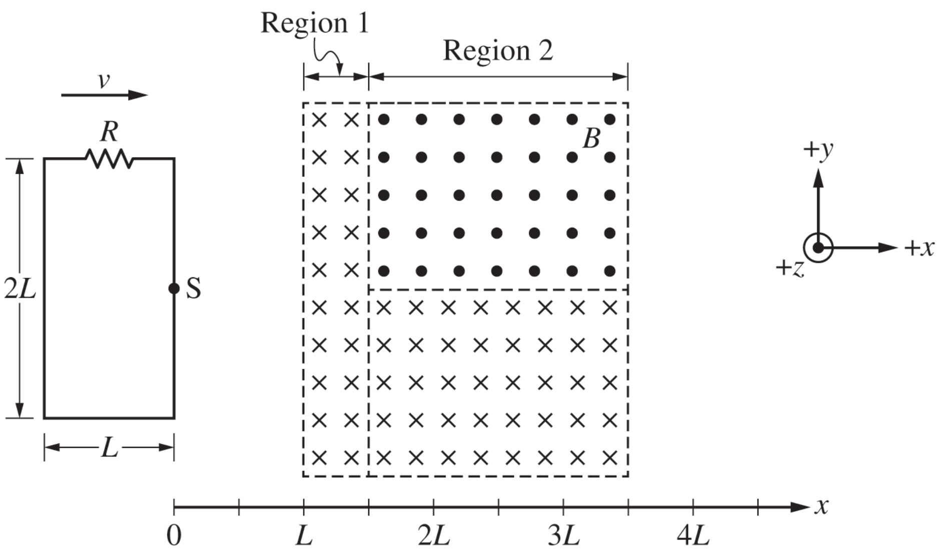


Figure 2

(c) **Indicate** whether E_{new} is greater than, less than, or equal to E_{original}.

______ $E_{\text{new}} > E_{\text{original}}$ ______ $E_{\text{new}} < E_{\text{original}}$ ______ $E_{\text{new}} = E_{\text{original}}$

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

The original magnetic fields are modified so that the region $L < x < 3.5L$ contains an external uniform magnetic field of magnitude B that is directed in the $-z$-direction.

A new wire is connected to a resistor of resistance R to form a rigid triangular loop with base length L and height $2L$. An external force is exerted on the loop so that the loop always moves with speed v in the $+x$-direction, as shown in Figure 3. Point S represents the lower-leading corner of the loop.

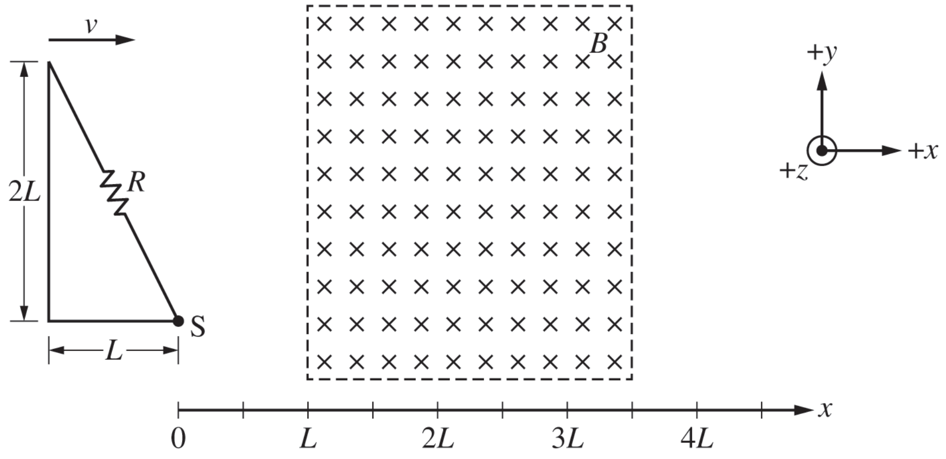
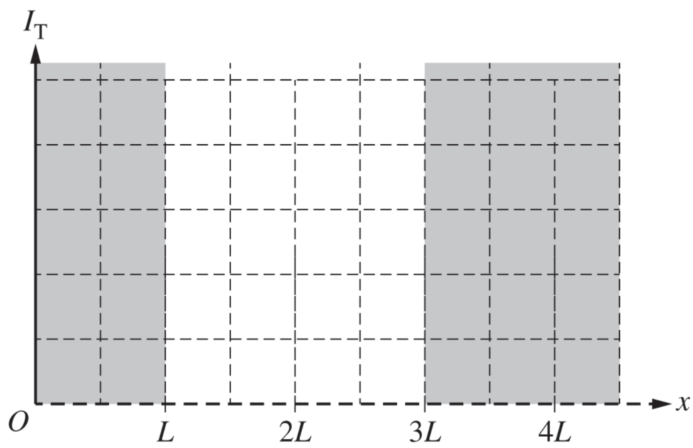


Figure 3

(d) On the following axes, **sketch** a graph of the induced current I_T in the triangular loop as Point S moves from $x = L$ to $x = 3L$.



GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

Begin your response to **QUESTION 3** on this page.

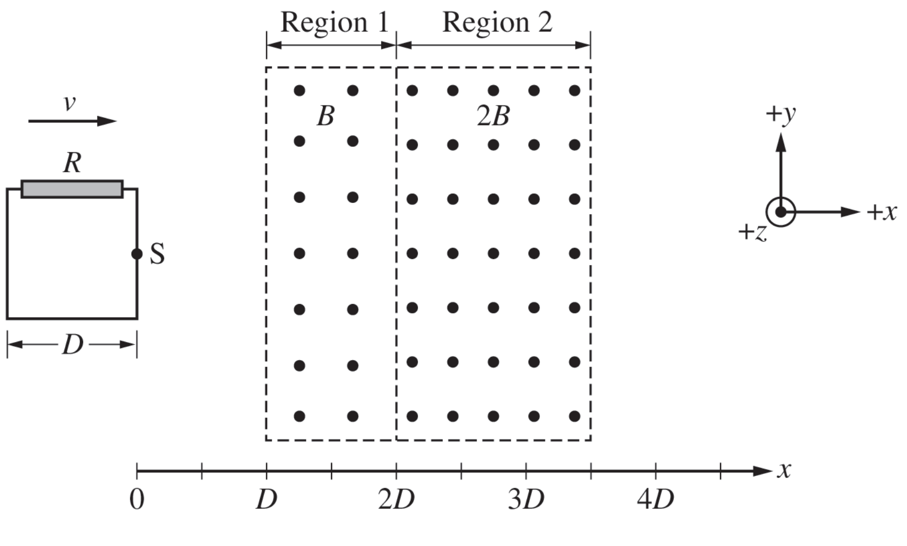


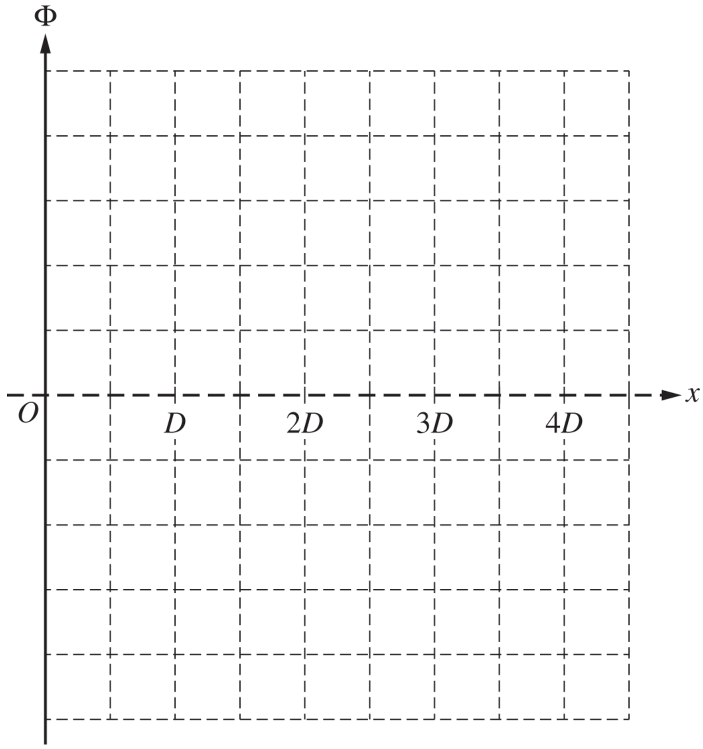
Figure 1

3. A wire is connected to a resistor of resistance R to form a rigid square loop of side length D. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$ direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $+z$-direction. Region 1 has boundaries $x = D$ and $x = 2D$. The loop later enters Region 2 of external uniform magnetic field of magnitude $2B$ that is directed in the $+z$-direction. Region 2 has boundaries $x = 2D$ and $x = 3.5D$. Point S is the midpoint of the leading edge of the loop.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(a) On the following axes, **sketch** a graph of the magnetic flux Φ through the square loop as a function of the position x of Point S from $x = 0$ to $x = 4.5D$. The $+z$-direction indicated in Figure 1 corresponds to $+\Phi$.



(b) Consider the instant when Point S reaches $x = 1.5D$.

i. **Indicate** whether the current I_S that is induced in the square loop when Point S reaches $x = 1.5D$ is clockwise, counterclockwise, or zero.

_____ Clockwise   _____ Counterclockwise   _____ Zero

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

ii. **Derive** an expression for I_S when Point S reaches $x = 1.5D$. If $I_S = 0$, indicate how the derived expression shows that $I_S = 0$. Express your answer in terms of R, D, v, B, and physical constants, as appropriate.

iii. **Derive** an expression for the power P dissipated by the resistor when Point S reaches $x = 1.5D$. Express your answer in terms of R, D, v, B, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

The total energy dissipated by the resistor in the square loop as Point S moves from $x = 0$ to $x = 4.5D$ is $E_{original}$.

The vertical boundary between regions 1 and 2 is now shifted to $x = 2.5D$. After the boundary is shifted, the square loop again moves with speed v in the $+x$-direction, as shown in Figure 2. The total energy dissipated by the resistor as Point S moves from $x = 0$ to $x = 4.5D$ is E_{new}.

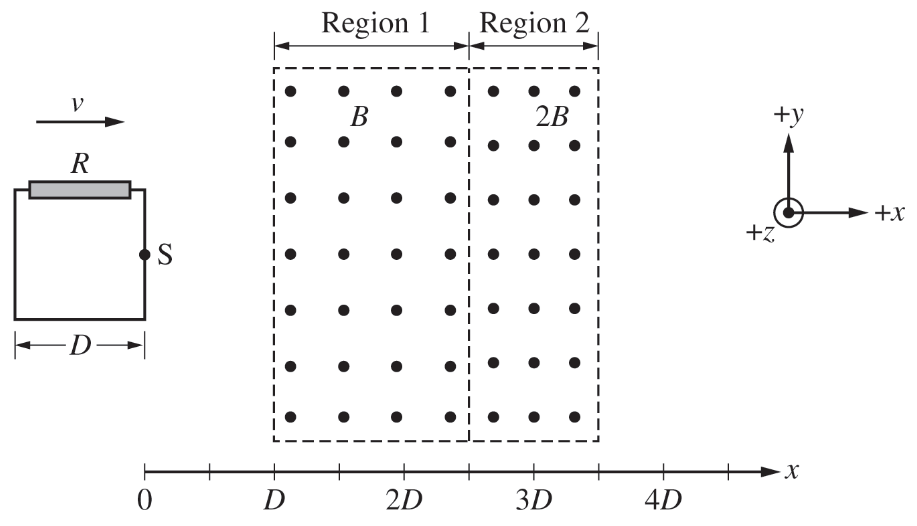


Figure 2

(c) **Indicate** whether E_{new} is greater than, less than, or equal to $E_{original}$.

_____ $E_{new} > E_{original}$ _____ $E_{new} < E_{original}$ _____ $E_{new} = E_{original}$

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

The original magnetic fields are modified so that the region $D < x < 3.5D$ contains an external uniform magnetic field of magnitude B that is directed in the $+z$-direction.

A new wire is connected to a resistor of resistance R to form a rigid triangular loop with base length D and height D. An external force is exerted on the loop so that the loop always moves with speed v in the $+x$-direction, as shown in Figure 3. Point S represents the upper-leading corner of the loop.

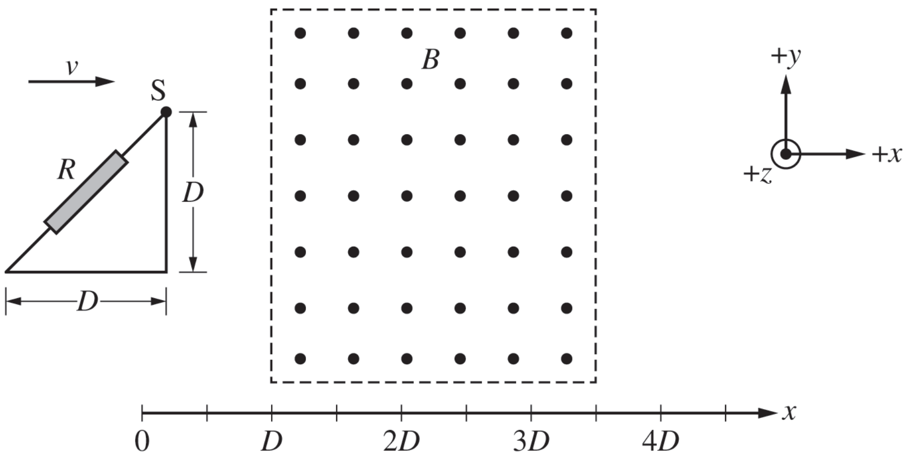
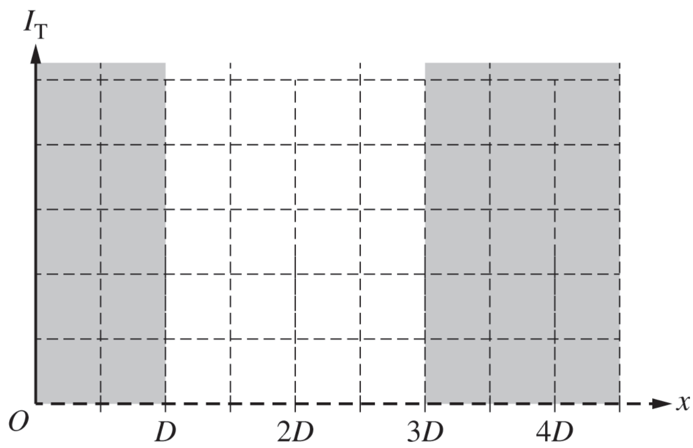


Figure 3

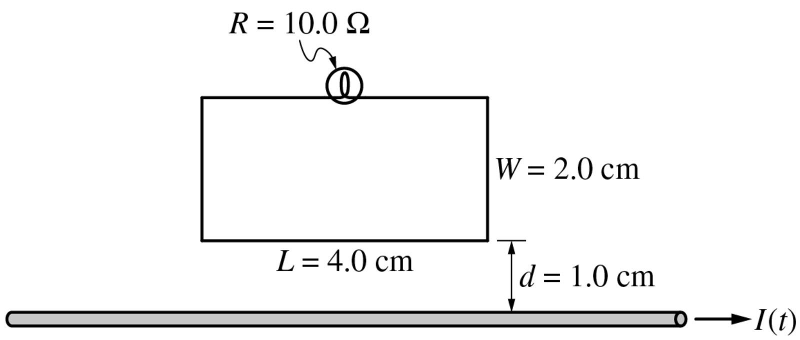
(d) On the following axes, **sketch** a graph of the induced current I_T in the loop as Point S moves from $x = D$ to $x = 3D$.



GO ON TO THE NEXT PAGE.

STOP

END OF EXAM



. A lightbulb of resistance $R = 10.0\ \Omega$ is connected to a rectangular loop of wire of negligible resistance near a very long current-carrying wire. The rectangular loop has a length $L = 4.0$ cm and a width $W = 2.0$ cm and is positioned so one of the longer sides of the loop is a distance $d = 1.0$ cm above and parallel to the long wire, as shown. The current in the long wire is initially flowing to the right and is given by $I(t) = C - Dt$, where $C = 10.0$ A and $D = 2.0$ A/s. At time $t = 5.0$ s, the current in the long wire is instantaneously zero as the current changes direction.

(a) What is the direction, if any, of the magnetic field produced by the induced current in the rectangular loop as the current in the long wire changes direction?

___ Into the page ___ Out of the page ___ No direction, because the field is zero

Justify your answer.

(b) Calculate the magnetic flux through the loop due to only the long wire at time $t = 3.0$ s.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(c) Calculate the current through the lightbulb at time $t = 3.0$ s.

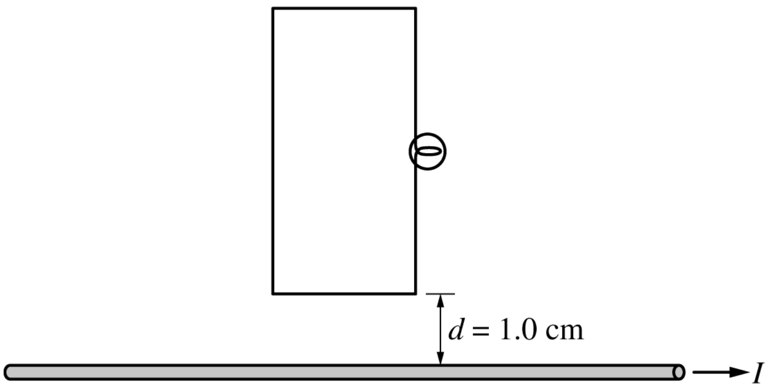
(d) A group of students attempts to experimentally verify whether the current through the lightbulb is consistent with the current calculation from part (c). The current in the rectangular loop is measured to be greater than the current calculated in part (c). Which of the following could explain this discrepancy? Select one answer.

___ The students did not account for Earth's magnetic field.
___ The rectangular loop is tilted and is not in the same plane as the wire.
___ The resistance of the lightbulb is greater than the recorded value.
___ The long side of the rectangular loop is shorter than the recorded value.
___ The current in the long wire changes at a faster rate than expected.

Briefly justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.



(e) Later, the same rectangular loop with lightbulb is rotated such that a short side of the loop is 1.0 cm above and parallel to the long current-carrying wire, as shown. The current in the wire is again initially flowing from left to right and given by $I(t) = C - Dt$, where $C = 10.0$ A and $D = 2.0$ A/s. The current through the lightbulb in the loop's new orientation at time $t = 3.0$ s is I_2. Which of the following correctly relates the current I_2 to I_1, the current through the lightbulb in part (c)?

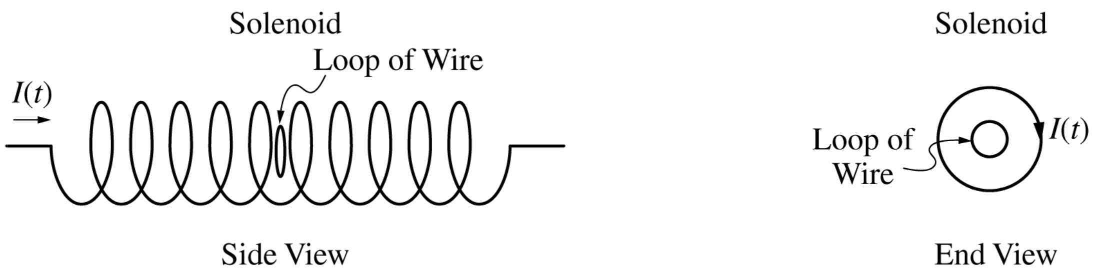
___ $I_2 < I_1$ ___ $I_2 = I_1$ ___ $I_2 > I_1$

Justify your answer.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM



<u>Note:</u> Figures not drawn to scale.

. A single loop of wire with resistance $3.0\,\Omega$ and radius 0.10 m is placed inside a solenoid, with the normal to the loop parallel to the axis of the solenoid. The solenoid has 500 turns, is 0.25 m long, and is connected to a power supply that is not shown. At time $t = 0$, the power supply is turned on, and the current I in the solenoid as a function of t is given by the equation $I(t) = \beta t$, where $\beta = 5.0\text{ A/s}$. The direction of the current in the solenoid is clockwise, as shown in the end view.

(a) At time $t = 2.0\text{ s}$, is the induced current in the loop, as seen from the end view shown, clockwise, counterclockwise, or zero?

___ Clockwise   ___ Counterclockwise   ___ Zero

Justify your answer.

(b) Calculate the current in the <u>loop of wire</u> at time $t = 2.0\text{ s}$.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(c) Calculate the total energy dissipated by the loop of wire from time $t = 0$ to time $t = 2.0\text{ s}$.

(d) A group of students attempts to verify experimentally the calculation of the current from part (b). The current in the inner circular loop at time $t = 2.0\text{ s}$ is measured to be less than the current calculated in part (b). Which of the following could explain this discrepancy? Select one answer.

___ The experiment did not account for Earth's magnetic field.
___ The plane of the loop is not perpendicular to the axis of the solenoid.
___ The center of the loop is not on the axis of the solenoid.
___ The resistance of the loop is less than the given value.
___ The radius of the loop is actually larger than 0.10 m.

Justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(e) The power supply is now turned off. The original loop of wire is then replaced with a second loop made from wire that has the same thickness and is made from the same material as the original loop of wire. The second loop has radius 0.20 m, is placed in the same orientation as the original loop, and fits completely inside the solenoid. The power supply is turned on, and the current I in the solenoid as a function of t is again given by the equation $I(t) = \beta t$, where $\beta = 5.0$ A/s. Which of the following expressions correctly indicates the ratio $\frac{I_2}{I_1}$ where I_1 represents the current induced in the original loop of wire in part (b) and I_2 represents the current induced in the second loop of wire?

___ $\frac{I_2}{I_1} = 1$ ___ $1 < \frac{I_2}{I_1} < 2$ ___ $\frac{I_2}{I_1} = 2$ ___ $\frac{I_2}{I_1} > 2$

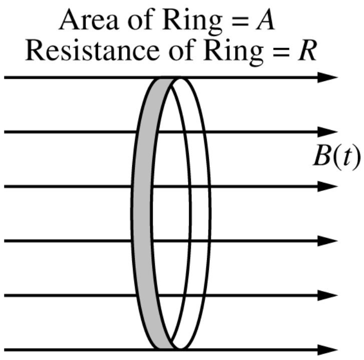
Justify your answer.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

Begin your response to **QUESTION 3** on this page.



