Week 13

AP Physics C: Electricity and Magnetism

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

AP Physics C: Electricity and Magnetism Name: ____________ Score: ______

1. Electric current is:

   ☐ charge times time

   ☐ the rate of charge flow, dQ/dt

   ☐ energy per charge

   ☐ force per charge

2. The EMF of a battery is:

   ☐ a force in newtons

   ☐ the energy given to each coulomb (volts)

   ☐ the current it carries

   ☐ its internal charge

3. Ohm's law relates voltage, current and resistance as:

   ☐ $V = I/R$

   ☐ $V = IR$

   ☐ $V = R/I$

   ☐ $V = I + R$

4. Electric power delivered to a component is:

   ☐ $P = IV$

   ☐ $P = I/V$

   ☐ $P = V/I$

   ☐ $P = I + V$

5. Two resistors in series have a total resistance of:

   ☐ $R_1 + R_2$

   ☐ less than either one

   ☐ $R_1 R_2$

   ☐ zero

6. If 10 C passes a point in 5 s, find the current (in A).

   ____________ A

7. Opening a switch anywhere in a simple loop stops the current everywhere.

☐ True ☐ False

8. A 9 V supply drives 3 A through a resistor. Find R (in Ω).

_________________ Ω

9. A device draws 4 A at 12 V. Find its power (in W).

_________________ W

10. Two 4 Ω resistors in parallel give what total resistance (in Ω)?

_________________ Ω

AP Physics C: Electricity and Magnetism

1. **the rate of charge flow,** dQ/dt

   $I = dQ/dt$ — charge passing a point per second.

2. **the energy given to each coulomb (volts)**

   EMF is energy per charge, in volts — not really a force.

3. $V = IR$

   $V = IR$ for an ohmic device.

4. $P = IV$

   $P = IV$ — current times voltage, in watts.

5. $R_1 + R_2$

   Series resistances add: $R = R_1 + R_2$.

6. **2 A**

   $I = Q/t = 10/5 = 2$ A.

7. **True**

   A break anywhere opens the loop, so no current flows at all.

8. **3** Ω

   $R = V/I = 9/3 = 3\ \Omega$.

9. **48 W**

   $P = IV = 4 \times 12 = 48$ W.

10. **2** Ω

    $1/R = 1/4 + 1/4 = 1/2$, so $R = 2\ \Omega$.

AP PHYSICS C: ELECTRICITY AND MAGNETISM • EXERCISE SHEET

11.1 Electric Current

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

KEY WORDS electric current 电流 • current density 电流密度 • drift velocity 漂移速度 • amperes 安培

Objective

Build the skills to answer exam questions on **electric current**.

You must be able to:

- define **electric current 电流** as the rate of flow of charge, $I = \dfrac{dQ}{dt}$
- distinguish **conventional current** from electron flow
- relate current to **current density 电流密度** and **drift velocity 漂移速度**
- apply **conservation of charge** to steady current

1 Worked examples

Study these first. Each one shows the method for a question type used later.

> **Current**
>
> $I = \dfrac{dQ}{dt}$, measured in amperes. Over a steady flow, $I = \dfrac{\Delta Q}{\Delta t}$.
>
> **Conventional current**
>
> The direction that **positive** charge would flow — opposite to the actual electron drift.
>
> **Microscopic picture**
>
> Current is carriers **drifting** slowly through the conductor; a potential difference across the conductor drives the flow, and charge is conserved.

2 Practice

2.1 A charge of 15 C flows past a point in 3.0 s. Find the current. [2]

2.2 State the direction of conventional current relative to the electron flow. [1]

2.3 State what drives a current through a conductor. [1]

---

3 Exam-style questions

3.1 Electric current is defined as [1]

- **A** Q/V
- **B** $\frac{dQ}{dt}$
- **C** It
- **D** V/R

3.2 Conventional current flows in the direction of [1]

- **A** electron flow
- **B** positive charge flow
- **C** neutron flow
- **D** no charge

3.3 A wire carries a steady current of 3.0 A.

(a) Find the charge that passes in 20 s. [2]

(b) State the conservation law obeyed by a steady current. [1]

Solutions

2.1 $I = \dfrac{\Delta Q}{\Delta t} = \dfrac{15}{3.0} = 5.0$ A.

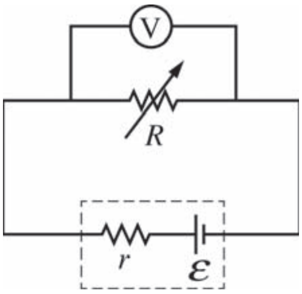
2.2 opposite to the electron flow.

2.3 a potential difference across it.

3.1 **B**.

3.2 **B**.

3.3 (a) $\Delta Q = I\,\Delta t = 3.0 \times 20 = 60$ C. (b) conservation of charge.



E&M.2.

A student performs an experiment to determine the emf $\mathcal{E}$ and internal resistance r of a given battery. The student connects the battery in series to a variable resistance R, with a voltmeter across the variable resistor, as shown in the figure above, and measures the voltmeter reading V as a function of the resistance R. The data are shown in the table below.

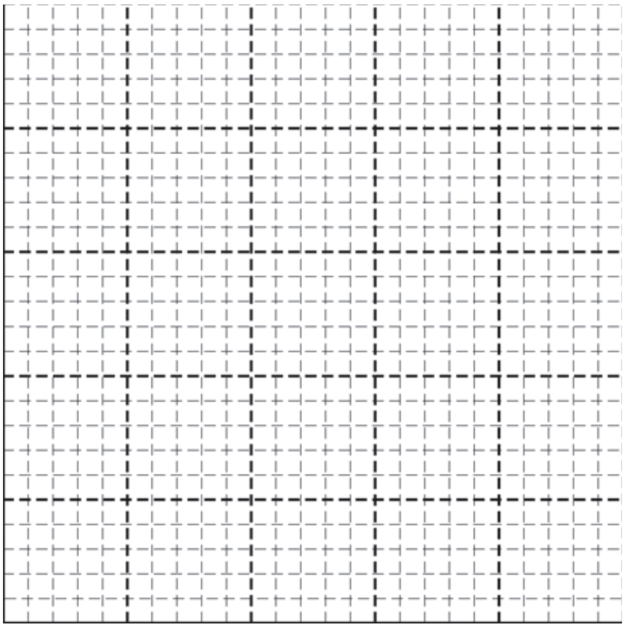
Trial #	Resistance (Ω)	Voltage (V)	$1/R$ ($1/\Omega$)	$1/V$ (1/V)
1	0.50	5.6	2.00	0.179
2	1.0	7.4	1.00	0.135
3	2.0	9.4	0.50	0.106
4	3.0	10.6	0.33	0.094
5	5.0	10.9	0.20	0.092
6	10	11.4	0.10	0.088

(a)

i. Derive an expression for the measured voltage V. Express your answer in terms of R, $\mathcal{E}$, r, and physical constants, as appropriate.

ii. Rewrite your expression from part (a)-i to express $1/V$ as a function of $1/R$.

(b) On the grid below, plot data points for the graph of $1/V$ as a function of $1/R$. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.



(c) Use the straight line from part (b) to obtain values for the following.

i. $\mathcal{E}$

ii. r

(d) Using the results of the experiment, calculate the maximum current that the battery can provide.

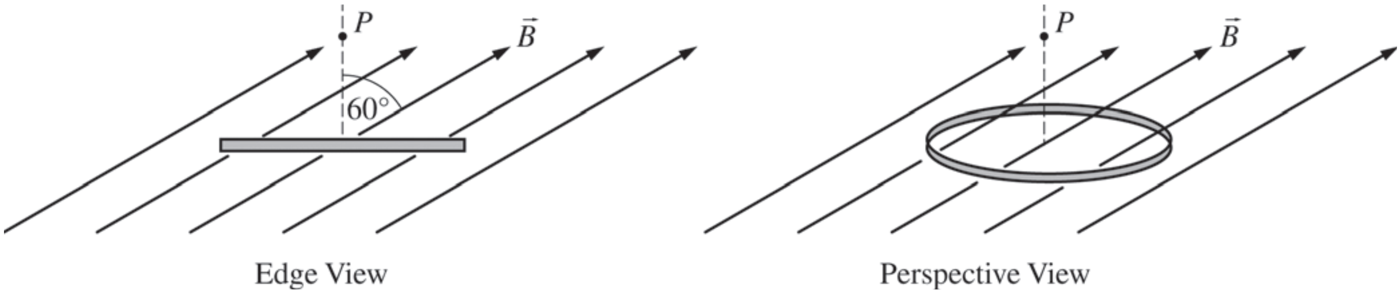
(e) A voltmeter is to be used to determine the emf of the battery after removing the battery from the circuit. Two voltmeters are available to take this measurement—one with low internal resistance and one with high internal resistance. Indicate which voltmeter will provide the most accurate measurement.

____ The voltmeter with low resistance will provide the most accurate measurement.

____ The voltmeter with high resistance will provide the most accurate measurement.

____ The two voltmeters will provide equal accuracy.

Justify your answer.



E&M. 3.

A circular wire loop with radius 0.10 m and resistance 50 Ω is held in place horizontally in a magnetic field $\vec{B}$ directed upward at an angle of 60° with the vertical, as shown in the figure above. The magnetic field in the direction shown is given as a function of time t by $B(t) = a(1 - bt)$, where $a = 4.0$ T and $b = 0.20\ s^{-1}$.

(a) Derive an expression for the magnetic flux through the loop as a function of time t.

(b) Calculate the numerical value of the induced emf in the loop.

(c)

i. Calculate the numerical value of the induced current in the loop.

ii. What is the direction of the induced current in the loop as viewed from point P ?

____ Clockwise ____ Counterclockwise

Justify your answer.

(d) Assuming the loop stays in its current position, calculate the energy dissipated in the loop in 4.0 seconds.

(e) Indicate whether the net magnetic force and net magnetic torque on the loop are zero or nonzero while the loop is in the magnetic field.

Net magnetic force: ____ Zero ____ Nonzero

Net magnetic torque: ____ Zero ____ Nonzero

Justify both of your answers.

STOP

END OF EXAM

11.2 Electric Circuits

Name: ______________ Class: __________ Date: ____________ **Total: 9 marks**

KEY WORDS series 串联 • parallel 并联 • source of emf 电动势源 • short circuit 短路 • resistance 电阻

Objective

Build the skills to answer exam questions on **electric circuits**.

You must be able to:

- identify the components of a circuit: **source of emf 电动势源**, resistors, wires, switches
- distinguish an **open**, a **closed**, and a **short circuit 短路**
- describe an **ideal battery** as a source of constant potential difference
- state how **ammeters** and **voltmeters** are connected

1 Worked examples

Study these first. Each one shows the method for a question type used later.

> ■ **Components**
>
> A source of emf, resistors, connecting wires, and switches — each with a standard schematic symbol.
>
> ■ **Circuit states**
>
> - **Open** — broken loop, no current.
> - **Closed** — complete loop, current flows.
> - **Short** — a very low-resistance path that lets a large current bypass a component.
>
> ■ **Meters**
>
> An **ammeter** goes **in series** (to read the current through a branch); a **voltmeter** goes **in parallel** (to read the potential difference across a component).

2 Practice

2.1 State how an ammeter is connected in a circuit. [1]

2.2 State how a voltmeter is connected. [1]

2.3 State the difference between an open and a closed circuit. [2]

3 Exam-style questions

3.1 An ammeter is connected in [1]

- **A** series
- **B** parallel
- **C** either way
- **D** neither way

3.2 A short circuit is a path of [1]

- **A** very high resistance
- **B** very low resistance
- **C** no current at all
- **D** infinite voltage

3.3 A circuit has a battery, a resistor, an ammeter, and a voltmeter across the resistor.

(a) State how the ammeter is connected. [1]

(b) State how the voltmeter is connected. [1]

(c) Explain why a voltmeter should have a very high resistance. [1]

Solutions

2.1 in series.

2.2 in parallel with the component.

2.3 a closed circuit is a complete loop that carries current; an open circuit is broken, so no current flows.

3.1 **A**.

3.2 **B**.

3.3 (a) in series. (b) in parallel with the resistor. (c) so it draws almost no current and does not disturb the circuit it measures.

Begin your response to **QUESTION 2** on this page.

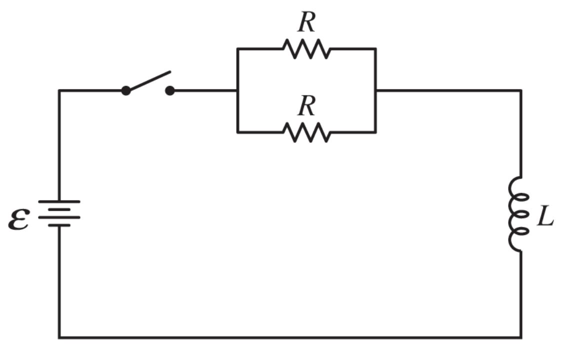


Figure 1

2. Students are asked to determine the resistance R of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that decreases with time to determine R.

(a)

i. On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that decreases with time.

Voltmeter Symbol

ii. **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using the measured quantity that decreases with time. Provide enough detail so that another student could replicate the experiment.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b)

i. On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.

Figure 2

ii. **Describe** how the information from the graph in part (b)(i) would be used to determine the experimental value for R.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(c) Starting with an appropriate application of Kirchhoff's loop rule, **derive**, but do NOT solve, a differential equation that can be used to determine the current I in the inductor at time t after the switch is closed. Express your answer in terms of R, $\mathcal{E}$, L, t, and physical constants, as appropriate.

After reaching steady state, the absolute value of the potential difference across the inductor is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across the new inductor is $|\Delta V_2|$.

(d) **Indicate** whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

____ $|\Delta V_2| > |\Delta V_1|$ ____ $|\Delta V_2| < |\Delta V_1|$ ____ $|\Delta V_2| = |\Delta V_1|$

Justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 2** on this page.

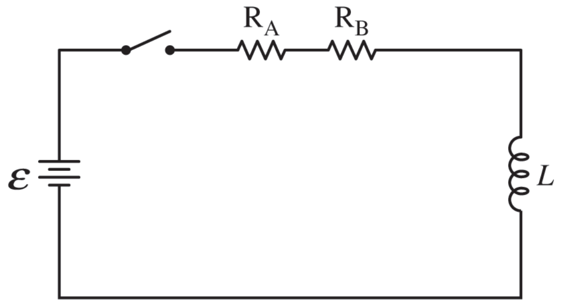


Figure 1

2. Students are asked to determine the resistance R of identical resistors R_A and R_B. The resistors are connected in series with each other, a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that increases with time to determine R.

(a)

i. On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that increases with time.

Voltmeter Symbol

ii. **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using a measured quantity that increases with time. Provide enough detail so that another student could replicate the experiment.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b)

i. On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.

Figure 2

ii. **Describe** how the information from the graph in part (b)(i) would be used to determine the experimental value for R.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(c) Starting with an appropriate application of Kirchhoff's loop rule, **derive,** but do NOT solve, a differential equation that can be used to determine the current I in the inductor at time t after the switch is closed. Express your answer in terms of R, $\mathcal{E}$, L, t, and physical constants, as appropriate.

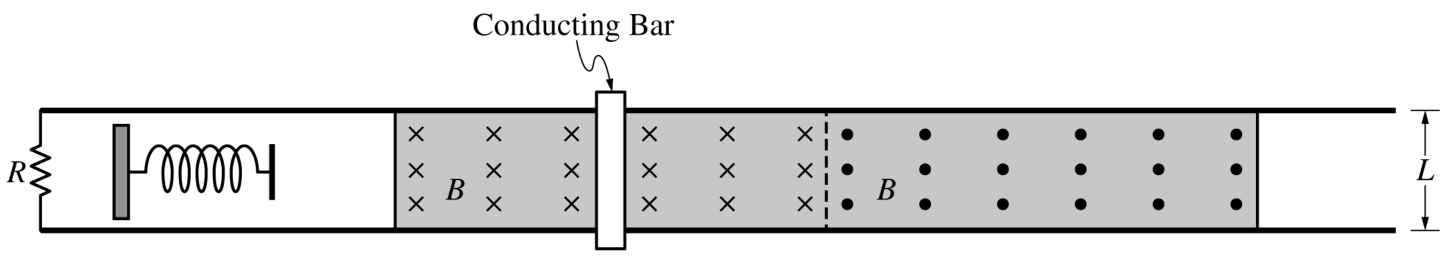
After reaching steady state, the absolute value of the potential difference across R_A is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across R_A is $|\Delta V_2|$.

(d) **Indicate** whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

____ $|\Delta V_2| > |\Delta V_1|$ ____ $|\Delta V_2| < |\Delta V_1|$ ____ $|\Delta V_2| = |\Delta V_1|$

Justify your answer.

GO ON TO THE NEXT PAGE.



Top View (at time t_B)

. Two horizontal, parallel, conducting rails are separated by distance $L = 0.40$ m. A resistor of resistance $R = 0.30\ \Omega$ connects the rails. A horizontal ideal spring is located between the rails. The right end of the spring is free to move and the left end is fixed in place. A conducting bar of mass $m = 0.23$ kg is placed on the rails and is in contact with the spring, which is initially compressed. Frictional forces and the resistance of the bar and rails are negligible.

- At time $t = 0$, the bar is released from rest and is pushed to the right by the spring.
- At time t_1, the bar loses contact with the spring and slides to the right.
- At time t_2, the bar enters and travels through a uniform magnetic field of magnitude $B = 0.50$ T that is directed into the page, as shown.
- At time t_3, the bar enters a region where the magnitude of the uniform magnetic field is still $B = 0.50$ T but is directed out of the page.
- At time t_4, the bar enters a region with no magnetic field.

:onsider time t_B such that $t_2 < t_B < t_3$.

(a) On the following diagram of the bar, draw an arrow indicating the direction of the net force F_{net} exerted on the bar at time t_B. If the net force is zero, write $F_{net} = 0$.

GO ON TO THE NEXT PAGE.

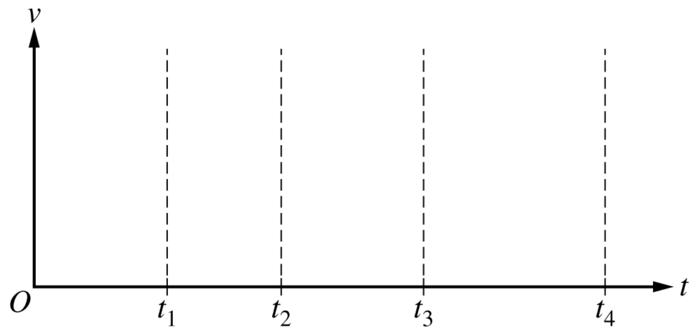
Continue your response to **QUESTION 2** on this page.

(b) At time t_B, the speed of the bar is $v = 2.5$ m/s.

i. Calculate the magnitude of the current in the bar at time t_B.

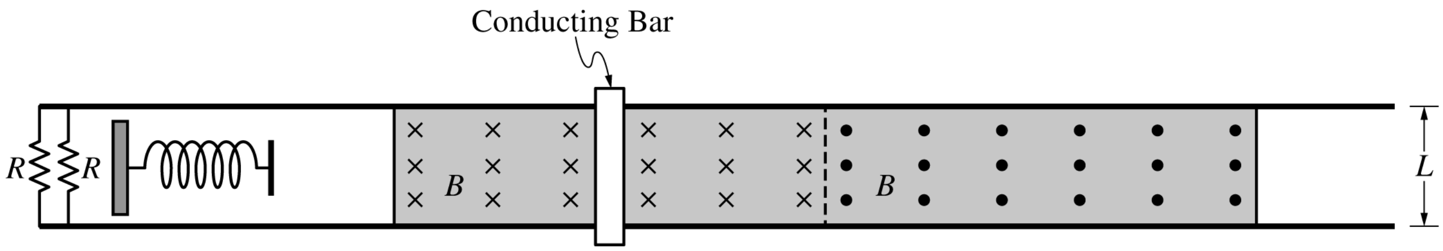
ii. Calculate the magnitude of the net force F_{net} exerted on the bar at time t_B.

(c) On the following axes, sketch a graph of the speed v of the bar as a function of time t between $t = 0$ and t_4.



GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.



Top View (at time t_B)

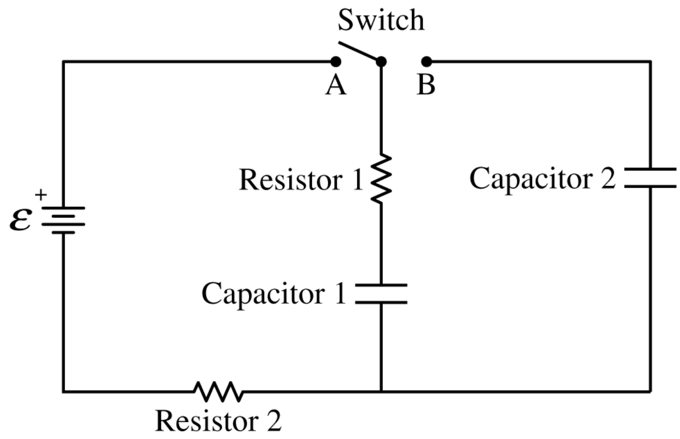
(d) The scenario is repeated but an additional resistor of resistance $R = 0.30\ \Omega$ is connected, as shown.

i. Determine the total resistance R_{total} of the closed circuit for the new scenario.

ii. In the original scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is $a_{original}$. In the new scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{new}. Is a_{new} greater than, less than, or equal to $a_{original}$? Justify your answer.

(e) Describe a modification to m, B, or L that will result in a smaller induced potential difference across the original resistor immediately after the bar enters the first uniform magnetic field. Justify your answer.

GO ON TO THE NEXT PAGE.



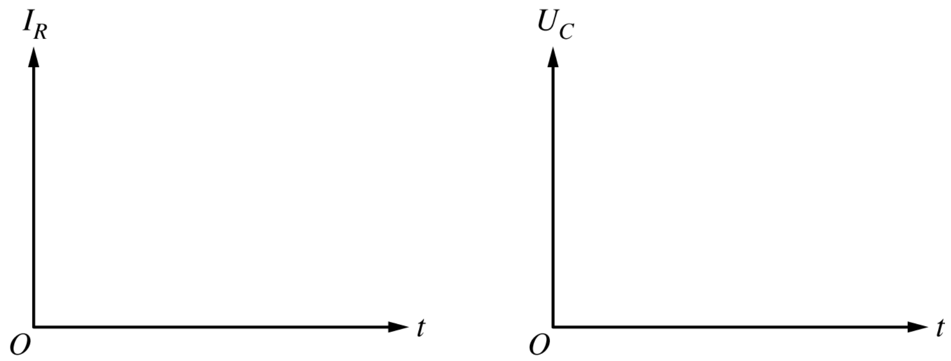
. The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

(b) On the axes shown, sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.



A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

(c) At time t_1, the switch is closed to Position B.

i. Immediately after t_1, is the direction of the current in Resistor 1 directed up, directed down, or is there no current? Briefly justify your answer.

ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

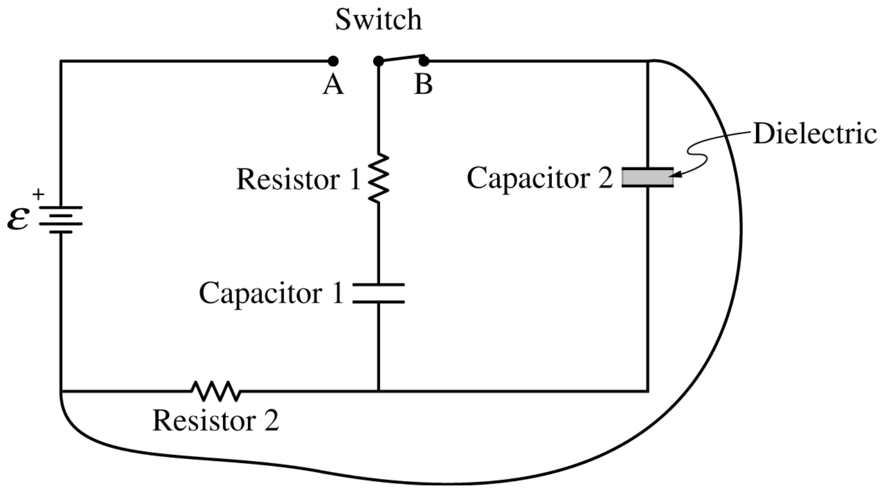
iii. Derive an expression for the total energy dissipated by Resistor 1 immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.

With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

(d) Determine the charge on the positive plate of Capacitor 2 a long time after the dielectric has been inserted. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.



With the switch still closed to Position B, a wire of negligible resistance is connected between two corners of the circuit, as shown.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.

i. Derive an expression for the current in Resistor 2 immediately after the wire is connected to the circuit.

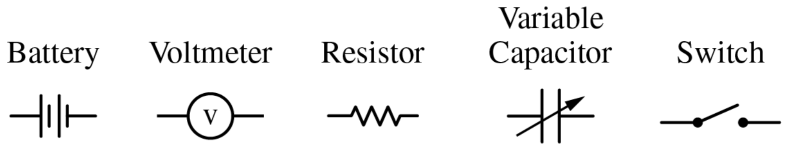
ii. Determine the current in Resistor 2 a long time after the wire is connected to the circuit.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

. The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.



(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the potential difference across the capacitor as it discharges through the resistor.

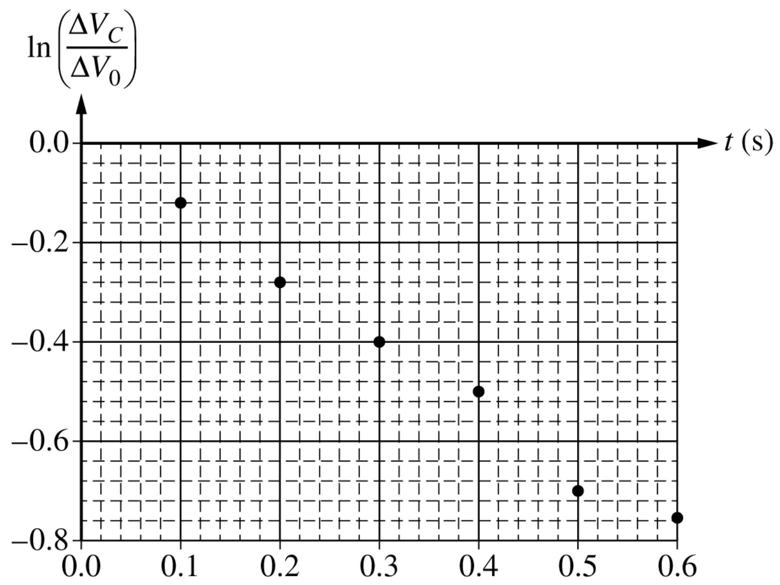
The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

(b) Show that the potential difference ΔV_C across the capacitor as a function of time t is $\Delta V_C(t) = \Delta V_0 e^{-\frac{t}{RC}}$ as the capacitor discharges.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(c) The experiment is performed using a resistor of $R = 150\,\text{k}\Omega$. Data for the potential difference ΔV_C across the capacitor as a function of t are recorded and a plot of $\ln\left(\frac{\Delta V_C}{\Delta V_0}\right)$ as a function of t is created on the graph below.



i. Draw the best-fit line for the data.

ii. Using the best-fit line, calculate a value for the unknown capacitance C.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) The capacitor is adjusted so that the surface area of the plates is increased, and the experiment is repeated. Would the slope of the best-fit line in the second experiment be more steep, less steep, or unchanged compared to the slope of the best-fit line in part (c)?

___ More steep ___ Less steep ___ Unchanged

Briefly justify your answer.

(e) The ideal battery is then replaced with a non-ideal battery with internal resistance r, and the experiment is repeated.

i. Would the slope of the graph in this final experiment change compared to the graph in part (c)?

___ Yes ___ No

Briefly justify your answer.

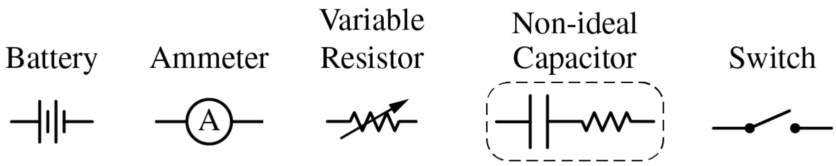
ii. Would the vertical intercept of the graph in this final experiment change compared to the graph in part (c)?

___ Yes ___ No

Briefly justify your answer.

GO ON TO THE NEXT PAGE.

. A non-ideal capacitor has internal resistance that can be modeled as an ideal capacitor in series with a small resistor of resistance r_C. A group of students performs an experiment to determine the internal resistance of a capacitor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single ammeter, a single variable resistor of resistance R, a single uncharged non-ideal capacitor of capacitance C, and one or more switches as needed.