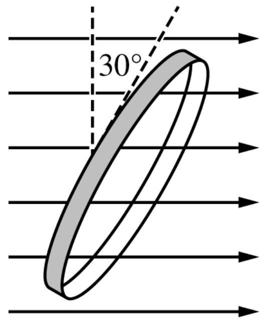
3. A thin, conducting ring of area A and resistance R is aligned in a uniform magnetic field directed to the right and perpendicular to the plane of the ring, as shown. At time $t = 0$, the magnitude of the magnetic field is B_0. At $t = 1$ s, the magnitude of the magnetic field begins to decrease according to the equation $B(t) = \dfrac{\beta}{t}$, where β has units of T·s.

(a) Derive an equation for the magnitude of the induced current I in the ring as a function of t for $t > 1$ s. Express your answer in terms of β, A, R, t, and physical constants, as appropriate.

Assume $A = 0.50\,\text{m}^2$, $R = 2.0\,\Omega$, and $\beta = 0.50\,\text{T}\cdot\text{s}$.

(b) Calculate the electrical energy dissipated in the ring from $t = 1$ s to $t = 2$ s.

GO ON TO THE NEXT PAGE.



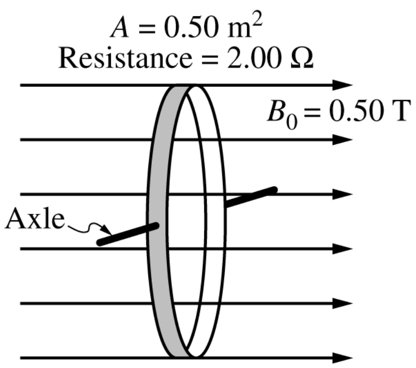
The ring is then rotated so that the plane of the ring is aligned at a 30° angle to the magnetic field, as shown.

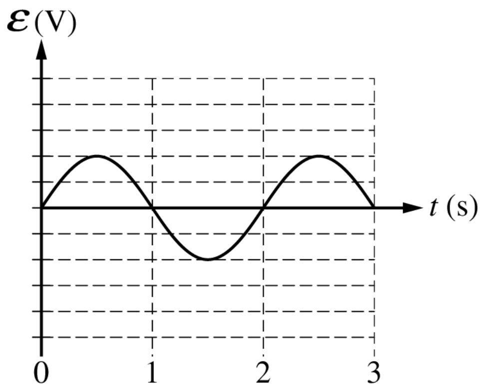
The magnitude of the magnetic field is reset to a magnitude of B_0 at a new time $t = 0$ and again begins to decrease at $t = 1\text{ s}$ according to the equation $B(t) = \dfrac{\beta}{t}$, where β has units of T·s.

(c) Will the amount of energy dissipated in the ring from $t = 1\text{ s}$ to $t = 2\text{ s}$ be greater than, less than, or equal to the energy dissipated in part (b) ?

_____ Greater than   _____ Less than   _____ Equal to

Justify your answer.





The ring is now mounted on an axle that is perpendicular to the magnetic field. The magnitude of the magnetic field is now held at a constant $B_0 = 0.50\text{ T}$, as shown. The ring rotates about the axle, and the emf ε induced in the ring as a function of time t is shown on the graph.

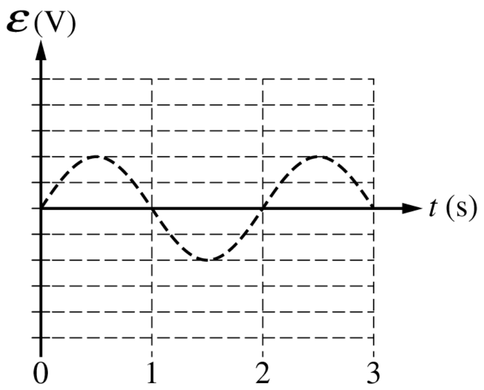
(d) Calculate the angular speed ω of the rotating ring in rad/s.

GO ON TO THE NEXT PAGE.

(e) Calculate the magnitude of the maximum emf ε_{MAX} induced in the ring.

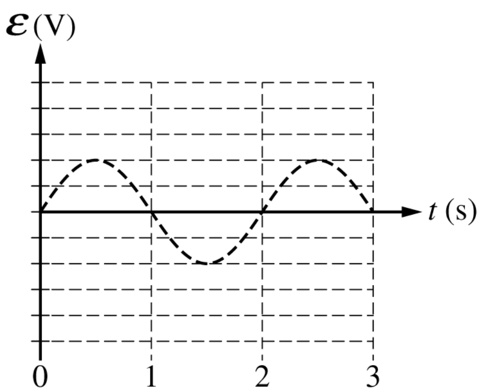
The ring now begins to rotate at an angular speed 2ω.

(f) On the graph below, draw a curve to indicate the new induced emf ε in the ring. The dashed curve shows the emf induced under the original conditions.



Justify your sketch, specifically identifying and addressing any similarities or differences between the sketch and the original graph.

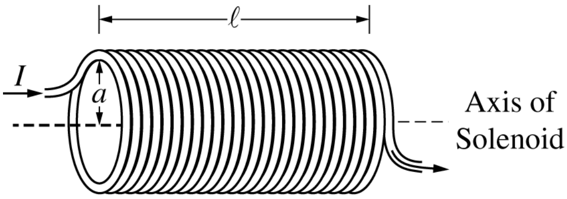
PRACTICE GRAPH - Use the graph below to practice your sketch for part (f). Any work shown on the graph below will NOT be graded.



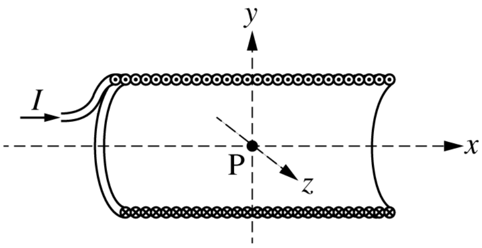
GO ON TO THE NEXT PAGE.

STOP

END OF EXAM



Solenoid



Cutaway View

Note: Figures not drawn to scale.

3. A solenoid is used to generate a magnetic field. The solenoid has an inner radius a, length ℓ, and N total turns of wire. A power supply, not shown, is connected to the solenoid and generates current I, as shown in the figure on the left above. The x-axis runs along the axis of the solenoid. Point P is in the middle of the solenoid at the origin of the xyz-coordinate system, as shown in the cutaway view on the right above. Assume $\ell >> a$.

(a) Select the correct direction of the magnetic field at point P.

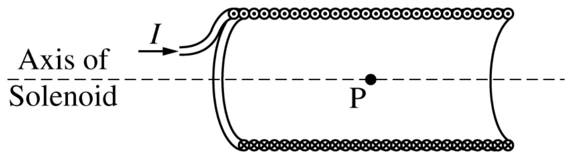
____ $+x$-direction ____ $+y$-direction ____ $+z$-direction

____ $-x$-direction ____ $-y$-direction ____ $-z$-direction

Justify your selection.

(b)

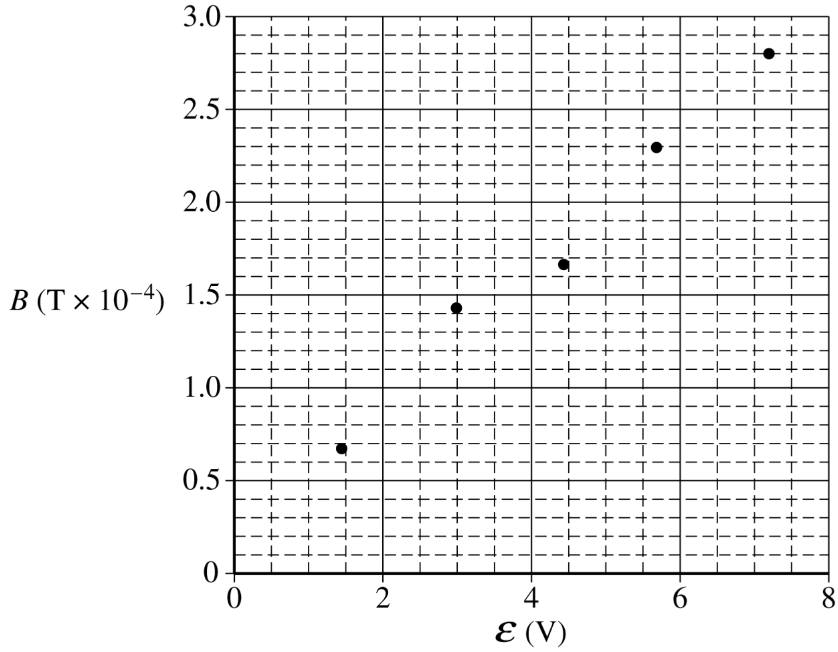
i. On the cutaway view below, clearly draw an Amperian loop that can be used to determine the magnetic field at point P at the center of the solenoid.



Cutaway View

ii. Use Ampere's law to derive an expression for the magnetic field strength at point P. Express your answer in terms of I, ℓ, N, a, and physical constants, as appropriate.

Some physics students conduct an experiment to determine the resistance R_S of a solenoid with radius $a = 0.015$ m, total turns $N = 100$, and total length $\ell = 0.40$ m. The students connect the solenoid to a variable power supply. A magnetic field sensor is used to measure the magnetic field strength along the central axis at the center of the solenoid. The plot of the magnetic field strength B as a function of the emf $\mathcal{E}$ of the power supply is shown below.



(c)

i. On the graph above, draw a best-fit line for the data.

ii. Use the straight line to determine the resistance R_S of the solenoid used in the experiment.

(d) One of the students notes that the horizontal component of the magnetic field of Earth is 2.5×10^{-5} T.

i. Is there evidence from the graph that the horizontal orientation of the solenoid affects the measured values for B ?

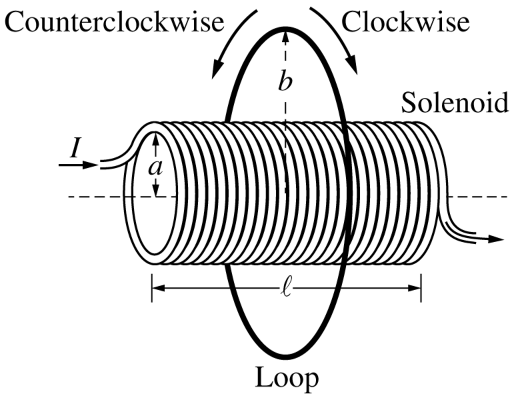
____ Yes ____ No

Justify your answer.

ii. Would the horizontal orientation of the solenoid affect the calculated value for R_S ?

____ Yes ____ No

Justify your answer.



A thin conducting loop of radius b and resistance R_L is placed concentric with the solenoid, as shown above. The current in the solenoid is decreased from I to zero over time Δt.

(e)

i. Is the direction of the induced current in the loop clockwise or counterclockwise during the time period that the current in the solenoid is decreasing?

____ Clockwise ____ Counterclockwise

Justify your answer.

ii. Derive an equation for the average induced current i_{IND} in the loop during the time period that the current in the solenoid is decreasing. Express your answer in terms of I, ℓ, N, a, b, R_L, R_S, Δt, and physical constants, as appropriate.

STOP

END OF EXAM

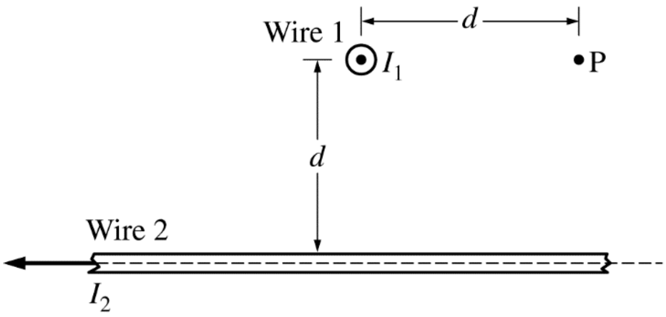


Figure 1. Side view

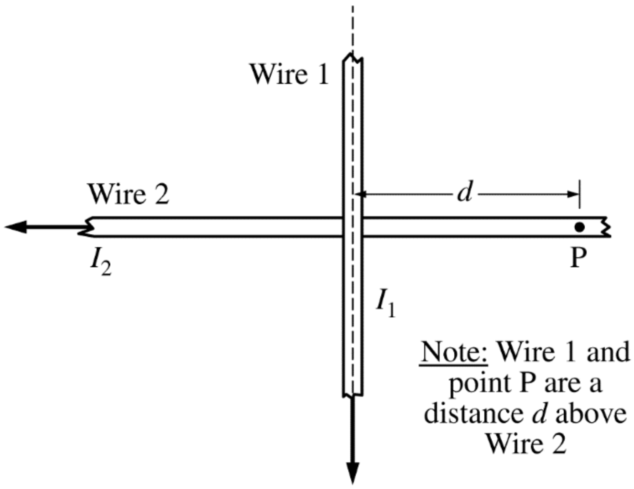


Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I, d, and physical constants, as appropriate.

   (a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

   (b) Derive an expression for the magnitude of the net magnetic field at point P.

   (c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

   (d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2 ? Assume gravitational effects are negligible.

   ____ Wire 1 will not move.

   ____ Wire 1 will move upward as viewed in Figure 1.

   ____ Wire 1 will move downward as viewed in Figure 1.

   ____ Wire 1 will rotate clockwise as viewed in Figure 2.

   ____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

   Justify your answer.

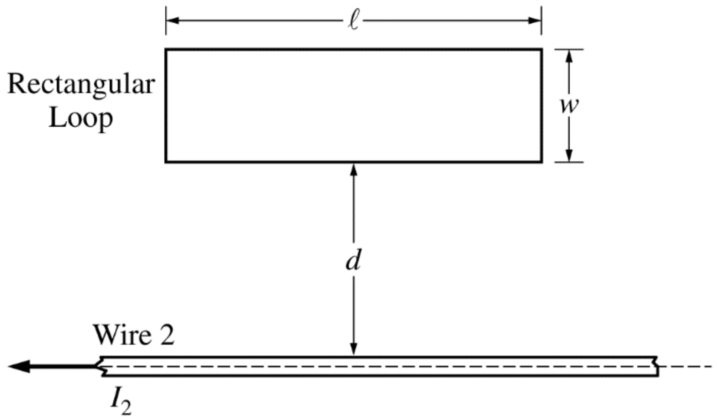


Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1}.

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

(f) Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \frac{\mu_0 I_0 \ell (1-kt)}{\pi} \ln\left(\frac{d+w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

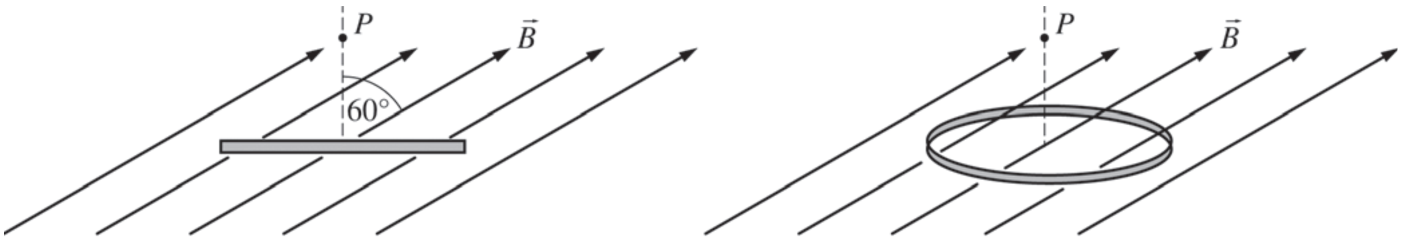
____ Clockwise ____ Counterclockwise

____ Undefined, because there is no current induced in the loop

Justify your answer.

STOP

END OF EXAM



Edge View Perspective View

E&M. 3.

A circular wire loop with radius 0.10 m and resistance 50 Ω is held in place horizontally in a magnetic field $\vec{B}$ directed upward at an angle of 60° with the vertical, as shown in the figure above. The magnetic field in the direction shown is given as a function of time t by $B(t) = a(1 - bt)$, where $a = 4.0$ T and $b = 0.20\ s^{-1}$.

(a) Derive an expression for the magnetic flux through the loop as a function of time t.

(b) Calculate the numerical value of the induced emf in the loop.

(c)

i. Calculate the numerical value of the induced current in the loop.

ii. What is the direction of the induced current in the loop as viewed from point P ?

____ Clockwise ____ Counterclockwise

Justify your answer.

(d) Assuming the loop stays in its current position, calculate the energy dissipated in the loop in 4.0 seconds.

(e) Indicate whether the net magnetic force and net magnetic torque on the loop are zero or nonzero while the loop is in the magnetic field.

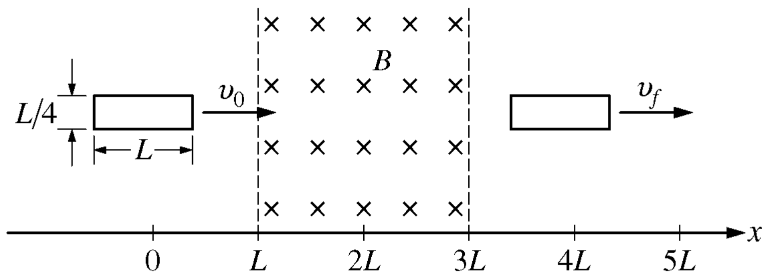
Net magnetic force: ____ Zero ____ Nonzero

Net magnetic torque: ____ Zero ____ Nonzero

Justify both of your answers.

STOP

END OF EXAM



E&M. 2.

The rectangular loop of wire shown on the left in the figure above has mass M, length L, width $L/4$, and resistance R. It is initially moving to the right at constant speed v_0, with no net force acting on it. At time $t = 0$ the loop enters a region of length $2L$ that contains a uniform magnetic field of magnitude B directed into the page. The loop emerges from the field at time t_f with final speed v_f. Express all algebraic answers to the following in terms of M, L, R, B, v_0, and fundamental constants, as appropriate.

(a) Let x represent the position of the right end of the loop. Place a check mark in the appropriate box in each column in the table below to indicate whether the speed of the loop increases, decreases, or stays the same as the loop moves to the right.

	Position of Right End of Loop			
Speed of Loop	$L < x < 2L$	$2L < x < 3L$	$3L < x < 4L$	$4L < x < 5L$
Increases				
Decreases				
Stays the same				

(b) Derive an expression for the magnitude of the current induced in the loop as its right edge enters the field.

(c) What is the direction of the induced current determined in part (b) ?

____ Clockwise ____ Counterclockwise

Justify your answer.

(d) Write, but do not solve, a differential equation for the speed v as a function of time as the loop enters the field.

(e) What is the direction of the acceleration of the loop just before its left edge leaves the field?

____ Left ___ Right ____ Up ____ Down

Justify your answer.

13.2 Electromagnetic Induction

Name: ______________ Class: __________ Date: ____________ **Total: 10 marks**

KEY WORDS induced current 感应电流 • emf 电动势 • lenz's law 楞次定律 • faraday's law 法拉第定律 • motional emf 动生电动势

Objective

Build the skills to answer exam questions on **electromagnetic induction**.

You must be able to:

- state **Faraday's law 法拉第定律**, $\varepsilon = -\frac{d\Phi_B}{dt}$
- apply **Lenz's law 楞次定律** to find the direction of an induced current
- explain Lenz's law as **conservation of energy**
- calculate the **motional emf 动生电动势** of a moving conductor, $\varepsilon = BLv$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Faraday's law

$\varepsilon = -\frac{d\Phi_B}{dt}$; for a coil of N turns, $\varepsilon = -N\frac{d\Phi_B}{dt}$. A faster flux change gives a bigger emf.

■ Lenz's law

The induced current flows so as to **oppose** the change in flux (the minus sign) — a consequence of energy conservation.

■ Motional emf

A rod of length L moving at speed v across a field B generates $\varepsilon = BLv$.

2 Practice

2.1 State Faraday's law. [1]

2.2 A rod of length 0.20 m moves at 3.0 m s^{-1} perpendicular to a 0.50 T field. Find the motional emf. [2]

2.3 State Lenz's law. [1]

3 Exam-style questions

3.1 Faraday's law says the induced emf equals [1]

- **A** $-\frac{d\Phi_B}{dt}$
- **B** BIL
- **C** $\mu_0 nI$
- **D** qvB

3.2 Lenz's law is a consequence of conservation of [1]

- **A** charge
- **B** energy
- **C** momentum
- **D** flux

3.3 A coil of 200 turns has its flux change from 0.020 Wb to 0.005 Wb in 0.10 s.

(a) Find the size of the induced emf. [3]

(b) State what determines the direction of the induced current. [1]

Solutions

2.1 the induced emf equals the negative rate of change of magnetic flux.

2.2 $\varepsilon = BLv = 0.50 \times 0.20 \times 3.0 = 0.30$ V.

2.3 the induced current opposes the change in flux that produces it.

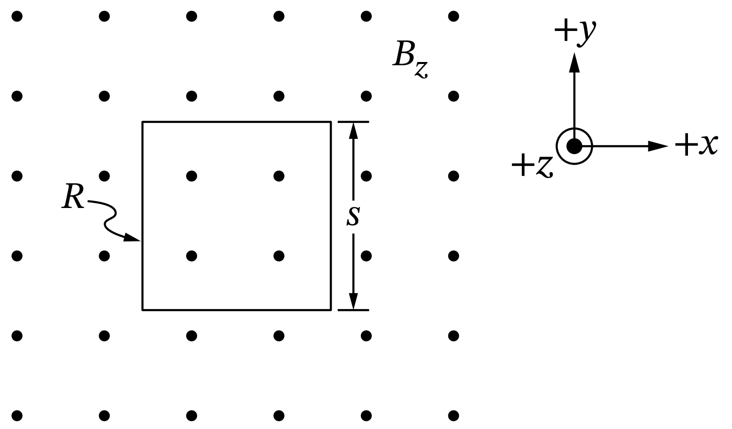
3.1 **A**.

3.2 **B**.

3.3 (a) $|\varepsilon| = N \left| \frac{\Delta\Phi_B}{\Delta t} \right| = 200 \times \frac{0.020 - 0.005}{0.10} = 30$ V. (b) Lenz's law (it opposes the change).

Question 4

4. In Scenario 1, a square conducting loop with side length s has total resistance R. The loop is held fixed in the xy-plane in an external, uniform, time-varying magnetic field that is initially directed in the $+z$-direction, as shown. The z-component of the magnetic field changes as a function of time t as described by $B_z = B_0 \cos\left(2\pi \frac{t}{t_1}\right)$, where B_0 and t_1 are positive constants.