(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the current through the capacitor as it discharges through the resistor.

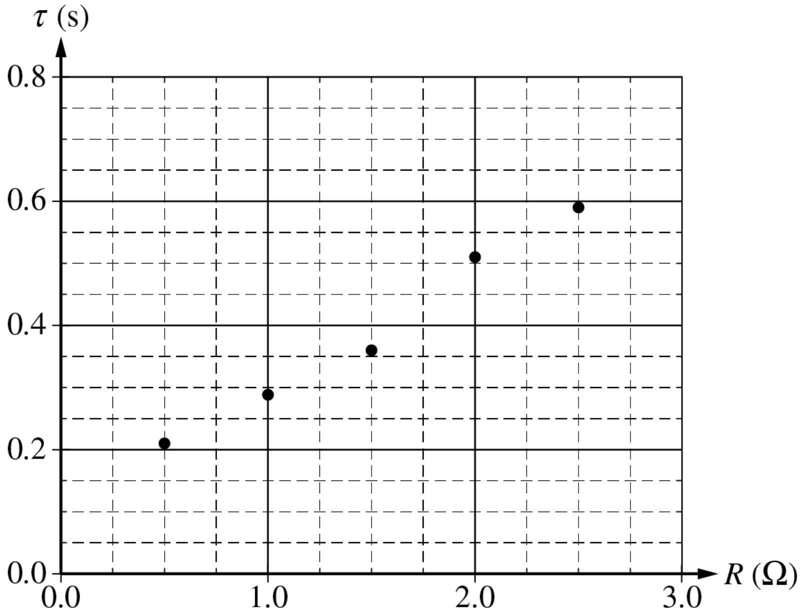
The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

(b) Show that the current I through the capacitor as a function of time t is $I(t) = I_0 e^{\frac{-t}{(R+r_C)C}}$ as the capacitor discharges.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(c) The students determine the time constant τ for the circuit as a function of the resistance R. The students' data are shown in the following graph.



i. Draw the best-fit line for the data.

ii. Using the best-fit line, calculate a value for the internal resistance r_C of the capacitor.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) The ammeter is found to be nonideal. Is the actual value for the internal resistance r_C for the capacitor greater than, less than, or equal to the experimental internal resistance of the capacitor calculated in part (c)?

___ Greater than   ___ Less than   ___ Equal to

Briefly justify your answer using features of the graph in part (c).

(e) The values of the variable resistor in the original experiment ranged from $0.5\ \Omega$ to $2.5\ \Omega$. The experiment is repeated with values ranging from $3.0\ \Omega$ to $6.0\ \Omega$. Would the slope of the best-fit line be more steep, be less steep, or remain unchanged compared to the graph in part (c)?

___ More steep   ___ Less steep   ___ Remain unchanged

Briefly justify your answer.

GO ON TO THE NEXT PAGE.

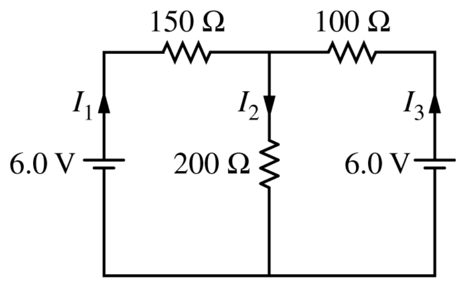


Figure 1

2. The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(a)

i. Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

ii. Calculate the current in the $200\ \Omega$ resistor.

iii. Calculate the power dissipated by the $200\ \Omega$ resistor.

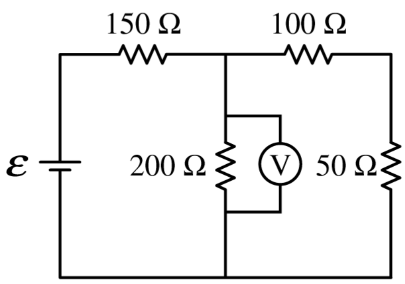


Figure 2

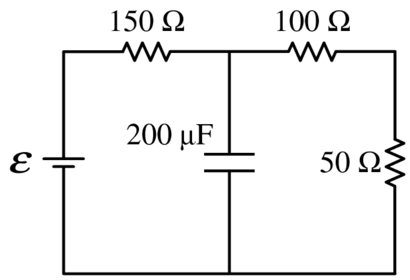
The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

(b) Calculate the current through the $50\ \Omega$ resistor.

(c) Calculate the voltage $\mathcal{E}$ of the battery.

(d)

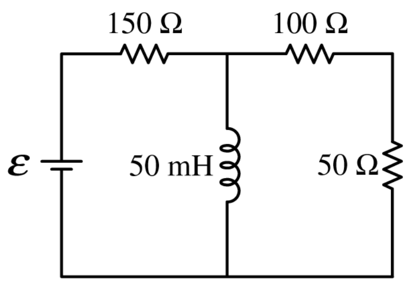
i. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu\text{F}$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.

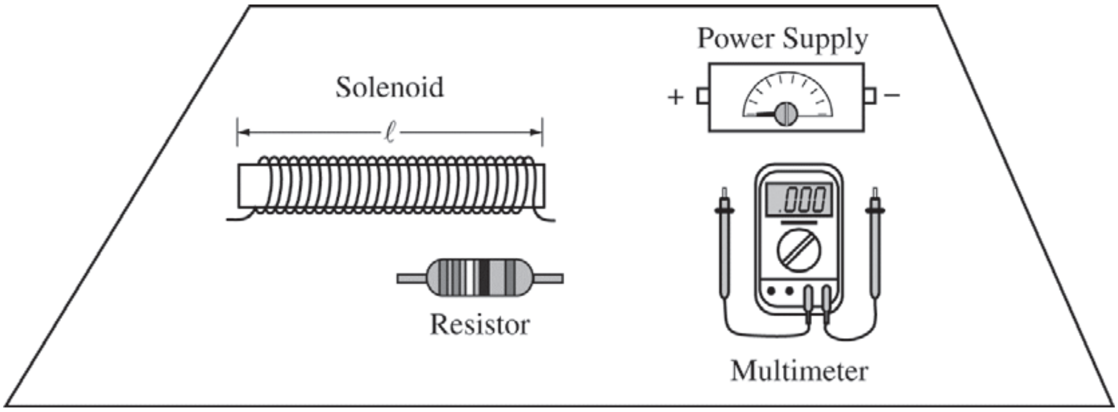


ii. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b) ?

____ Greater than ____ Less than ____ Equal to

Justify your answer.





3. When studying Ampere's law, students collect data on the magnetic field of two different solenoids in order to determine the magnetic permeability of free space μ_0. The solenoids are created by wrapping wire around a hollow plastic tube. The solenoids of length ℓ with N turns of wire will be connected in series to a power supply and resistor. A multimeter will be used as an ammeter to measure the magnitude of the current I through the solenoids. The main components for the setup with one of the solenoids are shown in the figure above.

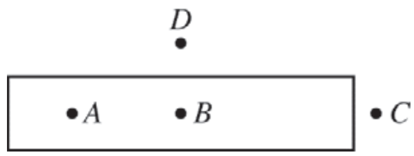
(a)

i. On the figure above, draw wire connections between the solenoid, power supply, resistor, and multimeter that will complete the circuit and allow students to measure the magnitude of the current through the solenoid.

ii. Using the connections you made in part (a)i above, what will be the direction of the magnetic field inside the solenoid?

____ Toward the top of the page	_____ To the left	____ Out of the page
____ Toward the bottom of the page	_____ To the right	____ Into the page

The rectangle shown below represents the solenoid (the loops of wire are not shown). Points A, B, and C are along the central axis of the solenoid with point B at the middle of the solenoid. Point D is directly above point B.

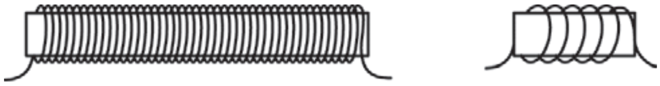


iii. From the choices below, select the point where you would place a magnetic field probe (a probe that can measure the magnitude of the magnetic field) to best measure the strength of the magnetic field of the solenoid in order to determine the magnetic permeability of free space μ_0.

____ A ____ B ____ C ____ D

Justify your answer based on the model for a simple solenoid.

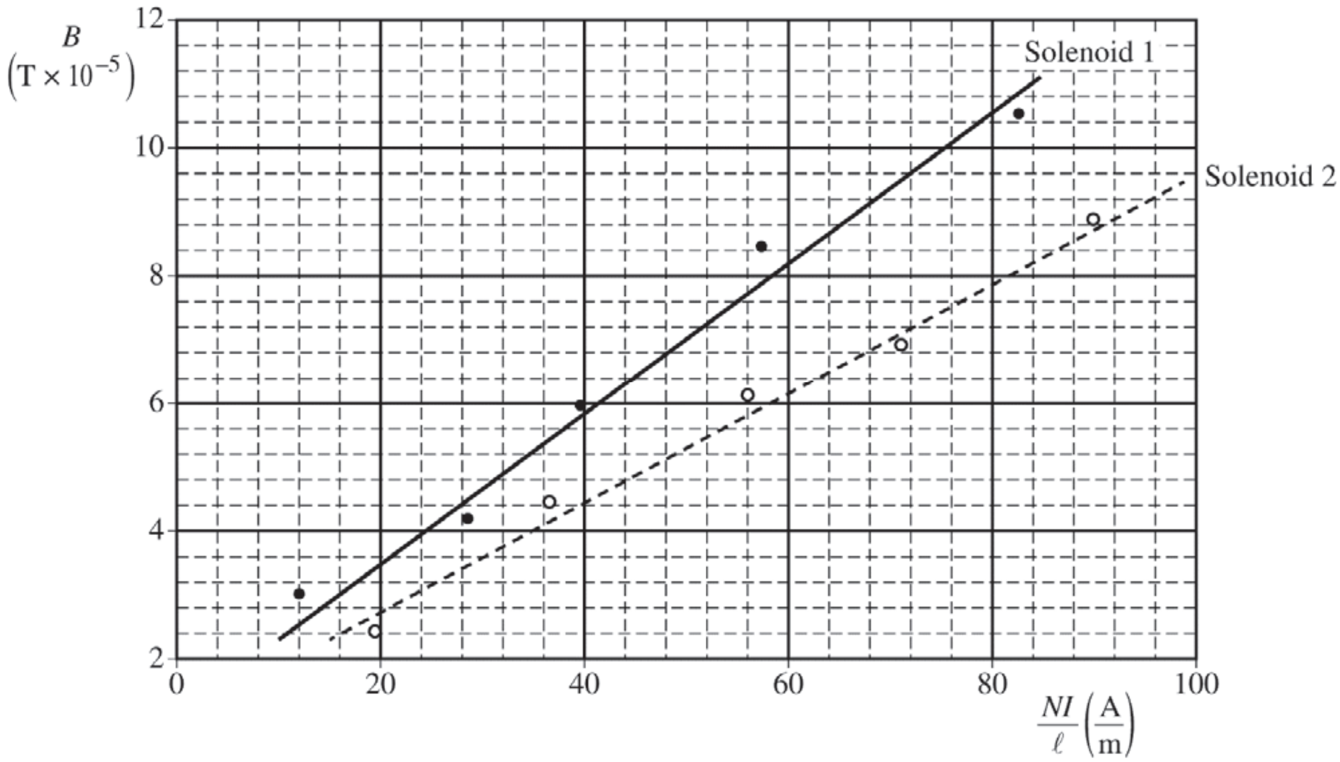
The figures below show two different solenoids that will be connected in the circuit above. Solenoid 1 has a length $\ell = 25$ cm with $N = 100$ turns. Solenoid 2 has a length $\ell = 5.0$ cm with $N = 5$ turns.



Solenoid 1 Solenoid 2

Note: Figures not drawn to scale.

A graph of the magnitude of the magnetic field B as a function of NI/ℓ is shown below. The best-fit lines for the data are shown as a solid line for solenoid 1 and as a dashed line for solenoid 2.



(b) Which solenoid's best-fit line would give the best results for determining a value for the magnetic permeability of free space μ_0?

____ Solenoid 1 ____ Solenoid 2

Justify your answer.

(c)

i. Use the slope of the best-fit line for the solenoid chosen in part (b) to calculate the magnetic permeability of free space μ_0.

ii. Calculate the percent error for the experimental value of the magnetic permeability of free space μ_0 determined in part (c)i.

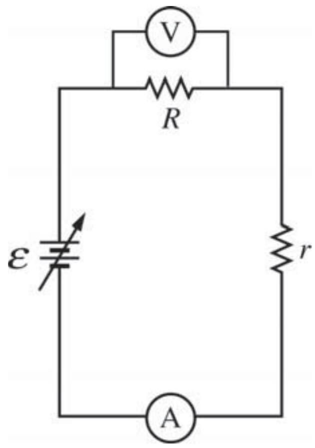
(d)

i. What is a reasonable physical explanation for a best-fit line that does not pass through the origin?

ii. Suppose a student connects the solenoid in a closed circuit similar to the circuit in part (a)i but without the resistor. The student notices the multimeter stops functioning after the power supply is turned on. Explain what causes the failure of the multimeter.

STOP

END OF EXAM



E&M.2.

The circuit shown above consists of a source of variable emf $\mathcal{E}$, an ideal ammeter A, an ideal voltmeter V, a resistor of resistance *R*, and a sample of wire with resistance *r*.

(a) How does the current through the wire sample compare with the current through the resistor *R* ?

_____ It is greater through *R*. _____ It is greater through the sample.

_____ It is the same through both. _____ It depends on the resistance of the sample.

Justify your answer.

(b) How does the potential difference across the wire sample compare with the potential difference across the resistor *R* ?

_____ It is greater across *R*. _____ It is greater across the sample.

_____ It is the same across both. _____ It depends on the resistance of the sample.

Justify your answer.

With the sample of wire in place, the emf of the source is set to a given value. The current through and potential difference across the resistor *R* are measured. This is repeated for several values of emf, and the data are recorded in the table below.

$\mathcal{E}$ (V)	V_R (V)	I_R (A)		
0.250	0.179	0.162		
0.500	0.335	0.327		
0.750	0.520	0.490		
1.000	0.670	0.687		

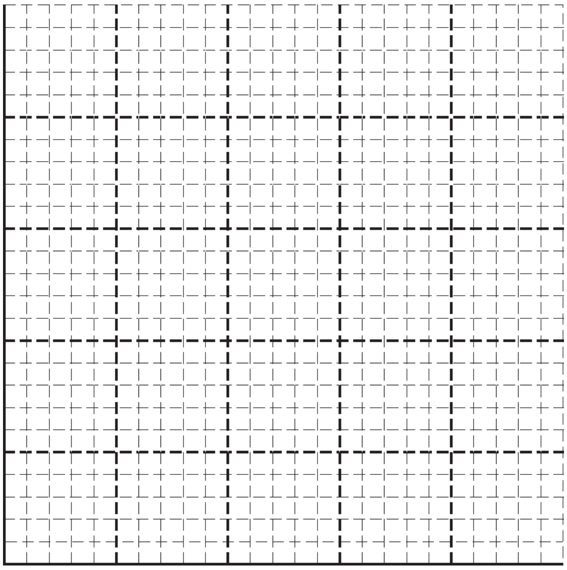
(c) Indicate below which quantities should be graphed to yield a straight line that could be used to calculate a numerical value for the resistance of the wire sample.

Horizontal axis: _____________

Vertical axis: _____________

You may use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given.

(d) On the grid below, plot the straight line data points from part (c). Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(e) Use your straight line to calculate the value of the resistance of the wire sample.

(f) The wire sample has a length of 3.00 m and a radius of 1.00×10^{-3} m . Calculate the resistivity of the material from which the wire sample is made.

(g)

i. Suppose the ammeter used to collect these data was not ideal. Would the actual value of the resistance of the wire sample be greater than, less than, or equal to that calculated in part (e) ?

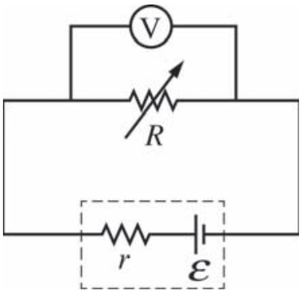
____ Greater than ____ Less than ____ Equal to

Justify your answer.

ii. If the ideal voltmeter is replaced by a voltmeter that is not ideal and the experiment is repeated, would the readings of the ideal ammeter be greater than, less than, or equal to those in the data chart before part (c) ?

____ Greater than ____ Less than ____ Equal to

Justify your answer.



E&M.2.

A student performs an experiment to determine the emf $\mathcal{E}$ and internal resistance r of a given battery. The student connects the battery in series to a variable resistance R, with a voltmeter across the variable resistor, as shown in the figure above, and measures the voltmeter reading V as a function of the resistance R. The data are shown in the table below.

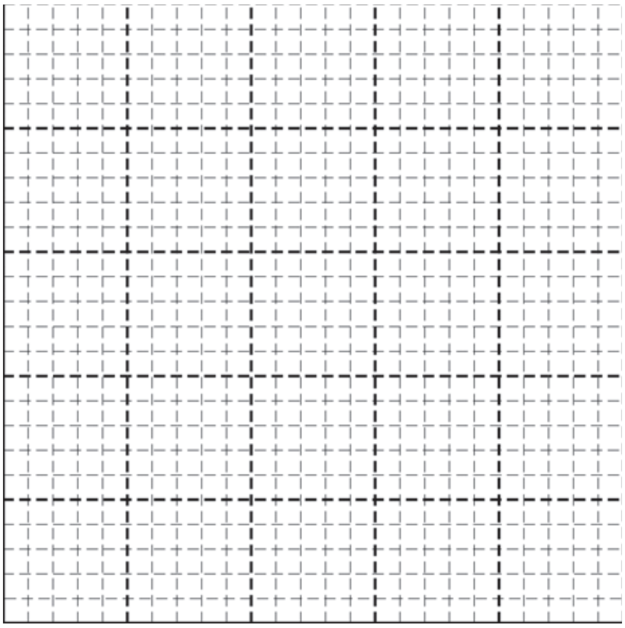
Trial #	Resistance (Ω)	Voltage (V)	$1/R$ (1/Ω)	$1/V$ (1/V)
1	0.50	5.6	2.00	0.179
2	1.0	7.4	1.00	0.135
3	2.0	9.4	0.50	0.106
4	3.0	10.6	0.33	0.094
5	5.0	10.9	0.20	0.092
6	10	11.4	0.10	0.088

(a)

i. Derive an expression for the measured voltage V. Express your answer in terms of R, $\mathcal{E}$, r, and physical constants, as appropriate.

ii. Rewrite your expression from part (a)-i to express $1/V$ as a function of $1/R$.

(b) On the grid below, plot data points for the graph of $1/V$ as a function of $1/R$. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.



(c) Use the straight line from part (b) to obtain values for the following.

i. $\mathcal{E}$

ii. r

(d) Using the results of the experiment, calculate the maximum current that the battery can provide.

(e) A voltmeter is to be used to determine the emf of the battery after removing the battery from the circuit. Two voltmeters are available to take this measurement—one with low internal resistance and one with high internal resistance. Indicate which voltmeter will provide the most accurate measurement.

____ The voltmeter with low resistance will provide the most accurate measurement.

____ The voltmeter with high resistance will provide the most accurate measurement.

____ The two voltmeters will provide equal accuracy.

Justify your answer.

11.3 Resistance, Resistivity, and Ohm's Law

Name: ____________ Class: ________ Date: __________ **Total: 11 marks**

KEY WORDS resistance 电阻 • resistivity 电阻率 • ohm's law 欧姆定律 • ohmic 欧姆性 • series 串联

Objective

Build the skills to answer exam questions on **resistance, resistivity, and Ohm's law**.

You must be able to:

- state **Ohm's law** 欧姆定律, $V = IR$, and identify ohmic behaviour
- relate **resistance** 电阻 to geometry, $R = \dfrac{\rho L}{A}$
- distinguish **resistivity** 电阻率 (a material property) from resistance (an object property)
- calculate **equivalent resistance** in series and parallel

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Ohm's law

$V = IR$. A device is **ohmic** if R is constant (a straight I-V line).

■ Resistance and resistivity

$R = \dfrac{\rho L}{A}$: **resistivity** ρ is a property of the **material**; **resistance** R also depends on the length L and area A of the object.

■ Combining

Series: $R = R_1 + R_2 + \ldots$; parallel: $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \ldots$

2 Practice

2.1 A 12 V supply drives 3.0 A through a resistor. Find its resistance. [2]

2.2 State how doubling a wire's length changes its resistance (same material and area). [1]

2.3 State the difference between resistivity and resistance. [2]

3 Exam-style questions

3.1 Resistance relates to geometry by [1]

- **A** $\frac{\rho A}{L}$
- **B** $\frac{\rho L}{A}$
- **C** $\frac{L}{\rho A}$
- **D** $\rho L A$

3.2 Resistivity is a property of the [1]

- **A** object's shape
- **B** material
- **C** current
- **D** voltage

3.3 A wire has resistivity $1.7 \times 10^{-8}\ \Omega\,\mathrm{m}$, length 2.0 m, and area $1.0 \times 10^{-6}\ \mathrm{m}^2$.

(a) Find its resistance. [2]

(b) The wire is stretched to twice its length (its volume unchanged, so its area halves). Find the new resistance. [2]

Solutions

2.1 $R = \dfrac{V}{I} = \dfrac{12}{3.0} = 4.0\ \Omega$.

2.2 it doubles ($R \propto L$).

2.3 resistivity is a property of the material alone; resistance also depends on the object's length and cross-sectional area.

3.1 **B**.

3.2 **B**.

3.3 (a) $R = \dfrac{\rho L}{A} = \dfrac{(1.7 \times 10^{-8})(2.0)}{1.0 \times 10^{-6}} = 0.034\ \Omega$. (b) length $\times 2$ and area $\div 2$ give $R \times 4 = 0.136\ \Omega$.

Question 2: Version J

2. A rotating, circular, conducting loop of area A and resistance R is in an external uniform magnetic field of magnitude B that is directed in the $-z$-direction. At time $t = 0$, the magnetic field is perpendicular to the plane of the loop, as shown in Figure 1. The loop is rotating with constant angular speed ω and period T about the dashed line that is along the diameter of the loop. The value of the magnetic flux through the loop as a function of time t is $\Phi = BA\cos(\omega t)$.

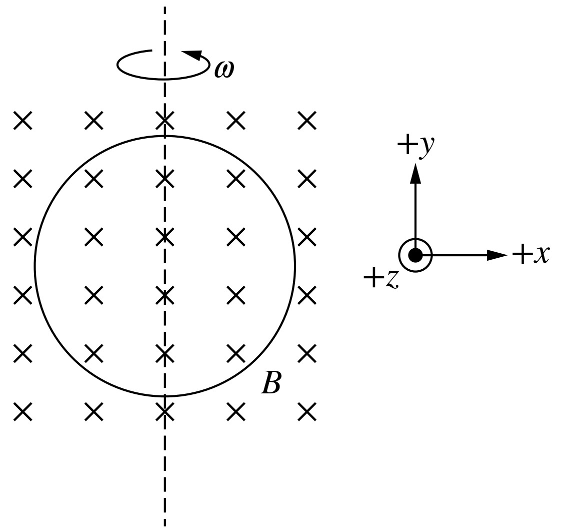


Figure 1

A. The absolute value of the induced emf in the loop is $|\varepsilon|$. The partially completed bar chart in Figure 2 shows a bar that represents $|\varepsilon|$ at $t = \frac{3}{4}T$. In Figure 2, **draw** bars to represent $|\varepsilon|$ at times $t = 0$, $\frac{1}{4}T$, and $\frac{1}{2}T$ relative to $|\varepsilon|$ shown at $\frac{3}{4}T$. If $|\varepsilon| = 0$, **write** a “0” in that column.

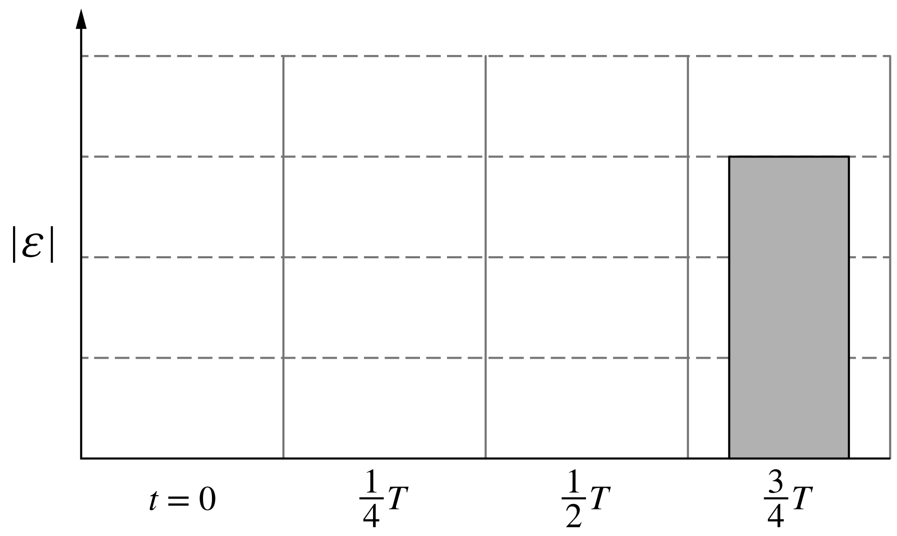


Figure 2

B. Derive an expression for the maximum induced current in the loop in terms of A, R, B, ω, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

C. On the axes shown in Figure 3, **sketch** a graph of the instantaneous power P dissipated by the loop as a function of t during the time interval $0 \le t \le T$.

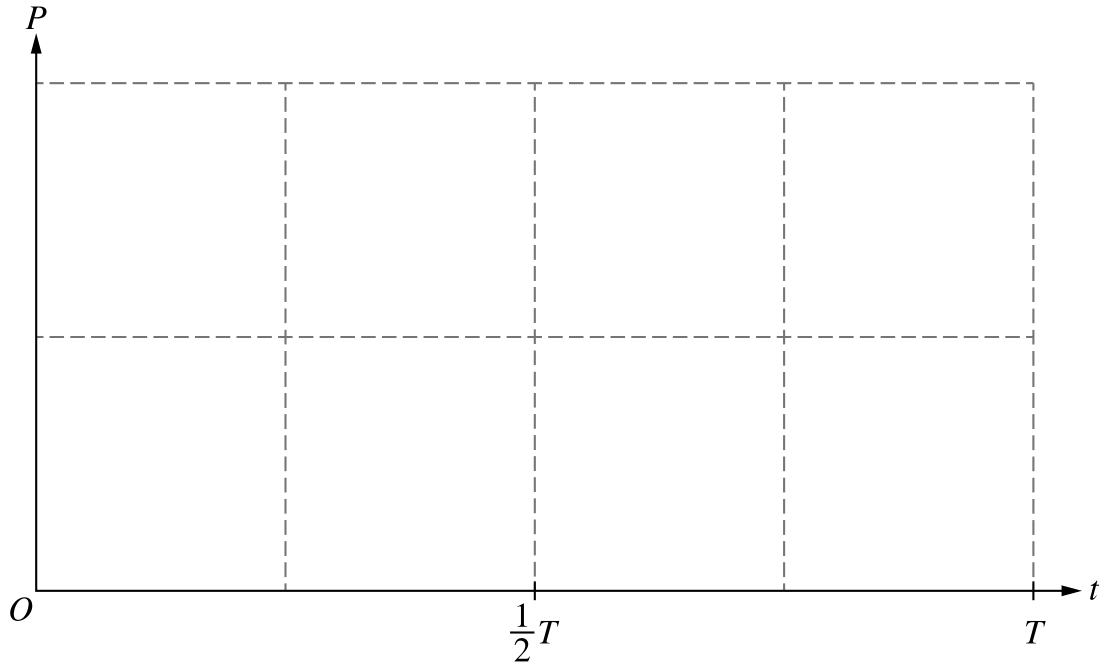


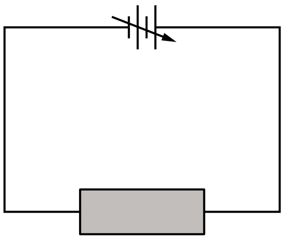
Figure 3

D. Indicate whether the sketch you drew in part C is or is not consistent with the bars that you drew in part A. Briefly **justify** your answer by referencing the functional dependence between P and $|\varepsilon|$.

Question 3

3. In Experiment 1, students are asked to use a graph to determine the resistivity ρ_1 of a circuit element that is connected to a variable power supply, as shown in Figure 1. The circuit element is cylindrical and has uniform resistivity. The students have access to a voltmeter, an ammeter, and a ruler.

Figure 1



A. Describe a procedure for collecting data that would allow the students to use a graph to determine ρ_1, including any steps necessary to reduce experimental uncertainty.