Part A

Indicate whether the induced current I in the loop is clockwise, counterclockwise, or zero during the time interval $0 < t < \frac{t_1}{4}$ by writing one of the following.

- I is clockwise.
- I is counterclockwise.
- I is zero.

Justify your answer without only manipulating equations.

Part B

Derive an expression for I as a function of t in terms of s, R, B_0, t_1, t, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

In Scenario 2, a new square conducting loop with side length $2s$ and total resistance $2R$ is placed in the same external, uniform, time-varying magnetic field as in Scenario 1.

Indicate whether the maximum induced current I_2 in Scenario 2 is greater than, less than, or equal to the maximum induced current I_1 in Scenario 1 by writing one of the following.

- $I_2 > I_1$
- $I_2 < I_1$
- $I_2 = I_1$

Briefly **justify** your answer by referencing the expression you derived in part B.

STOP

END OF EXAM

Question 2: Version J

2. A rotating, circular, conducting loop of area A and resistance R is in an external uniform magnetic field of magnitude B that is directed in the $-z$-direction. At time $t = 0$, the magnetic field is perpendicular to the plane of the loop, as shown in Figure 1. The loop is rotating with constant angular speed ω and period T about the dashed line that is along the diameter of the loop. The value of the magnetic flux through the loop as a function of time t is $\Phi = BA\cos(\omega t)$.

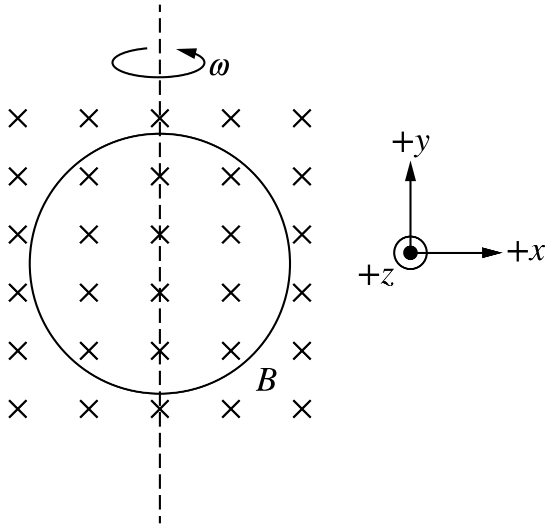


Figure 1

A. The absolute value of the induced emf in the loop is $|\varepsilon|$. The partially completed bar chart in Figure 2 shows a bar that represents $|\varepsilon|$ at $t = \frac{3}{4}T$. In Figure 2, **draw** bars to represent $|\varepsilon|$ at times $t = 0$, $\frac{1}{4}T$, and $\frac{1}{2}T$ relative to $|\varepsilon|$ shown at $\frac{3}{4}T$. If $|\varepsilon| = 0$, **write** a “0” in that column.

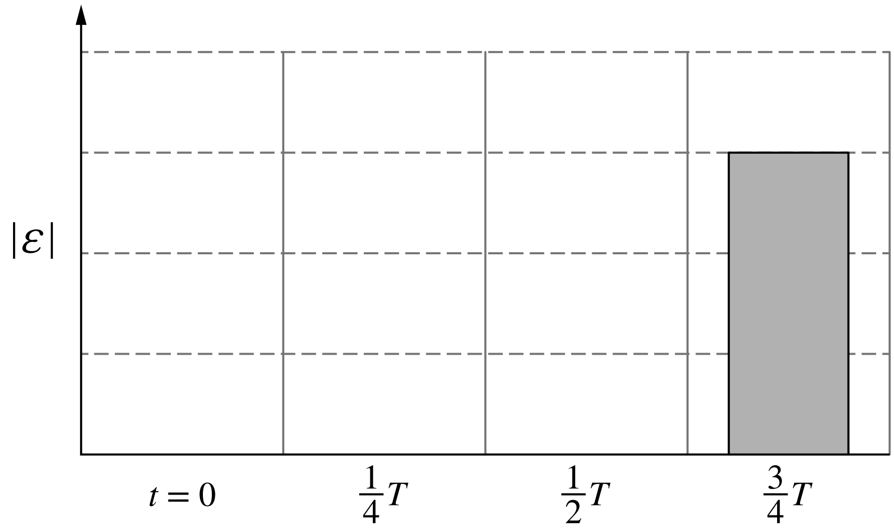


Figure 2

B. **Derive** an expression for the maximum induced current in the loop in terms of A, R, B, ω, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

C. On the axes shown in Figure 3, **sketch** a graph of the instantaneous power P dissipated by the loop as a function of t during the time interval $0 \le t \le T$.

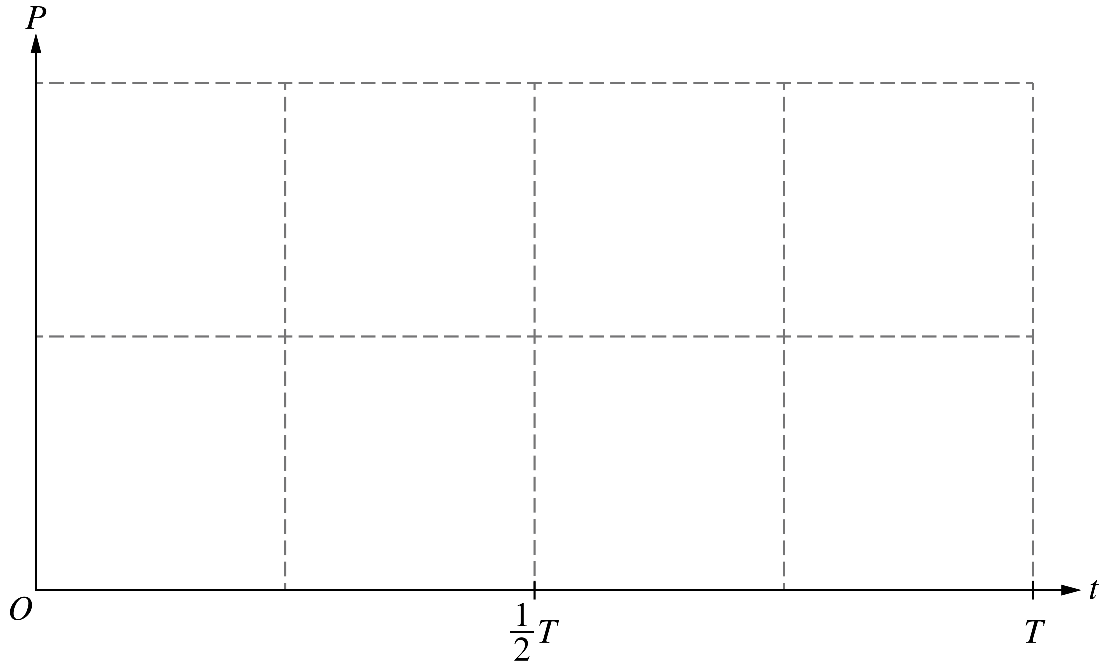


Figure 3

D. **Indicate** whether the sketch you drew in part C is or is not consistent with the bars that you drew in part A. Briefly **justify** your answer by referencing the functional dependence between P and $|\varepsilon|$.

Begin your response to **QUESTION 3** on this page.

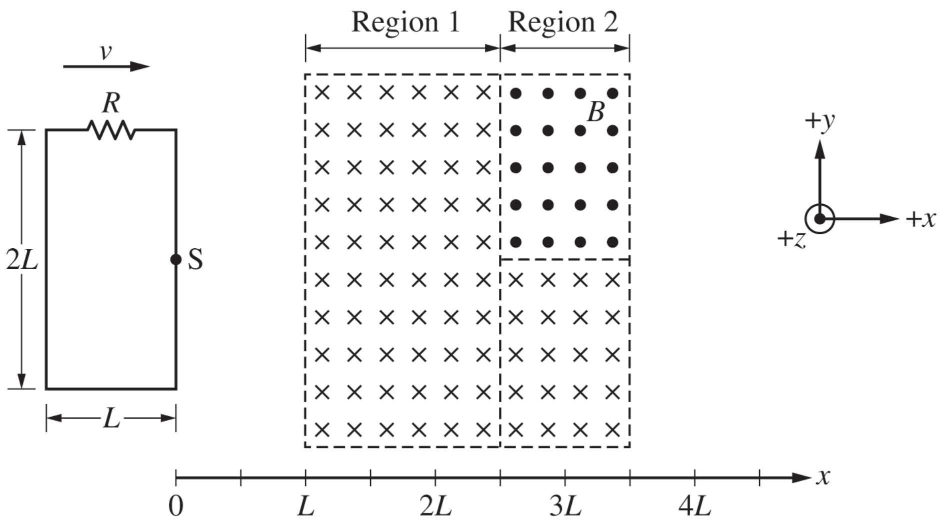


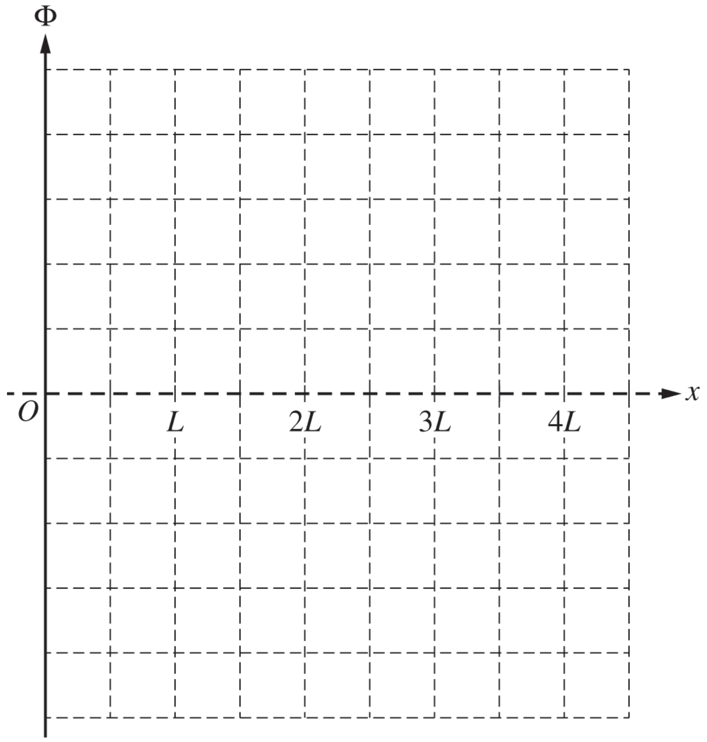
Figure 1

3. A wire is connected to a resistor of resistance R to form a rigid rectangular loop of width L and height $2L$. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$-direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $-z$-direction. Region 1 has boundaries $x = L$ and $x = 2.5L$. The loop later enters Region 2 with two external, uniform magnetic fields, each of magnitude B, that are parallel but are directed in opposite z-directions. Region 2 has boundaries $x = 2.5L$ and $x = 3.5L$. Point S is the midpoint of the leading edge of the loop and is aligned with the horizontal boundary in Region 2 that separates the two magnetic fields.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(a) On the following axes, **sketch** a graph of the magnetic flux Φ through the rectangular loop as a function of the position x of Point S from $x = 0$ to $x = 4.5L$. The $+z$-direction indicated in Figure 1 corresponds to $+\Phi$.



(b) Consider the instant when Point S reaches $x = 1.5L$.

i. **Indicate** whether the current I_R that is induced in the rectangular loop when Point S reaches $x = 1.5L$ is clockwise, counterclockwise, or zero.

_____ Clockwise _____ Counterclockwise _____ Zero

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

ii. **Derive** an expression for I_R when Point S reaches $x = 1.5L$. If $I_R = 0$, indicate how the derived expression shows that $I_R = 0$. Express your answer in terms of R, L, v, B, and physical constants, as appropriate.

iii. **Derive** an expression for the power P dissipated by the resistor when Point S reaches $x = 1.5L$. Express your answer in terms of R, L, v, B, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

The total energy dissipated by the resistor in the rectangular loop as Point S moves from $x = 0$ to $x = 4.5L$ is E_{original}.

The vertical boundary between regions 1 and 2 is now shifted to $x = 1.5L$. After the boundary is shifted, the rectangular loop again moves with speed v in the $+x$-direction, as shown in Figure 2. The total energy dissipated by the resistor as Point S moves from $x = 0$ to $x = 4.5L$ is E_{new}.

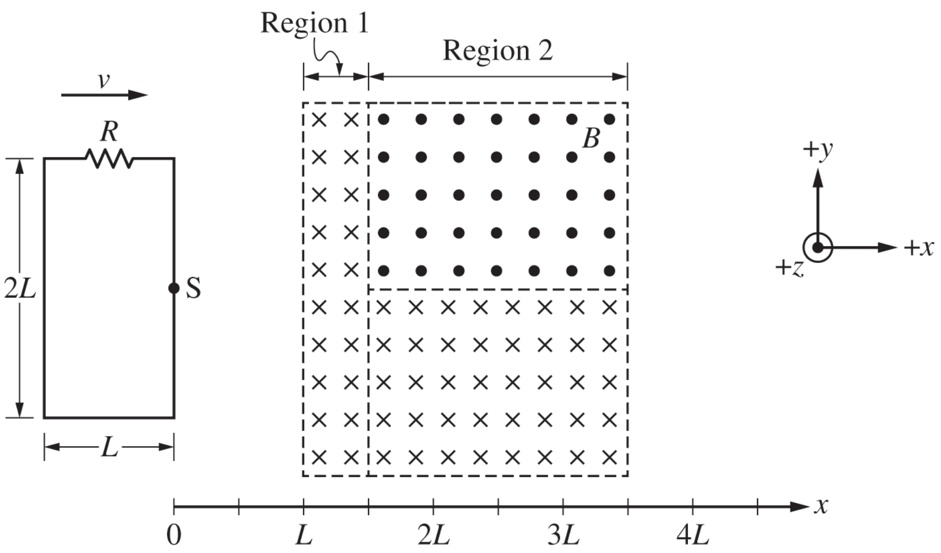


Figure 2

(c) **Indicate** whether E_{new} is greater than, less than, or equal to E_{original}.

______ $E_{\text{new}} > E_{\text{original}}$ ______ $E_{\text{new}} < E_{\text{original}}$ ______ $E_{\text{new}} = E_{\text{original}}$

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

The original magnetic fields are modified so that the region $L < x < 3.5L$ contains an external uniform magnetic field of magnitude B that is directed in the $-z$-direction.

A new wire is connected to a resistor of resistance R to form a rigid triangular loop with base length L and height $2L$. An external force is exerted on the loop so that the loop always moves with speed v in the $+x$-direction, as shown in Figure 3. Point S represents the lower-leading corner of the loop.

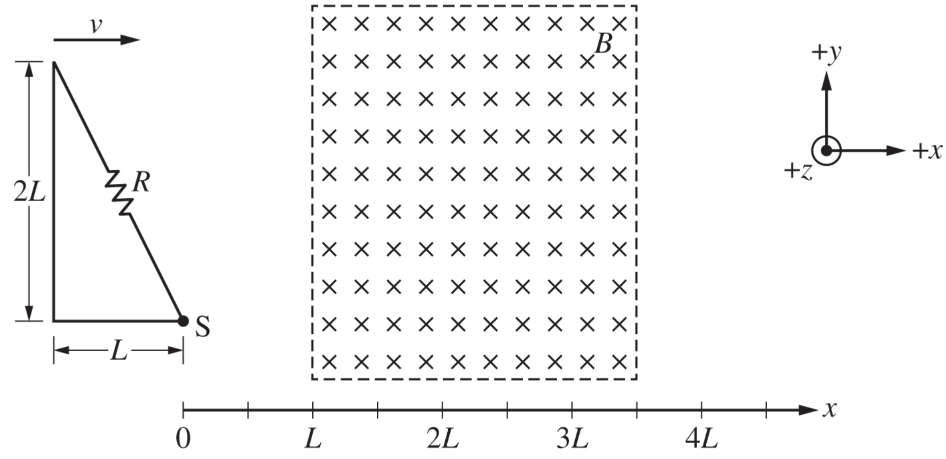
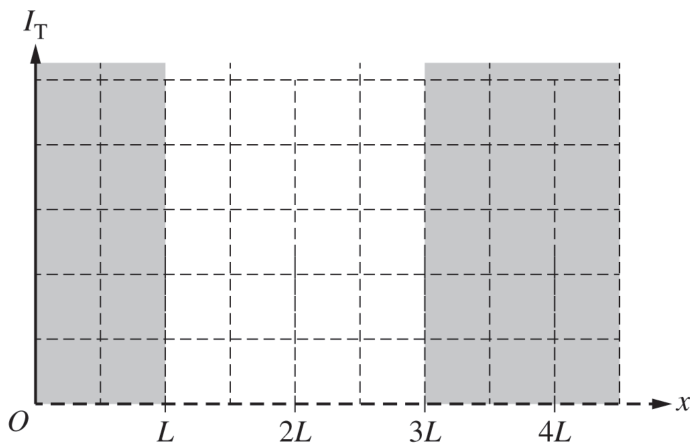


Figure 3

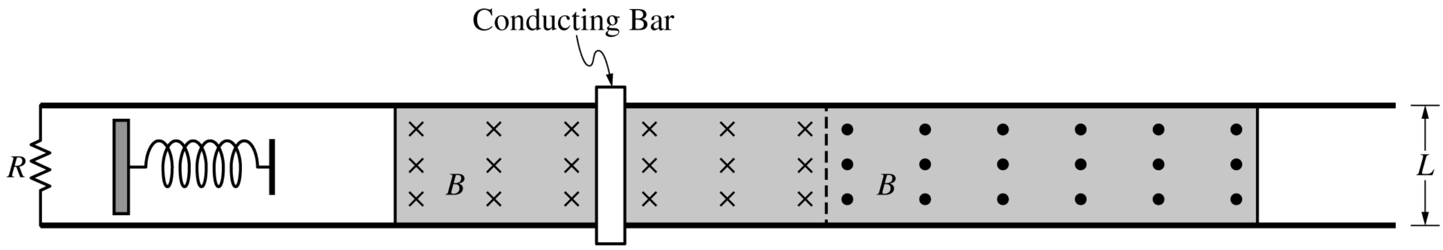
(d) On the following axes, **sketch** a graph of the induced current I_T in the triangular loop as Point S moves from $x = L$ to $x = 3L$.



GO ON TO THE NEXT PAGE.

STOP

END OF EXAM



Top View (at time t_B)

. Two horizontal, parallel, conducting rails are separated by distance $L = 0.40$ m. A resistor of resistance $R = 0.30\ \Omega$ connects the rails. A horizontal ideal spring is located between the rails. The right end of the spring is free to move and the left end is fixed in place. A conducting bar of mass $m = 0.23$ kg is placed on the rails and is in contact with the spring, which is initially compressed. Frictional forces and the resistance of the bar and rails are negligible.

- At time $t = 0$, the bar is released from rest and is pushed to the right by the spring.
- At time t_1, the bar loses contact with the spring and slides to the right.
- At time t_2, the bar enters and travels through a uniform magnetic field of magnitude $B = 0.50$ T that is directed into the page, as shown.
- At time t_3, the bar enters a region where the magnitude of the uniform magnetic field is still $B = 0.50$ T but is directed out of the page.
- At time t_4, the bar enters a region with no magnetic field.

'onsider time t_B such that $t_2 < t_B < t_3$.

(a) On the following diagram of the bar, draw an arrow indicating the direction of the net force F_{net} exerted on the bar at time t_B. If the net force is zero, write $F_{net} = 0$.

GO ON TO THE NEXT PAGE.

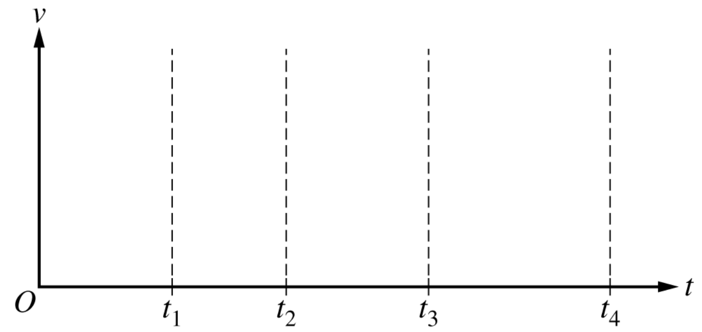
Continue your response to **QUESTION 2** on this page.

(b) At time t_B, the speed of the bar is $v = 2.5$ m/s.

i. Calculate the magnitude of the current in the bar at time t_B.

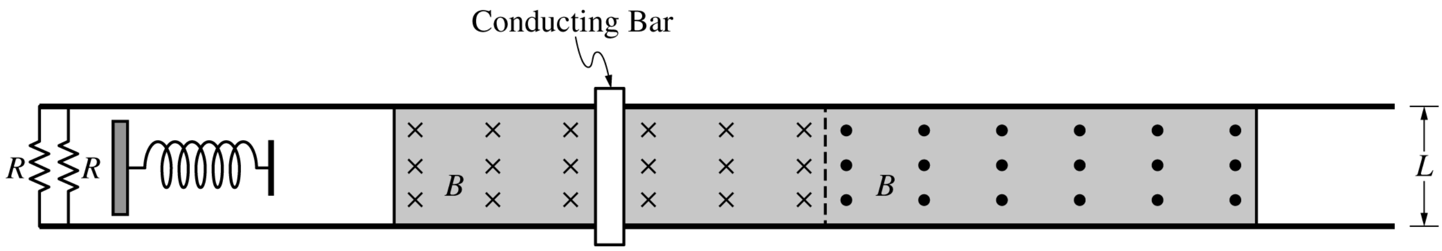
ii. Calculate the magnitude of the net force F_{net} exerted on the bar at time t_B.

(c) On the following axes, sketch a graph of the speed v of the bar as a function of time t between $t = 0$ and t_4.



GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.