B. Describe how the collected data could be graphed and how that graph would be analyzed to determine ρ_1.

In Experiment 2, the students are asked to use a graph to determine the resistivity ρ_2 of solid, cylindrical resistors made of the same material but of different lengths L. The cross-sectional area of each resistor is $5.0 \times 10^{-6}\ \mathrm{m}^2$. The students directly measure the resistance R between the ends of each resistor. Table 1 provides L and R for each resistor.

Table 1

L (m)	R (Ω)
0.010	0.90
0.020	1.6
0.030	2.5
0.040	3.2
0.050	4.0

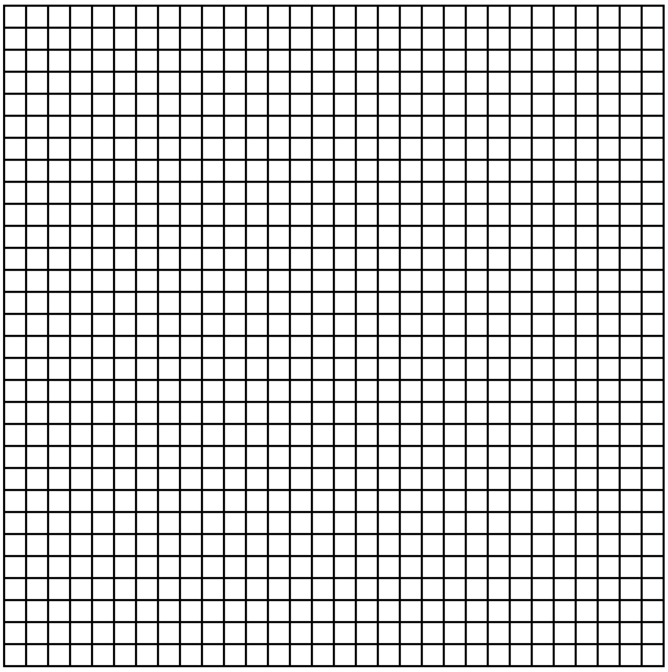
C.

i. **Indicate** two quantities, either measured quantities from Table 1 or additional calculated quantities, that could be graphed to produce a straight line that could be used to determine ρ_2.

Vertical axis: ______ Horizontal axis: ______

ii. On the grid provided, create a graph of the quantities indicated in part C (i).

- Use Table 2 to record the measured or calculated quantities that you will plot.
- Clearly **label** the axes, including units as appropriate.
- **Plot** the points you recorded in Table 2.



iii. **Draw** a best-fit line for the data graphed in part C (ii).

D. Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for ρ_2.

Begin your response to **QUESTION 2** on this page.

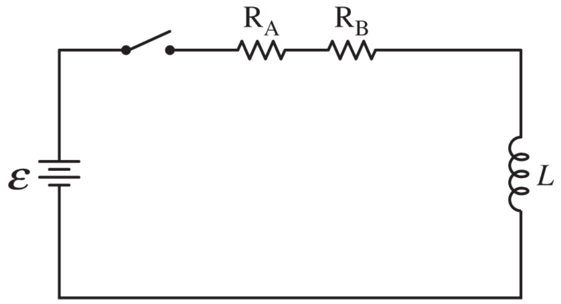


Figure 1

2. Students are asked to determine the resistance R of identical resistors R_A and R_B. The resistors are connected in series with each other, a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that increases with time to determine R.

(a)

i. On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that increases with time.

Voltmeter Symbol

ii. **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using a measured quantity that increases with time. Provide enough detail so that another student could replicate the experiment.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b)

i. On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.

Figure 2

ii. **Describe** how the information from the graph in part (b)(i) would be used to determine the experimental value for R.

GO ON TO THE NEXT PAGE.

(c) Starting with an appropriate application of Kirchhoff's loop rule, **derive,** but do NOT solve, a differential equation that can be used to determine the current I in the inductor at time t after the switch is closed. Express your answer in terms of R, $\mathcal{E}$, L, t, and physical constants, as appropriate.

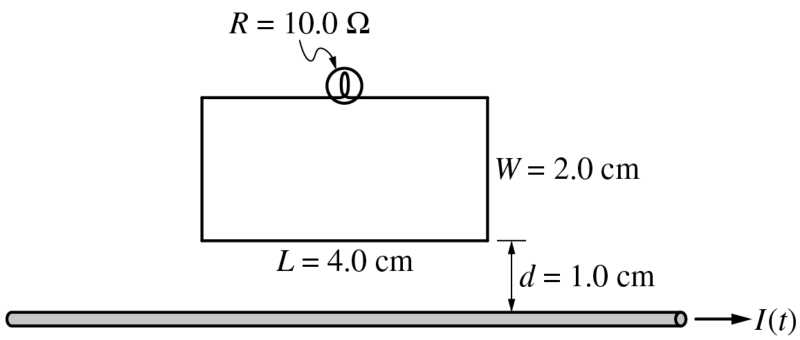
After reaching steady state, the absolute value of the potential difference across R_A is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across R_A is $|\Delta V_2|$.

(d) **Indicate** whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

____ $|\Delta V_2| > |\Delta V_1|$ ____ $|\Delta V_2| < |\Delta V_1|$ ____ $|\Delta V_2| = |\Delta V_1|$

Justify your answer.

GO ON TO THE NEXT PAGE.



. A lightbulb of resistance $R = 10.0\ \Omega$ is connected to a rectangular loop of wire of negligible resistance near a very long current-carrying wire. The rectangular loop has a length $L = 4.0$ cm and a width $W = 2.0$ cm and is positioned so one of the longer sides of the loop is a distance $d = 1.0$ cm above and parallel to the long wire, as shown. The current in the long wire is initially flowing to the right and is given by $I(t) = C - Dt$, where $C = 10.0$ A and $D = 2.0$ A/s. At time $t = 5.0$ s, the current in the long wire is instantaneously zero as the current changes direction.

(a) What is the direction, if any, of the magnetic field produced by the induced current in the rectangular loop as the current in the long wire changes direction?

___ Into the page   ___ Out of the page   ___ No direction, because the field is zero

Justify your answer.

(b) Calculate the magnetic flux through the <u>loop</u> due to only the long wire at time $t = 3.0$ s.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(c) Calculate the current through the lightbulb at time $t = 3.0$ s.

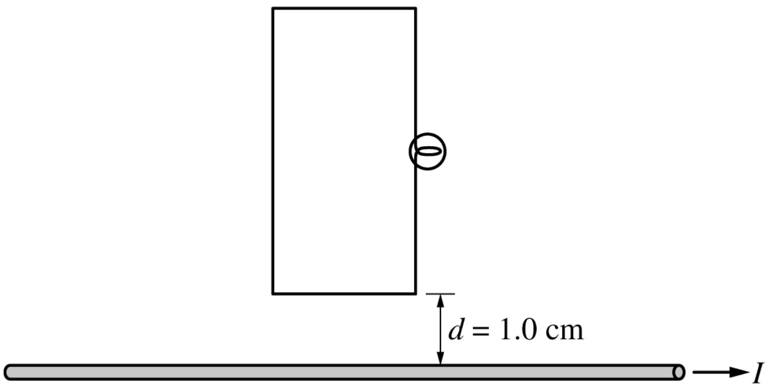
(d) A group of students attempts to experimentally verify whether the current through the lightbulb is consistent with the current calculation from part (c). The current in the rectangular loop is measured to be greater than the current calculated in part (c). Which of the following could explain this discrepancy? Select one answer.

___ The students did not account for Earth's magnetic field.
___ The rectangular loop is tilted and is not in the same plane as the wire.
___ The resistance of the lightbulb is greater than the recorded value.
___ The long side of the rectangular loop is shorter than the recorded value.
___ The current in the long wire changes at a faster rate than expected.

Briefly justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.



(e) Later, the same rectangular loop with lightbulb is rotated such that a short side of the loop is 1.0 cm above and parallel to the long current-carrying wire, as shown. The current in the wire is again initially flowing from left to right and given by $I(t) = C - Dt$, where $C = 10.0\ \text{A}$ and $D = 2.0\ \text{A/s}$. The current through the lightbulb in the loop's new orientation at time $t = 3.0\,\text{s}$ is I_2. Which of the following correctly relates the current I_2 to I_1, the current through the lightbulb in part (c)?

___ $I_2 < I_1$ ___ $I_2 = I_1$ ___ $I_2 > I_1$

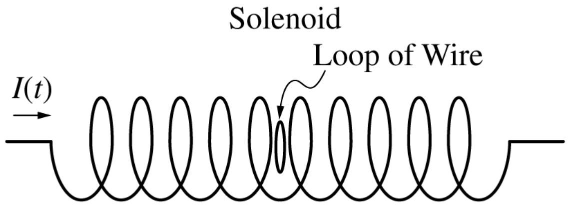
Justify your answer.

GO ON TO THE NEXT PAGE.

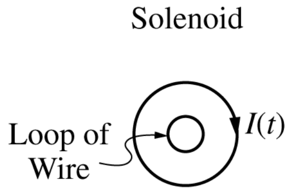
STOP

END OF EXAM

Begin your response to **QUESTION 3** on this page.



Side View



End View

Note: Figures not drawn to scale.

. A single loop of wire with resistance $3.0\,\Omega$ and radius 0.10 m is placed inside a solenoid, with the normal to the loop parallel to the axis of the solenoid. The solenoid has 500 turns, is 0.25 m long, and is connected to a power supply that is not shown. At time $t = 0$, the power supply is turned on, and the current I in the solenoid as a function of t is given by the equation $I(t) = \beta t$, where $\beta = 5.0\text{ A/s}$. The direction of the current in the solenoid is clockwise, as shown in the end view.

(a) At time $t = 2.0\text{ s}$, is the induced current in the loop, as seen from the end view shown, clockwise, counterclockwise, or zero?

___ Clockwise ___ Counterclockwise ___ Zero

Justify your answer.

(b) Calculate the current in the loop of wire at time $t = 2.0\text{ s}$.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(c) Calculate the total energy dissipated by the loop of wire from time $t = 0$ to time $t = 2.0$ s.

(d) A group of students attempts to verify experimentally the calculation of the current from part (b). The current in the inner circular loop at time $t = 2.0$ s is measured to be less than the current calculated in part (b). Which of the following could explain this discrepancy? Select one answer.

___ The experiment did not account for Earth’s magnetic field.
___ The plane of the loop is not perpendicular to the axis of the solenoid.
___ The center of the loop is not on the axis of the solenoid.
___ The resistance of the loop is less than the given value.
___ The radius of the loop is actually larger than 0.10 m.

Justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(e) The power supply is now turned off. The original loop of wire is then replaced with a second loop made from wire that has the same thickness and is made from the same material as the original loop of wire. The second loop has radius 0.20 m, is placed in the same orientation as the original loop, and fits completely inside the solenoid. The power supply is turned on, and the current I in the solenoid as a function of t is again given by the equation $I(t) = \beta t$, where $\beta = 5.0$ A/s. Which of the following expressions correctly indicates the ratio $\frac{I_2}{I_1}$ where I_1 represents the current induced in the original loop of wire in part (b) and I_2 represents the current induced in the second loop of wire?

___ $\frac{I_2}{I_1} = 1$ ___ $1 < \frac{I_2}{I_1} < 2$ ___ $\frac{I_2}{I_1} = 2$ ___ $\frac{I_2}{I_1} > 2$

Justify your answer.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

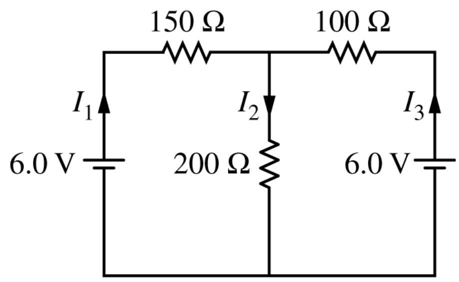


Figure 1

2. The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(a)

i. Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

ii. Calculate the current in the $200\ \Omega$ resistor.

iii. Calculate the power dissipated by the $200\ \Omega$ resistor.

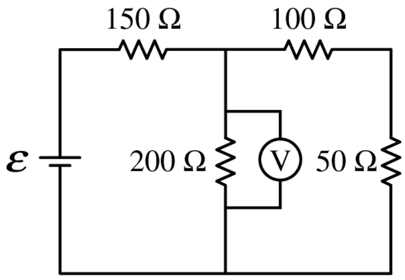


Figure 2

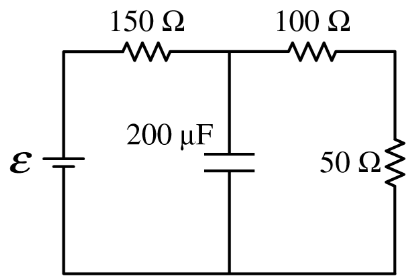
The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

(b) Calculate the current through the $50\ \Omega$ resistor.

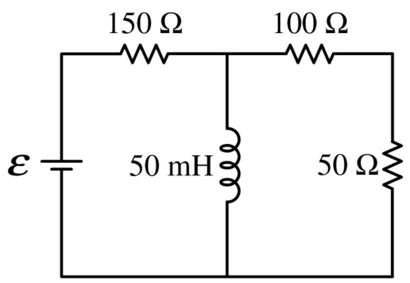
(c) Calculate the voltage $\mathcal{E}$ of the battery.

(d)

i. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu F$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.

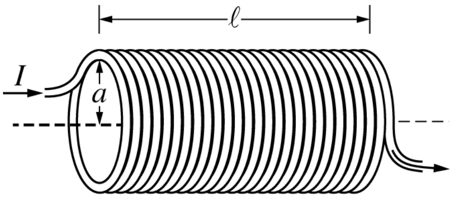


ii. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b) ?

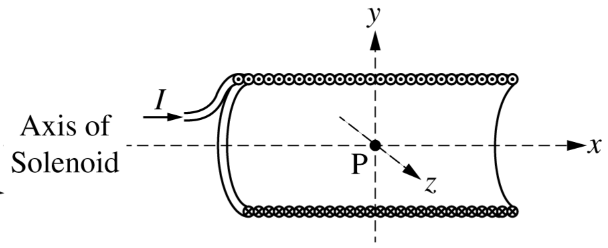


____ Greater than ____ Less than ____ Equal to

Justify your answer.



Solenoid



Cutaway View

Note: Figures not drawn to scale.

3. A solenoid is used to generate a magnetic field. The solenoid has an inner radius a, length ℓ, and N total turns of wire. A power supply, not shown, is connected to the solenoid and generates current I, as shown in the figure on the left above. The x-axis runs along the axis of the solenoid. Point P is in the middle of the solenoid at the origin of the xyz-coordinate system, as shown in the cutaway view on the right above. Assume $\ell >> a$.

(a) Select the correct direction of the magnetic field at point P.

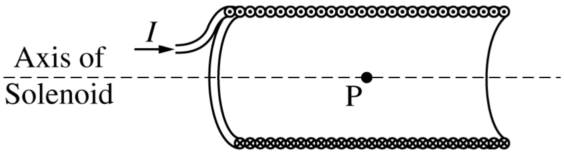
____ $+x$-direction ____ $+y$-direction ____ $+z$-direction

____ $-x$-direction ____ $-y$-direction ____ $-z$-direction

Justify your selection.

(b)

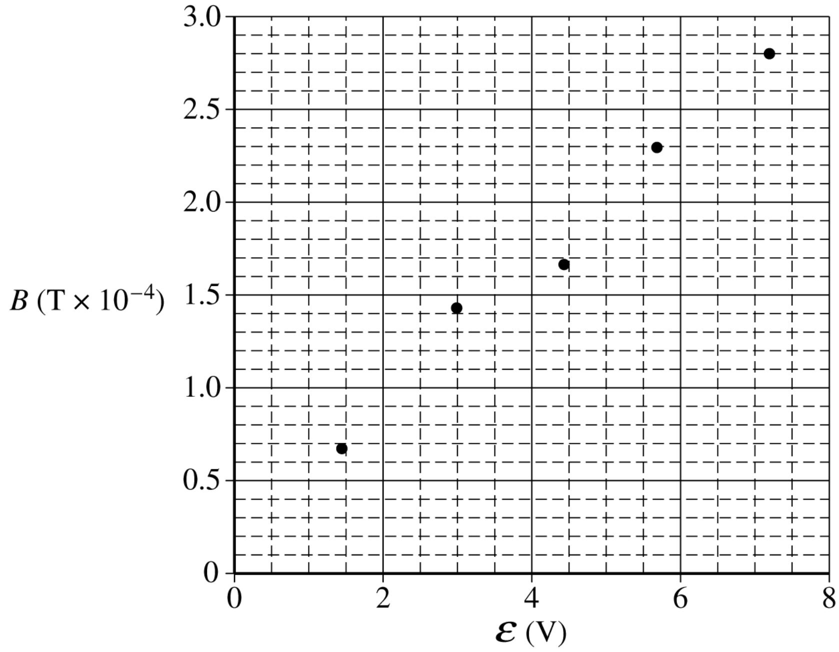
i. On the cutaway view below, clearly draw an Amperian loop that can be used to determine the magnetic field at point P at the center of the solenoid.



Cutaway View

ii. Use Ampere's law to derive an expression for the magnetic field strength at point P. Express your answer in terms of I, ℓ, N, a, and physical constants, as appropriate.

Some physics students conduct an experiment to determine the resistance R_S of a solenoid with radius $a = 0.015$ m, total turns $N = 100$, and total length $\ell = 0.40$ m. The students connect the solenoid to a variable power supply. A magnetic field sensor is used to measure the magnetic field strength along the central axis at the center of the solenoid. The plot of the magnetic field strength B as a function of the emf $\mathcal{E}$ of the power supply is shown below.



(c)

i. On the graph above, draw a best-fit line for the data.

ii. Use the straight line to determine the resistance R_S of the solenoid used in the experiment.

(d) One of the students notes that the horizontal component of the magnetic field of Earth is 2.5×10^{-5} T.

i. Is there evidence from the graph that the horizontal orientation of the solenoid affects the measured values for B ?

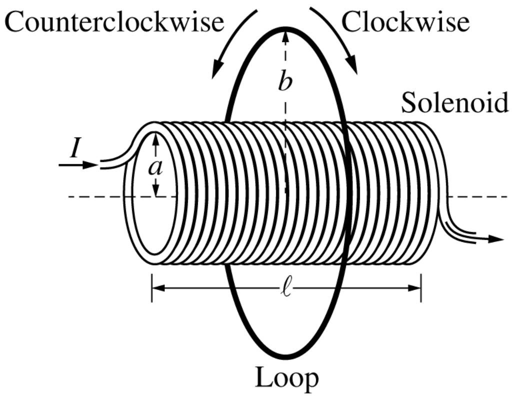
____ Yes ____ No

Justify your answer.

ii. Would the horizontal orientation of the solenoid affect the calculated value for R_S ?

____ Yes ____ No

Justify your answer.



A thin conducting loop of radius b and resistance R_L is placed concentric with the solenoid, as shown above. The current in the solenoid is decreased from I to zero over time Δt.

(e)

i. Is the direction of the induced current in the loop clockwise or counterclockwise during the time period that the current in the solenoid is decreasing?

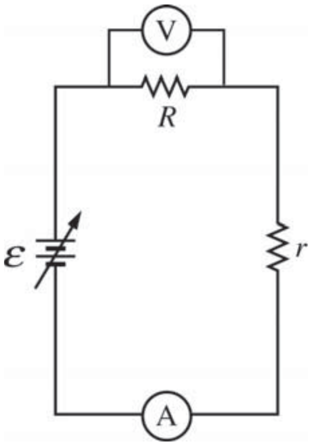
____ Clockwise ____ Counterclockwise

Justify your answer.

ii. Derive an equation for the average induced current i_{IND} in the loop during the time period that the current in the solenoid is decreasing. Express your answer in terms of I, ℓ, N, a, b, R_L, R_S, Δt, and physical constants, as appropriate.

STOP

END OF EXAM



E&M.2.

The circuit shown above consists of a source of variable emf $\mathcal{E}$, an ideal ammeter A, an ideal voltmeter V, a resistor of resistance R, and a sample of wire with resistance r.

(a) How does the current through the wire sample compare with the current through the resistor R ?

_____ It is greater through R. _____ It is greater through the sample.

_____ It is the same through both. _____ It depends on the resistance of the sample.

Justify your answer.

(b) How does the potential difference across the wire sample compare with the potential difference across the resistor R ?

_____ It is greater across R. _____ It is greater across the sample.

_____ It is the same across both. _____ It depends on the resistance of the sample.

Justify your answer.

With the sample of wire in place, the emf of the source is set to a given value. The current through and potential difference across the resistor R are measured. This is repeated for several values of emf, and the data are recorded in the table below.

$\mathcal{E}$ (V)	V_R (V)	I_R (A)		
0.250	0.179	0.162		
0.500	0.335	0.327		
0.750	0.520	0.490		
1.000	0.670	0.687		

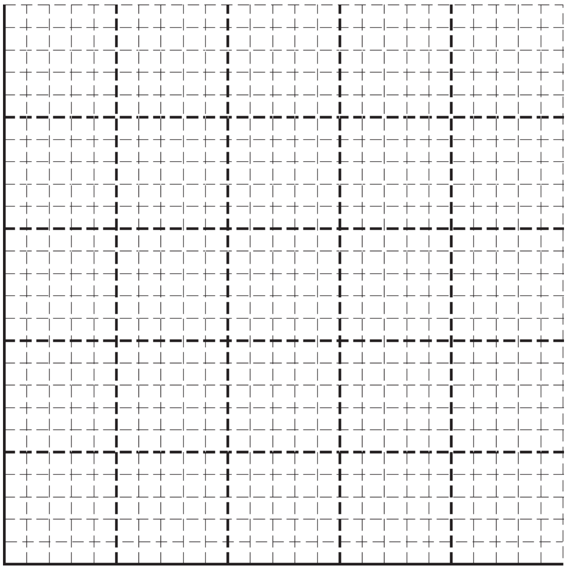
(c) Indicate below which quantities should be graphed to yield a straight line that could be used to calculate a numerical value for the resistance of the wire sample.

Horizontal axis: ____________

Vertical axis: ____________

You may use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given.

(d) On the grid below, plot the straight line data points from part (c). Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(e) Use your straight line to calculate the value of the resistance of the wire sample.

(f) The wire sample has a length of 3.00 m and a radius of 1.00×10^{-3} m . Calculate the resistivity of the material from which the wire sample is made.

(g)

i. Suppose the ammeter used to collect these data was not ideal. Would the actual value of the resistance of the wire sample be greater than, less than, or equal to that calculated in part (e) ?

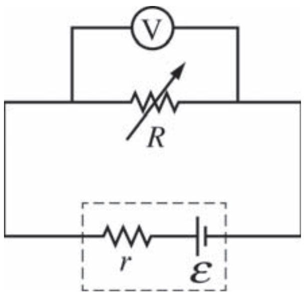
____ Greater than   ____ Less than   ____ Equal to

Justify your answer.

ii. If the ideal voltmeter is replaced by a voltmeter that is not ideal and the experiment is repeated, would the readings of the ideal ammeter be greater than, less than, or equal to those in the data chart before part (c) ?

____ Greater than   ____ Less than   ____ Equal to

Justify your answer.



E&M.2.

A student performs an experiment to determine the emf $\mathcal{E}$ and internal resistance r of a given battery. The student connects the battery in series to a variable resistance R, with a voltmeter across the variable resistor, as shown in the figure above, and measures the voltmeter reading V as a function of the resistance R. The data are shown in the table below.

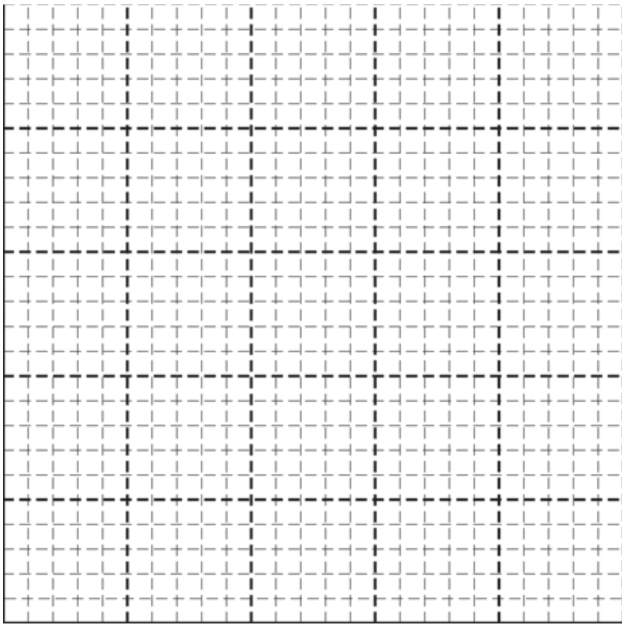
Trial #	Resistance (Ω)	Voltage (V)	$1/R$ $(1/\Omega)$	$1/V$ $(1/\mathrm{V})$
1	0.50	5.6	2.00	0.179
2	1.0	7.4	1.00	0.135
3	2.0	9.4	0.50	0.106
4	3.0	10.6	0.33	0.094
5	5.0	10.9	0.20	0.092
6	10	11.4	0.10	0.088

(a)

i. Derive an expression for the measured voltage V. Express your answer in terms of R, $\mathcal{E}$, r, and physical constants, as appropriate.

ii. Rewrite your expression from part (a)-i to express $1/V$ as a function of $1/R$.

(b) On the grid below, plot data points for the graph of $1/V$ as a function of $1/R$. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.



(c) Use the straight line from part (b) to obtain values for the following.

i. $\mathcal{E}$

ii. r

(d) Using the results of the experiment, calculate the maximum current that the battery can provide.

(e) A voltmeter is to be used to determine the emf of the battery after removing the battery from the circuit. Two voltmeters are available to take this measurement—one with low internal resistance and one with high internal resistance. Indicate which voltmeter will provide the most accurate measurement.

____ The voltmeter with low resistance will provide the most accurate measurement.

____ The voltmeter with high resistance will provide the most accurate measurement.

____ The two voltmeters will provide equal accuracy.

Justify your answer.

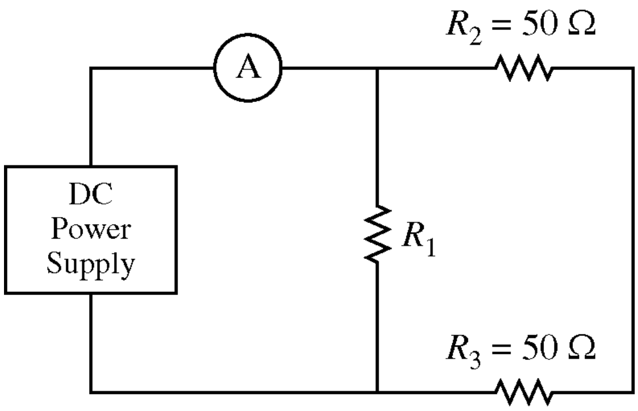
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

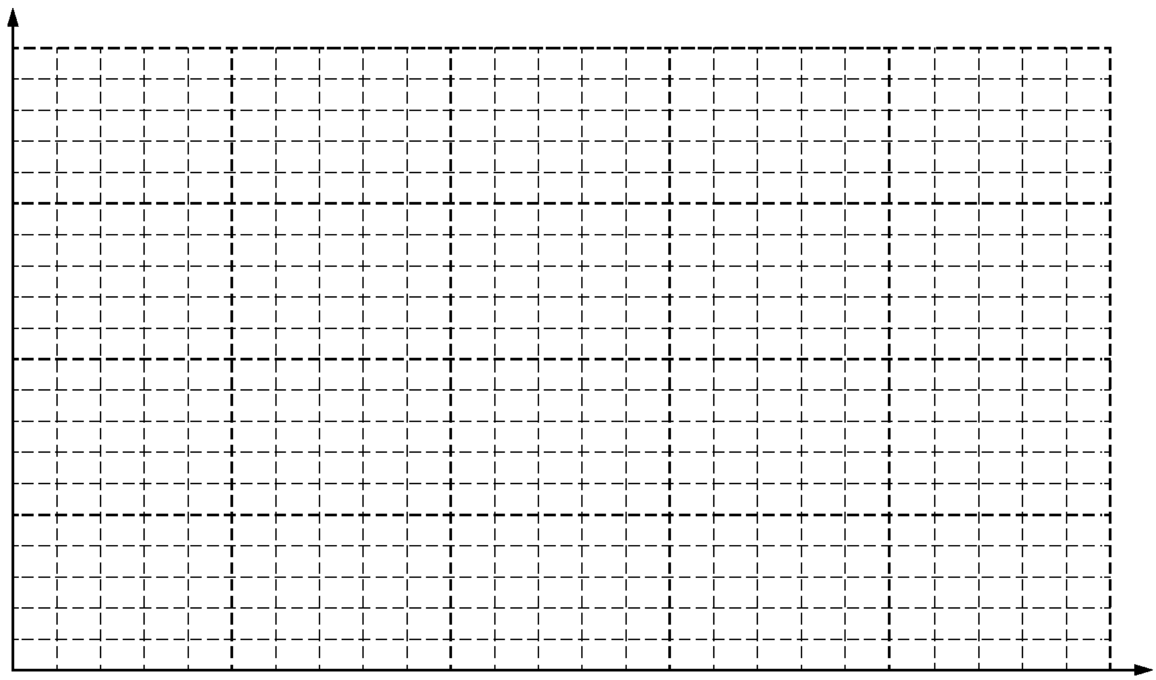


E&M. 1.

Physics students are analyzing the circuit above. A variable DC power supply is connected to an ammeter and three resistors. The resistances of two of the resistors are known to be $R_2 = R_3 = 50\ \Omega$, but the resistance of the third resistor is unknown. The students collect data on the potential difference across the power supply and the current measured by the ammeter, as follows.

Potential Difference (V)	2	4	6	8	10
Current (mA)	40	55	97	138	155

(a) On the grid below, plot the data points for the current as a function of the potential difference. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



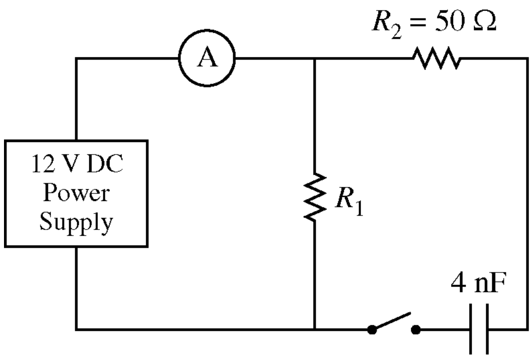
(b) Using the straight line from part (a), calculate the total resistance of the three-resistor combination.

(c) Calculate the value of R_1.

The power supply is now fixed at 12 V.

(d) Calculate the current through R_2.

(e) Resistor 3 is now removed and replaced by an open switch in series with an uncharged 4 nF capacitor, as shown below. The power supply is still fixed at 12 V.



i. Calculate the current in R_2 immediately after the switch is closed.

ii. A long time after the switch is closed, will the magnitude of the current in R_2 be greater than, less than, or equal to the current through R_2 found in part (d) ?

__ Greater than __ Less than __ Equal to

Justify your answer.

(f) The 4 nF capacitor is replaced with an uncharged 10 nF capacitor. Will the magnitude of the current in R_2 immediately after the switch is closed be greater than, less than, or equal to the current in part (e)i?

__ Greater than __ Less than __ Equal to

Justify your answer.

11.4 Electric Power

Name: ______________ Class: __________ Date: ____________ **Total: 11 marks**

KEY WORDS joule heating 焦耳热 • electric power 电功率 • resistance 电阻

Objective

Build the skills to answer exam questions on **electric power**.

You must be able to:

- define **electric power 电功率** as $P = IV$
- express the power in a resistor as $P = I^2R = \dfrac{V^2}{R}$
- describe **Joule heating 焦耳热** (electrical energy to thermal energy)
- find the total energy delivered, $E = Pt$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Power

$P = IV$, in watts. For a resistor, using $V = IR$:

$$P = I^2R = \frac{V^2}{R}.$$

■ Joule heating

A resistor turns electrical energy into **thermal energy** at rate I^2R.

■ Energy

$E = Pt$ over a time interval.

2 Practice

2.1 A device draws 2.5 A at 12 V. Find the power. [2]

2.2 A 10 Ω resistor carries 3.0 A. Find the power dissipated. [2]

2.3 A 100 W device runs for 5.0 minutes. Find the energy it uses. [2]

3 Exam-style questions

3.1 The power dissipated in a resistor is [1]

- **A** I^2R
- **B** IR^2
- **C** I/R
- **D** I^2/R

3.2 Joule heating converts electrical energy into [1]

- **A** light
- **B** thermal energy
- **C** sound
- **D** chemical energy

3.3 A resistor dissipates 48 W while carrying 4.0 A.

(a) Find its resistance. [2]

(b) Find the voltage across it. [1]

Solutions

2.1 $P = IV = 2.5 \times 12 = 30$ W.

2.2 $P = I^2R = (3.0)^2 \times 10 = 90$ W.

2.3 $E = Pt = 100 \times (5.0 \times 60) = 3.0 \times 10^4$ J.

3.1 **A**.

3.2 **B**.

3.3 (a) $R = \dfrac{P}{I^2} = \dfrac{48}{16} = 3.0\ \Omega$. (b) $V = IR = 4.0 \times 3.0 = 12$ V.

Question 2: Version J

2. A rotating, circular, conducting loop of area A and resistance R is in an external uniform magnetic field of magnitude B that is directed in the $-z$-direction. At time $t = 0$, the magnetic field is perpendicular to the plane of the loop, as shown in Figure 1. The loop is rotating with constant angular speed ω and period T about the dashed line that is along the diameter of the loop. The value of the magnetic flux through the loop as a function of time t is $\Phi = BA\cos(\omega t)$.

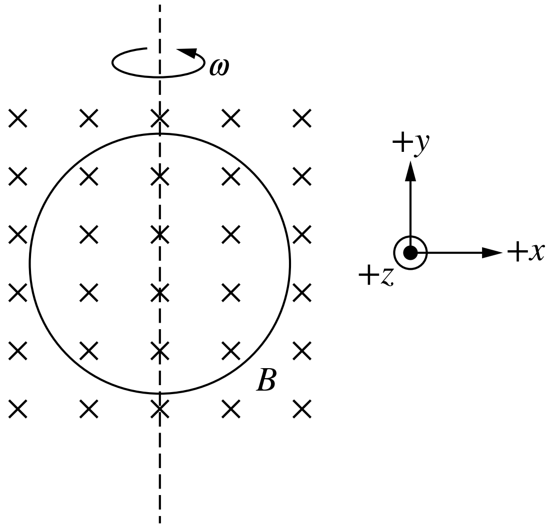


Figure 1

A. The absolute value of the induced emf in the loop is $|\varepsilon|$. The partially completed bar chart in Figure 2 shows a bar that represents $|\varepsilon|$ at $t = \frac{3}{4}T$. In Figure 2, **draw** bars to represent $|\varepsilon|$ at times $t = 0$, $\frac{1}{4}T$, and $\frac{1}{2}T$ relative to $|\varepsilon|$ shown at $\frac{3}{4}T$. If $|\varepsilon| = 0$, **write** a “0” in that column.

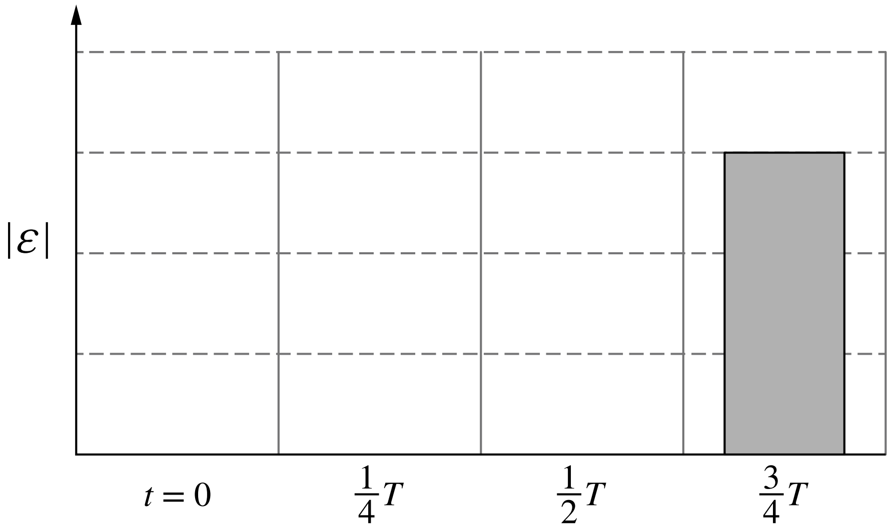


Figure 2

B. Derive an expression for the maximum induced current in the loop in terms of A, R, B, ω, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

C. On the axes shown in Figure 3, **sketch** a graph of the instantaneous power P dissipated by the loop as a function of t during the time interval $0 \leq t \leq T$.

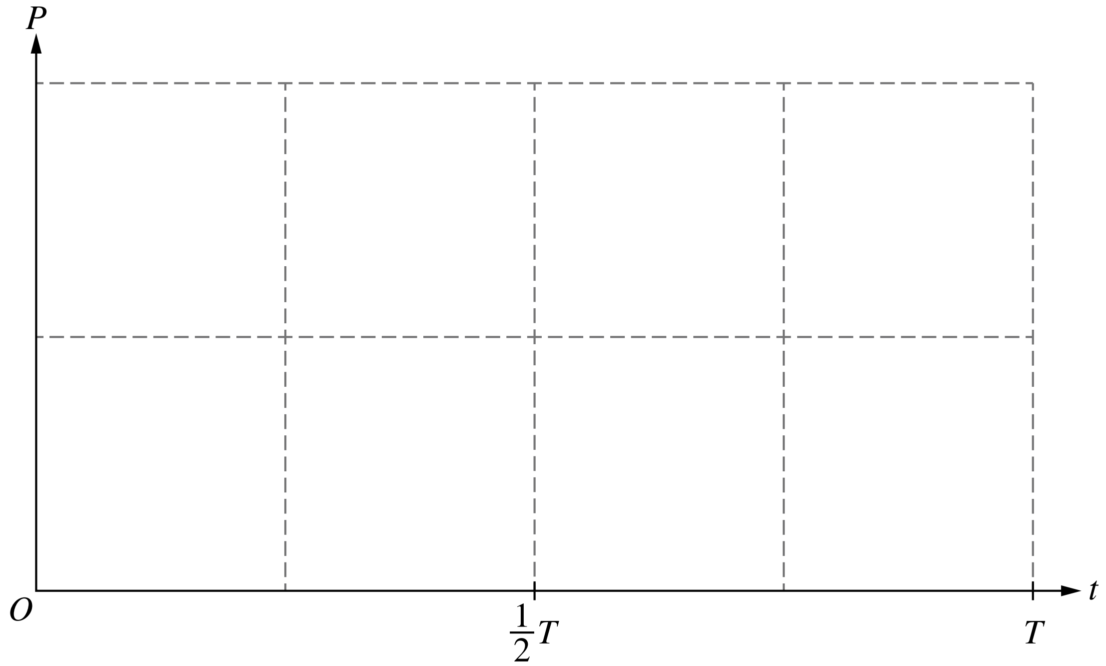


Figure 3

D. Indicate whether the sketch you drew in part C is or is not consistent with the bars that you drew in part A. Briefly **justify** your answer by referencing the functional dependence between P and $|\varepsilon|$.

Begin your response to **QUESTION 3** on this page.

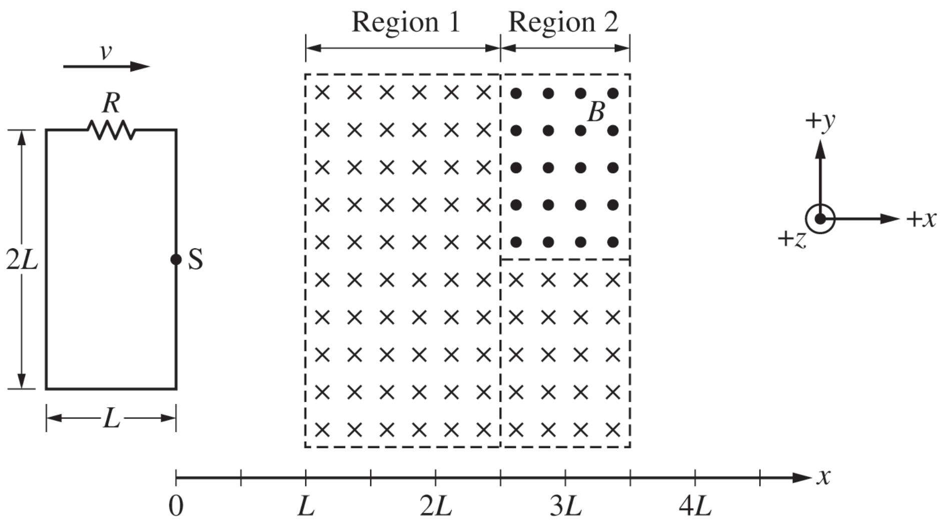


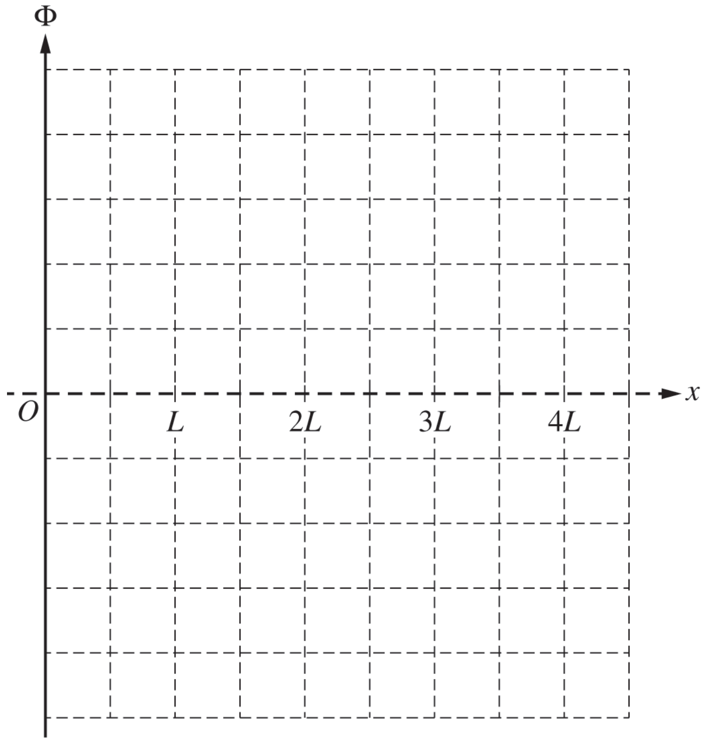
Figure 1

3. A wire is connected to a resistor of resistance R to form a rigid rectangular loop of width L and height $2L$. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$-direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $-z$-direction. Region 1 has boundaries $x = L$ and $x = 2.5L$. The loop later enters Region 2 with two external, uniform magnetic fields, each of magnitude B, that are parallel but are directed in opposite z-directions. Region 2 has boundaries $x = 2.5L$ and $x = 3.5L$. Point S is the midpoint of the leading edge of the loop and is aligned with the horizontal boundary in Region 2 that separates the two magnetic fields.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(a) On the following axes, **sketch** a graph of the magnetic flux Φ through the rectangular loop as a function of the position x of Point S from $x = 0$ to $x = 4.5L$. The $+z$-direction indicated in Figure 1 corresponds to $+\Phi$.



(b) Consider the instant when Point S reaches $x = 1.5L$.

i. **Indicate** whether the current I_R that is induced in the rectangular loop when Point S reaches $x = 1.5L$ is clockwise, counterclockwise, or zero.

_____ Clockwise _____ Counterclockwise _____ Zero

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

ii. **Derive** an expression for I_R when Point S reaches $x = 1.5L$. If $I_R = 0$, indicate how the derived expression shows that $I_R = 0$. Express your answer in terms of R, L, v, B, and physical constants, as appropriate.

iii. **Derive** an expression for the power P dissipated by the resistor when Point S reaches $x = 1.5L$. Express your answer in terms of R, L, v, B, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

The total energy dissipated by the resistor in the rectangular loop as Point S moves from $x = 0$ to $x = 4.5L$ is E_{original}.

The vertical boundary between regions 1 and 2 is now shifted to $x = 1.5L$. After the boundary is shifted, the rectangular loop again moves with speed v in the $+x$-direction, as shown in Figure 2. The total energy dissipated by the resistor as Point S moves from $x = 0$ to $x = 4.5L$ is E_{new}.

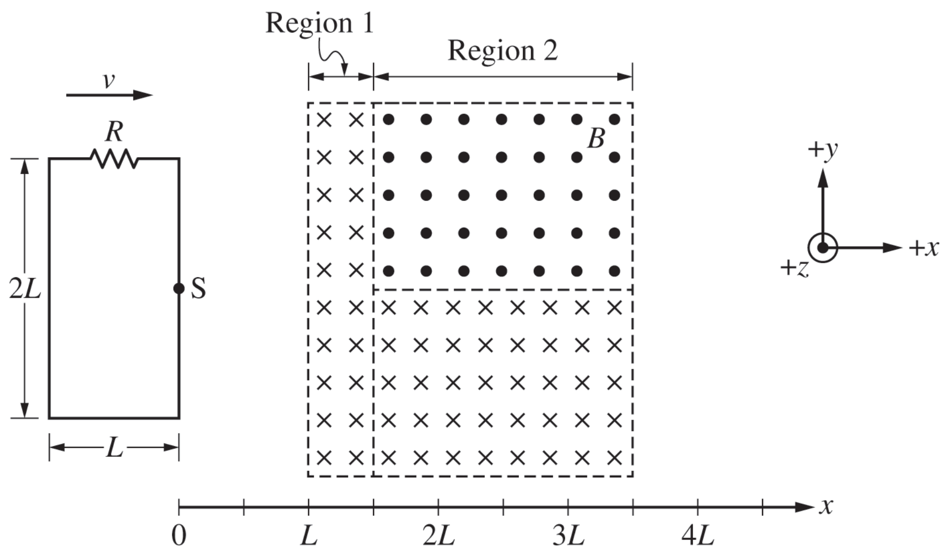


Figure 2

(c) **Indicate** whether E_{new} is greater than, less than, or equal to E_{original}.

____ $E_{\text{new}} > E_{\text{original}}$ ____ $E_{\text{new}} < E_{\text{original}}$ ____ $E_{\text{new}} = E_{\text{original}}$

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

The original magnetic fields are modified so that the region $L < x < 3.5L$ contains an external uniform magnetic field of magnitude B that is directed in the $-z$-direction.

A new wire is connected to a resistor of resistance R to form a rigid triangular loop with base length L and height $2L$. An external force is exerted on the loop so that the loop always moves with speed v in the $+x$-direction, as shown in Figure 3. Point S represents the lower-leading corner of the loop.

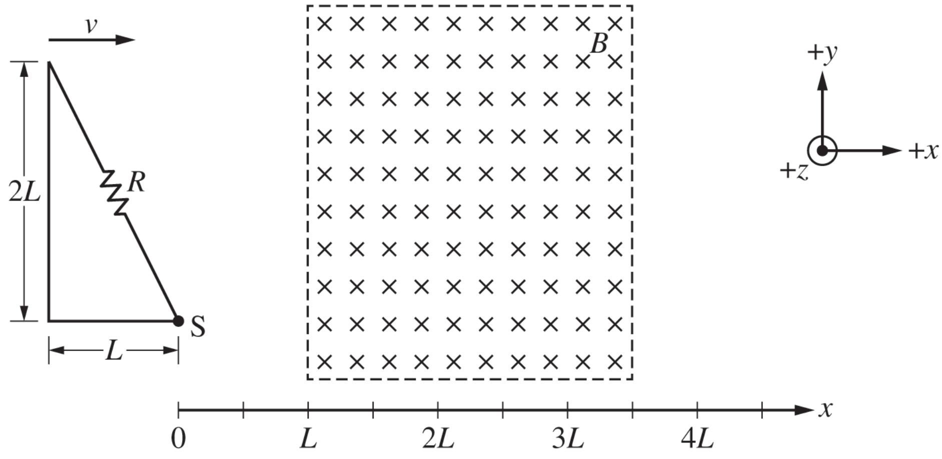
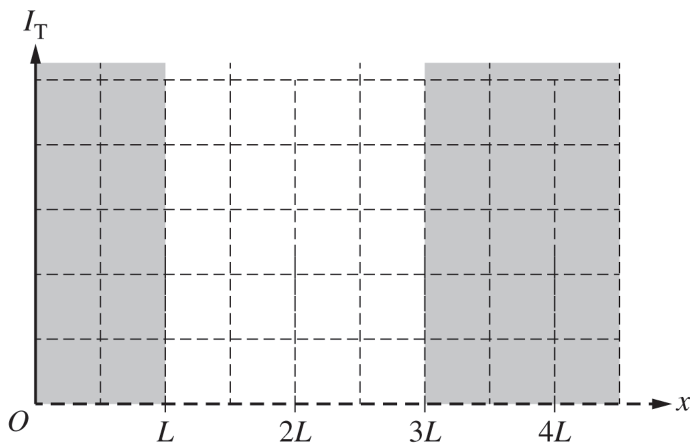


Figure 3

(d) On the following axes, **sketch** a graph of the induced current I_T in the triangular loop as Point S moves from $x = L$ to $x = 3L$.



STOP

END OF EXAM

Begin your response to **QUESTION 3** on this page.

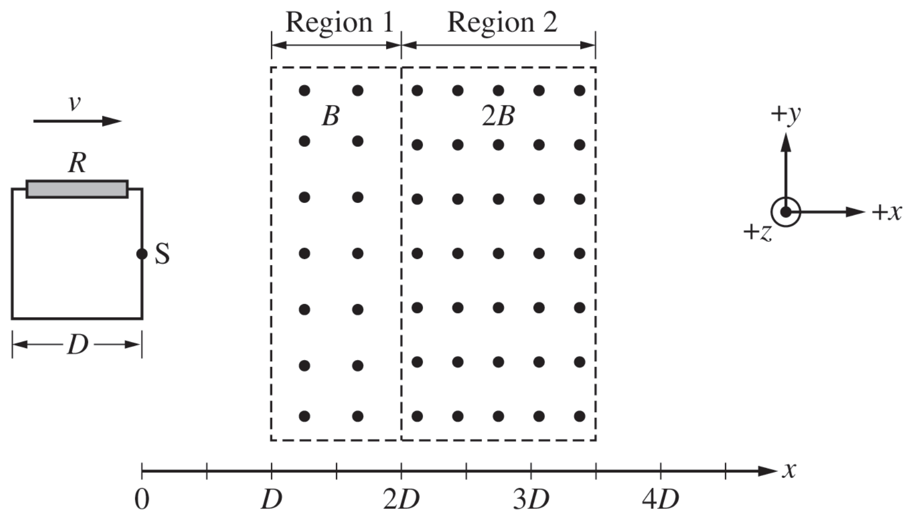


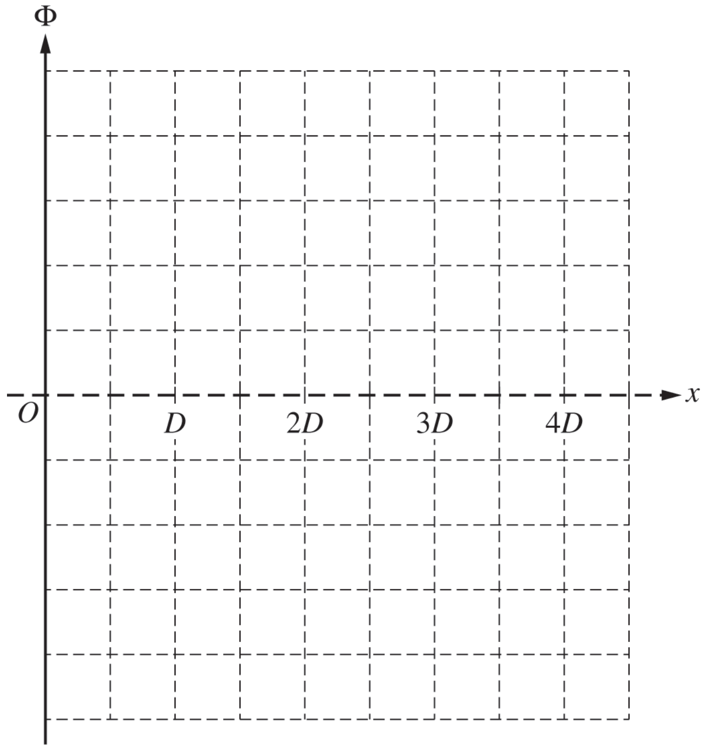
Figure 1

3. A wire is connected to a resistor of resistance R to form a rigid square loop of side length D. An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$ direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $+z$-direction. Region 1 has boundaries $x = D$ and $x = 2D$. The loop later enters Region 2 of external uniform magnetic field of magnitude $2B$ that is directed in the $+z$-direction. Region 2 has boundaries $x = 2D$ and $x = 3.5D$. Point S is the midpoint of the leading edge of the loop.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(a) On the following axes, **sketch** a graph of the magnetic flux Φ through the square loop as a function of the position x of Point S from $x = 0$ to $x = 4.5D$. The $+z$-direction indicated in Figure 1 corresponds to $+\Phi$.



(b) Consider the instant when Point S reaches $x = 1.5D$.

i. **Indicate** whether the current I_S that is induced in the square loop when Point S reaches $x = 1.5D$ is clockwise, counterclockwise, or zero.

_____ Clockwise _____ Counterclockwise _____ Zero

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

ii. **Derive** an expression for I_S when Point S reaches $x = 1.5D$. If $I_S = 0$, indicate how the derived expression shows that $I_S = 0$. Express your answer in terms of R, D, v, B, and physical constants, as appropriate.

iii. **Derive** an expression for the power P dissipated by the resistor when Point S reaches $x = 1.5D$. Express your answer in terms of R, D, v, B, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

The total energy dissipated by the resistor in the square loop as Point S moves from $x = 0$ to $x = 4.5D$ is E_{original}.

The vertical boundary between regions 1 and 2 is now shifted to $x = 2.5D$. After the boundary is shifted, the square loop again moves with speed v in the $+x$-direction, as shown in Figure 2. The total energy dissipated by the resistor as Point S moves from $x = 0$ to $x = 4.5D$ is E_{new}.

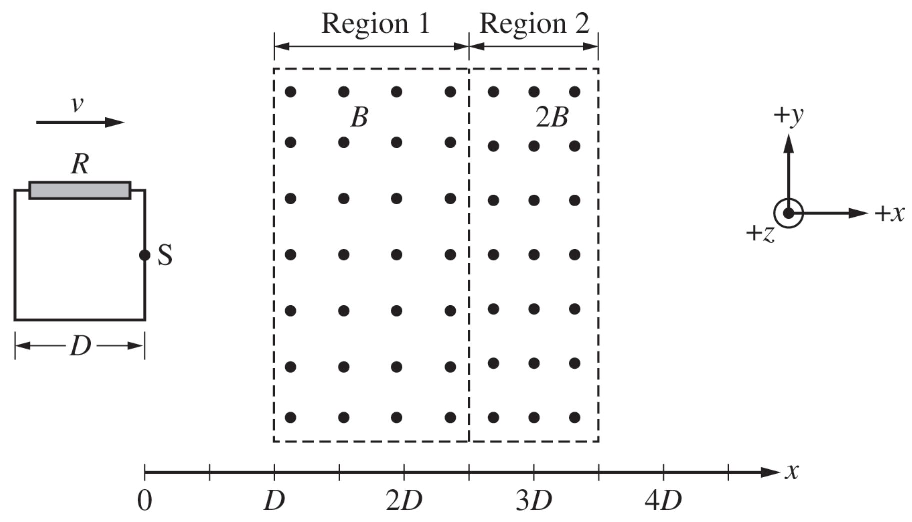


Figure 2

(c) **Indicate** whether E_{new} is greater than, less than, or equal to E_{original}.

_____ $E_{\text{new}} > E_{\text{original}}$ _____ $E_{\text{new}} < E_{\text{original}}$ _____ $E_{\text{new}} = E_{\text{original}}$

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

The original magnetic fields are modified so that the region $D < x < 3.5D$ contains an external uniform magnetic field of magnitude B that is directed in the $+z$-direction.

A new wire is connected to a resistor of resistance R to form a rigid triangular loop with base length D and height D. An external force is exerted on the loop so that the loop always moves with speed v in the $+x$-direction, as shown in Figure 3. Point S represents the upper-leading corner of the loop.