Top View (at time t_B)

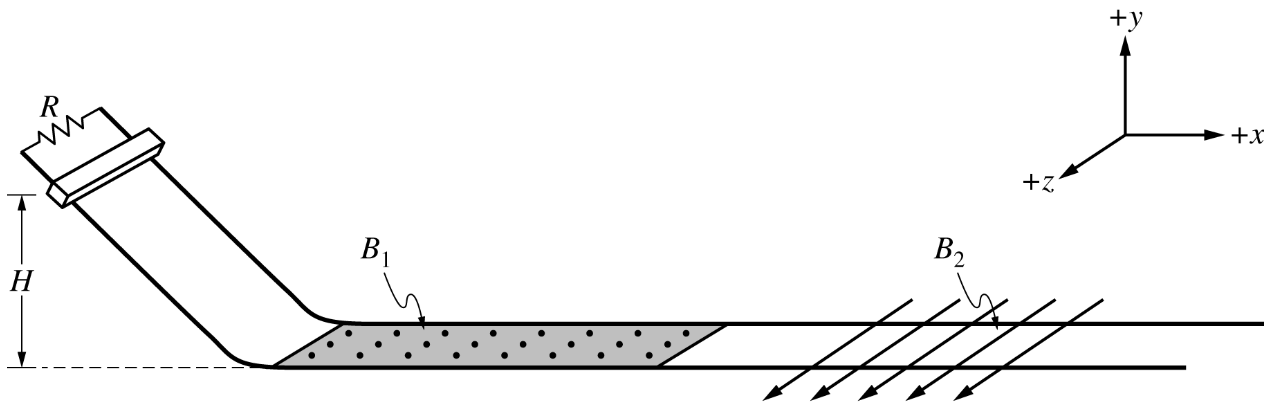
(d) The scenario is repeated but an additional resistor of resistance $R = 0.30\,\Omega$ is connected, as shown.

i. Determine the total resistance R_{total} of the closed circuit for the new scenario.

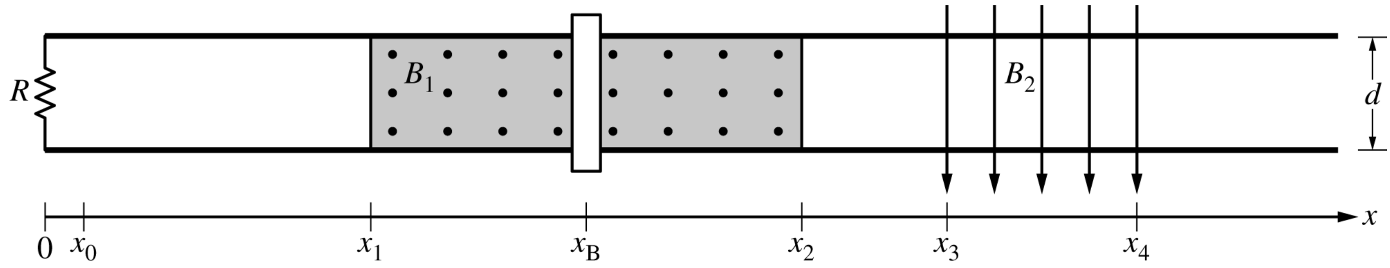
ii. In the original scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is $a_{original}$. In the new scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{new}. Is a_{new} greater than, less than, or equal to $a_{original}$? Justify your answer.

(e) Describe a modification to m, B, or L that will result in a smaller induced potential difference across the original resistor immediately after the bar enters the first uniform magnetic field. Justify your answer.

GO ON TO THE NEXT PAGE.



Perspective View (at time $t = 0$)



Top View (at time t_B)

. Two parallel conducting rails are separated by distance $d = 0.30\,\text{m}$. A resistor of resistance $R = 0.20\,\Omega$ connects the rails. A conducting bar is placed on a sloped section of the rails at height H above the horizontal section of the rails. Frictional forces and the resistances of the bar and rails are negligible.

- At time $t = 0$, the bar is released from rest from position x_0 and slides down the sloped section of the rails, as shown in the Perspective View.
- At time t_1, the bar reaches position x_1 and smoothly transitions to the horizontal section of the rails and enters a uniform magnetic field of magnitude $B_1 = 0.40\,\text{T}$ that is directed in the $+y$-direction.
- At time t_2, the bar reaches position x_2 and enters a region with no magnetic field.
- At time t_3, the bar reaches position x_3 and enters a uniform magnetic field of magnitude $B_2 = 0.60\,\text{T}$ that is directed in the $+z$-direction.
- At time t_4, the bar reaches position x_4 and enters a region with no magnetic field.

'he bar is at position x_B (shown in Top View) at time t_B such that $t_1 < t_B < t_2$.

GO ON TO THE NEXT PAGE.

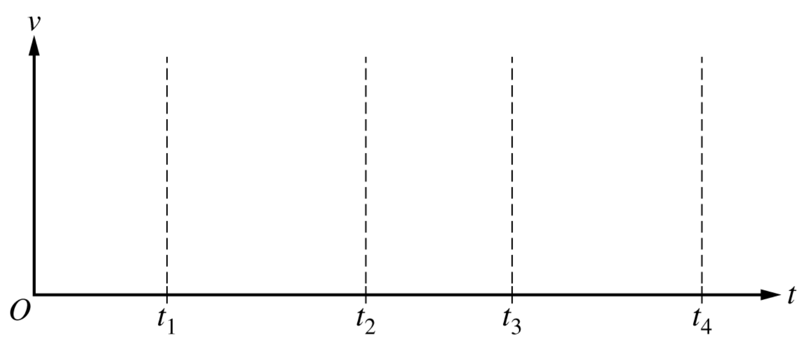
(a) On the following diagram of the bar, as observed from the Top View, draw an arrow indicating the direction of the net force F_{net} exerted on the bar at time t_B. If the net force is zero, write $F_{net} = 0$.

(b) At time t_B, the speed of the bar is $v = 2.5$ m/s.

i. Calculate the magnitude of the current in the bar at time t_B.

ii. Calculate the magnitude of the net force F_{net} exerted on the bar at time t_B.

(c) On the following axes, sketch a graph of the speed v of the bar as a function of time t between $t = 0$ and t_4.



GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

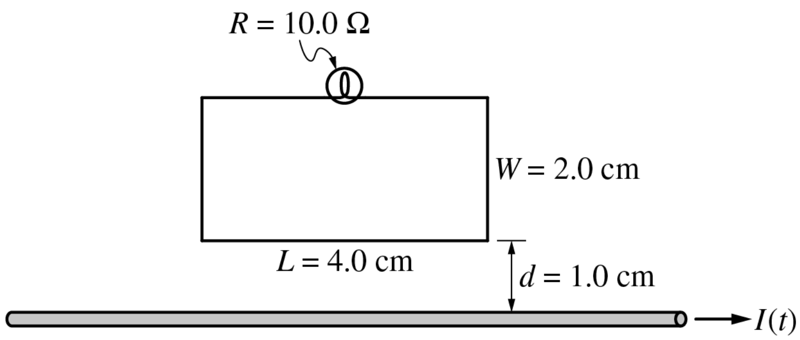
(d) The original scenario is repeated but with a new bar that has the same mass but with a nonnegligible resistance $R = 0.20\ \Omega$. The new bar is released from rest and smoothly transitions to the horizontal section of the rails and enters the first uniform magnetic field.

i. Determine the total resistance of the closed circuit.

ii. In the original scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{original}. In the new scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{new}. Is a_{new} greater than, less than, or equal to a_{original} ? Justify your answer.

(e) Describe a modification to H, B_1, or d that will result in a larger induced current in the new bar immediately after the bar enters the first uniform magnetic field. Justify your answer.

GO ON TO THE NEXT PAGE.



. A lightbulb of resistance $R = 10.0\ \Omega$ is connected to a rectangular loop of wire of negligible resistance near a very long current-carrying wire. The rectangular loop has a length $L = 4.0$ cm and a width $W = 2.0$ cm and is positioned so one of the longer sides of the loop is a distance $d = 1.0$ cm above and parallel to the long wire, as shown. The current in the long wire is initially flowing to the right and is given by $I(t) = C - Dt$, where $C = 10.0$ A and $D = 2.0$ A/s. At time $t = 5.0$ s, the current in the long wire is instantaneously zero as the current changes direction.

(a) What is the direction, if any, of the magnetic field produced by the induced current in the rectangular loop as the current in the long wire changes direction?

___ Into the page   ___ Out of the page   ___ No direction, because the field is zero

Justify your answer.

(b) Calculate the magnetic flux through the <u>loop</u> due to only the long wire at time $t = 3.0$ s.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

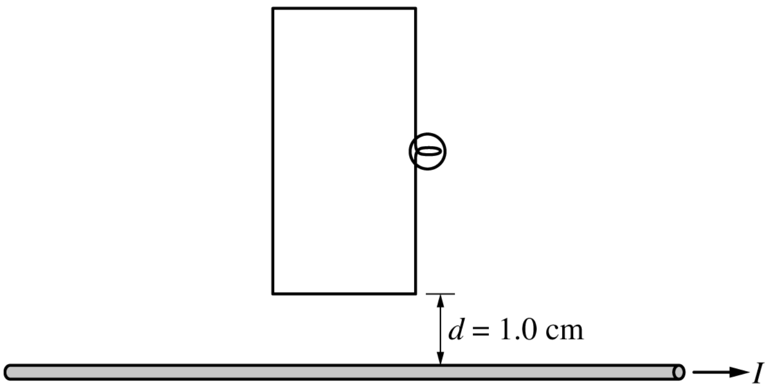
(c) Calculate the current through the lightbulb at time $t = 3.0$ s.

(d) A group of students attempts to experimentally verify whether the current through the lightbulb is consistent with the current calculation from part (c). The current in the rectangular loop is measured to be greater than the current calculated in part (c). Which of the following could explain this discrepancy? Select one answer.

___ The students did not account for Earth's magnetic field.
___ The rectangular loop is tilted and is not in the same plane as the wire.
___ The resistance of the lightbulb is greater than the recorded value.
___ The long side of the rectangular loop is shorter than the recorded value.
___ The current in the long wire changes at a faster rate than expected.

Briefly justify your answer.

GO ON TO THE NEXT PAGE.



(e) Later, the same rectangular loop with lightbulb is rotated such that a short side of the loop is 1.0 cm above and parallel to the long current-carrying wire, as shown. The current in the wire is again initially flowing from left to right and given by $I(t) = C - Dt$, where $C = 10.0$ A and $D = 2.0$ A/s. The current through the lightbulb in the loop's new orientation at time $t = 3.0$ s is I_2. Which of the following correctly relates the current I_2 to I_1, the current through the lightbulb in part (c)?

___ $I_2 < I_1$ ___ $I_2 = I_1$ ___ $I_2 > I_1$

Justify your answer.

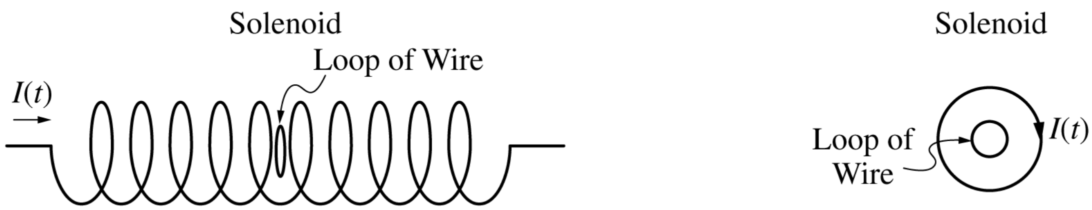
GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

66

Begin your response to **QUESTION** on this page.



Side View   End View

Note: Figures not drawn to scale.

. A single loop of wire with resistance $3.0\,\Omega$ and radius 0.10 m is placed inside a solenoid, with the normal to the loop parallel to the axis of the solenoid. The solenoid has 500 turns, is 0.25 m long, and is connected to a power supply that is not shown. At time $t = 0$, the power supply is turned on, and the current I in the solenoid as a function of t is given by the equation $I(t) = \beta t$, where $\beta = 5.0\text{ A/s}$. The direction of the current in the solenoid is clockwise, as shown in the end view.

(a) At time $t = 2.0\text{ s}$, is the induced current in the loop, as seen from the end view shown, clockwise, counterclockwise, or zero?

___ Clockwise   ___ Counterclockwise   ___ Zero

Justify your answer.

(b) Calculate the current in the loop of wire at time $t = 2.0\text{ s}$.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(c) Calculate the total energy dissipated by the loop of wire from time $t = 0$ to time $t = 2.0\text{ s}$.

(d) A group of students attempts to verify experimentally the calculation of the current from part (b). The current in the inner circular loop at time $t = 2.0\text{ s}$ is measured to be less than the current calculated in part (b). Which of the following could explain this discrepancy? Select one answer.

___ The experiment did not account for Earth's magnetic field.
___ The plane of the loop is not perpendicular to the axis of the solenoid.
___ The center of the loop is not on the axis of the solenoid.
___ The resistance of the loop is less than the given value.
___ The radius of the loop is actually larger than 0.10 m.

Justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(e) The power supply is now turned off. The original loop of wire is then replaced with a second loop made from wire that has the same thickness and is made from the same material as the original loop of wire. The second loop has radius 0.20 m, is placed in the same orientation as the original loop, and fits completely inside the solenoid. The power supply is turned on, and the current I in the solenoid as a function of t is again given by the equation $I(t) = \beta t$, where $\beta = 5.0$ A/s. Which of the following expressions correctly indicates the ratio $\frac{I_2}{I_1}$ where I_1 represents the current induced in the original loop of wire in part (b) and I_2 represents the current induced in the second loop of wire?

___ $\frac{I_2}{I_1} = 1$ ___ $1 < \frac{I_2}{I_1} < 2$ ___ $\frac{I_2}{I_1} = 2$ ___ $\frac{I_2}{I_1} > 2$

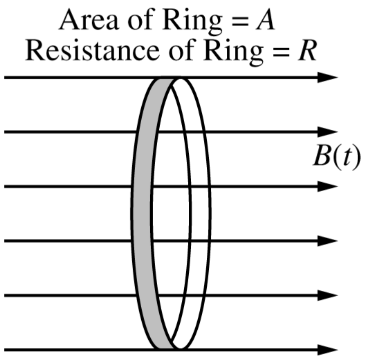
Justify your answer.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

Begin your response to **QUESTION 3** on this page.



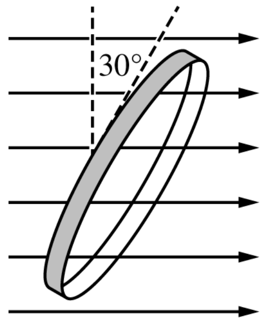
3. A thin, conducting ring of area A and resistance R is aligned in a uniform magnetic field directed to the right and perpendicular to the plane of the ring, as shown. At time $t = 0$, the magnitude of the magnetic field is B_0. At $t = 1\text{ s}$, the magnitude of the magnetic field begins to decrease according to the equation $B(t) = \dfrac{\beta}{t}$, where β has units of T·s.

(a) Derive an equation for the magnitude of the induced current I in the ring as a function of t for $t > 1\text{ s}$. Express your answer in terms of β, A, R, t, and physical constants, as appropriate.

Assume $A = 0.50\text{ m}^2$, $R = 2.0\,\Omega$, and $\beta = 0.50\text{ T}\cdot\text{s}$.

(b) Calculate the electrical energy dissipated in the ring from $t = 1\text{ s}$ to $t = 2\text{ s}$.

GO ON TO THE NEXT PAGE.



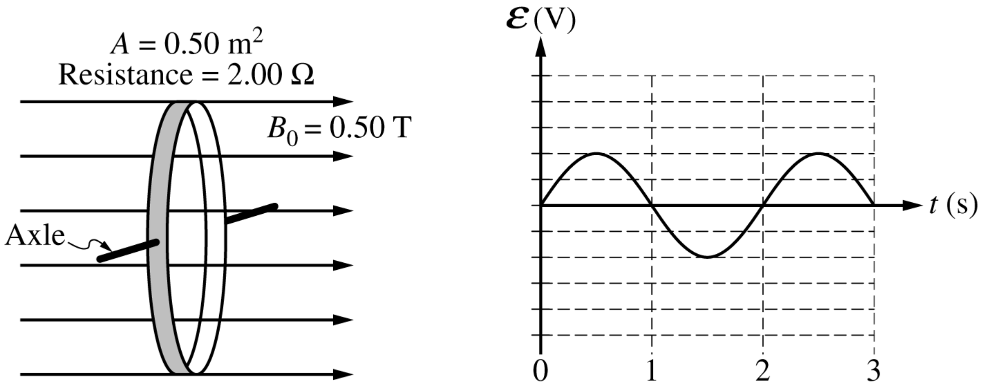
The ring is then rotated so that the plane of the ring is aligned at a 30° angle to the magnetic field, as shown.

The magnitude of the magnetic field is reset to a magnitude of B_0 at a new time $t = 0$ and again begins to decrease at $t = 1\text{ s}$ according to the equation $B(t) = \dfrac{\beta}{t}$, where β has units of T·s.

(c) Will the amount of energy dissipated in the ring from $t = 1\text{ s}$ to $t = 2\text{ s}$ be greater than, less than, or equal to the energy dissipated in part (b) ?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.



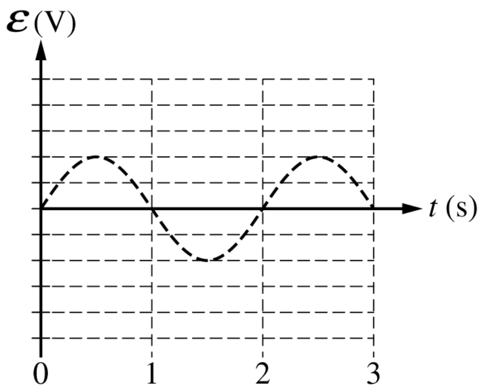
The ring is now mounted on an axle that is perpendicular to the magnetic field. The magnitude of the magnetic field is now held at a constant $B_0 = 0.50\text{ T}$, as shown. The ring rotates about the axle, and the emf ε induced in the ring as a function of time t is shown on the graph.

(d) Calculate the angular speed ω of the rotating ring in rad/s.

(e) Calculate the magnitude of the maximum emf ε_{MAX} induced in the ring.

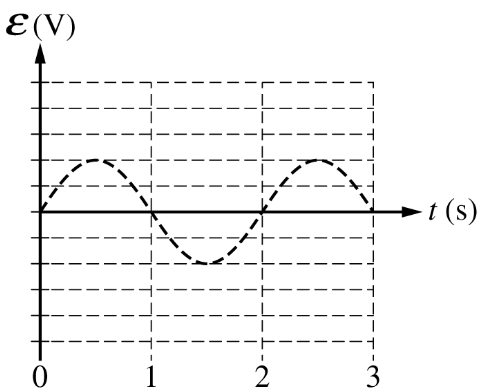
The ring now begins to rotate at an angular speed 2ω.

(f) On the graph below, draw a curve to indicate the new induced emf ε in the ring. The dashed curve shows the emf induced under the original conditions.



Justify your sketch, specifically identifying and addressing any similarities or differences between the sketch and the original graph.

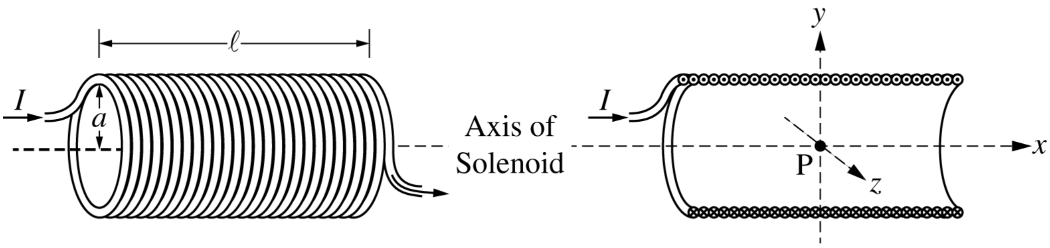
PRACTICE GRAPH - Use the graph below to practice your sketch for part (f). Any work shown on the graph below will NOT be graded.



GO ON TO THE NEXT PAGE.

STOP

END OF EXAM



Solenoid Cutaway View

Note: Figures not drawn to scale.

3. A solenoid is used to generate a magnetic field. The solenoid has an inner radius a, length ℓ, and N total turns of wire. A power supply, not shown, is connected to the solenoid and generates current I, as shown in the figure on the left above. The x-axis runs along the axis of the solenoid. Point P is in the middle of the solenoid at the origin of the xyz-coordinate system, as shown in the cutaway view on the right above. Assume $\ell >> a$.

   (a) Select the correct direction of the magnetic field at point P.

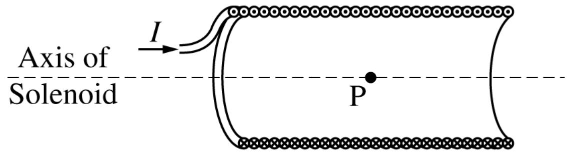
   ____ $+x$-direction ____ $+y$-direction ____ $+z$-direction

   ____ $-x$-direction ____ $-y$-direction ____ $-z$-direction

   Justify your selection.

   (b)

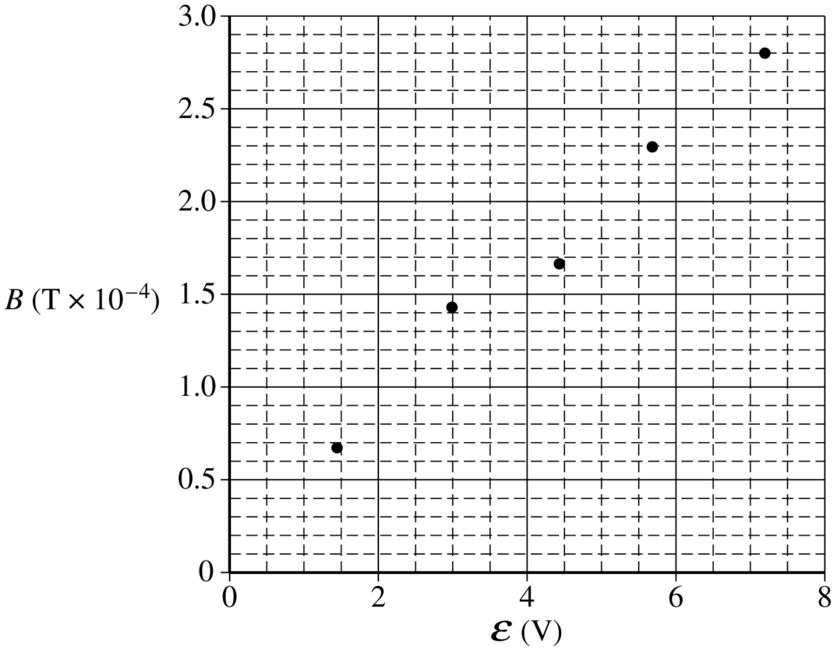
   i. On the cutaway view below, clearly draw an Amperian loop that can be used to determine the magnetic field at point P at the center of the solenoid.



Cutaway View

   ii. Use Ampere's law to derive an expression for the magnetic field strength at point P. Express your answer in terms of I, ℓ, N, a, and physical constants, as appropriate.