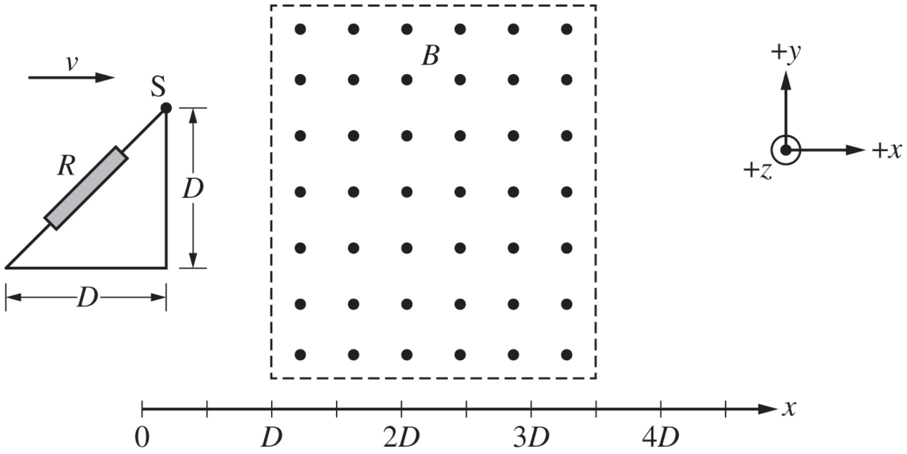
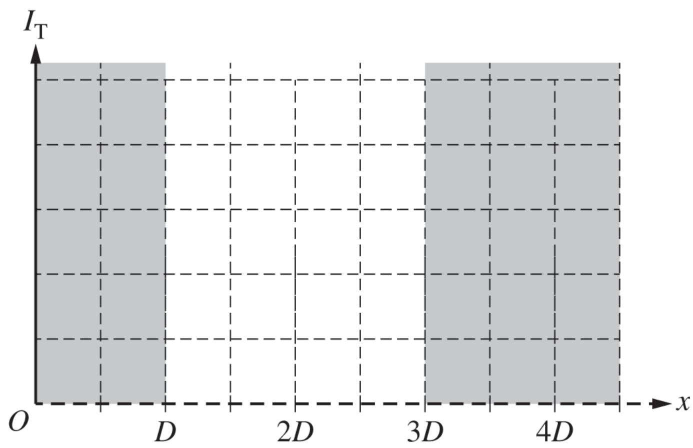


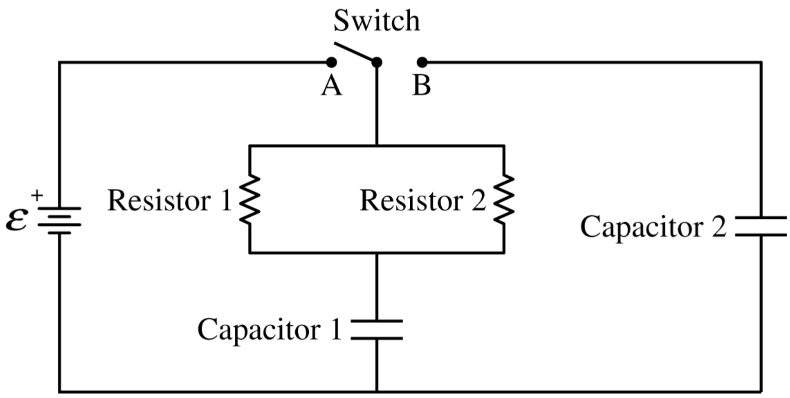
Figure 3

(d) On the following axes, **sketch** a graph of the induced current I_T in the loop as Point S moves from $x = D$ to $x = 3D$.



STOP

END OF EXAM



. The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

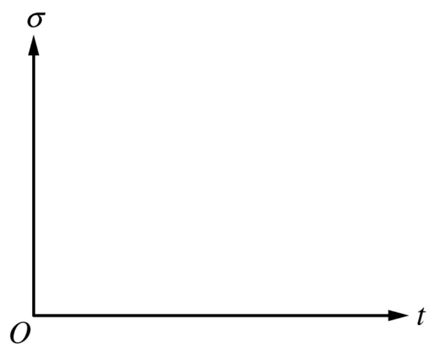
At time $t = 0$, the switch is closed to Position A.

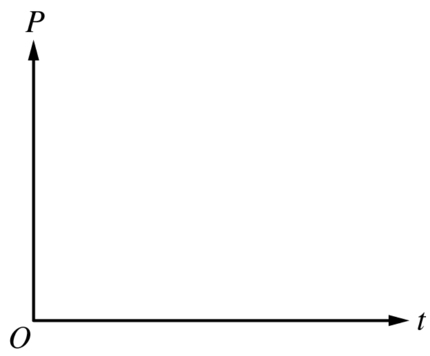
(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(b) On the axes shown, sketch graphs of the surface charge density σ on the positive plate of Capacitor 1 and the total power P dissipated by the resistors as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.





A long time after the switch is closed to Position A, the charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

(c) At time t_1, the switch is closed to Position B.

i. Immediately after time t_1, is the direction of the current in the switch directed toward the left, directed toward the right, or is there no current? Briefly justify your answer.

ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

iii. Derive an expression for the total energy E_R dissipated by resistors 1 and 2 from immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.

With the switch still closed to Position B, the parallel plates of Capacitor 2 are moved so that the separation distance increases by a factor of 2.

(d) Determine the ratio $\frac{U_2}{U_1}$ of the energy U_2 stored in Capacitor 2 to the energy U_1 stored in Capacitor 1 a long time after the plates of Capacitor 2 have been moved. Briefly justify your answer.

GO ON TO THE NEXT PAGE.

With the capacitors still charged as in part (d), the switch is now closed to Position A.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.

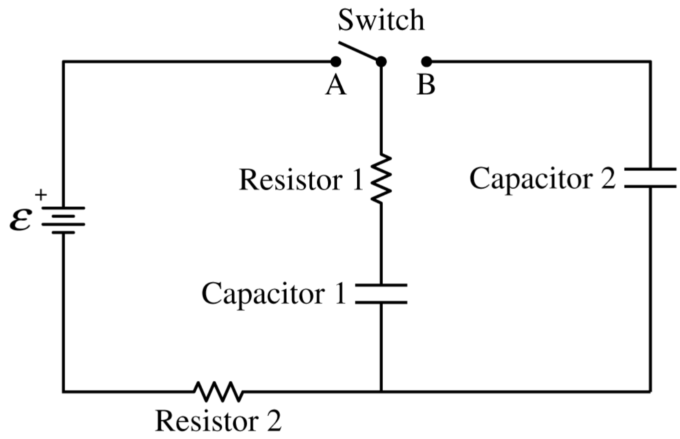
i. Derive an expression for the current I_0 from the battery immediately after the switch is closed to Position A.

ii. Determine the current I_∞ from the battery a long time after the switch is closed to Position A.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM



. The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

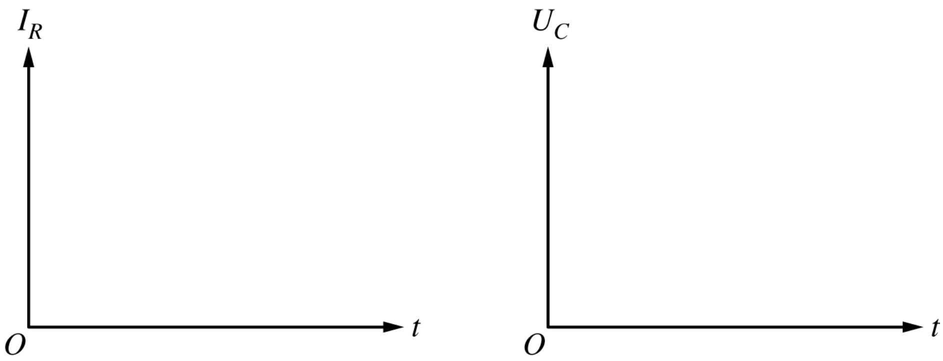
At time $t = 0$, the switch is closed to Position A.

(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(b) On the axes shown, sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.



A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

(c) At time t_1, the switch is closed to Position B.

i. Immediately after t_1, is the direction of the current in Resistor 1 directed up, directed down, or is there no current? Briefly justify your answer.

ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

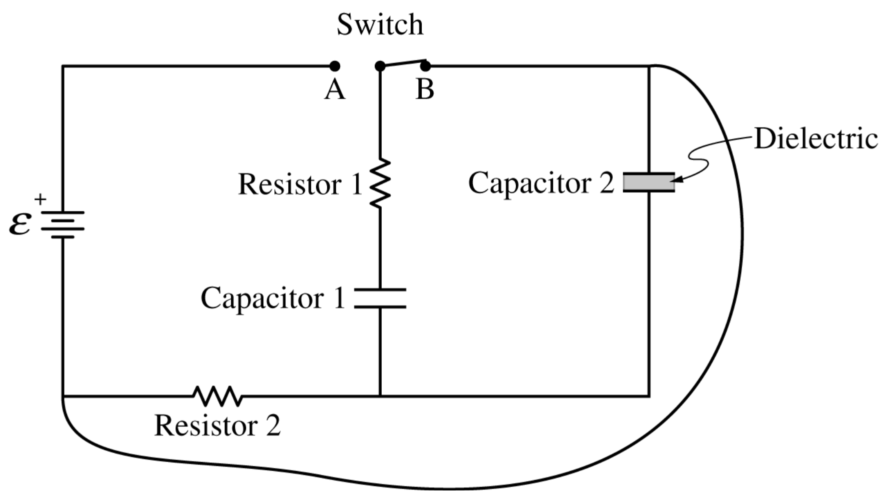
iii. Derive an expression for the total energy dissipated by Resistor 1 immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.

With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

(d) Determine the charge on the positive plate of Capacitor 2 a long time after the dielectric has been inserted. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.



With the switch still closed to Position B, a wire of negligible resistance is connected between two corners of the circuit, as shown.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.

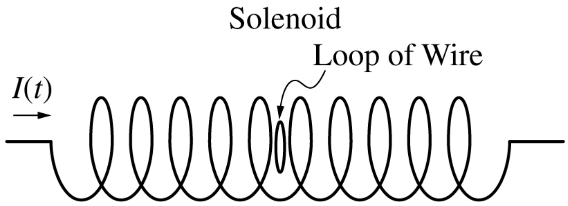
i. Derive an expression for the current in Resistor 2 immediately after the wire is connected to the circuit.

ii. Determine the current in Resistor 2 a long time after the wire is connected to the circuit.

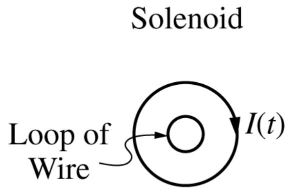
GO ON TO THE NEXT PAGE.

STOP

END OF EXAM



Side View



End View

Note: Figures not drawn to scale.

. A single loop of wire with resistance $3.0\,\Omega$ and radius 0.10 m is placed inside a solenoid, with the normal to the loop parallel to the axis of the solenoid. The solenoid has 500 turns, is 0.25 m long, and is connected to a power supply that is not shown. At time $t = 0$, the power supply is turned on, and the current I in the solenoid as a function of t is given by the equation $I(t) = \beta t$, where $\beta = 5.0\text{ A/s}$. The direction of the current in the solenoid is clockwise, as shown in the end view.

(a) At time $t = 2.0\text{ s}$, is the induced current in the loop, as seen from the end view shown, clockwise, counterclockwise, or zero?

___ Clockwise ___ Counterclockwise ___ Zero

Justify your answer.

(b) Calculate the current in the loop of wire at time $t = 2.0\text{ s}$.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(c) Calculate the total energy dissipated by the loop of wire from time $t = 0$ to time $t = 2.0\text{ s}$.

(d) A group of students attempts to verify experimentally the calculation of the current from part (b). The current in the inner circular loop at time $t = 2.0\text{ s}$ is measured to be less than the current calculated in part (b). Which of the following could explain this discrepancy? Select one answer.

___ The experiment did not account for Earth's magnetic field.
___ The plane of the loop is not perpendicular to the axis of the solenoid.
___ The center of the loop is not on the axis of the solenoid.
___ The resistance of the loop is less than the given value.
___ The radius of the loop is actually larger than 0.10 m.

Justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(e) The power supply is now turned off. The original loop of wire is then replaced with a second loop made from wire that has the same thickness and is made from the same material as the original loop of wire. The second loop has radius 0.20 m, is placed in the same orientation as the original loop, and fits completely inside the solenoid. The power supply is turned on, and the current I in the solenoid as a function of t is again given by the equation $I(t) = \beta t$, where $\beta = 5.0$ A/s. Which of the following expressions correctly indicates the ratio $\frac{I_2}{I_1}$ where I_1 represents the current induced in the original loop of wire in part (b) and I_2 represents the current induced in the second loop of wire?

___ $\frac{I_2}{I_1} = 1$ ___ $1 < \frac{I_2}{I_1} < 2$ ___ $\frac{I_2}{I_1} = 2$ ___ $\frac{I_2}{I_1} > 2$

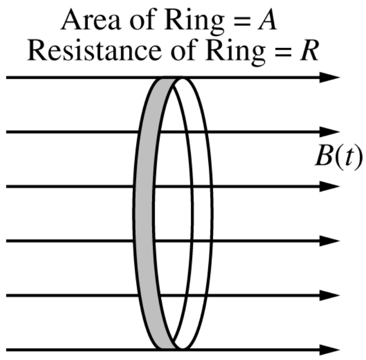
Justify your answer.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

Begin your response to **QUESTION 3** on this page.



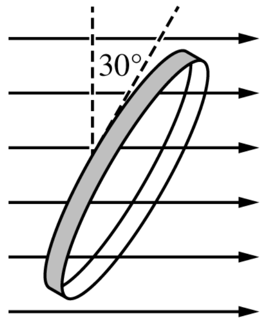
3. A thin, conducting ring of area A and resistance R is aligned in a uniform magnetic field directed to the right and perpendicular to the plane of the ring, as shown. At time $t = 0$, the magnitude of the magnetic field is B_0. At $t = 1$ s, the magnitude of the magnetic field begins to decrease according to the equation $B(t) = \frac{\beta}{t}$, where β has units of T·s.

(a) Derive an equation for the magnitude of the induced current I in the ring as a function of t for $t > 1$ s. Express your answer in terms of β, A, R, t, and physical constants, as appropriate.

Assume $A = 0.50\ \text{m}^2$, $R = 2.0\ \Omega$, and $\beta = 0.50\ \text{T}\cdot\text{s}$.

(b) Calculate the electrical energy dissipated in the ring from $t = 1$ s to $t = 2$ s.

GO ON TO THE NEXT PAGE.



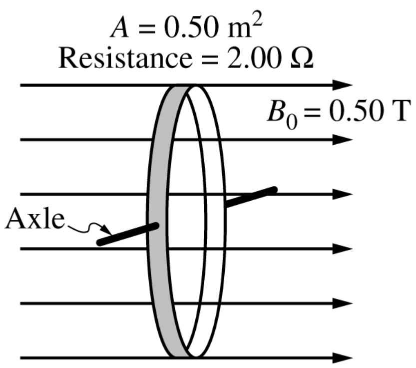
The ring is then rotated so that the plane of the ring is aligned at a 30° angle to the magnetic field, as shown.

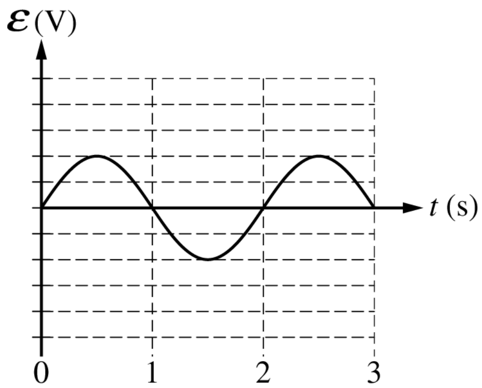
The magnitude of the magnetic field is reset to a magnitude of B_0 at a new time $t = 0$ and again begins to decrease at $t = 1\text{ s}$ according to the equation $B(t) = \dfrac{\beta}{t}$, where β has units of T·s.

(c) Will the amount of energy dissipated in the ring from $t = 1\text{ s}$ to $t = 2\text{ s}$ be greater than, less than, or equal to the energy dissipated in part (b) ?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.





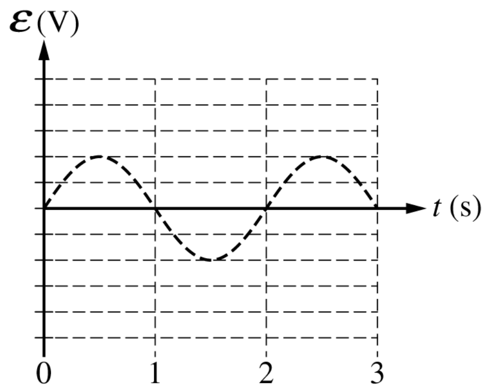
The ring is now mounted on an axle that is perpendicular to the magnetic field. The magnitude of the magnetic field is now held at a constant $B_0 = 0.50\text{ T}$, as shown. The ring rotates about the axle, and the emf ε induced in the ring as a function of time t is shown on the graph.

(d) Calculate the angular speed ω of the rotating ring in rad/s.

(e) Calculate the magnitude of the maximum emf ε_{MAX} induced in the ring.

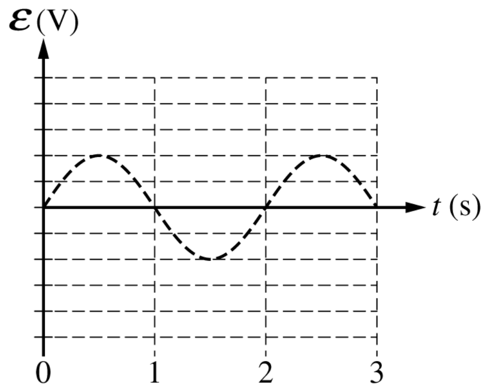
The ring now begins to rotate at an angular speed 2ω.

(f) On the graph below, draw a curve to indicate the new induced emf ε in the ring. The dashed curve shows the emf induced under the original conditions.



Justify your sketch, specifically identifying and addressing any similarities or differences between the sketch and the original graph.

PRACTICE GRAPH - Use the graph below to practice your sketch for part (f). Any work shown on the graph below will NOT be graded.



GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

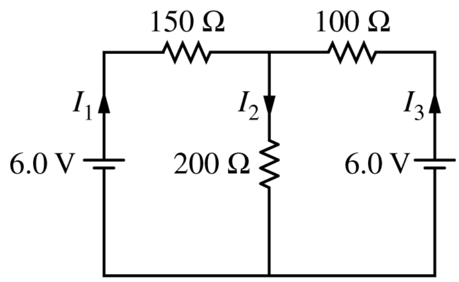


Figure 1

2. The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(a)

i. Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

ii. Calculate the current in the $200\ \Omega$ resistor.

iii. Calculate the power dissipated by the $200\ \Omega$ resistor.

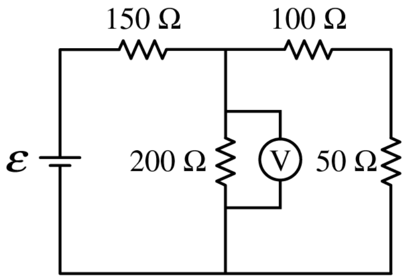


Figure 2

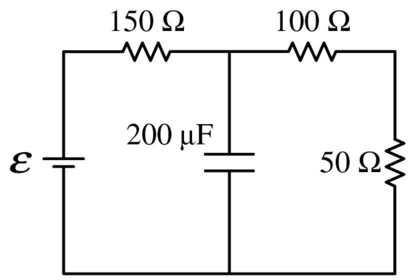
The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

(b) Calculate the current through the $50\ \Omega$ resistor.

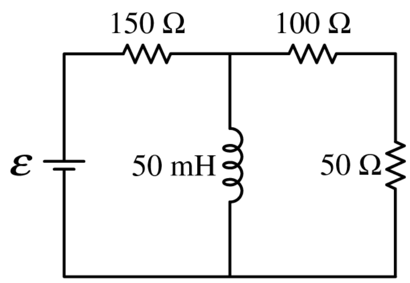
(c) Calculate the voltage $\mathcal{E}$ of the battery.

(d)

i. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu\text{F}$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.

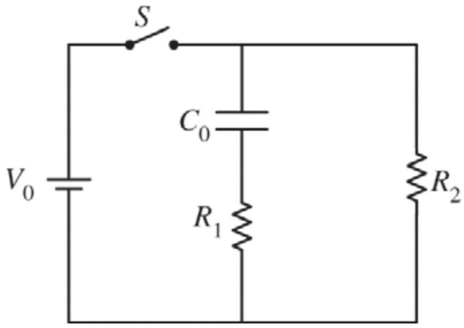


ii. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b) ?



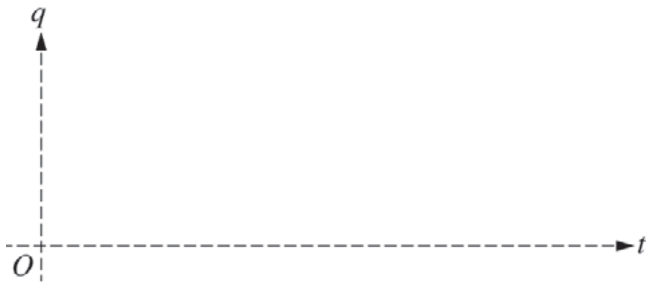
____ Greater than ____ Less than ____ Equal to

Justify your answer.



2. In the circuit above, an ideal battery of voltage V_0 is connected to a capacitor with capacitance C_0 and resistors with resistances R_1 and R_2, with $R_1 > R_2$. The switch S is open, and the capacitor is initially uncharged.

(a) The switch is closed at time $t = 0$. On the axes below, sketch the charge q on the capacitor as a function of time t. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



(b) On the axes below, sketch the current I through each resistor as a function of time t. Clearly label the two curves as I_1 and I_2, the currents through resistors R_1 and R_2, respectively. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

The circuit is constructed using an ideal 1.5 V battery, an 80 μF capacitor, and resistors $R_1 = 150\ \Omega$ and $R_2 = 100\ \Omega$. The switch is closed, allowing the capacitor to fully charge. The switch is then opened, allowing the capacitor to discharge.

(c) The time it takes to charge the capacitor to 50% of its maximum charge is Δt_C. The time it takes for the capacitor to discharge to 50% of its maximum charge is Δt_D. Which of the following correctly relates the two time intervals?

____ $\Delta t_C > \Delta t_D$ ____ $\Delta t_C = \Delta t_D$ ____ $\Delta t_C < \Delta t_D$

Justify your answer.

(d)

i. Calculate the current through resistor R_2 immediately after the switch is opened.

ii. Is the current through resistor R_2 increasing, decreasing, or constant immediately after the switch is opened?

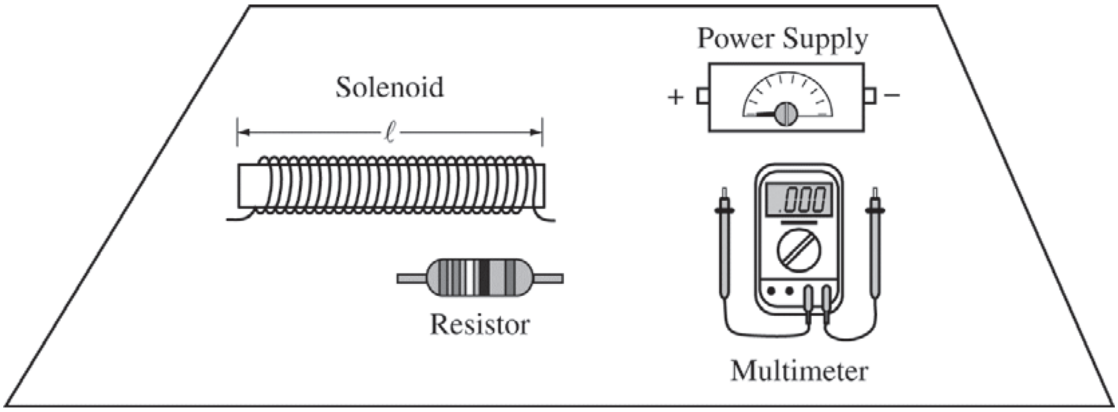
____ Increasing ____ Decreasing ____ Constant

Justify your answer.

(e)

i. Calculate the energy stored in the capacitor immediately after the switch is opened.

ii. Calculate the energy dissipated by resistor R_1 as the capacitor completely discharges.



3. When studying Ampere's law, students collect data on the magnetic field of two different solenoids in order to determine the magnetic permeability of free space μ_0. The solenoids are created by wrapping wire around a hollow plastic tube. The solenoids of length ℓ with N turns of wire will be connected in series to a power supply and resistor. A multimeter will be used as an ammeter to measure the magnitude of the current I through the solenoids. The main components for the setup with one of the solenoids are shown in the figure above.

(a)

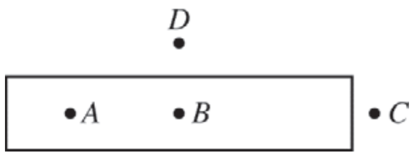
i. On the figure above, draw wire connections between the solenoid, power supply, resistor, and multimeter that will complete the circuit and allow students to measure the magnitude of the current through the solenoid.

ii. Using the connections you made in part (a)i above, what will be the direction of the magnetic field inside the solenoid?

____ Toward the top of the page ______ To the left ____ Out of the page

____ Toward the bottom of the page ______ To the right ____ Into the page

The rectangle shown below represents the solenoid (the loops of wire are not shown). Points A, B, and C are along the central axis of the solenoid with point B at the middle of the solenoid. Point D is directly above point B.

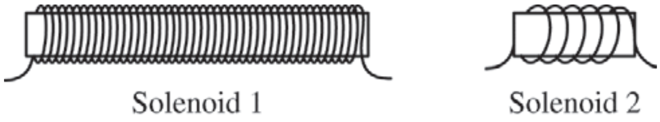


iii. From the choices below, select the point where you would place a magnetic field probe (a probe that can measure the magnitude of the magnetic field) to best measure the strength of the magnetic field of the solenoid in order to determine the magnetic permeability of free space μ_0.

____ A ____ B ____ C ____ D

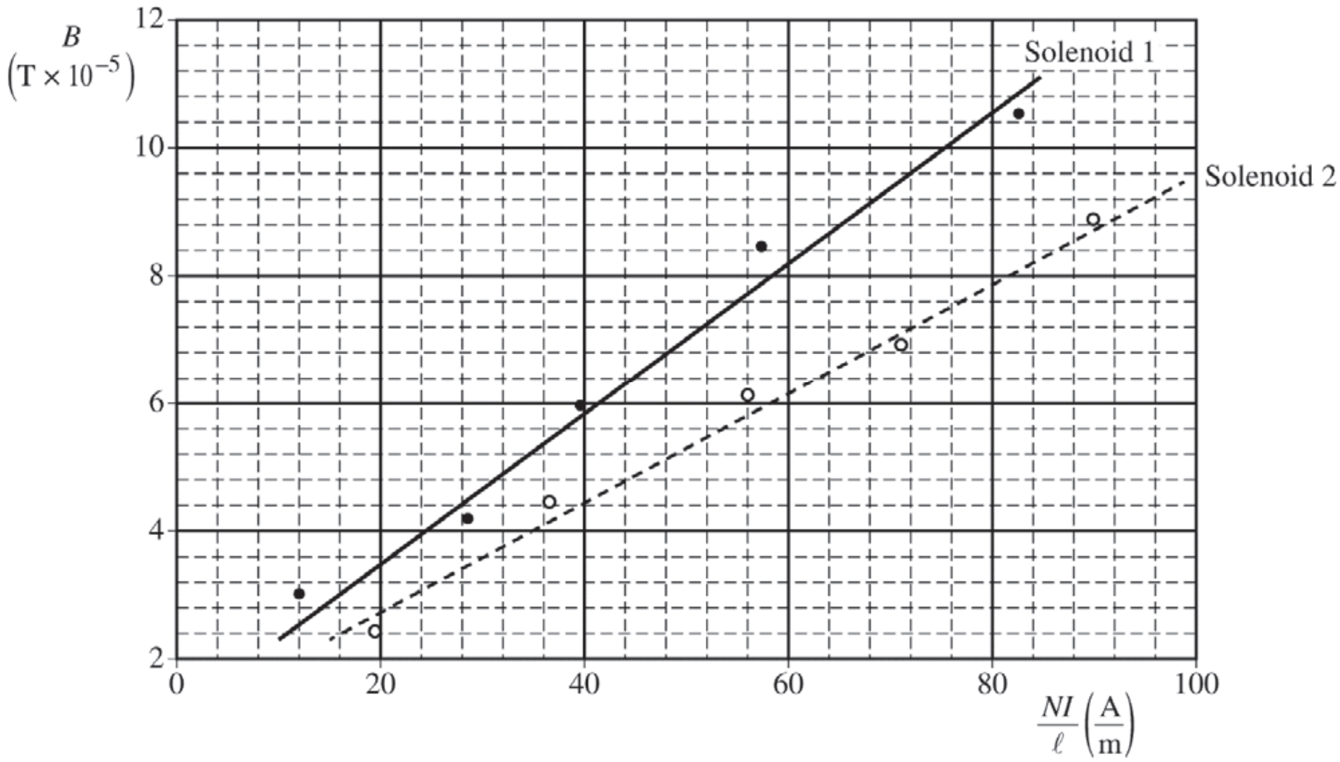
Justify your answer based on the model for a simple solenoid.

The figures below show two different solenoids that will be connected in the circuit above. Solenoid 1 has a length $\ell = 25$ cm with $N = 100$ turns. Solenoid 2 has a length $\ell = 5.0$ cm with $N = 5$ turns.



<u>Note:</u> Figures not drawn to scale.

A graph of the magnitude of the magnetic field B as a function of NI/ℓ is shown below. The best-fit lines for the data are shown as a solid line for solenoid 1 and as a dashed line for solenoid 2.



(b) Which solenoid's best-fit line would give the best results for determining a value for the magnetic permeability of free space μ_0?

____ Solenoid 1 ____ Solenoid 2

Justify your answer.

(c)

i. Use the slope of the best-fit line for the solenoid chosen in part (b) to calculate the magnetic permeability of free space μ_0.

ii. Calculate the percent error for the experimental value of the magnetic permeability of free space μ_0 determined in part (c)i.

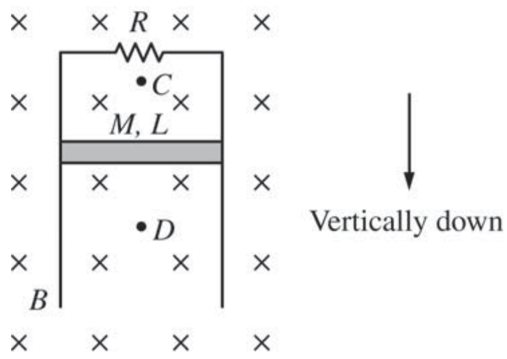
(d)

i. What is a reasonable physical explanation for a best-fit line that does not pass through the origin?

ii. Suppose a student connects the solenoid in a closed circuit similar to the circuit in part (a)i but without the resistor. The student notices the multimeter stops functioning after the power supply is turned on. Explain what causes the failure of the multimeter.

STOP

END OF EXAM



E&M.3.

A conducting bar of mass M, length L, and negligible resistance is connected to two long vertical conducting rails of negligible resistance. The two rails are connected by a resistor of resistance R at the top. The entire apparatus is located in a magnetic field of magnitude B directed into the page, as shown in the figure above. The bar is released from rest and slides without friction down the rails.

(a) What is the direction of the current in the resistor?

____ Left ____ Right

(b)

i. Is the magnitude of the net magnetic field above the bar at point C greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

ii. While the bar is above point D, is the magnitude of the net magnetic field at point D greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Express your answers to parts (c) and (d) in terms of M, L, R, B, and physical constants, as appropriate.

(c) Write, but do NOT solve, a differential equation that could be used to determine the velocity of the falling bar as a function of time t.

(d) Determine an expression for the terminal velocity v_T of the bar.

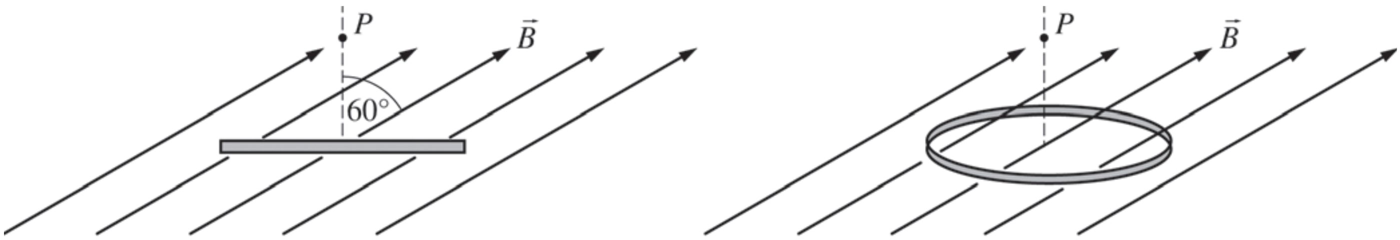
Express your answers to parts (e) and (f) in terms of v_T, M, L, R, B, and physical constants, as appropriate.

(e) Derive an expression for the power dissipated in the resistor when the bar is falling at terminal velocity.

(f) Using your differential equation from part (c), derive an expression for the speed of the falling bar $v(t)$ as a function of time t.

STOP

END OF EXAM



Edge View Perspective View

E&M. 3.

A circular wire loop with radius 0.10 m and resistance 50 Ω is held in place horizontally in a magnetic field $\vec{B}$ directed upward at an angle of 60° with the vertical, as shown in the figure above. The magnetic field in the direction shown is given as a function of time *t* by $B(t) = a(1 - bt)$, where $a = 4.0$ T and $b = 0.20\ s^{-1}$.

(a) Derive an expression for the magnetic flux through the loop as a function of time *t*.

(b) Calculate the numerical value of the induced emf in the loop.

(c)

i. Calculate the numerical value of the induced current in the loop.

ii. What is the direction of the induced current in the loop as viewed from point *P* ?

____ Clockwise ____ Counterclockwise

Justify your answer.

(d) Assuming the loop stays in its current position, calculate the energy dissipated in the loop in 4.0 seconds.

(e) Indicate whether the net magnetic force and net magnetic torque on the loop are zero or nonzero while the loop is in the magnetic field.

Net magnetic force: ____ Zero ____ Nonzero

Net magnetic torque: ____ Zero ____ Nonzero

Justify both of your answers.

STOP

END OF EXAM

11.5 Compound Direct Current Circuits

Name: ______________ Class: __________ Date: ____________ **Total: 11 marks**

KEY WORDS internal resistance 内阻 • terminal voltage 端电压 • ideal emf 电动势 • equivalent resistance 等效电阻 • series 串联

Objective

Build the skills to answer exam questions on **compound direct current circuits**.

You must be able to:

- reduce a network of **series and parallel** resistors to one equivalent resistance
- model a real battery as an **ideal emf 电动势** in series with an **internal resistance 内阻**
- distinguish a battery's **emf** from its **terminal voltage 端电压** under load
- find the current and voltage for each element

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Reducing a network

Combine resistors step by step: series add, parallel combine by $\dfrac{1}{R} = \sum \dfrac{1}{R_i}$, until one equivalent resistance remains.

■ Real battery

A real battery is an ideal emf ε in series with an internal resistance r, so the **terminal voltage** under load is

$$V = \varepsilon - Ir.$$

■ emf vs terminal voltage

The emf is the full ”push”; the terminal voltage is what is left across the external circuit once Ir is lost inside.

2 Practice

2.1 A battery of emf 9.0 V and internal resistance 1.0 Ω drives 2.0 A. Find the terminal voltage. [2]

2.2 Two 8.0 Ω resistors in parallel are in series with a 4.0 Ω resistor. Find the total resistance. [2]

2.3 State the difference between the emf and the terminal voltage of a battery. [1]

3 Exam-style questions

3.1 A real battery is modelled as an ideal emf in series with [1]

- **A** a capacitor
- **B** an internal resistance
- **C** an inductor
- **D** nothing

3.2 The terminal voltage of a battery under load is [1]

- **A** greater than the emf
- **B** less than the emf
- **C** equal to zero
- **D** infinite

3.3 A 12 V battery of internal resistance 0.50 Ω is connected to a 5.5 Ω resistor.

(a) Find the current. [2]

(b) Find the terminal voltage. [2]

Solutions

2.1 $V = \varepsilon - Ir = 9.0 - 2.0(1.0) = 7.0$ V.

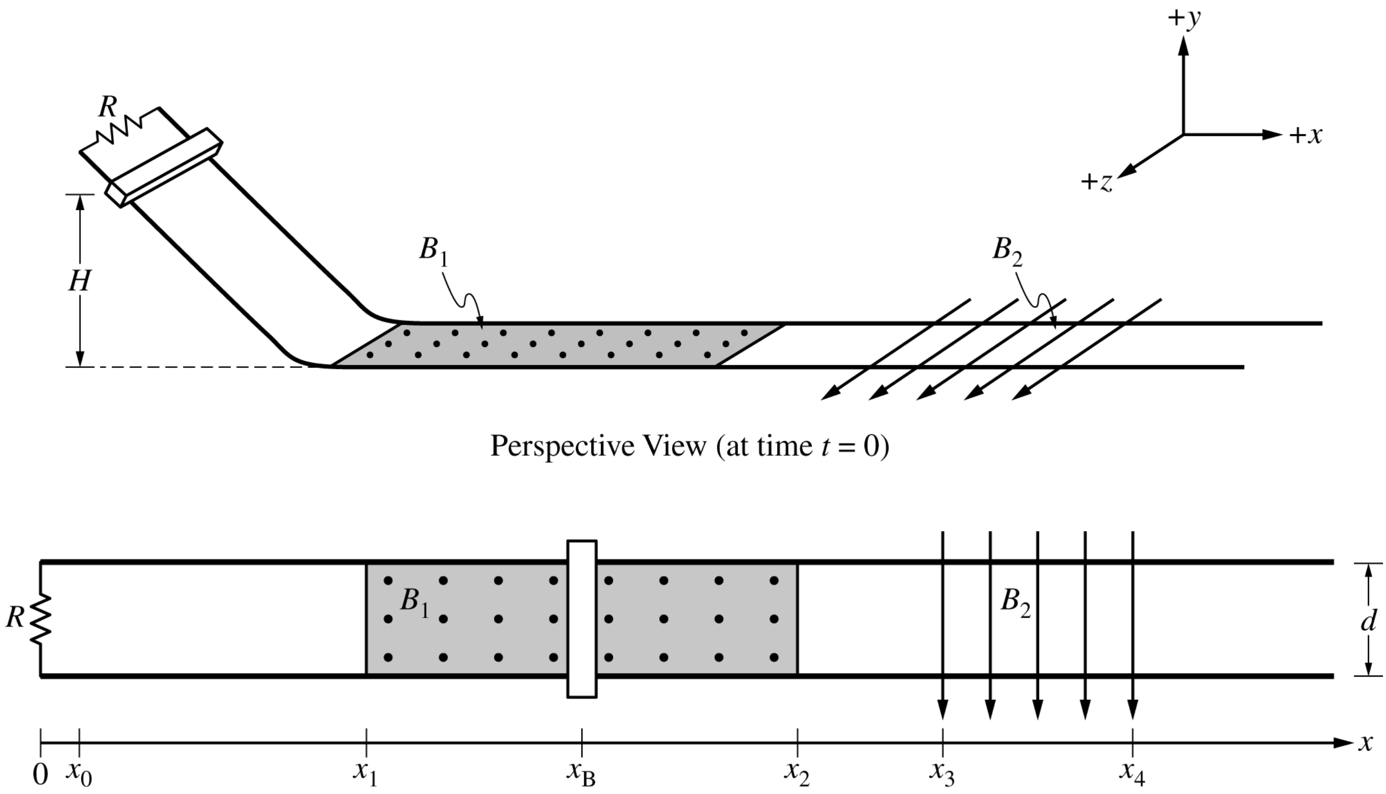
2.2 parallel pair: $\dfrac{8.0}{2} = 4.0\ \Omega$; then $4.0 + 4.0 = 8.0\ \Omega$.

2.3 the emf is the total energy per charge supplied; the terminal voltage is the smaller potential difference across the external circuit once Ir is lost internally.

3.1 **B**.

3.2 **B**.

3.3 (a) $I = \dfrac{\varepsilon}{R + r} = \dfrac{12}{5.5 + 0.50} = 2.0$ A. (b) $V = \varepsilon - Ir = 12 - 2.0(0.50) = 11$ V.



Perspective View (at time $t = 0$)

Top View (at time t_B)

. Two parallel conducting rails are separated by distance $d = 0.30\text{ m}$. A resistor of resistance $R = 0.20\,\Omega$ connects the rails. A conducting bar is placed on a sloped section of the rails at height H above the horizontal section of the rails. Frictional forces and the resistances of the bar and rails are negligible.

- At time $t = 0$, the bar is released from rest from position x_0 and slides down the sloped section of the rails, as shown in the Perspective View.
- At time t_1, the bar reaches position x_1 and smoothly transitions to the horizontal section of the rails and enters a uniform magnetic field of magnitude $B_1 = 0.40\text{ T}$ that is directed in the $+y$-direction.
- At time t_2, the bar reaches position x_2 and enters a region with no magnetic field.
- At time t_3, the bar reaches position x_3 and enters a uniform magnetic field of magnitude $B_2 = 0.60\text{ T}$ that is directed in the $+z$-direction.
- At time t_4, the bar reaches position x_4 and enters a region with no magnetic field.

'he bar is at position x_B (shown in Top View) at time t_B such that $t_1 < t_B < t_2$.

GO ON TO THE NEXT PAGE.

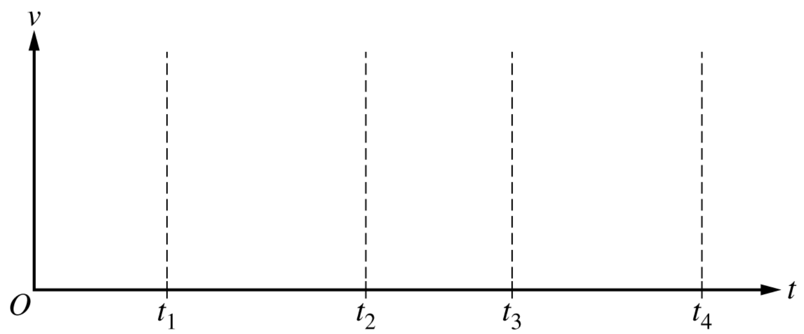
(a) On the following diagram of the bar, as observed from the Top View, draw an arrow indicating the direction of the net force F_{net} exerted on the bar at time t_B. If the net force is zero, write $F_{net} = 0$.

(b) At time t_B, the speed of the bar is $v = 2.5$ m/s.

i. Calculate the magnitude of the current in the bar at time t_B.

ii. Calculate the magnitude of the net force F_{net} exerted on the bar at time t_B.

(c) On the following axes, sketch a graph of the speed v of the bar as a function of time t between $t = 0$ and t_4.



GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) The original scenario is repeated but with a new bar that has the same mass but with a nonnegligible resistance $R = 0.20\,\Omega$. The new bar is released from rest and smoothly transitions to the horizontal section of the rails and enters the first uniform magnetic field.

i. Determine the total resistance of the closed circuit.

ii. In the original scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is $a_{original}$. In the new scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{new}. Is a_{new} greater than, less than, or equal to $a_{original}$? Justify your answer.

(e) Describe a modification to H, B_1, or d that will result in a larger induced current in the new bar immediately after the bar enters the first uniform magnetic field. Justify your answer.

GO ON TO THE NEXT PAGE.

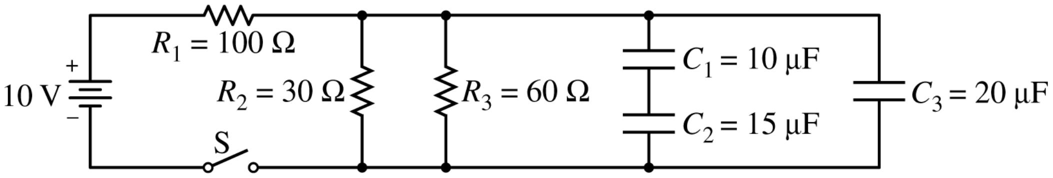
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. The circuit shown above is composed of an ideal 10 V battery, three resistors and three capacitors with the values shown, and an open switch S. The capacitors are initially uncharged. Switch S is now closed.

(a) Calculate the current through R_1 immediately after switch S is closed.

Switch S has been closed for a long time, and the circuit has reached a steady state.

(b) Calculate the potential difference across R_1.

(c)
i. Calculate the charge stored on the positive plate of capacitor C_2.

ii. Is the charge stored on capacitor C_3 greater than, less than, or equal to the charge stored on capacitor C_2 ?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

GO ON TO THE NEXT PAGE.

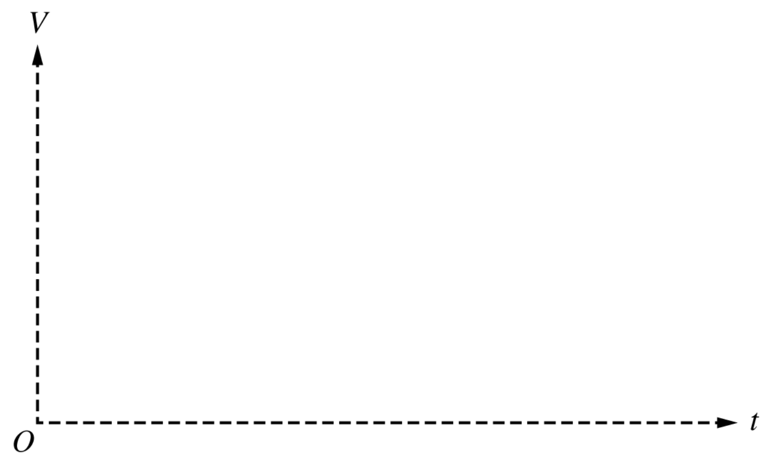
Switch S is then opened.

(d)

i. Determine the current through R_1 immediately after the switch is opened.

ii. Calculate the current through R_2 immediately after the switch is opened.

(e) On the axes below, sketch a graph of the potential difference V across capacitor C_2 as a function of time t if switch S is opened at time $t = 0$. Label the maximum value.



Capacitor C_3 is replaced by two $10\,\mu F$ capacitors connected in series, switch S is closed, and the circuit reaches equilibrium. Switch S is then opened at time $t = 0$.

(f) For $t > 0$, would the sketch of a graph of the new voltage across C_2 as a function of time be above, below, or the same as the sketch for part (e) ?

_____ Above   _____ Below   _____ The same

Justify your answer.

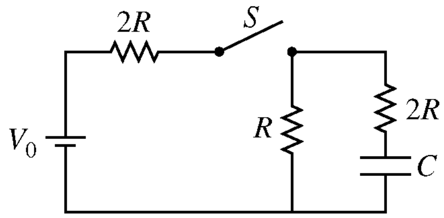
GO ON TO THE NEXT PAGE.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II
Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

   (a) Derive an expression for the steady-state current supplied by the battery.

   (b) Derive an expression for the charge on the capacitor.

   (c) Derive an expression for the energy stored in the capacitor.

   Now the switch is opened at time $t = 0$.

   (d) Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time t after the switch is opened.

(e)

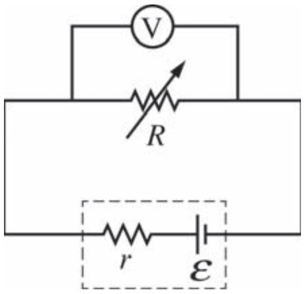
i. Calculate the current in resistor R immediately after the switch is opened.

ii. On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.

(f) Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c) ?

____ Greater than ____ Less than ____ Equal to

Justify your answer.



E&M.2.

A student performs an experiment to determine the emf $\mathcal{E}$ and internal resistance r of a given battery. The student connects the battery in series to a variable resistance R, with a voltmeter across the variable resistor, as shown in the figure above, and measures the voltmeter reading V as a function of the resistance R. The data are shown in the table below.

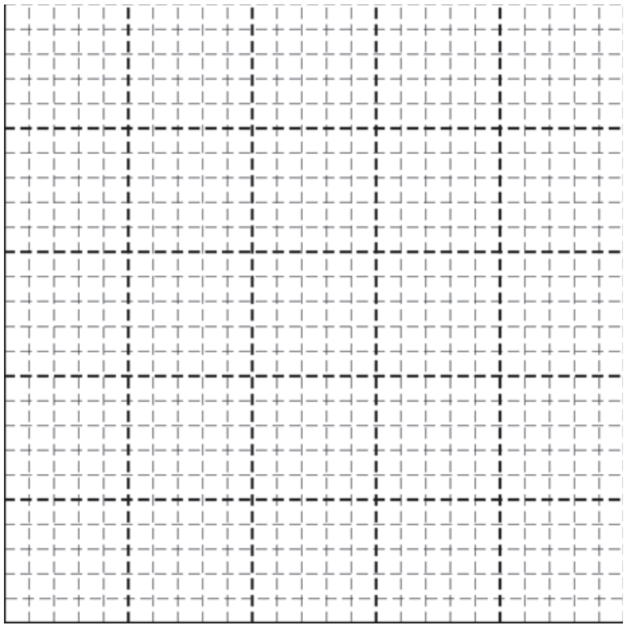
Trial #	Resistance (Ω)	Voltage (V)	$1/R$ $(1/\Omega)$	$1/V$ $(1/\text{V})$
1	0.50	5.6	2.00	0.179
2	1.0	7.4	1.00	0.135
3	2.0	9.4	0.50	0.106
4	3.0	10.6	0.33	0.094
5	5.0	10.9	0.20	0.092
6	10	11.4	0.10	0.088

(a)

i. Derive an expression for the measured voltage V. Express your answer in terms of R, $\mathcal{E}$, r, and physical constants, as appropriate.

ii. Rewrite your expression from part (a)-i to express $1/V$ as a function of $1/R$.

(b) On the grid below, plot data points for the graph of $1/V$ as a function of $1/R$. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.



(c) Use the straight line from part (b) to obtain values for the following.

i. $\mathcal{E}$

ii. r

(d) Using the results of the experiment, calculate the maximum current that the battery can provide.

(e) A voltmeter is to be used to determine the emf of the battery after removing the battery from the circuit. Two voltmeters are available to take this measurement—one with low internal resistance and one with high internal resistance. Indicate which voltmeter will provide the most accurate measurement.

____ The voltmeter with low resistance will provide the most accurate measurement.

____ The voltmeter with high resistance will provide the most accurate measurement.

____ The two voltmeters will provide equal accuracy.

Justify your answer.

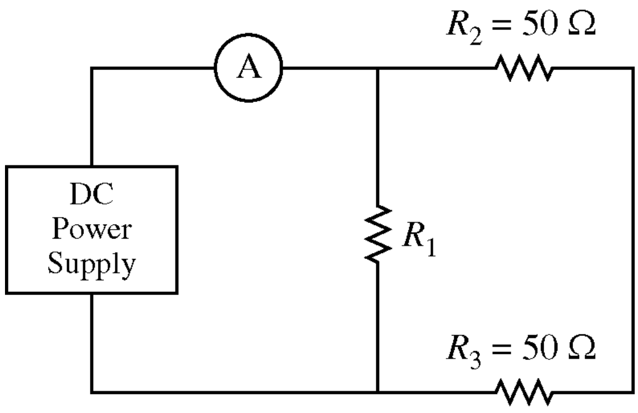
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

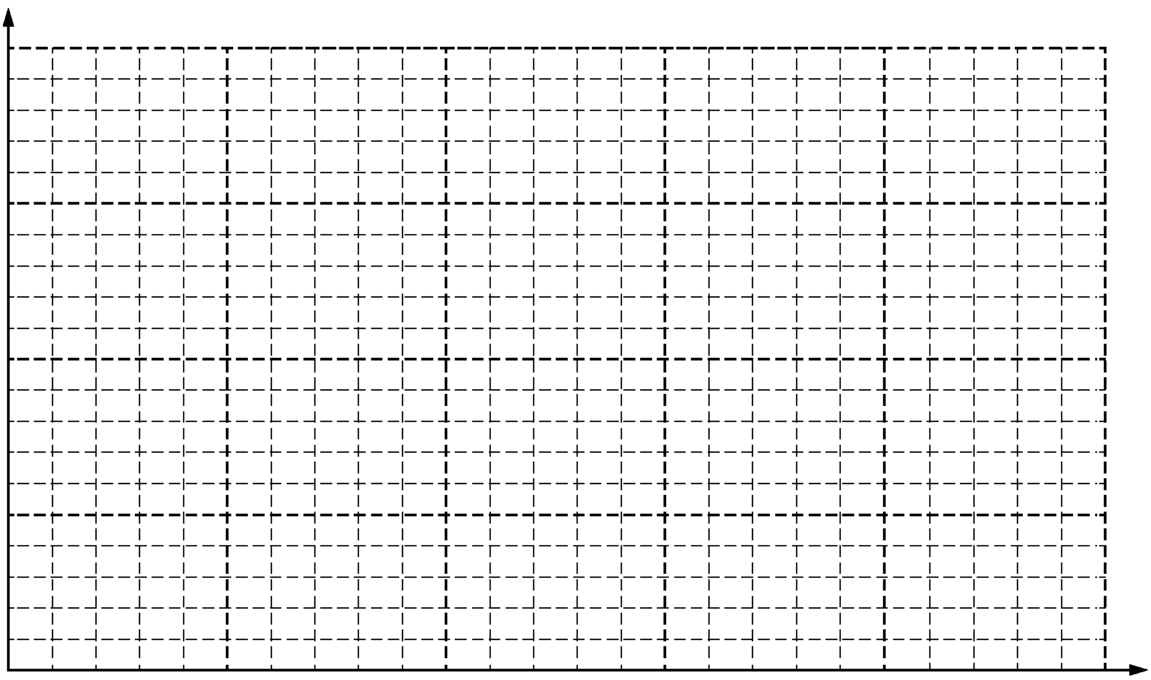


E&M. 1.

Physics students are analyzing the circuit above. A variable DC power supply is connected to an ammeter and three resistors. The resistances of two of the resistors are known to be $R_2 = R_3 = 50\ \Omega$, but the resistance of the third resistor is unknown. The students collect data on the potential difference across the power supply and the current measured by the ammeter, as follows.

Potential Difference (V)	2	4	6	8	10
Current (mA)	40	55	97	138	155

(a) On the grid below, plot the data points for the current as a function of the potential difference. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



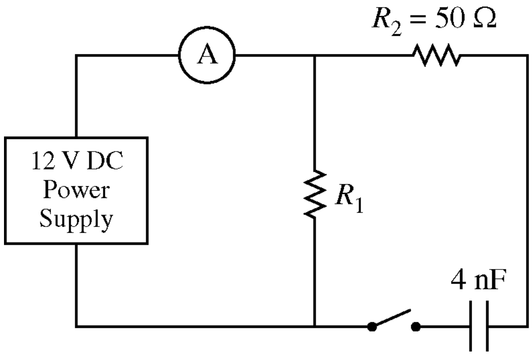
(b) Using the straight line from part (a), calculate the total resistance of the three-resistor combination.

(c) Calculate the value of R_1.

The power supply is now fixed at 12 V.

(d) Calculate the current through R_2.

(e) Resistor 3 is now removed and replaced by an open switch in series with an uncharged 4 nF capacitor, as shown below. The power supply is still fixed at 12 V.



i. Calculate the current in R_2 immediately after the switch is closed.

ii. A long time after the switch is closed, will the magnitude of the current in R_2 be greater than, less than, or equal to the current through R_2 found in part (d) ?

__ Greater than __ Less than __ Equal to

Justify your answer.

(f) The 4 nF capacitor is replaced with an uncharged 10 nF capacitor. Will the magnitude of the current in R_2 immediately after the switch is closed be greater than, less than, or equal to the current in part (e)i?

__ Greater than __ Less than __ Equal to

Justify your answer.

11.6 Kirchhoff's Loop Rule

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

KEY WORDS kirchhoff's loop rule 基尔霍夫电压定律 • conservation of energy 能量守恒 • series 串联

Objective

Build the skills to answer exam questions on **Kirchhoff's loop rule**.

You must be able to:

- state **Kirchhoff's loop rule** 基尔霍夫电压定律, $\sum \Delta V = 0$ around any closed loop
- explain it as a consequence of **conservation of energy** 能量守恒
- assign consistent **sign conventions** for emf sources and resistors
- write and solve **loop equations**

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ The loop rule

Around any closed loop, $\sum \Delta V = 0$ — the rises across the emf sources balance the drops across the resistors.

■ Why it holds

A charge returning to its starting point has the **same** potential energy, so the potential changes around the loop sum to zero (conservation of energy).

■ Signs

Going around the loop: a resistor traversed with the current is a **drop** ($-IR$); an emf traversed from $-$ to $+$ is a **rise** ($+\varepsilon$).

2 Practice

2.1 State Kirchhoff's loop rule. [1]

2.2 State the conservation law the loop rule expresses. [1]

2.3 A single loop has a 10 V battery and resistors 2.0 Ω and 3.0 Ω in series. Find the current. [2]

---

3 Exam-style questions

3.1 Kirchhoff's loop rule sums the potential differences around a closed loop to [1]

- **A** the emf
- **B** zero
- **C** a maximum
- **D** infinity

3.2 The loop rule expresses conservation of [1]

- **A** charge
- **B** energy
- **C** momentum
- **D** mass

3.3 A single loop contains a 12 V battery and resistors of 3.0 Ω and 1.0 Ω in series.

(a) Write the loop equation. [2]

(b) Find the current. [1]

Solutions

2.1 the sum of the potential differences around any closed loop is zero.

2.2 conservation of energy.