Some physics students conduct an experiment to determine the resistance R_S of a solenoid with radius $a = 0.015$ m, total turns $N = 100$, and total length $\ell = 0.40$ m. The students connect the solenoid to a variable power supply. A magnetic field sensor is used to measure the magnetic field strength along the central axis at the center of the solenoid. The plot of the magnetic field strength B as a function of the emf $\mathcal{E}$ of the power supply is shown below.



(c)

i. On the graph above, draw a best-fit line for the data.

ii. Use the straight line to determine the resistance R_S of the solenoid used in the experiment.

(d) One of the students notes that the horizontal component of the magnetic field of Earth is 2.5×10^{-5} T.

i. Is there evidence from the graph that the horizontal orientation of the solenoid affects the measured values for B ?

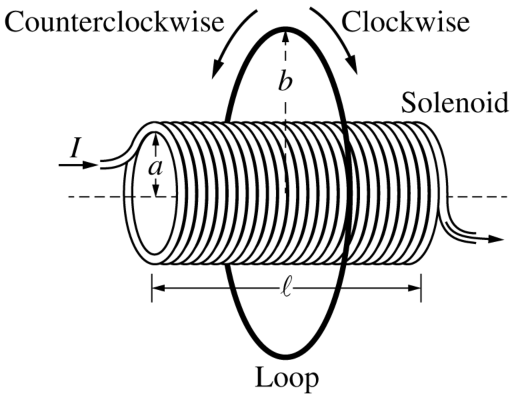
____ Yes ____ No

Justify your answer.

ii. Would the horizontal orientation of the solenoid affect the calculated value for R_S ?

____ Yes ____ No

Justify your answer.



A thin conducting loop of radius b and resistance R_L is placed concentric with the solenoid, as shown above. The current in the solenoid is decreased from I to zero over time Δt.

(e)

i. Is the direction of the induced current in the loop clockwise or counterclockwise during the time period that the current in the solenoid is decreasing?

____ Clockwise ____ Counterclockwise

Justify your answer.

ii. Derive an equation for the average induced current i_{IND} in the loop during the time period that the current in the solenoid is decreasing. Express your answer in terms of I, ℓ, N, a, b, R_L, R_S, Δt, and physical constants, as appropriate.

STOP

END OF EXAM

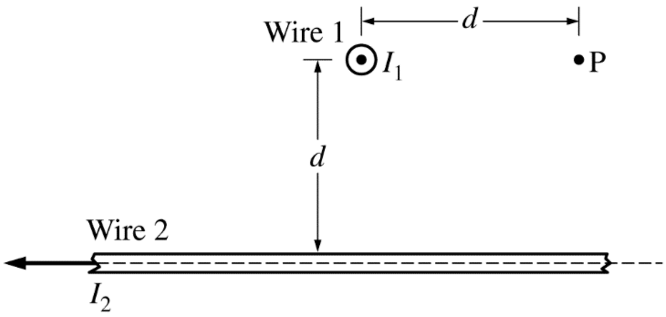


Figure 1. Side view

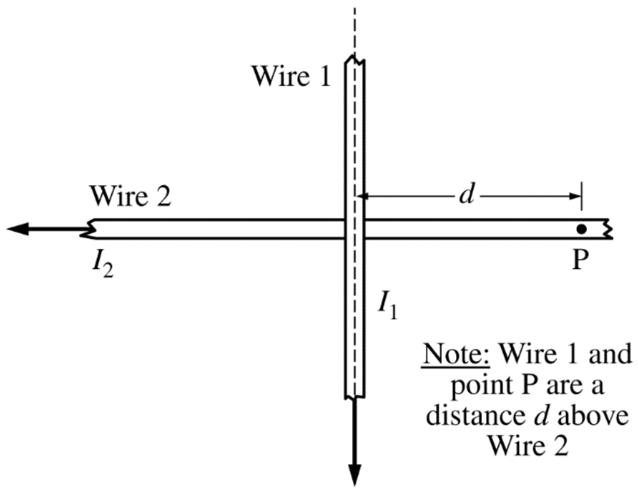


Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I, d, and physical constants, as appropriate.

   (a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

   (b) Derive an expression for the magnitude of the net magnetic field at point P.

   (c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

   (d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2 ? Assume gravitational effects are negligible.

   ____ Wire 1 will not move.

   ____ Wire 1 will move upward as viewed in Figure 1.

   ____ Wire 1 will move downward as viewed in Figure 1.

   ____ Wire 1 will rotate clockwise as viewed in Figure 2.

   ____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

   Justify your answer.

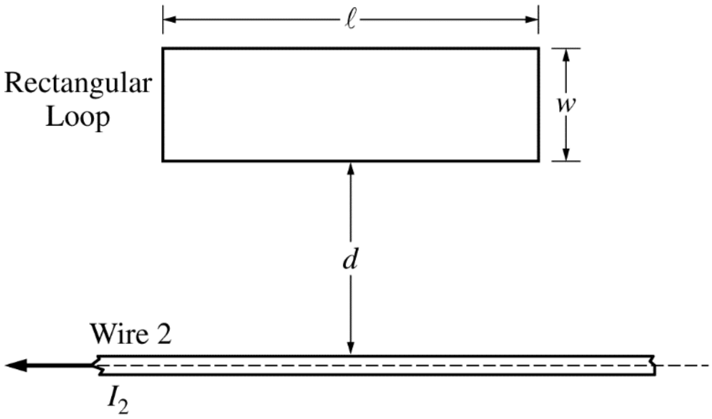


Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1}.

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

____ $\Phi = \displaystyle\int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$ ____ $\Phi = \displaystyle\int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\Phi = \displaystyle\int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$ ____ $\Phi = \displaystyle\int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

(f) Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \dfrac{\mu_0 I_0 \ell(1-kt)}{\pi} \ln\left(\dfrac{d+w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

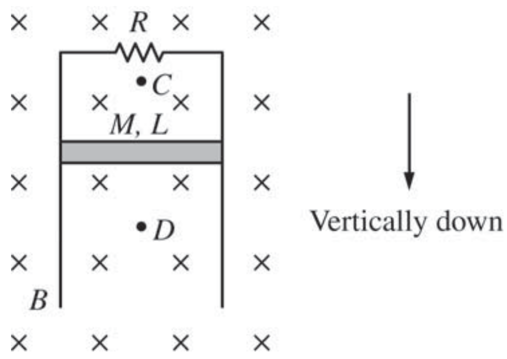
____ Clockwise ____ Counterclockwise

____ Undefined, because there is no current induced in the loop

Justify your answer.

STOP

END OF EXAM



E&M.3.

A conducting bar of mass M, length L, and negligible resistance is connected to two long vertical conducting rails of negligible resistance. The two rails are connected by a resistor of resistance R at the top. The entire apparatus is located in a magnetic field of magnitude B directed into the page, as shown in the figure above. The bar is released from rest and slides without friction down the rails.

(a) What is the direction of the current in the resistor?

____ Left ____ Right

(b)

i. Is the magnitude of the net magnetic field above the bar at point C greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

ii. While the bar is above point D, is the magnitude of the net magnetic field at point D greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Express your answers to parts (c) and (d) in terms of M, L, R, B, and physical constants, as appropriate.

(c) Write, but do NOT solve, a differential equation that could be used to determine the velocity of the falling bar as a function of time t.

(d) Determine an expression for the terminal velocity v_T of the bar.

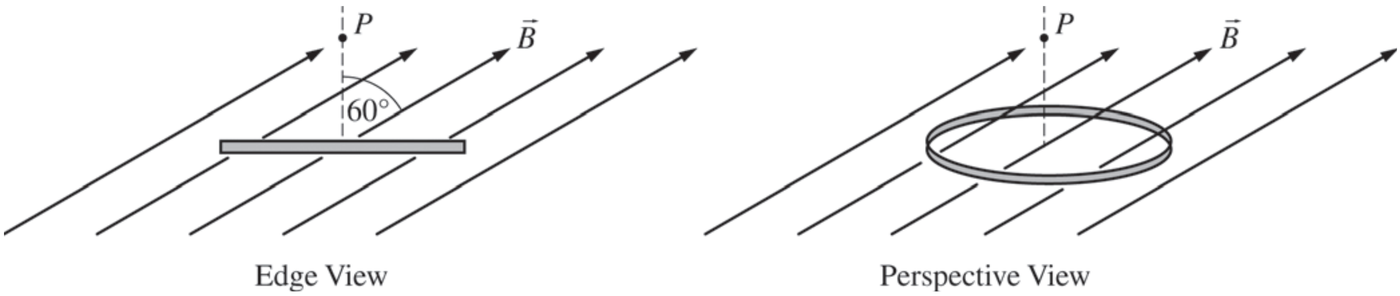
Express your answers to parts (e) and (f) in terms of v_T, M, L, R, B, and physical constants, as appropriate.

(e) Derive an expression for the power dissipated in the resistor when the bar is falling at terminal velocity.

(f) Using your differential equation from part (c), derive an expression for the speed of the falling bar $v(t)$ as a function of time t.

STOP

END OF EXAM



E&M. 3.

A circular wire loop with radius 0.10 m and resistance 50 Ω is held in place horizontally in a magnetic field $\vec{B}$ directed upward at an angle of 60° with the vertical, as shown in the figure above. The magnetic field in the direction shown is given as a function of time t by $B(t) = a(1 - bt)$, where $a = 4.0$ T and $b = 0.20\ s^{-1}$.

(a) Derive an expression for the magnetic flux through the loop as a function of time t.

(b) Calculate the numerical value of the induced emf in the loop.

(c)

i. Calculate the numerical value of the induced current in the loop.

ii. What is the direction of the induced current in the loop as viewed from point P ?

____ Clockwise ____ Counterclockwise

Justify your answer.

(d) Assuming the loop stays in its current position, calculate the energy dissipated in the loop in 4.0 seconds.

(e) Indicate whether the net magnetic force and net magnetic torque on the loop are zero or nonzero while the loop is in the magnetic field.

Net magnetic force: ____ Zero ____ Nonzero

Net magnetic torque: ____ Zero ____ Nonzero

Justify both of your answers.

STOP

END OF EXAM

13.3 Induced Currents and Magnetic Forces

Name: ____________ Class: ________ Date: __________ **Total: 10 marks**

KEY WORDS eddy currents 涡电流 • retarding force 阻碍力 • induced current 感应电流 • lenz's law 楞次定律

Objective

Build the skills to answer exam questions on **induced currents and magnetic forces**.

You must be able to:

- determine the direction and size of a current **induced** by a changing flux
- explain why an induced current creates a **retarding force** 阻碍力
- analyse the energy transfer (mechanical → electrical → thermal)
- describe **eddy currents** 涡电流

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Induced current and retarding force

A changing flux drives an induced current whose direction (Lenz's law) **opposes** the change. In a moving conductor this makes a force that **resists** the motion.

Energy transfer

To keep the conductor moving you must do **work**; that mechanical energy becomes electrical energy, then thermal energy in the resistance.

Eddy currents

In a solid conductor, a changing flux drives swirling **eddy currents** — used in magnetic braking, and a cause of heating in transformer cores.

2 Practice

2.1 State the rule that gives the direction of an induced current. [1]

2.2 Explain why an induced current opposes the motion that produces it. [2]

2.3 Name the swirling induced currents in a solid conductor. [1]

3 Exam-style questions

3.1 An induced current always opposes the [1]

- **A** applied voltage
- **B** change that produces it
- **C** resistance of the wire
- **D** shape of the wire

3.2 Eddy currents are used in [1]

- **A** light bulbs
- **B** magnetic braking
- **C** batteries
- **D** fixed resistors

3.3 A metal loop is pulled out of a magnetic field region.

(a) State whether a current is induced in the loop. [1]

(b) State the direction of the magnetic force on the loop relative to its motion. [1]

(c) Describe the energy transfer as the loop is pulled out. [2]

Solutions

2.1 Lenz's law.

2.2 the induced current opposes the change in flux, so the force it feels acts against the motion that is causing the change.

2.3 eddy currents.

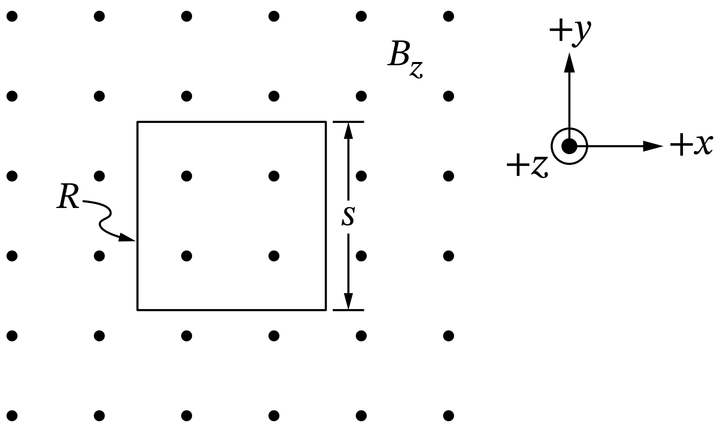
3.1 B.

3.2 B.

3.3 (a) yes. (b) opposite to the motion (it resists being pulled out). (c) the work done pulling the loop becomes electrical energy in the induced current, then thermal energy in the loop's resistance.

Question 4

4. In Scenario 1, a square conducting loop with side length s has total resistance R. The loop is held fixed in the xy-plane in an external, uniform, time-varying magnetic field that is initially directed in the $+z$-direction, as shown. The z-component of the magnetic field changes as a function of time t as described by $B_z = B_0 \cos\left(2\pi \frac{t}{t_1}\right)$, where B_0 and t_1 are positive constants.



Part A

Indicate whether the induced current I in the loop is clockwise, counterclockwise, or zero during the time interval $0 < t < \frac{t_1}{4}$ by writing one of the following.

- I is clockwise.
- I is counterclockwise.
- I is zero.

Justify your answer without only manipulating equations.

Part B

Derive an expression for I as a function of t in terms of s, R, B_0, t_1, t, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

In Scenario 2, a new square conducting loop with side length $2s$ and total resistance $2R$ is placed in the same external, uniform, time-varying magnetic field as in Scenario 1.

Indicate whether the maximum induced current I_2 in Scenario 2 is greater than, less than, or equal to the maximum induced current I_1 in Scenario 1 by writing one of the following.

- $I_2 > I_1$
- $I_2 < I_1$
- $I_2 = I_1$

Briefly **justify** your answer by referencing the expression you derived in part B.

STOP

END OF EXAM

Begin your response to **QUESTION 3** on this page.

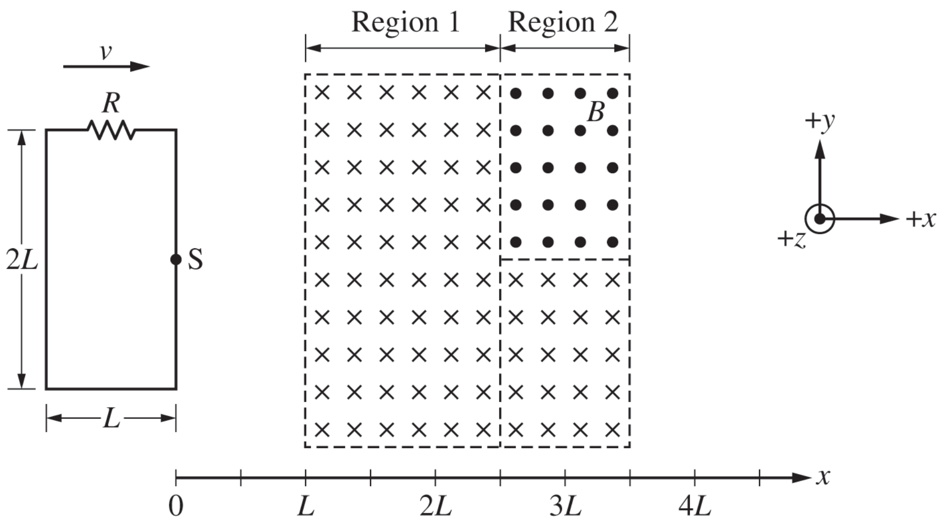


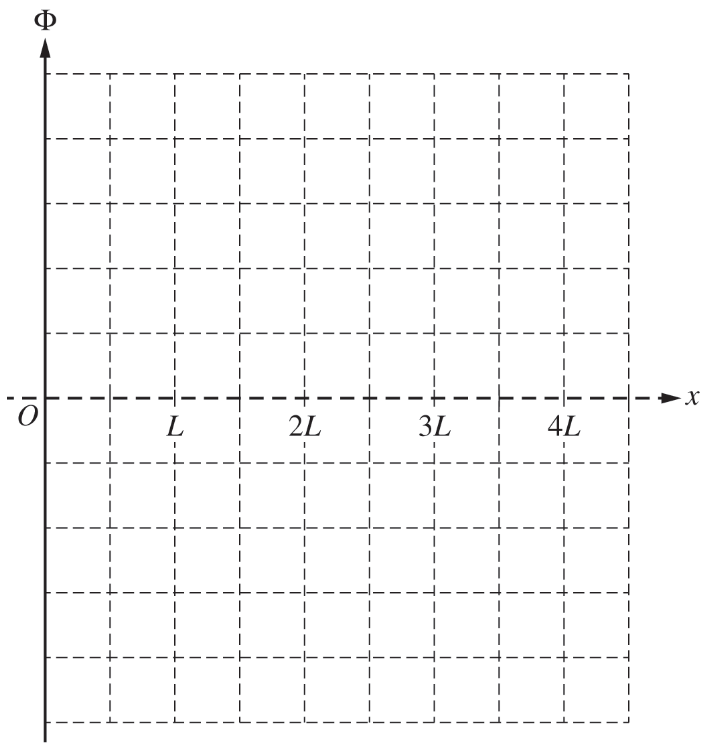
Figure 1

3. A wire is connected to a resistor of resistance R to form a rigid rectangular loop of width L and height $2L$. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$-direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $-z$-direction. Region 1 has boundaries $x = L$ and $x = 2.5L$. The loop later enters Region 2 with two external, uniform magnetic fields, each of magnitude B, that are parallel but are directed in opposite z-directions. Region 2 has boundaries $x = 2.5L$ and $x = 3.5L$. Point S is the midpoint of the leading edge of the loop and is aligned with the horizontal boundary in Region 2 that separates the two magnetic fields.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(a) On the following axes, **sketch** a graph of the magnetic flux Φ through the rectangular loop as a function of the position x of Point S from $x = 0$ to $x = 4.5L$. The $+z$-direction indicated in Figure 1 corresponds to $+\Phi$.



(b) Consider the instant when Point S reaches $x = 1.5L$.

i. **Indicate** whether the current I_R that is induced in the rectangular loop when Point S reaches $x = 1.5L$ is clockwise, counterclockwise, or zero.

_____ Clockwise _____ Counterclockwise _____ Zero

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

ii. **Derive** an expression for I_R when Point S reaches $x = 1.5L$. If $I_R = 0$, indicate how the derived expression shows that $I_R = 0$. Express your answer in terms of R, L, v, B, and physical constants, as appropriate.

iii. **Derive** an expression for the power P dissipated by the resistor when Point S reaches $x = 1.5L$. Express your answer in terms of R, L, v, B, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

The total energy dissipated by the resistor in the rectangular loop as Point S moves from $x = 0$ to $x = 4.5L$ is E_{original}.

The vertical boundary between regions 1 and 2 is now shifted to $x = 1.5L$. After the boundary is shifted, the rectangular loop again moves with speed v in the $+x$-direction, as shown in Figure 2. The total energy dissipated by the resistor as Point S moves from $x = 0$ to $x = 4.5L$ is E_{new}.

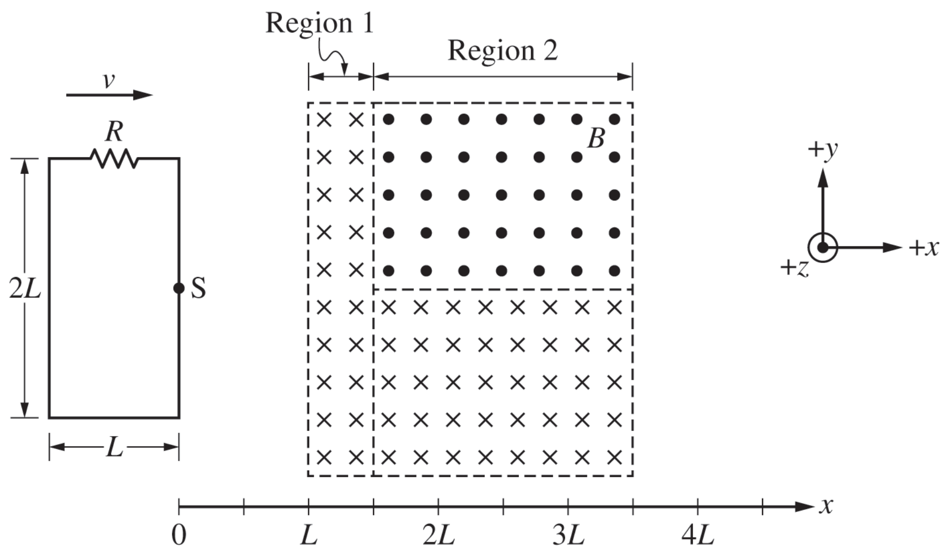


Figure 2

(c) **Indicate** whether E_{new} is greater than, less than, or equal to E_{original}.

_____ $E_{\text{new}} > E_{\text{original}}$ _____ $E_{\text{new}} < E_{\text{original}}$ _____ $E_{\text{new}} = E_{\text{original}}$

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

The original magnetic fields are modified so that the region $L < x < 3.5L$ contains an external uniform magnetic field of magnitude B that is directed in the $-z$-direction.

A new wire is connected to a resistor of resistance R to form a rigid triangular loop with base length L and height $2L$. An external force is exerted on the loop so that the loop always moves with speed v in the $+x$-direction, as shown in Figure 3. Point S represents the lower-leading corner of the loop.