2.3 $10 = I(2.0 + 3.0) \Rightarrow I = 2.0$ A.

3.1 **B**.

3.2 **B**.

3.3 (a) $12 = I(3.0) + I(1.0)$. (b) $I = \dfrac{12}{4.0} = 3.0$ A.

Begin your response to **QUESTION 2** on this page.

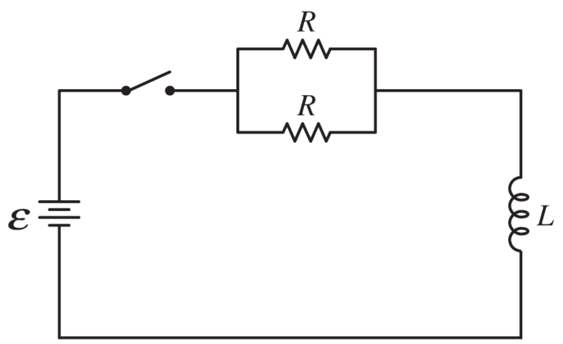


Figure 1

2. Students are asked to determine the resistance R of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that decreases with time to determine R.

(a)

i. On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that decreases with time.

Voltmeter Symbol

ii. **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using the measured quantity that decreases with time. Provide enough detail so that another student could replicate the experiment.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b)

i. On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.

Figure 2

ii. **Describe** how the information from the graph in part (b)(i) would be used to determine the experimental value for R.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(c) Starting with an appropriate application of Kirchhoff's loop rule, **derive**, but do NOT solve, a differential equation that can be used to determine the current I in the inductor at time t after the switch is closed. Express your answer in terms of R, $\mathcal{E}$, L, t, and physical constants, as appropriate.

After reaching steady state, the absolute value of the potential difference across the inductor is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across the new inductor is $|\Delta V_2|$.

(d) **Indicate** whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

____ $|\Delta V_2| > |\Delta V_1|$ ____ $|\Delta V_2| < |\Delta V_1|$ ____ $|\Delta V_2| = |\Delta V_1|$

Justify your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 2** on this page.

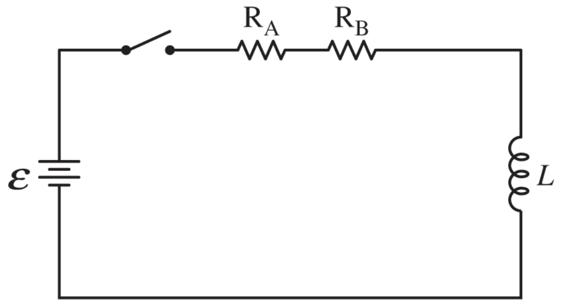


Figure 1

2. Students are asked to determine the resistance R of identical resistors R_A and R_B. The resistors are connected in series with each other, a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that <u>increases</u> with time to determine R.

(a)

i. On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that <u>increases</u> with time.

Voltmeter Symbol

ii. **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using a measured quantity that <u>increases</u> with time. Provide enough detail so that another student could replicate the experiment.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b)

i. On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.

Figure 2

ii. **Describe** how the information from the graph in part (b)(i) would be used to determine the experimental value for R.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(c) Starting with an appropriate application of Kirchhoff's loop rule, **derive,** but do NOT solve, a differential equation that can be used to determine the current I in the inductor at time t after the switch is closed. Express your answer in terms of R, $\mathcal{E}$, L, t, and physical constants, as appropriate.

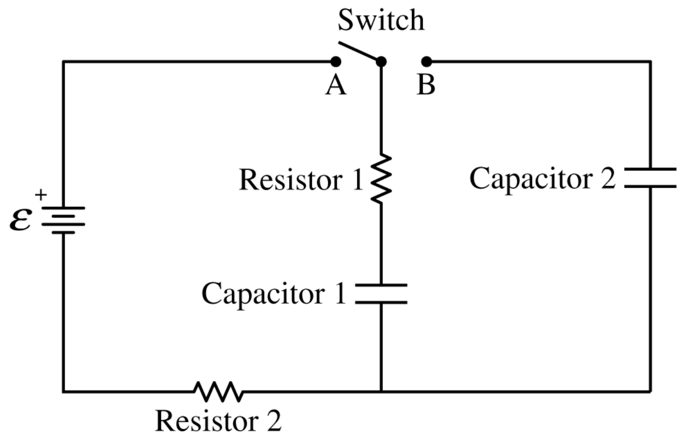
After reaching steady state, the absolute value of the potential difference across R_A is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across R_A is $|\Delta V_2|$.

(d) **Indicate** whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

____ $|\Delta V_2| > |\Delta V_1|$ ____ $|\Delta V_2| < |\Delta V_1|$ ____ $|\Delta V_2| = |\Delta V_1|$

Justify your answer.

GO ON TO THE NEXT PAGE.



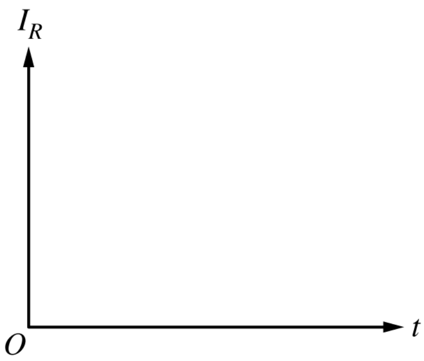
. The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

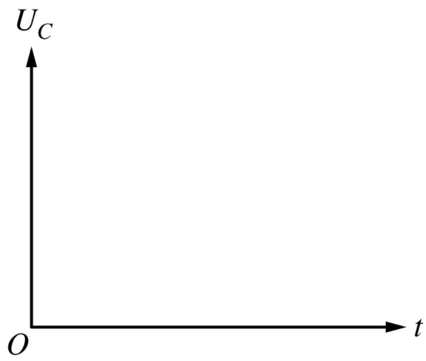
At time $t = 0$, the switch is closed to Position A.

(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

(b) On the axes shown, sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.





A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

(c) At time t_1, the switch is closed to Position B.

i. Immediately after t_1, is the direction of the current in Resistor 1 directed up, directed down, or is there no current? Briefly justify your answer.

ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

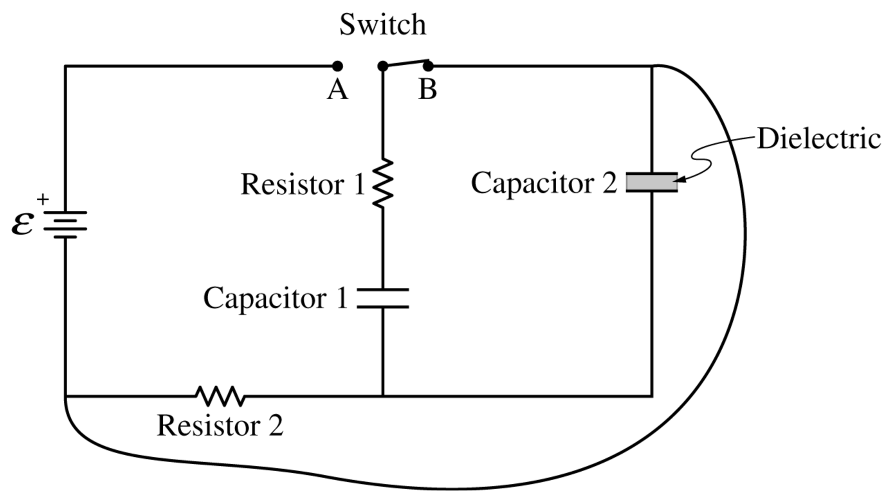
iii. Derive an expression for the total energy dissipated by Resistor 1 immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.

With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

(d) Determine the charge on the positive plate of Capacitor 2 a long time after the dielectric has been inserted. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.



With the switch still closed to Position B, a wire of negligible resistance is connected between two corners of the circuit, as shown.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.

i. Derive an expression for the current in Resistor 2 immediately after the wire is connected to the circuit.

ii. Determine the current in Resistor 2 a long time after the wire is connected to the circuit.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

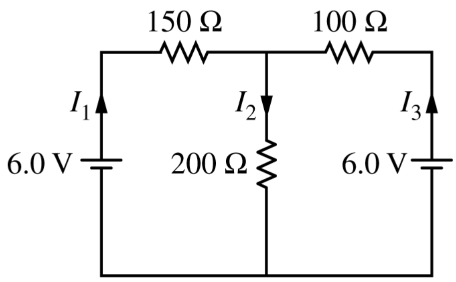


Figure 1

2. The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

   (a)

   i. Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

   ii. Calculate the current in the $200\ \Omega$ resistor.

   iii. Calculate the power dissipated by the $200\ \Omega$ resistor.

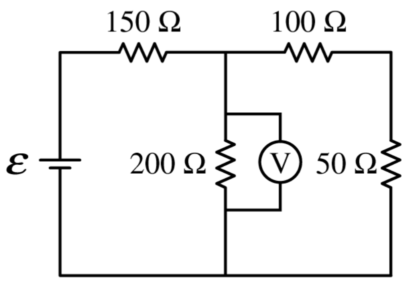


Figure 2

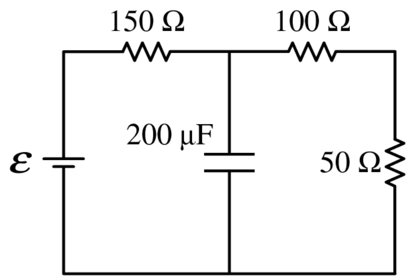
The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

(b) Calculate the current through the $50\ \Omega$ resistor.

(c) Calculate the voltage $\mathcal{E}$ of the battery.

(d)

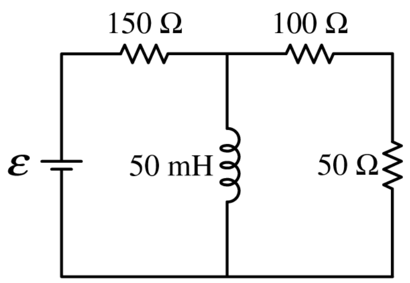
i. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu F$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.



ii. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b) ?

____ Greater than ____ Less than ____ Equal to

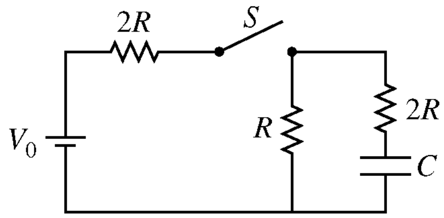
Justify your answer.



PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

   (a) Derive an expression for the steady-state current supplied by the battery.

   (b) Derive an expression for the charge on the capacitor.

   (c) Derive an expression for the energy stored in the capacitor.

   Now the switch is opened at time $t = 0$.

   (d) Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time t after the switch is opened.

(e)

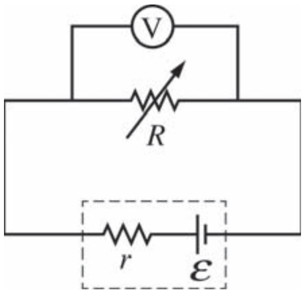
i. Calculate the current in resistor R immediately after the switch is opened.

ii. On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.

(f) Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c) ?

____ Greater than ____ Less than ____ Equal to

Justify your answer.



E&M.2.

A student performs an experiment to determine the emf $\mathcal{E}$ and internal resistance r of a given battery. The student connects the battery in series to a variable resistance R, with a voltmeter across the variable resistor, as shown in the figure above, and measures the voltmeter reading V as a function of the resistance R. The data are shown in the table below.

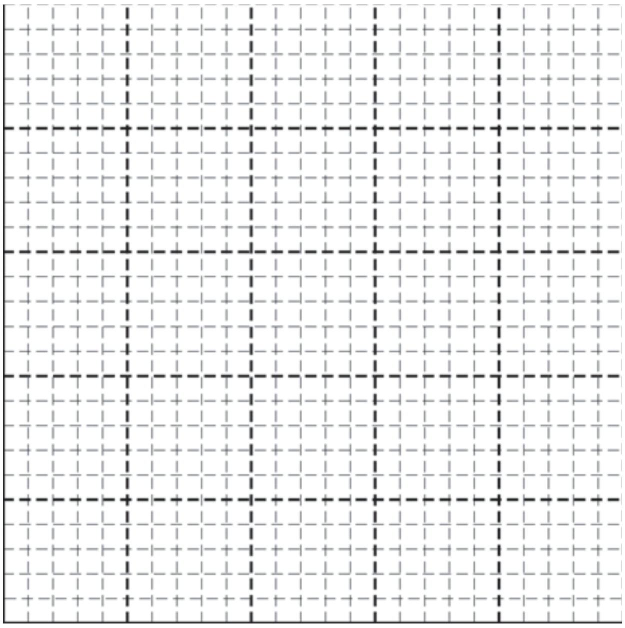
Trial #	Resistance (Ω)	Voltage (V)	$1/R$ $(1/\Omega)$	$1/V$ $(1/\mathrm{V})$
1	0.50	5.6	2.00	0.179
2	1.0	7.4	1.00	0.135
3	2.0	9.4	0.50	0.106
4	3.0	10.6	0.33	0.094
5	5.0	10.9	0.20	0.092
6	10	11.4	0.10	0.088

(a)

i. Derive an expression for the measured voltage V. Express your answer in terms of R, $\mathcal{E}$, r, and physical constants, as appropriate.

ii. Rewrite your expression from part (a)-i to express $1/V$ as a function of $1/R$.

(b) On the grid below, plot data points for the graph of $1/V$ as a function of $1/R$. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.



(c) Use the straight line from part (b) to obtain values for the following.

i. $\mathcal{E}$

ii. r

(d) Using the results of the experiment, calculate the maximum current that the battery can provide.

(e) A voltmeter is to be used to determine the emf of the battery after removing the battery from the circuit. Two voltmeters are available to take this measurement—one with low internal resistance and one with high internal resistance. Indicate which voltmeter will provide the most accurate measurement.

____ The voltmeter with low resistance will provide the most accurate measurement.

____ The voltmeter with high resistance will provide the most accurate measurement.

____ The two voltmeters will provide equal accuracy.

Justify your answer.

11.7 Kirchhoff's Junction Rule

Name: ______________ Class: __________ Date: ____________ **Total: 9 marks**

KEY WORDS junction 节点 • kirchhoff's junction rule 基尔霍夫电流定律 • conservation of charge 电荷守恒

Objective

Build the skills to answer exam questions on **Kirchhoff's junction rule**.

You must be able to:

- state **Kirchhoff's junction rule** 基尔霍夫电流定律, $\sum I_{\text{in}} = \sum I_{\text{out}}$
- explain it as a consequence of **conservation of charge** 电荷守恒
- write **junction equations** for the nodes of a circuit
- combine it with the loop rule to find **branch currents**

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ The junction rule

At any junction, $\sum I_{\text{in}} = \sum I_{\text{out}}$ — the current arriving equals the current leaving.

■ Why it holds

Charge cannot build up at a point, so it is conserved at every node.

■ Using it

Write a junction equation at each node and a loop equation for each loop, then solve the set for the unknown branch currents.

2 Practice

2.1 State Kirchhoff's junction rule. [1]

2.2 State the conservation law the junction rule expresses. [1]

2.3 At a junction, 5.0 A enters and two branches carry 2.0 A and I out. Find I. [2]

3 Exam-style questions

3.1 By the junction rule, the current into a junction equals the current [1]

- **A** stored there
- **B** out of it
- **C** squared
- **D** halved

3.2 The junction rule expresses conservation of [1]

- **A** energy
- **B** charge
- **C** momentum
- **D** power

3.3 At a node, 8.0 A enters by one wire; three branches leave carrying 3.0 A, 2.0 A, and I.

(a) Find I. [2]

(b) Name the law used. [1]

Solutions

2.1 the total current into a junction equals the total current out.

2.2 conservation of charge.

2.3 $I = 5.0 - 2.0 = 3.0$ A.

3.1 **B**.

3.2 **B**.

3.3 (a) $I = 8.0 - 3.0 - 2.0 = 3.0$ A. (b) Kirchhoff's junction rule.

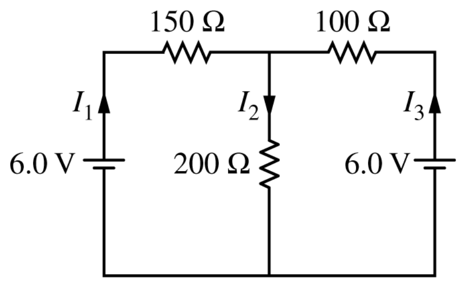


Figure 1

2. The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(a)

i. Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

ii. Calculate the current in the $200\ \Omega$ resistor.

iii. Calculate the power dissipated by the $200\ \Omega$ resistor.

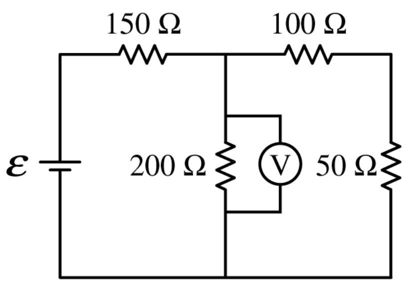


Figure 2

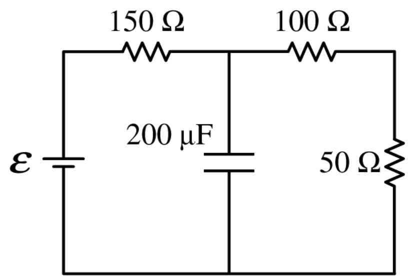
The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

(b) Calculate the current through the $50\ \Omega$ resistor.

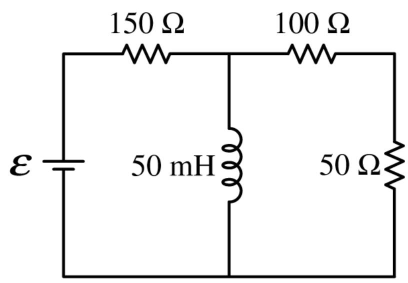
(c) Calculate the voltage $\mathcal{E}$ of the battery.

(d)

i. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu\text{F}$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.



ii. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b) ?

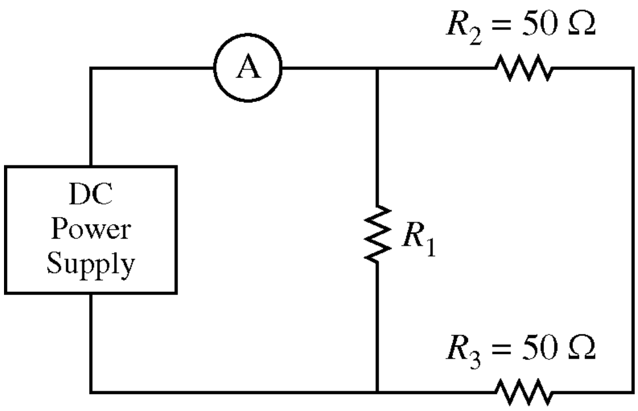


____ Greater than ____ Less than ____ Equal to

Justify your answer.

PHYSICS C: ELECTRICITY AND MAGNETISM
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

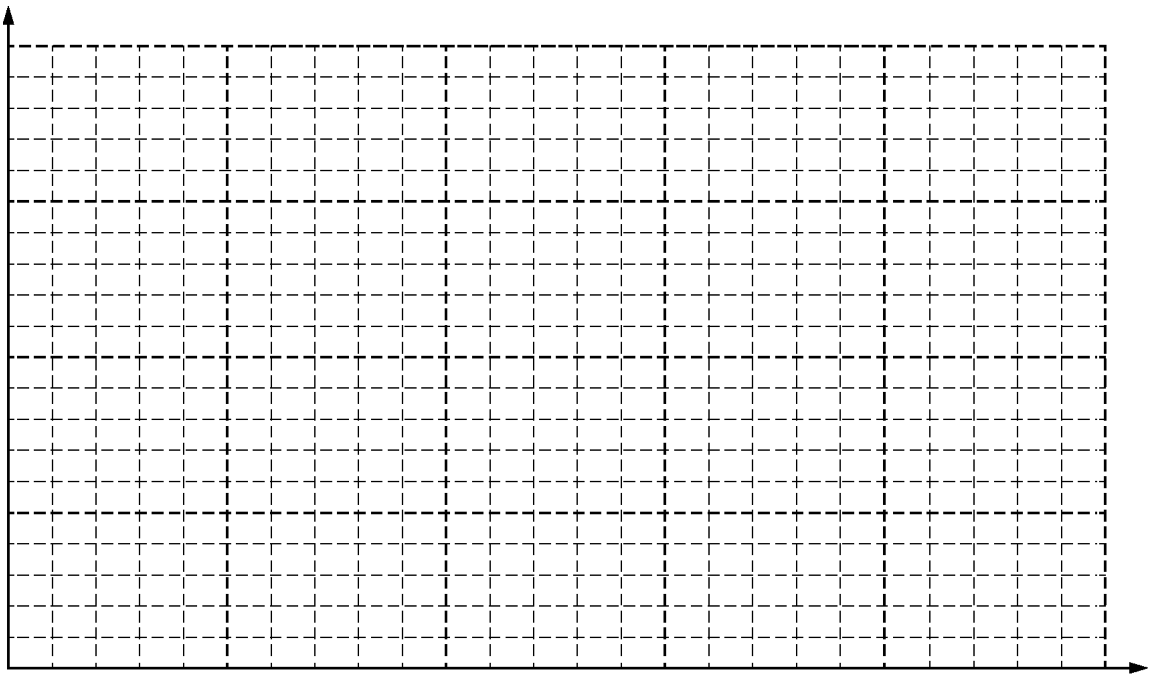


E&M. 1.

Physics students are analyzing the circuit above. A variable DC power supply is connected to an ammeter and three resistors. The resistances of two of the resistors are known to be $R_2 = R_3 = 50\ \Omega$, but the resistance of the third resistor is unknown. The students collect data on the potential difference across the power supply and the current measured by the ammeter, as follows.

Potential Difference (V)	2	4	6	8	10
Current (mA)	40	55	97	138	155

(a) On the grid below, plot the data points for the current as a function of the potential difference. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



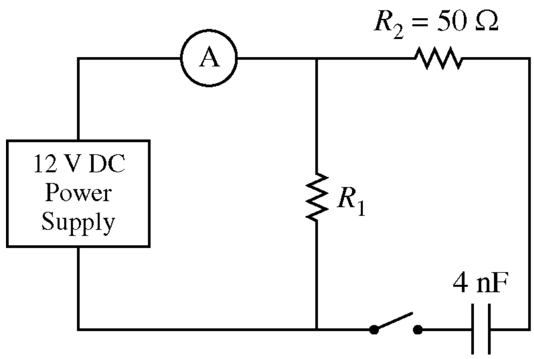
(b) Using the straight line from part (a), calculate the total resistance of the three-resistor combination.

(c) Calculate the value of R_1.

The power supply is now fixed at 12 V.

(d) Calculate the current through R_2.

(e) Resistor 3 is now removed and replaced by an open switch in series with an uncharged 4 nF capacitor, as shown below. The power supply is still fixed at 12 V.



i. Calculate the current in R_2 immediately after the switch is closed.

ii. A long time after the switch is closed, will the magnitude of the current in R_2 be greater than, less than, or equal to the current through R_2 found in part (d) ?

__ Greater than __ Less than __ Equal to

Justify your answer.

(f) The 4 nF capacitor is replaced with an uncharged 10 nF capacitor. Will the magnitude of the current in R_2 immediately after the switch is closed be greater than, less than, or equal to the current in part (e)i?

__ Greater than __ Less than __ Equal to

Justify your answer.

11.8 Resistor-Capacitor (RC) Circuits

Name: ______________ Class: __________ Date: ____________ **Total: 10 marks**

KEY WORDS time constant 时间常数 • steady state 稳态 • rc circuit RC 电路 • resistance 电阻

Objective

Build the skills to answer exam questions on **resistor-capacitor (RC) circuits**.

You must be able to:

- describe the **charging** and **discharging** of a capacitor through a resistor
- define the **time constant** 时间常数 $\tau = RC$
- use $q(t) = Q(1 - e^{-t/\tau})$ (charging) and $q(t) = Q\,e^{-t/\tau}$ (discharging)
- analyse the **initial** and **steady-state** behaviour

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **Charging and discharging**

$$q(t) = Q\left(1 - e^{-t/\tau}\right) \text{ (charging)}, \qquad q(t) = Q\,e^{-t/\tau} \text{ (discharging)}.$$

■ **Time constant**

$\tau = RC$ is the characteristic time; after one τ the charge reaches about 63% (charging) or falls to 37% (discharging).

■ **Start and steady state**

At $t = 0$ an uncharged capacitor acts like a **wire** (current $I_0 = V/R$); at steady state it is fully charged and carries **no** current.

2 Practice

2.1 Find the time constant of a 5.0 kΩ resistor and a 20 μF capacitor. [2]

2.2 State the current a long time after charging begins. [1]

2.3 Write the expression for the charge on a **discharging** capacitor. [1]

---

3 Exam-style questions

3.1 The time constant of an RC circuit is [1]

- **A** R/C
- **B** RC
- **C** $R+C$
- **D** $1/(RC)$

3.2 At $t=0$, an uncharged capacitor in an RC circuit behaves like a [1]

- **A** break in the circuit
- **B** plain wire
- **C** battery
- **D** resistor of infinite resistance

3.3 A 2.0 kΩ resistor charges a 50 μF capacitor from a 10 V supply.

(a) Find the time constant. [2]

(b) Find the initial current (at $t=0$). [2]

Solutions

2.1 $\tau = RC = (5.0 \times 10^3)(20 \times 10^{-6}) = 0.10$ s.

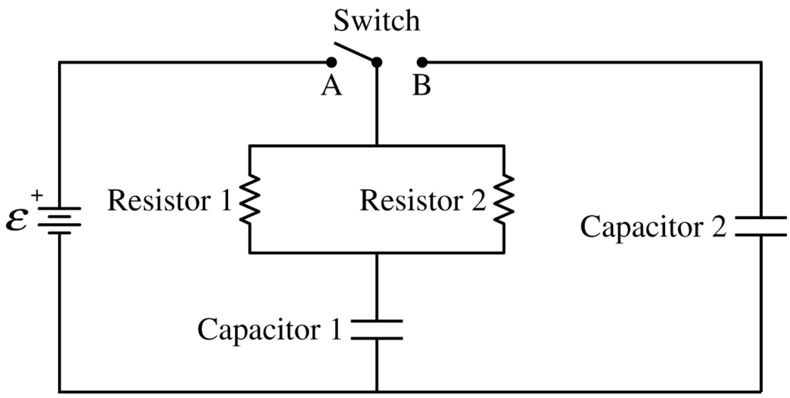
2.2 zero (the capacitor is fully charged).

2.3 $q(t) = Q\, e^{-t/\tau}$.

3.1 **B**.

3.2 **B**.

3.3 (a) $\tau = RC = (2.0\times10^3)(50\times10^{-6}) = 0.10$ s. (b) $I_0 = \dfrac{V}{R} = \dfrac{10}{2.0 \times 10^3} = 5.0\times10^{-3}$ A.



. The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

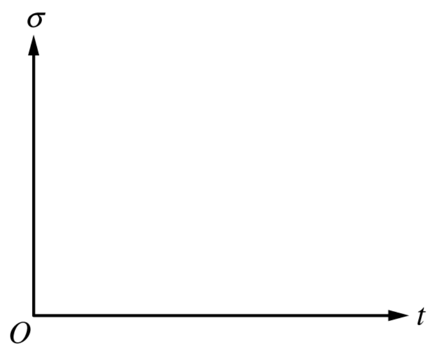
At time $t = 0$, the switch is closed to Position A.

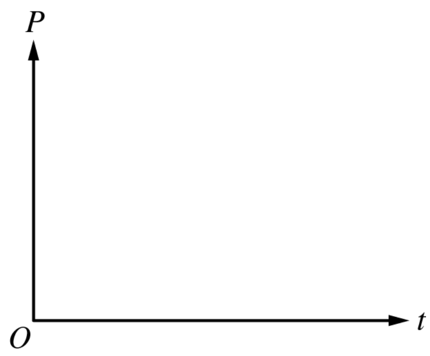
(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(b) On the axes shown, sketch graphs of the surface charge density σ on the positive plate of Capacitor 1 and the total power P dissipated by the resistors as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.





A long time after the switch is closed to Position A, the charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

(c) At time t_1, the switch is closed to Position B.

i. Immediately after time t_1, is the direction of the current in the switch directed toward the left, directed toward the right, or is there no current? Briefly justify your answer.

ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

iii. Derive an expression for the total energy E_R dissipated by resistors 1 and 2 from immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.

With the switch still closed to Position B, the parallel plates of Capacitor 2 are moved so that the separation distance increases by a factor of 2.

(d) Determine the ratio $\frac{U_2}{U_1}$ of the energy U_2 stored in Capacitor 2 to the energy U_1 stored in Capacitor 1 a long time after the plates of Capacitor 2 have been moved. Briefly justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

With the capacitors still charged as in part (d), the switch is now closed to Position A.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.

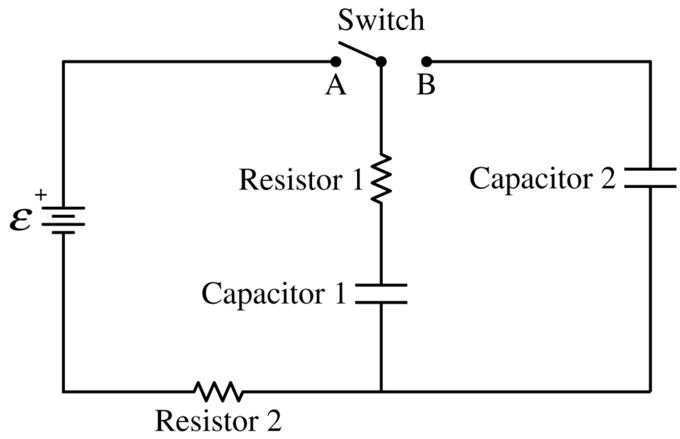
i. Derive an expression for the current I_0 from the battery immediately after the switch is closed to Position A.

ii. Determine the current I_∞ from the battery a long time after the switch is closed to Position A.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM



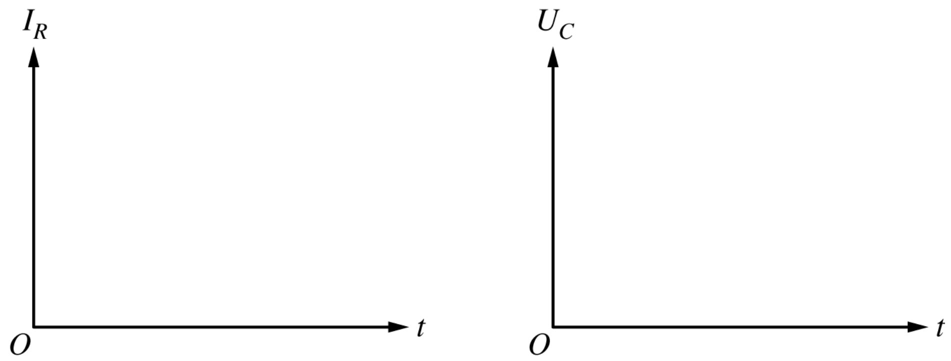
. The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

(b) On the axes shown, sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.



A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

(c) At time t_1, the switch is closed to Position B.

i. Immediately after t_1, is the direction of the current in Resistor 1 directed up, directed down, or is there no current? Briefly justify your answer.

ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

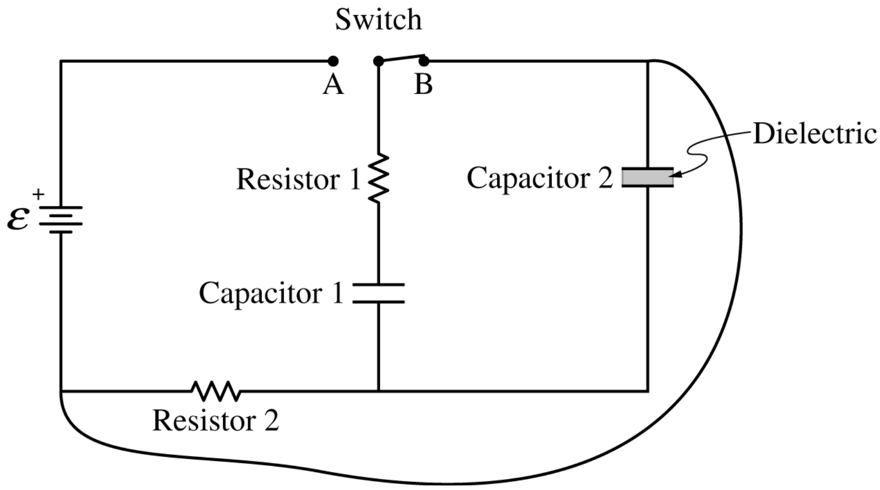
iii. Derive an expression for the total energy dissipated by Resistor 1 immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.

With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

(d) Determine the charge on the positive plate of Capacitor 2 a long time after the dielectric has been inserted. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.



With the switch still closed to Position B, a wire of negligible resistance is connected between two corners of the circuit, as shown.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.

i. Derive an expression for the current in Resistor 2 immediately after the wire is connected to the circuit.

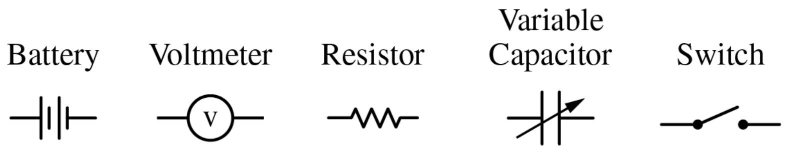
ii. Determine the current in Resistor 2 a long time after the wire is connected to the circuit.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

. The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.



(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the potential difference across the capacitor as it discharges through the resistor.

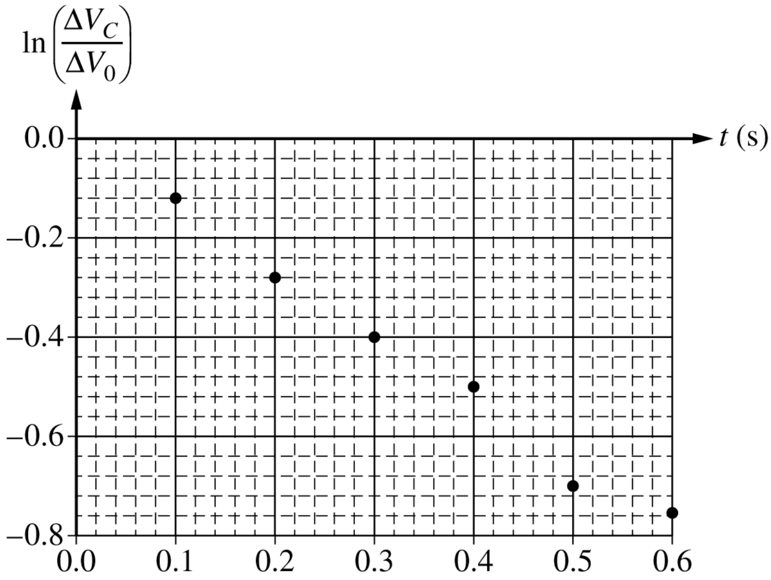
The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

(b) Show that the potential difference ΔV_C across the capacitor as a function of time t is $\Delta V_C(t) = \Delta V_0 e^{-\frac{t}{RC}}$ as the capacitor discharges.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(c) The experiment is performed using a resistor of $R = 150\ \text{k}\Omega$. Data for the potential difference ΔV_C across the capacitor as a function of t are recorded and a plot of $\ln\left(\frac{\Delta V_C}{\Delta V_0}\right)$ as a function of t is created on the graph below.



i. Draw the best-fit line for the data.

ii. Using the best-fit line, calculate a value for the unknown capacitance C.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) The capacitor is adjusted so that the surface area of the plates is increased, and the experiment is repeated. Would the slope of the best-fit line in the second experiment be more steep, less steep, or unchanged compared to the slope of the best-fit line in part (c)?

___ More steep ___ Less steep ___ Unchanged

Briefly justify your answer.

(e) The ideal battery is then replaced with a non-ideal battery with internal resistance r, and the experiment is repeated.

i. Would the slope of the graph in this final experiment change compared to the graph in part (c)?

___ Yes ___ No

Briefly justify your answer.

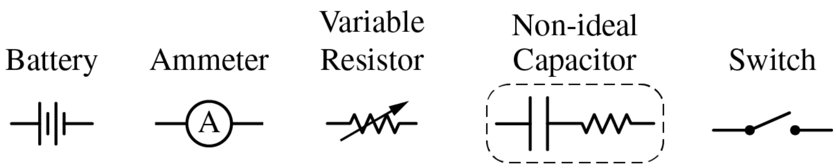
ii. Would the vertical intercept of the graph in this final experiment change compared to the graph in part (c)?

___ Yes ___ No

Briefly justify your answer.

GO ON TO THE NEXT PAGE.

. A non-ideal capacitor has internal resistance that can be modeled as an ideal capacitor in series with a small resistor of resistance r_C. A group of students performs an experiment to determine the internal resistance of a capacitor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single ammeter, a single variable resistor of resistance R, a single uncharged non-ideal capacitor of capacitance C, and one or more switches as needed.



(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the current through the capacitor as it discharges through the resistor.

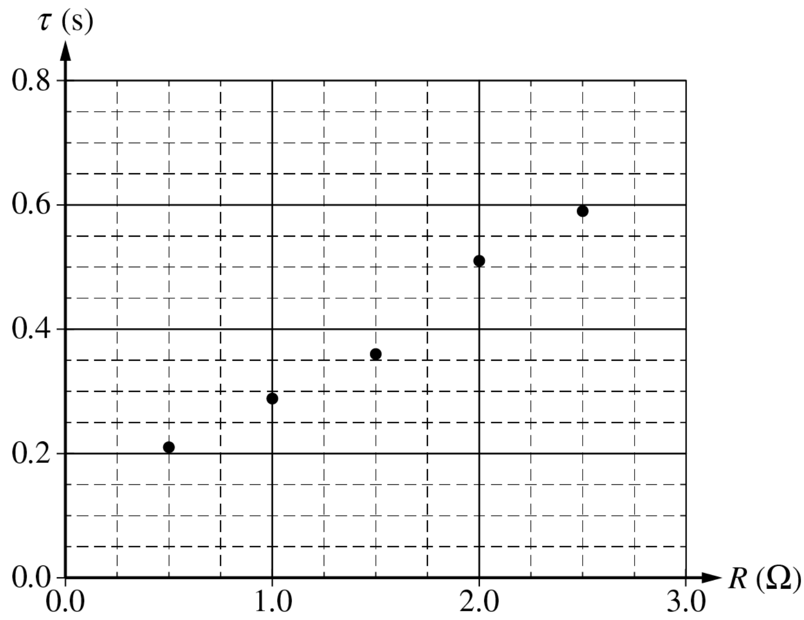
The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

(b) Show that the current I through the capacitor as a function of time t is $I(t) = I_0 e^{\frac{-t}{(R+r_C)C}}$ as the capacitor discharges.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(c) The students determine the time constant τ for the circuit as a function of the resistance R. The students' data are shown in the following graph.



i. Draw the best-fit line for the data.

ii. Using the best-fit line, calculate a value for the internal resistance r_C of the capacitor.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) The ammeter is found to be nonideal. Is the actual value for the internal resistance r_C for the capacitor greater than, less than, or equal to the experimental internal resistance of the capacitor calculated in part (c)?

___ Greater than ___ Less than ___ Equal to

Briefly justify your answer using features of the graph in part (c).

(e) The values of the variable resistor in the original experiment ranged from $0.5\ \Omega$ to $2.5\ \Omega$. The experiment is repeated with values ranging from $3.0\ \Omega$ to $6.0\ \Omega$. Would the slope of the best-fit line be more steep, be less steep, or remain unchanged compared to the graph in part (c)?

___ More steep ___ Less steep ___ Remain unchanged

Briefly justify your answer.

GO ON TO THE NEXT PAGE.

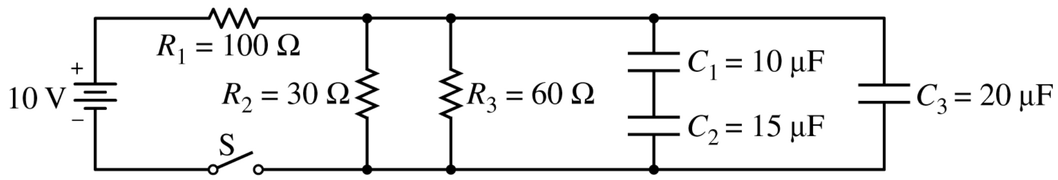
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. The circuit shown above is composed of an ideal 10 V battery, three resistors and three capacitors with the values shown, and an open switch S. The capacitors are initially uncharged. Switch S is now closed.

(a) Calculate the current through R_1 immediately after switch S is closed.

Switch S has been closed for a long time, and the circuit has reached a steady state.

(b) Calculate the potential difference across R_1.

(c)
i. Calculate the charge stored on the positive plate of capacitor C_2.

ii. Is the charge stored on capacitor C_3 greater than, less than, or equal to the charge stored on capacitor C_2 ?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

GO ON TO THE NEXT PAGE.

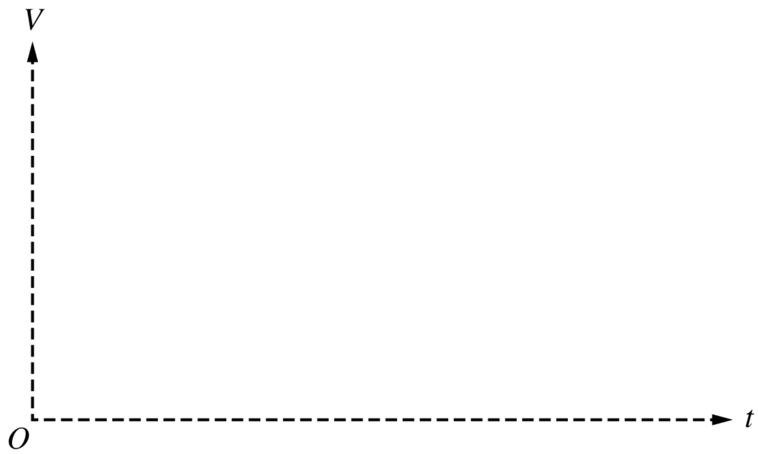
Switch S is then opened.

(d)

i. Determine the current through R_1 immediately after the switch is opened.

ii. Calculate the current through R_2 immediately after the switch is opened.

(e) On the axes below, sketch a graph of the potential difference V across capacitor C_2 as a function of time t if switch S is opened at time $t = 0$. Label the maximum value.



Capacitor C_3 is replaced by two $10\,\mu F$ capacitors connected in series, switch S is closed, and the circuit reaches equilibrium. Switch S is then opened at time $t = 0$.

(f) For $t > 0$, would the sketch of a graph of the new voltage across C_2 as a function of time be above, below, or the same as the sketch for part (e) ?

_____ Above   _____ Below   _____ The same

Justify your answer.

GO ON TO THE NEXT PAGE.

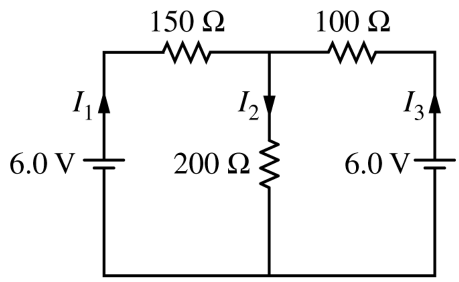


Figure 1

2. The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(a)

i. Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

ii. Calculate the current in the $200\ \Omega$ resistor.

iii. Calculate the power dissipated by the $200\ \Omega$ resistor.

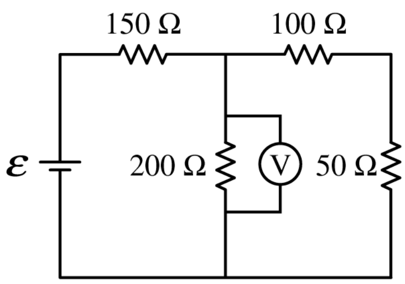


Figure 2

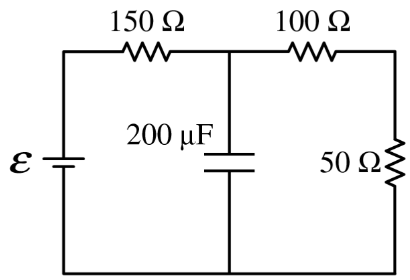
The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

(b) Calculate the current through the $50\ \Omega$ resistor.

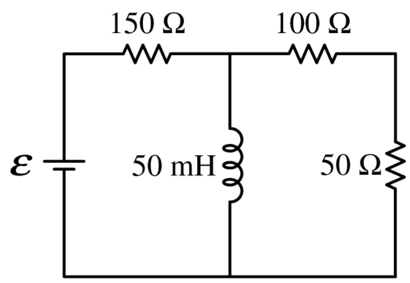
(c) Calculate the voltage $\mathcal{E}$ of the battery.

(d)

i. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu F$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.



ii. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b) ?



____ Greater than ____ Less than ____ Equal to

Justify your answer.

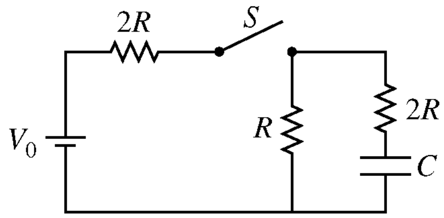
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(a) Derive an expression for the steady-state current supplied by the battery.

(b) Derive an expression for the charge on the capacitor.

(c) Derive an expression for the energy stored in the capacitor.

Now the switch is opened at time $t = 0$.

(d) Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time t after the switch is opened.

(e)

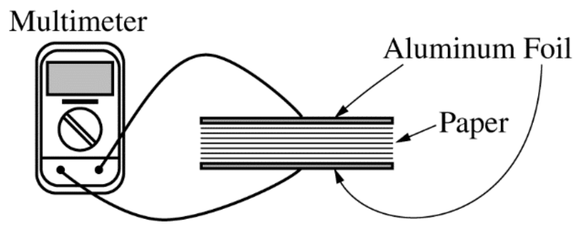
i. Calculate the current in resistor R immediately after the switch is opened.

ii. On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.

(f) Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c) ?

____ Greater than ____ Less than ____ Equal to

Justify your answer.



2. An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \text{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)		
500	0.045	6.5×10^{-11}		
400	0.036	7.4×10^{-11}		
300	0.027	8.9×10^{-11}		
200	0.018	11.9×10^{-11}		
100	0.010	21.0×10^{-11}		

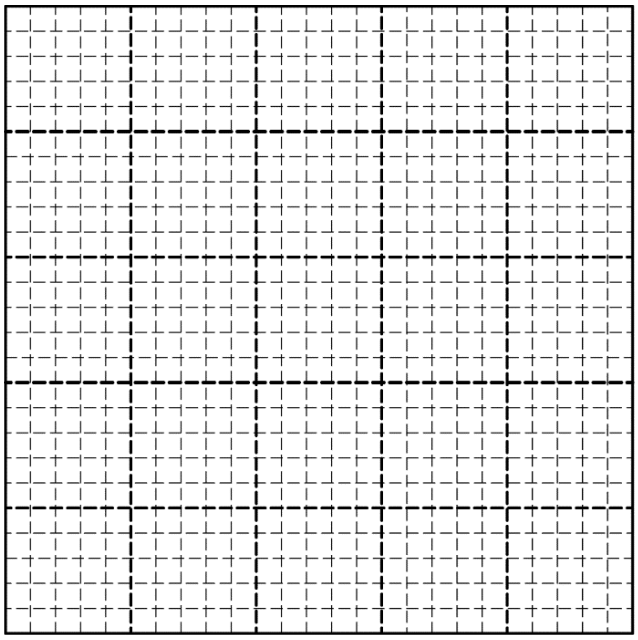
(a) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: ____________

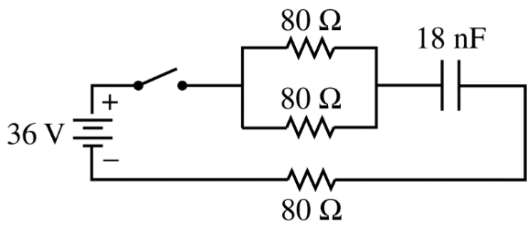
Horizontal axis: ____________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

(b) Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(c) Using the straight line, calculate a dielectric constant for the paper.



The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical $80\ \Omega$ resistors, and an open switch, as shown in the figure above.

(d) Calculate the current in the battery immediately after the switch is closed.

(e) Determine the time constant for this circuit.

(f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

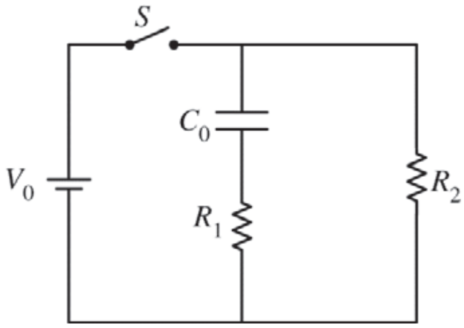
____ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

ii. Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

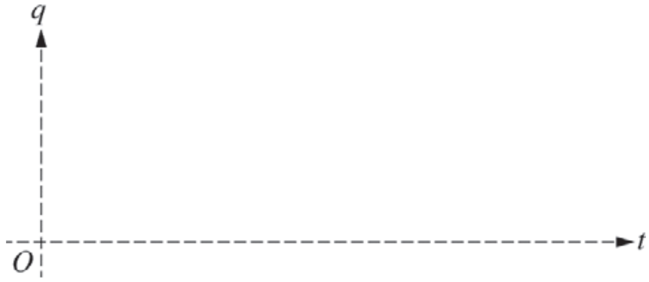
____ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

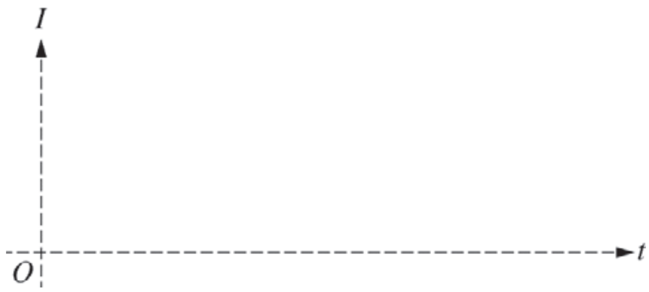


2. In the circuit above, an ideal battery of voltage V_0 is connected to a capacitor with capacitance C_0 and resistors with resistances R_1 and R_2, with $R_1 > R_2$. The switch S is open, and the capacitor is initially uncharged.

(a) The switch is closed at time $t = 0$. On the axes below, sketch the charge q on the capacitor as a function of time t. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



(b) On the axes below, sketch the current I through each resistor as a function of time t. Clearly label the two curves as I_1 and I_2, the currents through resistors R_1 and R_2, respectively. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



The circuit is constructed using an ideal 1.5 V battery, an 80 μF capacitor, and resistors $R_1 = 150\ \Omega$ and $R_2 = 100\ \Omega$. The switch is closed, allowing the capacitor to fully charge. The switch is then opened, allowing the capacitor to discharge.

(c) The time it takes to charge the capacitor to 50% of its maximum charge is Δt_C. The time it takes for the capacitor to discharge to 50% of its maximum charge is Δt_D. Which of the following correctly relates the two time intervals?

____ $\Delta t_C > \Delta t_D$ ____ $\Delta t_C = \Delta t_D$ ____ $\Delta t_C < \Delta t_D$

Justify your answer.

(d)

i. Calculate the current through resistor R_2 immediately after the switch is opened.

ii. Is the current through resistor R_2 increasing, decreasing, or constant immediately after the switch is opened?

____ Increasing ____ Decreasing ____ Constant

Justify your answer.

(e)

i. Calculate the energy stored in the capacitor immediately after the switch is opened.

ii. Calculate the energy dissipated by resistor R_1 as the capacitor completely discharges.

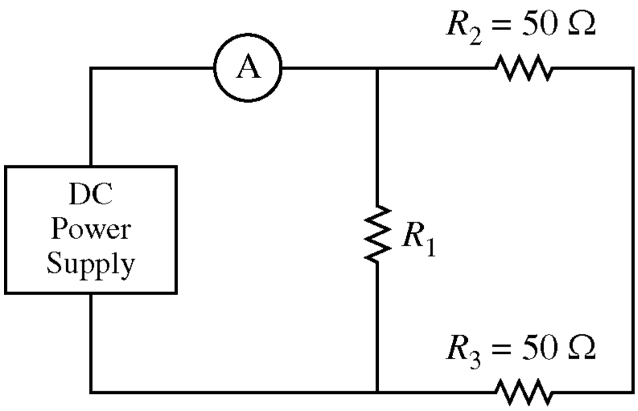
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

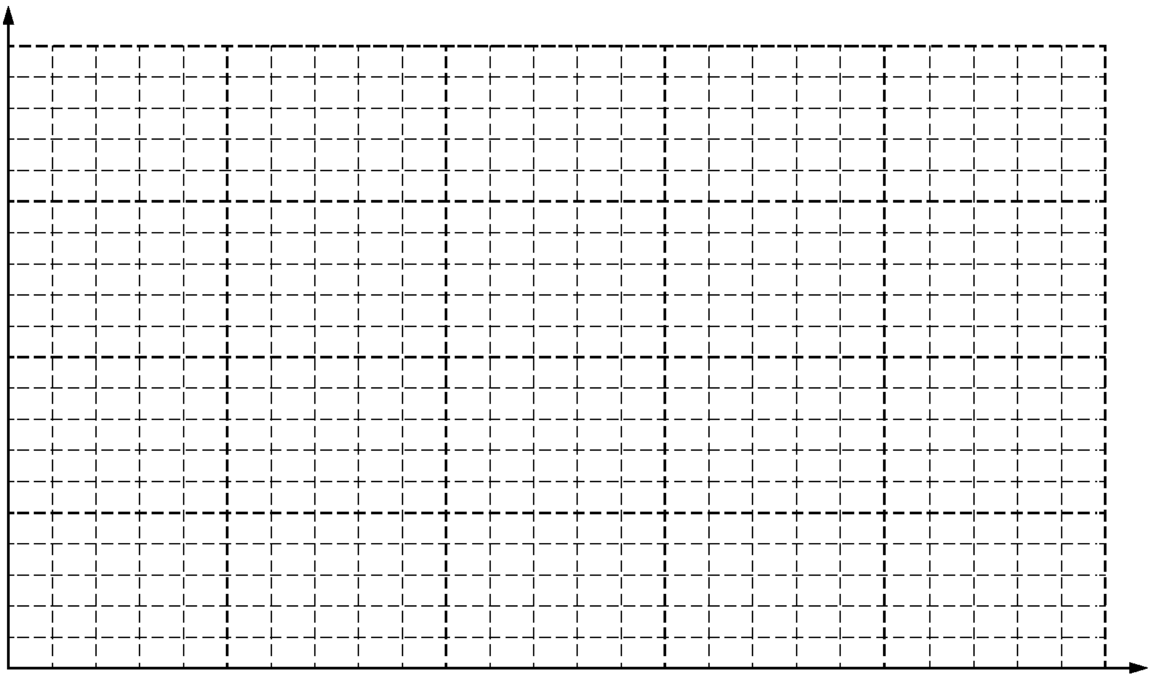


E&M. 1.

Physics students are analyzing the circuit above. A variable DC power supply is connected to an ammeter and three resistors. The resistances of two of the resistors are known to be $R_2 = R_3 = 50\ \Omega$, but the resistance of the third resistor is unknown. The students collect data on the potential difference across the power supply and the current measured by the ammeter, as follows.

Potential Difference (V)	2	4	6	8	10
Current (mA)	40	55	97	138	155

(a) On the grid below, plot the data points for the current as a function of the potential difference. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



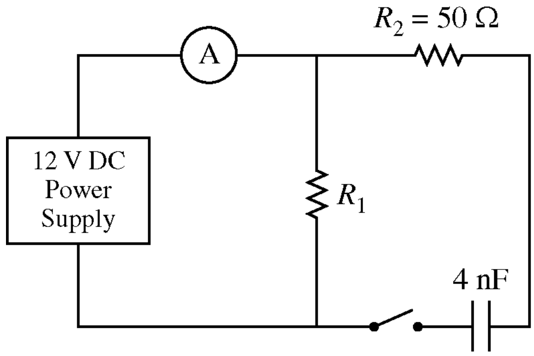
(b) Using the straight line from part (a), calculate the total resistance of the three-resistor combination.

(c) Calculate the value of R_1.

The power supply is now fixed at 12 V.

(d) Calculate the current through R_2.

(e) Resistor 3 is now removed and replaced by an open switch in series with an uncharged 4 nF capacitor, as shown below. The power supply is still fixed at 12 V.



i. Calculate the current in R_2 immediately after the switch is closed.

ii. A long time after the switch is closed, will the magnitude of the current in R_2 be greater than, less than, or equal to the current through R_2 found in part (d) ?

__ Greater than __ Less than __ Equal to

Justify your answer.

(f) The 4 nF capacitor is replaced with an uncharged 10 nF capacitor. Will the magnitude of the current in R_2 immediately after the switch is closed be greater than, less than, or equal to the current in part (e)i ?

__ Greater than __ Less than __ Equal to

Justify your answer.