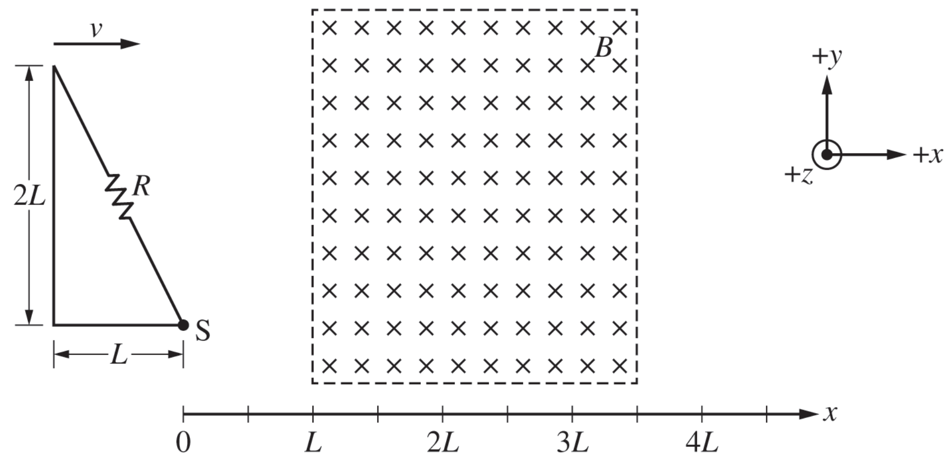
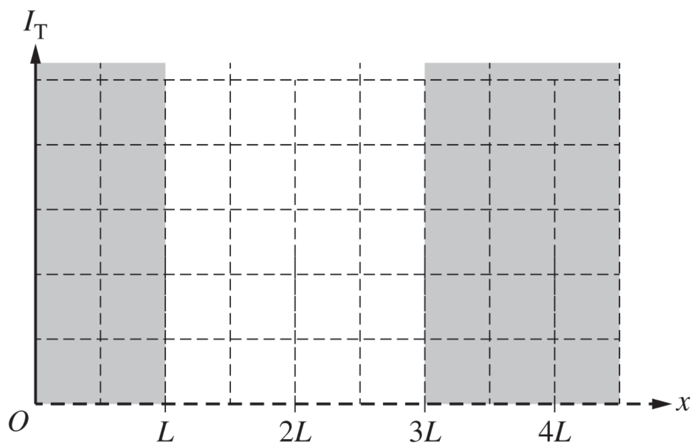


Figure 3

(d) On the following axes, **sketch** a graph of the induced current I_T in the triangular loop as Point S moves from $x = L$ to $x = 3L$.



GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

Begin your response to **QUESTION 3** on this page.

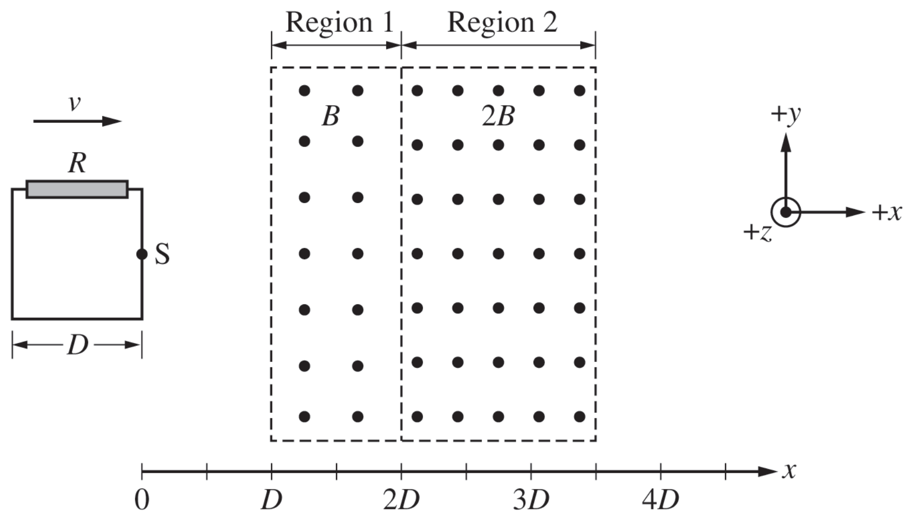


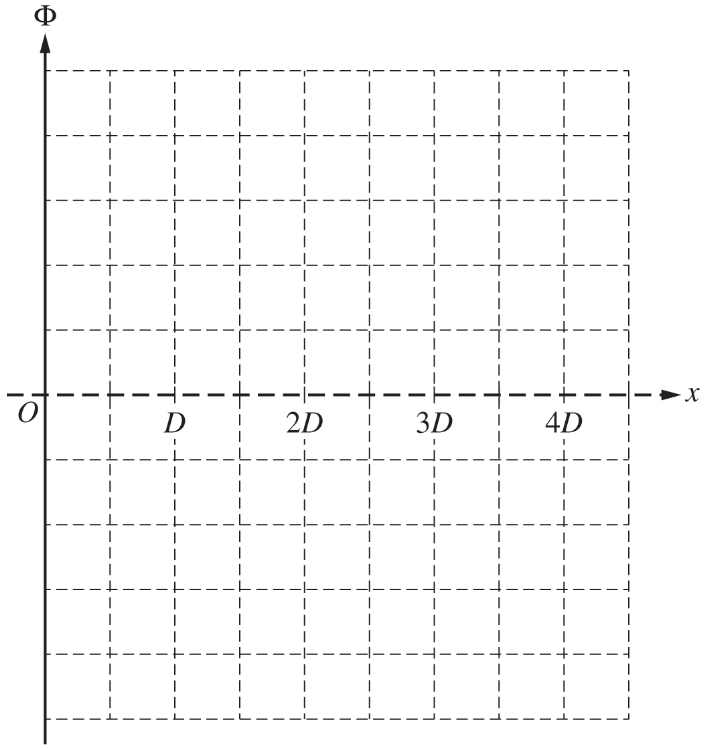
Figure 1

3. A wire is connected to a resistor of resistance R to form a rigid square loop of side length D. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$ direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $+z$-direction. Region 1 has boundaries $x = D$ and $x = 2D$. The loop later enters Region 2 of external uniform magnetic field of magnitude $2B$ that is directed in the $+z$-direction. Region 2 has boundaries $x = 2D$ and $x = 3.5D$. Point S is the midpoint of the leading edge of the loop.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(a) On the following axes, **sketch** a graph of the magnetic flux Φ through the square loop as a function of the position x of Point S from $x = 0$ to $x = 4.5D$. The $+z$-direction indicated in Figure 1 corresponds to $+\Phi$.



(b) Consider the instant when Point S reaches $x = 1.5D$.

i. **Indicate** whether the current I_S that is induced in the square loop when Point S reaches $x = 1.5D$ is clockwise, counterclockwise, or zero.

_____ Clockwise _____ Counterclockwise _____ Zero

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

ii. **Derive** an expression for I_S when Point S reaches $x = 1.5D$. If $I_S = 0$, indicate how the derived expression shows that $I_S = 0$. Express your answer in terms of R, D, v, B, and physical constants, as appropriate.

iii. **Derive** an expression for the power P dissipated by the resistor when Point S reaches $x = 1.5D$. Express your answer in terms of R, D, v, B, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

The total energy dissipated by the resistor in the square loop as Point S moves from $x = 0$ to $x = 4.5D$ is $E_{original}$.

The vertical boundary between regions 1 and 2 is now shifted to $x = 2.5D$. After the boundary is shifted, the square loop again moves with speed v in the $+x$-direction, as shown in Figure 2. The total energy dissipated by the resistor as Point S moves from $x = 0$ to $x = 4.5D$ is E_{new}.

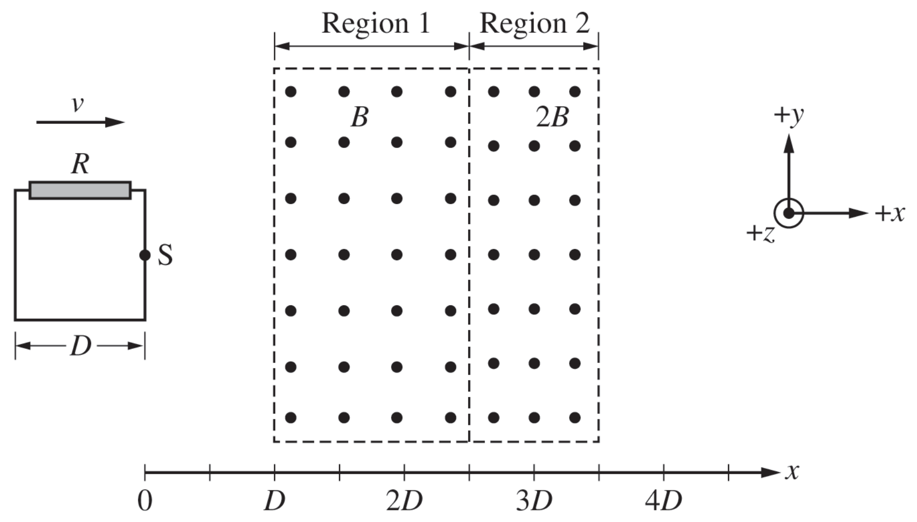


Figure 2

(c) **Indicate** whether E_{new} is greater than, less than, or equal to $E_{original}$.

_____ $E_{new} > E_{original}$ _____ $E_{new} < E_{original}$ _____ $E_{new} = E_{original}$

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

The original magnetic fields are modified so that the region $D < x < 3.5D$ contains an external uniform magnetic field of magnitude B that is directed in the $+z$-direction.

A new wire is connected to a resistor of resistance R to form a rigid triangular loop with base length D and height D. An external force is exerted on the loop so that the loop always moves with speed v in the $+x$-direction, as shown in Figure 3. Point S represents the upper-leading corner of the loop.

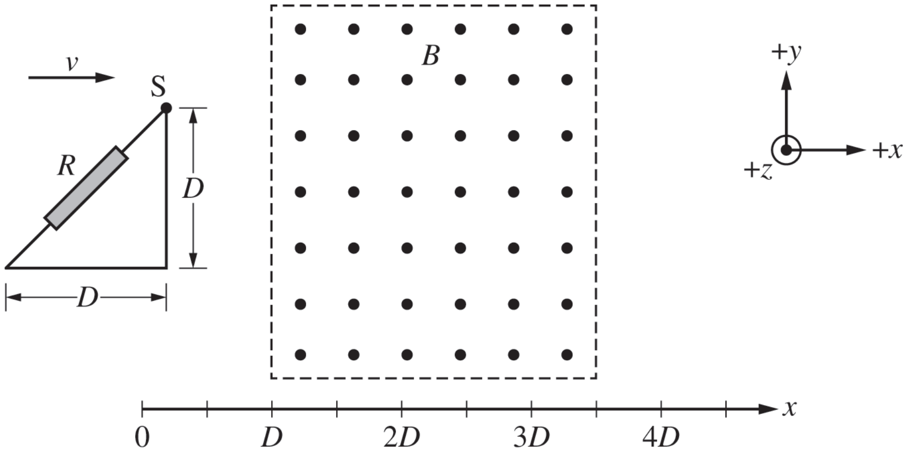
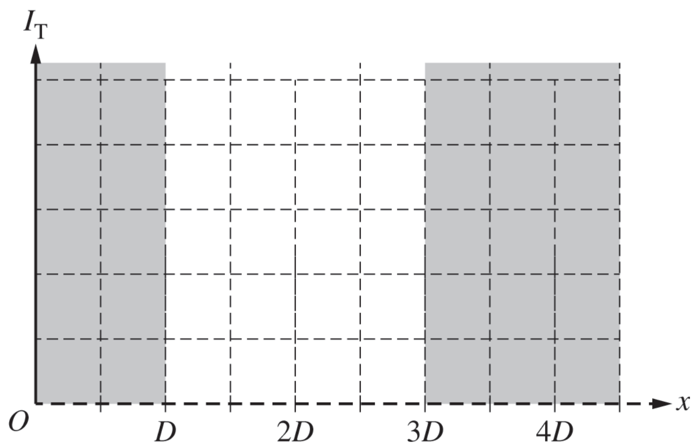


Figure 3

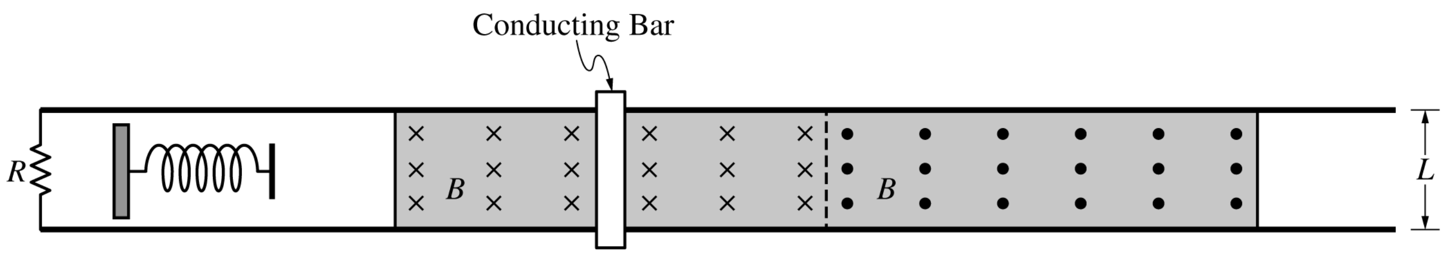
(d) On the following axes, **sketch** a graph of the induced current I_T in the loop as Point S moves from $x = D$ to $x = 3D$.



GO ON TO THE NEXT PAGE.

STOP

END OF EXAM



Top View (at time t_B)

. Two horizontal, parallel, conducting rails are separated by distance $L = 0.40$ m. A resistor of resistance $R = 0.30\,\Omega$ connects the rails. A horizontal ideal spring is located between the rails. The right end of the spring is free to move and the left end is fixed in place. A conducting bar of mass $m = 0.23$ kg is placed on the rails and is in contact with the spring, which is initially compressed. Frictional forces and the resistance of the bar and rails are negligible.

- At time $t = 0$, the bar is released from rest and is pushed to the right by the spring.
- At time t_1, the bar loses contact with the spring and slides to the right.
- At time t_2, the bar enters and travels through a uniform magnetic field of magnitude $B = 0.50$ T that is directed into the page, as shown.
- At time t_3, the bar enters a region where the magnitude of the uniform magnetic field is still $B = 0.50$ T but is directed out of the page.
- At time t_4, the bar enters a region with no magnetic field.

:onsider time t_B such that $t_2 < t_B < t_3$.

(a) On the following diagram of the bar, draw an arrow indicating the direction of the net force F_{net} exerted on the bar at time t_B. If the net force is zero, write $F_{net} = 0$.

GO ON TO THE NEXT PAGE.

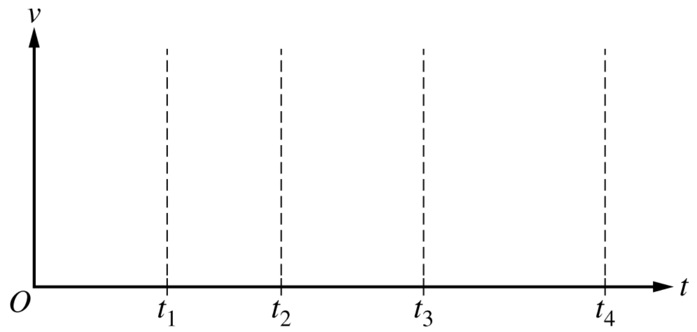
Continue your response to **QUESTION 2** on this page.

(b) At time t_B, the speed of the bar is $v = 2.5$ m/s.

i. Calculate the magnitude of the current in the bar at time t_B.

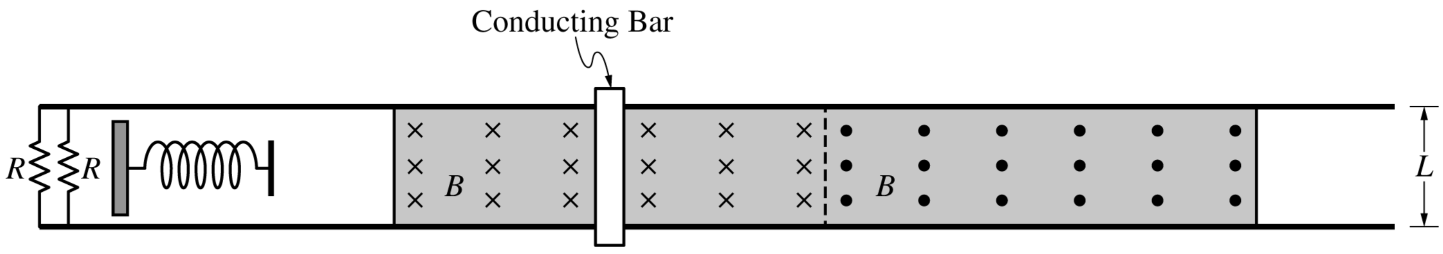
ii. Calculate the magnitude of the net force F_{net} exerted on the bar at time t_B.

(c) On the following axes, sketch a graph of the speed v of the bar as a function of time t between $t = 0$ and t_4.



GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.



Top View (at time t_B)

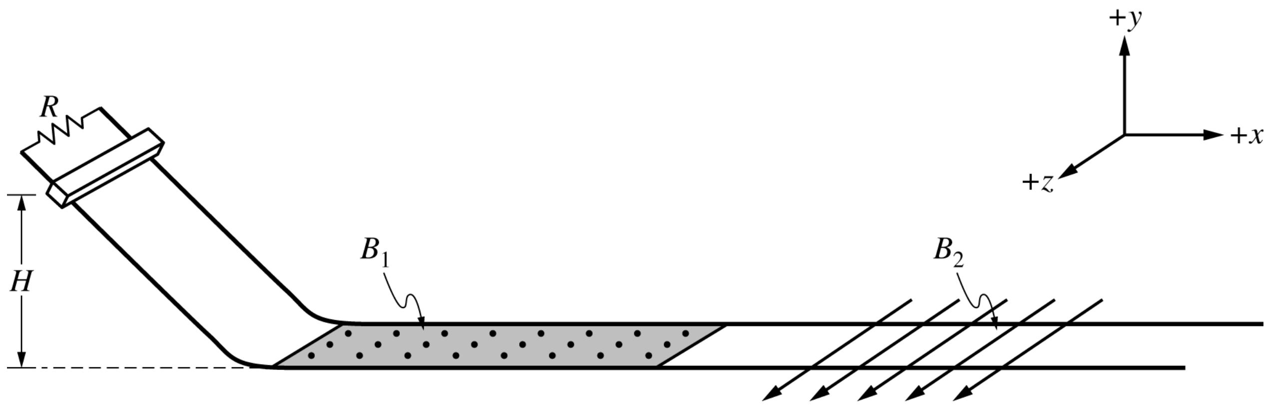
(d) The scenario is repeated but an additional resistor of resistance $R = 0.30\ \Omega$ is connected, as shown.

i. Determine the total resistance R_{total} of the closed circuit for the new scenario.

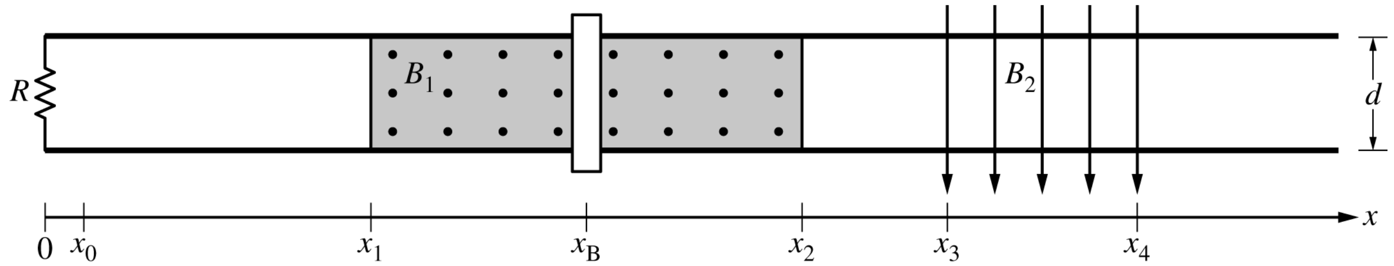
ii. In the original scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is $a_{original}$. In the new scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{new}. Is a_{new} greater than, less than, or equal to $a_{original}$? Justify your answer.

(e) Describe a modification to m, B, or L that will result in a smaller induced potential difference across the original resistor immediately after the bar enters the first uniform magnetic field. Justify your answer.

GO ON TO THE NEXT PAGE.



Perspective View (at time $t = 0$)



Top View (at time t_B)

. Two parallel conducting rails are separated by distance $d = 0.30\ \text{m}$. A resistor of resistance $R = 0.20\ \Omega$ connects the rails. A conducting bar is placed on a sloped section of the rails at height H above the horizontal section of the rails. Frictional forces and the resistances of the bar and rails are negligible.

- At time $t = 0$, the bar is released from rest from position x_0 and slides down the sloped section of the rails, as shown in the Perspective View.
- At time t_1, the bar reaches position x_1 and smoothly transitions to the horizontal section of the rails and enters a uniform magnetic field of magnitude $B_1 = 0.40\ \text{T}$ that is directed in the $+y$-direction.
- At time t_2, the bar reaches position x_2 and enters a region with no magnetic field.
- At time t_3, the bar reaches position x_3 and enters a uniform magnetic field of magnitude $B_2 = 0.60\ \text{T}$ that is directed in the $+z$-direction.
- At time t_4, the bar reaches position x_4 and enters a region with no magnetic field.

he bar is at position x_B (shown in Top View) at time t_B such that $t_1 < t_B < t_2$.

GO ON TO THE NEXT PAGE.

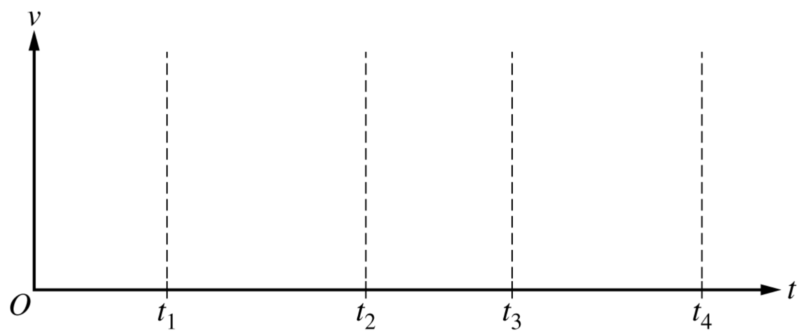
(a) On the following diagram of the bar, as observed from the Top View, draw an arrow indicating the direction of the net force F_{net} exerted on the bar at time t_B. If the net force is zero, write $F_{\text{net}} = 0$.

(b) At time t_B, the speed of the bar is $v = 2.5$ m/s.

i. Calculate the magnitude of the current in the bar at time t_B.

ii. Calculate the magnitude of the net force F_{net} exerted on the bar at time t_B.

(c) On the following axes, sketch a graph of the speed v of the bar as a function of time t between $t = 0$ and t_4.



GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) The original scenario is repeated but with a new bar that has the same mass but with a nonnegligible resistance $R = 0.20\,\Omega$. The new bar is released from rest and smoothly transitions to the horizontal section of the rails and enters the first uniform magnetic field.

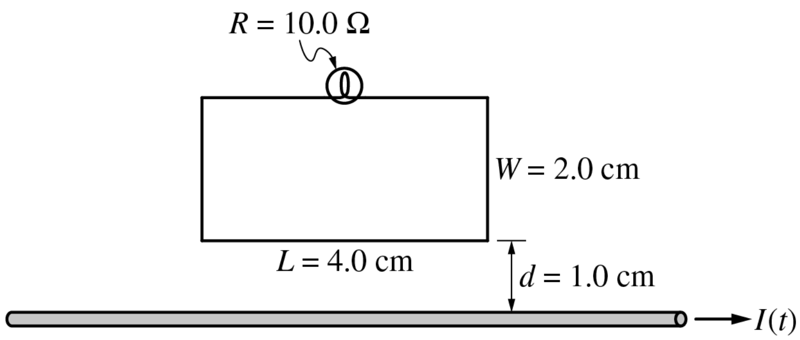
i. Determine the total resistance of the closed circuit.

ii. In the original scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is $a_{original}$. In the new scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{new}. Is a_{new} greater than, less than, or equal to $a_{original}$? Justify your answer.

(e) Describe a modification to H, B_1, or d that will result in a larger induced current in the new bar immediately after the bar enters the first uniform magnetic field. Justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 3** on this page.



. A lightbulb of resistance $R = 10.0\ \Omega$ is connected to a rectangular loop of wire of negligible resistance near a very long current-carrying wire. The rectangular loop has a length $L = 4.0$ cm and a width $W = 2.0$ cm and is positioned so one of the longer sides of the loop is a distance $d = 1.0$ cm above and parallel to the long wire, as shown. The current in the long wire is initially flowing to the right and is given by $I(t) = C - Dt$, where $C = 10.0$ A and $D = 2.0$ A/s. At time $t = 5.0$ s, the current in the long wire is instantaneously zero as the current changes direction.

(a) What is the direction, if any, of the magnetic field produced by the induced current in the rectangular loop as the current in the long wire changes direction?

___ Into the page ___ Out of the page ___ No direction, because the field is zero

Justify your answer.

(b) Calculate the magnetic flux through the loop due to only the long wire at time $t = 3.0$ s.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

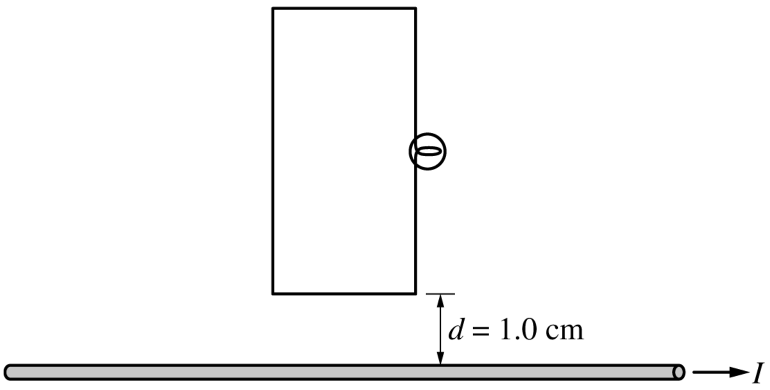
(c) Calculate the current through the lightbulb at time $t = 3.0$ s.

(d) A group of students attempts to experimentally verify whether the current through the lightbulb is consistent with the current calculation from part (c). The current in the rectangular loop is measured to be greater than the current calculated in part (c). Which of the following could explain this discrepancy? Select one answer.

___ The students did not account for Earth's magnetic field.
___ The rectangular loop is tilted and is not in the same plane as the wire.
___ The resistance of the lightbulb is greater than the recorded value.
___ The long side of the rectangular loop is shorter than the recorded value.
___ The current in the long wire changes at a faster rate than expected.

Briefly justify your answer.

GO ON TO THE NEXT PAGE.



(e) Later, the same rectangular loop with lightbulb is rotated such that a short side of the loop is 1.0 cm above and parallel to the long current-carrying wire, as shown. The current in the wire is again initially flowing from left to right and given by $I(t) = C - Dt$, where $C = 10.0\text{ A}$ and $D = 2.0\text{ A/s}$. The current through the lightbulb in the loop's new orientation at time $t = 3.0\text{s}$ is I_2. Which of the following correctly relates the current I_2 to I_1, the current through the lightbulb in part (c)?

___ $I_2 < I_1$ ___ $I_2 = I_1$ ___ $I_2 > I_1$

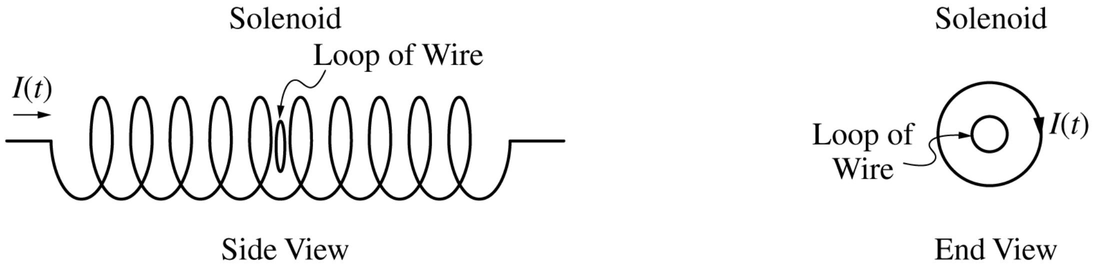
Justify your answer.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

Begin your response to **QUESTION 3** on this page.



Note: Figures not drawn to scale.

. A single loop of wire with resistance $3.0\,\Omega$ and radius 0.10 m is placed inside a solenoid, with the normal to the loop parallel to the axis of the solenoid. The solenoid has 500 turns, is 0.25 m long, and is connected to a power supply that is not shown. At time $t = 0$, the power supply is turned on, and the current I in the solenoid as a function of t is given by the equation $I(t) = \beta t$, where $\beta = 5.0\text{ A/s}$. The direction of the current in the solenoid is clockwise, as shown in the end view.

(a) At time $t = 2.0\text{ s}$, is the induced current in the loop, as seen from the end view shown, clockwise, counterclockwise, or zero?

___ Clockwise ___ Counterclockwise ___ Zero

Justify your answer.

(b) Calculate the current in the loop of wire at time $t = 2.0\text{ s}$.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(c) Calculate the total energy dissipated by the loop of wire from time $t = 0$ to time $t = 2.0\ \text{s}$.

(d) A group of students attempts to verify experimentally the calculation of the current from part (b). The current in the inner circular loop at time $t = 2.0\ \text{s}$ is measured to be less than the current calculated in part (b). Which of the following could explain this discrepancy? Select one answer.

___ The experiment did not account for Earth's magnetic field.
___ The plane of the loop is not perpendicular to the axis of the solenoid.
___ The center of the loop is not on the axis of the solenoid.
___ The resistance of the loop is less than the given value.
___ The radius of the loop is actually larger than $0.10\ \text{m}$.

Justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(e) The power supply is now turned off. The original loop of wire is then replaced with a second loop made from wire that has the same thickness and is made from the same material as the original loop of wire. The second loop has radius 0.20 m, is placed in the same orientation as the original loop, and fits completely inside the solenoid. The power supply is turned on, and the current I in the solenoid as a function of t is again given by the equation $I(t) = \beta t$, where $\beta = 5.0$ A/s. Which of the following expressions correctly indicates the ratio $\frac{I_2}{I_1}$ where I_1 represents the current induced in the original loop of wire in part (b) and I_2 represents the current induced in the second loop of wire?

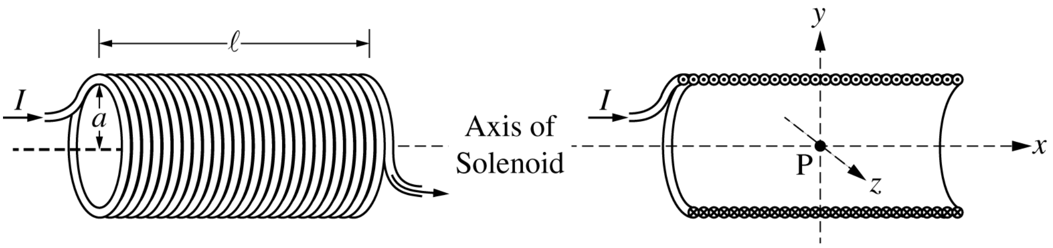
___ $\frac{I_2}{I_1} = 1$ ___ $1 < \frac{I_2}{I_1} < 2$ ___ $\frac{I_2}{I_1} = 2$ ___ $\frac{I_2}{I_1} > 2$

Justify your answer.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM



Solenoid Cutaway View

Note: Figures not drawn to scale.

3. A solenoid is used to generate a magnetic field. The solenoid has an inner radius a, length ℓ, and N total turns of wire. A power supply, not shown, is connected to the solenoid and generates current I, as shown in the figure on the left above. The x-axis runs along the axis of the solenoid. Point P is in the middle of the solenoid at the origin of the xyz-coordinate system, as shown in the cutaway view on the right above. Assume $\ell >> a$.

(a) Select the correct direction of the magnetic field at point P.

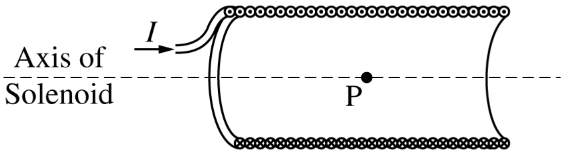
____ $+x$-direction ____ $+y$-direction ____ $+z$-direction

____ $-x$-direction ____ $-y$-direction ____ $-z$-direction

Justify your selection.

(b)

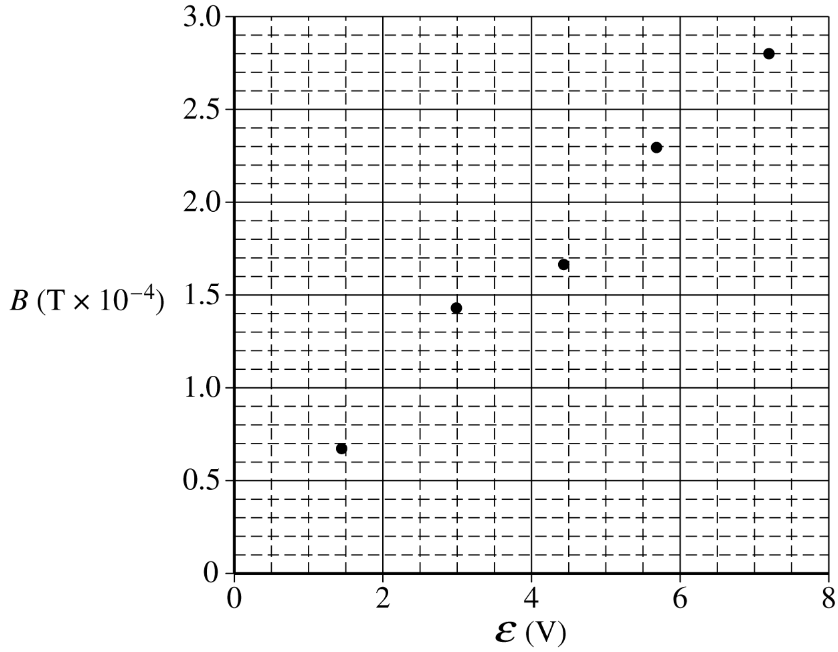
i. On the cutaway view below, clearly draw an Amperian loop that can be used to determine the magnetic field at point P at the center of the solenoid.



Cutaway View

ii. Use Ampere's law to derive an expression for the magnetic field strength at point P. Express your answer in terms of I, ℓ, N, a, and physical constants, as appropriate.

Some physics students conduct an experiment to determine the resistance R_S of a solenoid with radius $a = 0.015$ m, total turns $N = 100$, and total length $\ell = 0.40$ m. The students connect the solenoid to a variable power supply. A magnetic field sensor is used to measure the magnetic field strength along the central axis at the center of the solenoid. The plot of the magnetic field strength B as a function of the emf $\mathcal{E}$ of the power supply is shown below.



(c)

i. On the graph above, draw a best-fit line for the data.

ii. Use the straight line to determine the resistance R_S of the solenoid used in the experiment.

(d) One of the students notes that the horizontal component of the magnetic field of Earth is 2.5×10^{-5} T.

i. Is there evidence from the graph that the horizontal orientation of the solenoid affects the measured values for B ?

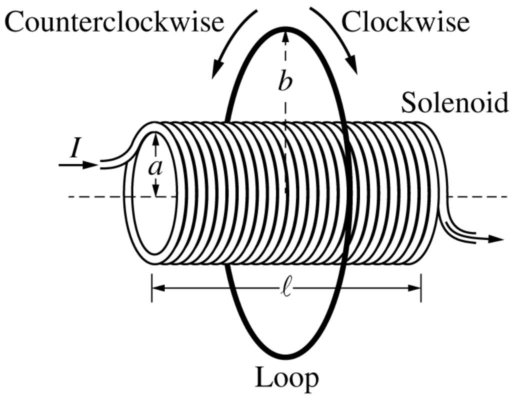
____ Yes ____ No

Justify your answer.

ii. Would the horizontal orientation of the solenoid affect the calculated value for R_S ?

____ Yes ____ No

Justify your answer.



A thin conducting loop of radius b and resistance R_L is placed concentric with the solenoid, as shown above. The current in the solenoid is decreased from I to zero over time Δt.

(e)

i. Is the direction of the induced current in the loop clockwise or counterclockwise during the time period that the current in the solenoid is decreasing?

____ Clockwise ____ Counterclockwise

Justify your answer.

ii. Derive an equation for the average induced current i_{IND} in the loop during the time period that the current in the solenoid is decreasing. Express your answer in terms of I, ℓ, N, a, b, R_L, R_S, Δt, and physical constants, as appropriate.

STOP

END OF EXAM

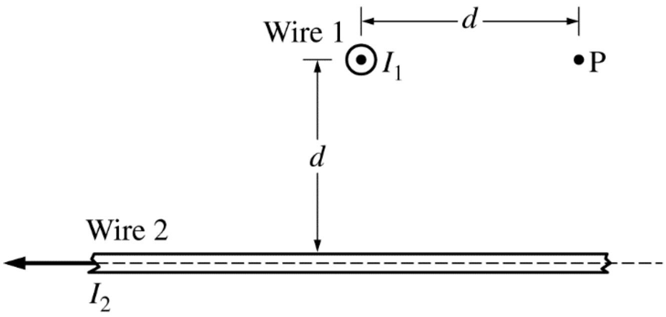


Figure 1. Side view

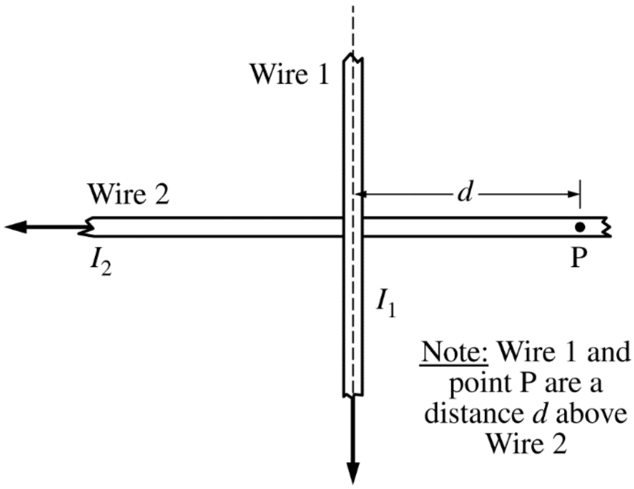


Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I, d, and physical constants, as appropriate.

(a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

(b) Derive an expression for the magnitude of the net magnetic field at point P.

(c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

(d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2 ? Assume gravitational effects are negligible.

____ Wire 1 will not move.

____ Wire 1 will move upward as viewed in Figure 1.

____ Wire 1 will move downward as viewed in Figure 1.

____ Wire 1 will rotate clockwise as viewed in Figure 2.

____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

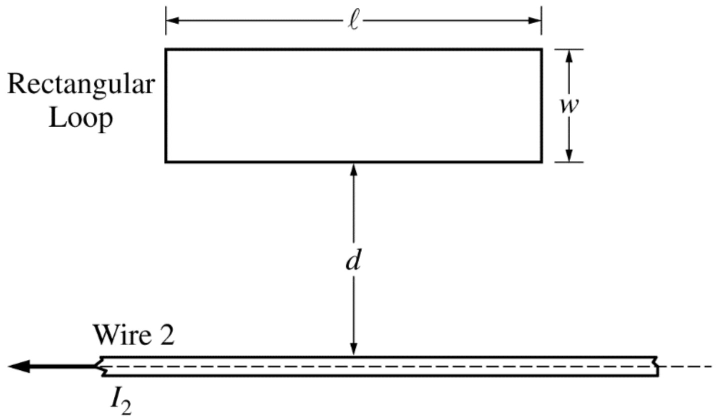


Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1}.

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0 (2I_0)(1 - kt)\ell w}{2\pi} dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0 (2I_0)(1 - kt)\ell w}{2\pi} dr$

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0 (2I_0)(1 - kt)}{2\pi r} \ell dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0 (2I_0)(1 - kt)}{2\pi r} \ell dr$

(f) Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \frac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\frac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

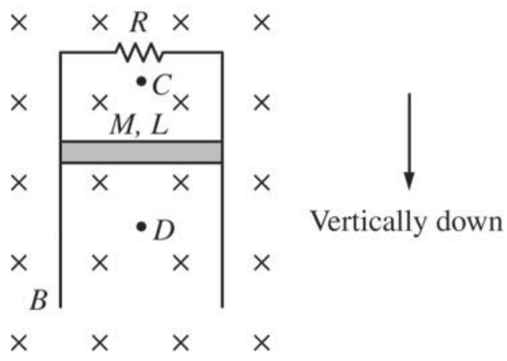
____ Clockwise ____ Counterclockwise

____ Undefined, because there is no current induced in the loop

Justify your answer.

STOP

END OF EXAM



E&M.3.

A conducting bar of mass M, length L, and negligible resistance is connected to two long vertical conducting rails of negligible resistance. The two rails are connected by a resistor of resistance R at the top. The entire apparatus is located in a magnetic field of magnitude B directed into the page, as shown in the figure above. The bar is released from rest and slides without friction down the rails.

(a) What is the direction of the current in the resistor?

____ Left ____ Right

(b)

i. Is the magnitude of the net magnetic field above the bar at point C greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

ii. While the bar is above point D, is the magnitude of the net magnetic field at point D greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Express your answers to parts (c) and (d) in terms of M, L, R, B, and physical constants, as appropriate.

(c) Write, but do NOT solve, a differential equation that could be used to determine the velocity of the falling bar as a function of time t.

(d) Determine an expression for the terminal velocity v_T of the bar.

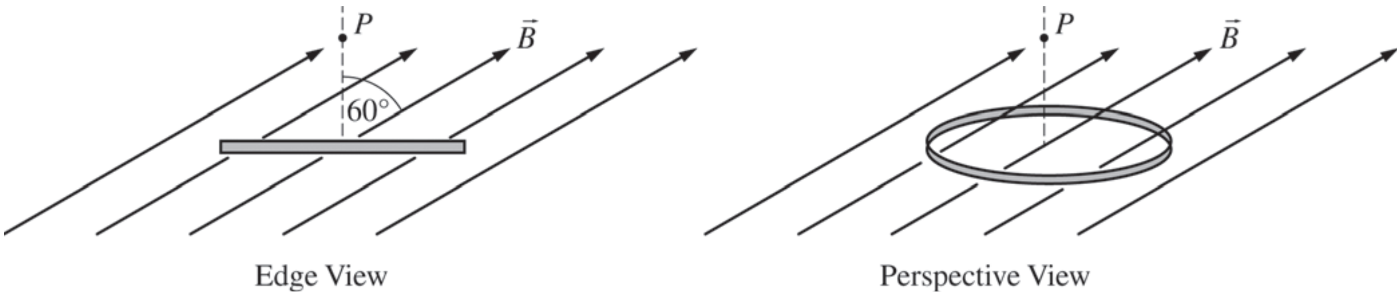
Express your answers to parts (e) and (f) in terms of v_T, M, L, R, B, and physical constants, as appropriate.

(e) Derive an expression for the power dissipated in the resistor when the bar is falling at terminal velocity.

(f) Using your differential equation from part (c), derive an expression for the speed of the falling bar $v(t)$ as a function of time t.

STOP

END OF EXAM



E&M. 3.

A circular wire loop with radius 0.10 m and resistance 50 Ω is held in place horizontally in a magnetic field $\vec{B}$ directed upward at an angle of 60° with the vertical, as shown in the figure above. The magnetic field in the direction shown is given as a function of time t by $B(t) = a(1 - bt)$, where $a = 4.0$ T and $b = 0.20\ s^{-1}$.

(a) Derive an expression for the magnetic flux through the loop as a function of time t.

(b) Calculate the numerical value of the induced emf in the loop.

(c)

i. Calculate the numerical value of the induced current in the loop.

ii. What is the direction of the induced current in the loop as viewed from point P ?

____ Clockwise ____ Counterclockwise

Justify your answer.

(d) Assuming the loop stays in its current position, calculate the energy dissipated in the loop in 4.0 seconds.

(e) Indicate whether the net magnetic force and net magnetic torque on the loop are zero or nonzero while the loop is in the magnetic field.

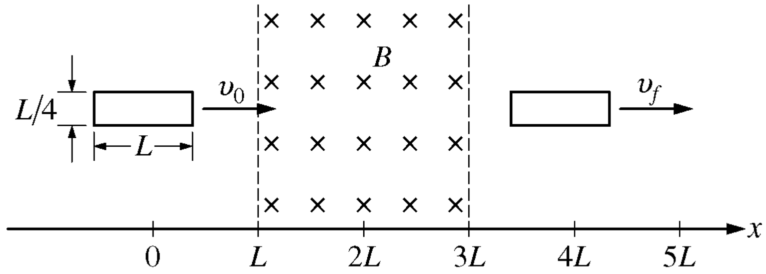
Net magnetic force: ____ Zero ____ Nonzero

Net magnetic torque: ____ Zero ____ Nonzero

Justify both of your answers.

STOP

END OF EXAM



E&M. 2.

The rectangular loop of wire shown on the left in the figure above has mass M, length L, width $L/4$, and resistance R. It is initially moving to the right at constant speed v_0, with no net force acting on it. At time $t = 0$ the loop enters a region of length $2L$ that contains a uniform magnetic field of magnitude B directed into the page. The loop emerges from the field at time t_f with final speed v_f. Express all algebraic answers to the following in terms of M, L, R, B, v_0, and fundamental constants, as appropriate.

(a) Let x represent the position of the right end of the loop. Place a check mark in the appropriate box in each column in the table below to indicate whether the speed of the loop increases, decreases, or stays the same as the loop moves to the right.

	Position of Right End of Loop			
Speed of Loop	$L < x < 2L$	$2L < x < 3L$	$3L < x < 4L$	$4L < x < 5L$
Increases				
Decreases				
Stays the same				

(b) Derive an expression for the magnitude of the current induced in the loop as its right edge enters the field.

(c) What is the direction of the induced current determined in part (b) ?

____ Clockwise ____ Counterclockwise

Justify your answer.

(d) Write, but do not solve, a differential equation for the speed v as a function of time as the loop enters the field.

(e) What is the direction of the acceleration of the loop just before its left edge leaves the field?

____ Left ___ Right ____ Up ____ Down

Justify your answer.

13.4 Inductance

Name: ______________ Class: __________ Date: ____________ **Total: 11 marks**

KEY WORDS inductor 电感器 • inductance 电感 • back emf 反电动势 • henry 亨利 • emf 电动势

Objective

Build the skills to answer exam questions on **inductance**.

You must be able to:

- define **inductance 电感** and its unit the **henry 亨利**
- relate the **back emf 反电动势** to the rate of change of current, $\varepsilon = -L\dfrac{dI}{dt}$
- calculate the **energy stored** in an inductor, $U = \frac{1}{2}LI^2$
- relate stored energy to the magnetic **energy density** $u = \dfrac{B^2}{2\mu_0}$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Inductance

$L = \dfrac{N\Phi_B}{I}$, measured in henries (H). It measures how much flux linkage a current produces.

■ Back emf

$\varepsilon = -L\dfrac{dI}{dt}$ — an inductor opposes **changes** in current with an induced (back) emf.

■ Energy stored

$U = \frac{1}{2}LI^2$, held in the magnetic field (energy density $u = \dfrac{B^2}{2\mu_0}$).

2 Practice

2.1 An inductor produces a back emf of 6.0 V when the current changes at 3.0 A s^{-1}. Find its inductance. [2]

2.2 Find the energy stored in a 0.50 H inductor carrying 4.0 A. [2]

2.3 State the SI unit of inductance. [1]

3 Exam-style questions

3.1 The back emf across an inductor is [1]

- **A** LI
- **B** $-L\dfrac{dI}{dt}$
- **C** $\frac{1}{2}LI^2$
- **D** I/L

3.2 The energy stored in an inductor is [1]

- **A** LI
- **B** $\frac{1}{2}LI^2$
- **C** $\frac{1}{2}L^2I$
- **D** LI^2

3.3 A 0.20 H inductor carries a current rising at 5.0 A s^{-1}.

(a) Find the back emf. [2]

(b) Find the energy stored when the current reaches 3.0 A. [2]

Solutions

2.1 $L = \dfrac{|\varepsilon|}{|dI/dt|} = \dfrac{6.0}{3.0} = 2.0$ H.

2.2 $U = \frac{1}{2}LI^2 = \frac{1}{2}(0.50)(4.0)^2 = 4.0$ J.

2.3 the henry (H).

3.1 **B**.

3.2 **B**.

3.3 (a) $|\varepsilon| = L\dfrac{dI}{dt} = 0.20 \times 5.0 = 1.0$ V. (b) $U = \frac{1}{2}(0.20)(3.0)^2 = 0.90$ J.

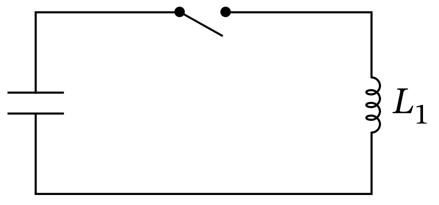
Question 3

The following information applies to parts A and B.

---

3. In Experiment 1, students are asked to collect data that can be graphed to determine the inductance L_1 of an inductor. The students have access to multiple charged capacitors, each of a different known capacitance, and a voltmeter that measures potential difference as a function of time. In each trial, the inductor is connected to a charged capacitor and an initially open switch, as shown in Figure 1.

Figure 1



Part A

i. **Indicate** quantities that could be measured by the students that would allow them to determine L_1 using a linear graph.

ii. Briefly **describe** a method to reduce experimental uncertainty.

Part B

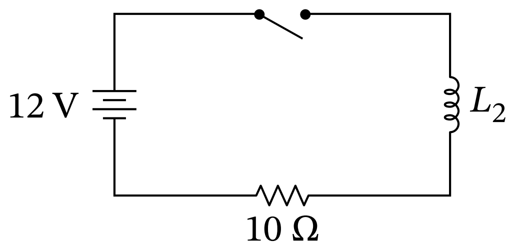
i. **Indicate** what quantities the students could graph on the horizontal and vertical axes to create a linear graph that can be used to determine L_1.

ii. Briefly **describe** the relationship between L_1 and a feature of the graph from part B (i). Your answer may include an equation that relates L_1 and the chosen feature of the graph.

The following information applies to parts C and D.

In Experiment 2, the students are asked to use a graph to determine the inductance L_2 of a new inductor. The inductor is connected to a 12 V battery, a 10 Ω resistor, and an initially open switch, as shown in Figure 2.

Figure 2



After the switch is closed, the current I in the circuit is measured at different times and the corresponding rate of change of the current, $\frac{dI}{dt}$, is determined. The collected data are shown in Table 1.

Table 1

I (A)	$\frac{dI}{dt}$ (A/s)
0.40	280
0.50	230
0.75	150
0.90	100
1.00	68

Part C

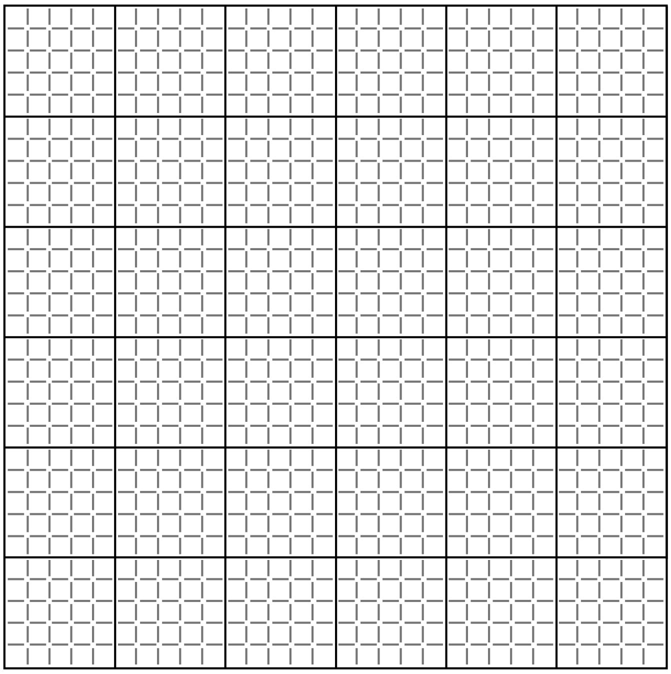
i. **Label** the axes of the grid provided with measured or calculated quantities. Include units, as appropriate. The graphed quantities should yield a linear graph that can be used to determine L_2.

ii. On the grid provided, create a graph of the quantities indicated in part C (i).

- Clearly **label** each axis with a numerical scale.
- **Plot** the corresponding data points on the grid.
- Table 2 is provided in your booklet for scratch work and will not be scored.

Quantity (units, if appropriate)

__________ __________



Quantity (units, if appropriate)

__________ __________

iii. **Draw** a best-fit line for the data plotted in part C (ii).

Part D

Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for L_2.

13.5 Circuits with Resistors and Inductors (LR Circuits)

Name: ______________ Class: __________ Date: ____________ **Total: 10 marks**

KEY WORDS time constant 时间常数 • inductor 电感器 • emf 电动势

Objective

Build the skills to answer exam questions on **circuits with resistors and inductors (LR circuits)**.

You must be able to:

- describe the **growth** and **decay** of current in an LR circuit
- explain how an inductor **opposes changes** in current
- define the **time constant** 时间常数 $\tau = \dfrac{L}{R}$
- analyse the **initial** and **steady-state** behaviour

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **Growth and decay**

$$I(t) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right) \text{ (rising)}, \qquad I(t) = I_0\, e^{-t/\tau} \text{ (falling)}.$$

■ **Time constant**

$\tau = \dfrac{L}{R}$ sets how quickly the current settles.

■ **Start and steady state**

At $t = 0$ the inductor **opposes** the sudden change, so the current is 0 (it acts like a break); at steady state it passes current freely (like a plain wire), and $I = \varepsilon/R$.

2 Practice

2.1 Find the time constant of a 0.40 H inductor and a 20 Ω resistor. [2]

2.2 State the current in an LR circuit immediately after the switch closes. [1]

2.3 State the steady-state current for an emf of 12 V and $R = 6.0\ \Omega$. [1]

---

3 Exam-style questions

3.1 The time constant of an LR circuit is [1]

- **A** RC
- **B** L/R
- **C** LR
- **D** R/L

3.2 At $t = 0$ in an LR circuit, an inductor behaves like [1]

- **A** a plain wire
- **B** a break (no current)
- **C** a battery
- **D** a capacitor

3.3 An LR circuit has $\varepsilon = 10$ V, $R = 5.0\ \Omega$, and $L = 0.25$ H.

(a) Find the time constant. [2]

(b) Find the final (steady-state) current. [2]

Solutions

2.1 $\tau = \dfrac{L}{R} = \dfrac{0.40}{20} = 0.020$ s.

2.2 zero (the inductor opposes the sudden change).

2.3 $I = \dfrac{\varepsilon}{R} = \dfrac{12}{6.0} = 2.0$ A.

3.1 **B**.

3.2 **B**.

3.3 (a) $\tau = \dfrac{L}{R} = \dfrac{0.25}{5.0} = 0.050$ s. (b) $I = \dfrac{\varepsilon}{R} = \dfrac{10}{5.0} = 2.0$ A.

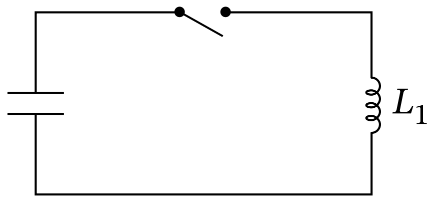
Question 3

The following information applies to parts A and B.

---

3. In Experiment 1, students are asked to collect data that can be graphed to determine the inductance L_1 of an inductor. The students have access to multiple charged capacitors, each of a different known capacitance, and a voltmeter that measures potential difference as a function of time. In each trial, the inductor is connected to a charged capacitor and an initially open switch, as shown in Figure 1.

Figure 1



Part A

i. **Indicate** quantities that could be measured by the students that would allow them to determine L_1 using a linear graph.

ii. Briefly **describe** a method to reduce experimental uncertainty.

Part B

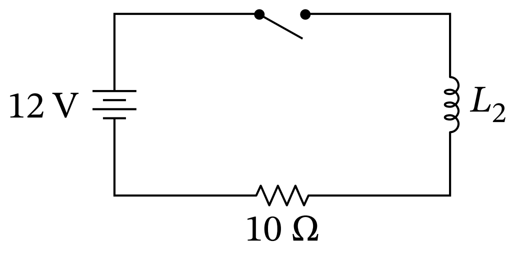
i. **Indicate** what quantities the students could graph on the horizontal and vertical axes to create a linear graph that can be used to determine L_1.

ii. Briefly **describe** the relationship between L_1 and a feature of the graph from part B (i). Your answer may include an equation that relates L_1 and the chosen feature of the graph.

The following information applies to parts C and D.

In Experiment 2, the students are asked to use a graph to determine the inductance L_2 of a new inductor. The inductor is connected to a $12\ \text{V}$ battery, a $10\ \Omega$ resistor, and an initially open switch, as shown in Figure 2.

Figure 2



After the switch is closed, the current I in the circuit is measured at different times and the corresponding rate of change of the current, $\frac{dI}{dt}$, is determined. The collected data are shown in Table 1.

Table 1

I (A)	$\frac{dI}{dt}$ (A/s)
0.40	280
0.50	230
0.75	150
0.90	100
1.00	68

Part C

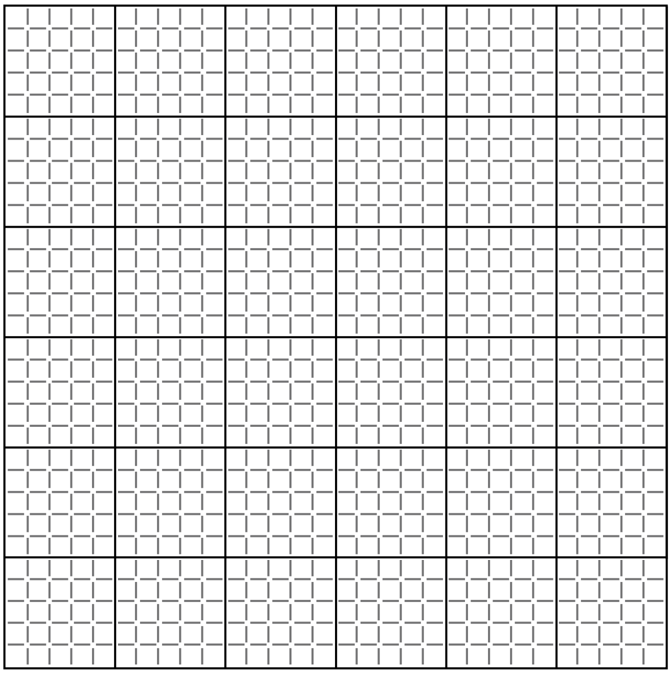
i. **Label** the axes of the grid provided with measured or calculated quantities. Include units, as appropriate. The graphed quantities should yield a linear graph that can be used to determine L_2.

ii. On the grid provided, create a graph of the quantities indicated in part C (i).

- Clearly **label** each axis with a numerical scale.
- **Plot** the corresponding data points on the grid.
- Table 2 is provided in your booklet for scratch work and will not be scored.

Quantity (units, if appropriate)

__________ __________



Quantity (units, if appropriate)

__________ __________

iii. **Draw** a best-fit line for the data plotted in part C (ii).

Part D

Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for L_2.

Begin your response to **QUESTION 2** on this page.

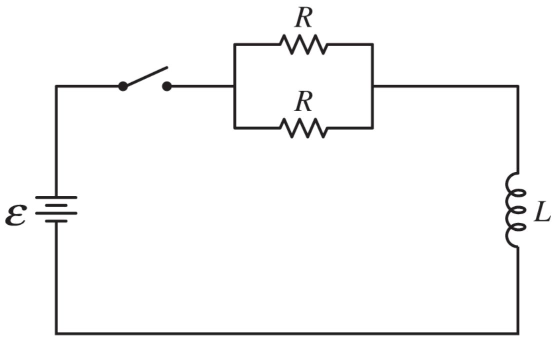


Figure 1

2. Students are asked to determine the resistance R of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that decreases with time to determine R.

(a)

i. On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that decreases with time.

Voltmeter Symbol

ii. **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using the measured quantity that decreases with time. Provide enough detail so that another student could replicate the experiment.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b)

i. On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.

Figure 2

ii. **Describe** how the information from the graph in part (b)(i) would be used to determine the experimental value for R.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(c) Starting with an appropriate application of Kirchhoff's loop rule, **derive**, but do NOT solve, a differential equation that can be used to determine the current I in the inductor at time t after the switch is closed. Express your answer in terms of R, $\mathcal{E}$, L, t, and physical constants, as appropriate.

After reaching steady state, the absolute value of the potential difference across the inductor is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across the new inductor is $|\Delta V_2|$.

(d) **Indicate** whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

____ $|\Delta V_2| > |\Delta V_1|$ ____ $|\Delta V_2| < |\Delta V_1|$ ____ $|\Delta V_2| = |\Delta V_1|$

Justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 2** on this page.

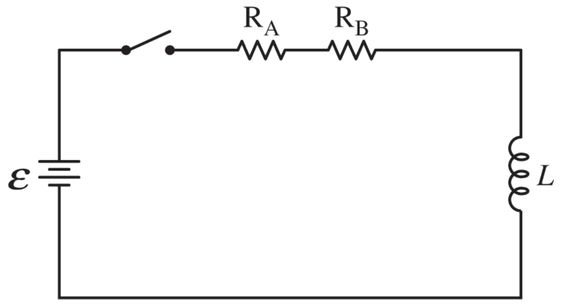


Figure 1

2. Students are asked to determine the resistance R of identical resistors R_A and R_B. The resistors are connected in series with each other, a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that increases with time to determine R.

(a)

i. On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that increases with time.

Voltmeter Symbol

ii. **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using a measured quantity that increases with time. Provide enough detail so that another student could replicate the experiment.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b)

i. On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.

Figure 2

ii. **Describe** how the information from the graph in part (b)(i) would be used to determine the experimental value for R.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(c) Starting with an appropriate application of Kirchhoff's loop rule, **derive,** but do NOT solve, a differential equation that can be used to determine the current I in the inductor at time t after the switch is closed. Express your answer in terms of R, $\mathcal{E}$, L, t, and physical constants, as appropriate.

After reaching steady state, the absolute value of the potential difference across R_A is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across R_A is $|\Delta V_2|$.

(d) **Indicate** whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

____ $|\Delta V_2| > |\Delta V_1|$ ____ $|\Delta V_2| < |\Delta V_1|$ ____ $|\Delta V_2| = |\Delta V_1|$

Justify your answer.

GO ON TO THE NEXT PAGE.

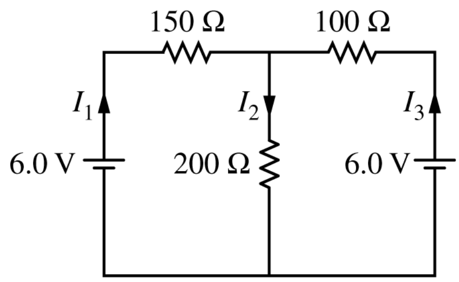


Figure 1

2. The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(a)

i. Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

ii. Calculate the current in the $200\ \Omega$ resistor.

iii. Calculate the power dissipated by the $200\ \Omega$ resistor.

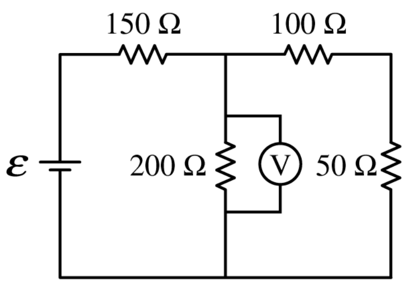


Figure 2

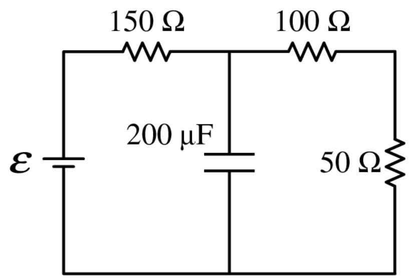
The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

(b) Calculate the current through the $50\ \Omega$ resistor.

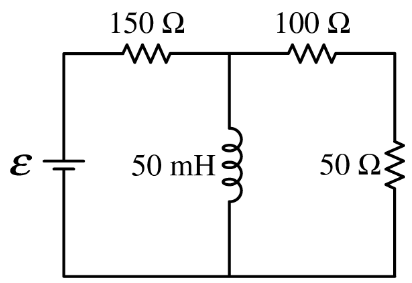
(c) Calculate the voltage $\mathcal{E}$ of the battery.

(d)

i. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu\text{F}$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.



ii. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b) ?



____ Greater than ____ Less than ____ Equal to

Justify your answer.

13.6 Circuits with Capacitors and Inductors (LC Circuits)

Name: ______________ Class: __________ Date: ____________ **Total: 10 marks**

KEY WORDS lc circuit LC 电路 • angular frequency 角频率 • oscillation 振荡 • simple harmonic motion 简谐运动 • inductor 电感器

Objective

Build the skills to answer exam questions on **circuits with capacitors and inductors (LC circuits)**.

You must be able to:

- describe the **oscillation 振荡** of charge and current in an LC circuit
- explain the **energy exchange** between the capacitor's electric field and the inductor's magnetic field
- relate the **angular frequency 角频率** to the elements, $\omega = \dfrac{1}{\sqrt{LC}}$
- compare LC oscillations with the **simple harmonic motion** of a mass on a spring

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ LC oscillation

With no resistance, the charge and current **oscillate**: energy sloshes back and forth between the capacitor's electric field and the inductor's magnetic field.

■ Angular frequency

$$\omega = \frac{1}{\sqrt{LC}}.$$

■ Like a mass on a spring

The LC circuit is the electrical analogue of simple harmonic motion (L plays the role of mass, $1/C$ the role of stiffness). Conservation of energy links the maximum charge to the maximum current: $\frac{1}{2}\dfrac{Q^2}{C} = \frac{1}{2}LI^2$.

2 Practice

2.1 Find the angular frequency of an LC circuit with $L = 2.0$ H and $C = 8.0$ μF. [2]

2.2 State what oscillates in an LC circuit. [1]

2.3 State the mechanical system that an LC circuit is analogous to. [1]

3 Exam-style questions

3.1 The angular frequency of an LC circuit is [1]

- **A** $\dfrac{1}{LC}$
- **B** $\dfrac{1}{\sqrt{LC}}$
- **C** $\sqrt{LC}$
- **D** LC

3.2 In an LC circuit, energy is exchanged between the [1]

- **A** resistor and the battery
- **B** capacitor and the inductor
- **C** two resistors
- **D** two batteries

3.3 An LC circuit has $L = 0.50$ H and $C = 20$ μF.

(a) Find the angular frequency of oscillation. [3]

(b) State the mechanical system this behaves like. [1]

Solutions

2.1 $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(2.0)(8.0 \times 10^{-6})}} = 250 \text{ rad s}^{-1}$.

2.2 the charge on the capacitor and the current in the inductor.

2.3 a mass on a spring (simple harmonic motion).

3.1 **B**.

3.2 **B**.

3.3 (a) $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.50)(20 \times 10^{-6})}} = \dfrac{1}{\sqrt{1.0 \times 10^{-5}}} = 316 \text{ rad s}^{-1}$. (b) a mass oscillating on a spring.