Week 10

AP Physics C: Electricity and Magnetism

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

10. Conductors and Capacitors

AP Physics C: Electricity and Magnetism Name: ____________________ Score: ______

1. Inside a conductor in electrostatic equilibrium, the electric field is:

   ☐ very large

   ☐ zero

   ☐ equal to the surface field

   ☐ pointing inward

2. When two conductors are connected, charge flows until they have equal:

   ☐ charge

   ☐ potential

   ☐ radius

   ☐ mass

3. Capacitance is defined as:

   ☐ Q/V

   ☐ QV

   ☐ V/Q

   ☐ $Q+V$

4. Inserting a dielectric of constant κ changes the capacitance to:

   ☐ C_0/κ

   ☐ κC_0

   ☐ $C_0-\kappa$

   ☐ unchanged

5. Excess charge on a solid conductor sits:

   ☐ spread through the volume

   ☐ at the exact centre

   ☐ entirely on the surface

   ☐ nowhere — it vanishes

6. At equal potential, the larger sphere holds more charge ($Q \propto R$).

   ☐ True ☐ False

7. A dielectric polarises and weakens the field between the plates.

☐ True ☐ False

8. A hollow conductor shields its inside from external fields (a Faraday cage).

☐ True ☐ False

9. A radius-$3R$ sphere shares charge with a radius-R sphere. The big one holds how many times the charge of the small one?

____

10. A 2 F capacitor is charged to 3 V. Find the stored energy (in J).

__________________ J

AP Physics C: Electricity and Magnetism

1. **zero**

   If it were not zero, free charges would move — so at equilibrium $E = 0$ inside.

2. **potential**

   Charge flows until the potentials are equal — no push remains.

3. Q/V

   $C = Q/V$, charge stored per volt, in farads.

4. κC_0

   $C = \kappa C_0$, and $\kappa > 1$, so capacitance rises.

5. **entirely on the surface**

   With zero interior field, all excess charge must lie on the surface.

6. **True**

   $V = kQ/R$ equal means $Q \propto R$ — bigger holds more.

7. **True**

   Its aligned charges oppose the plates' field, weakening the net field.

8. **True**

   The conductor cancels the field inside the cavity — a Faraday cage.

9. **3**

   $Q \propto R$, so the $3R$ sphere holds $3\times$ the charge.

10. **9 J**

    $U = \frac{1}{2}CV^2 = \frac{1}{2}(2)(3^2) = 9$ J.

10.1 Electrostatics with Conductors

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

KEY WORDS outer surface 外表面 • conductor 导体 • perpendicular 垂直 • electrostatic equilibrium 静电平衡 • equipotential 等势体

Objective

Build the skills to answer exam questions on **electrostatics with conductors**.

You must be able to:

- explain why the **electric field is zero** inside a conductor in electrostatic equilibrium
- show that excess charge resides on the **outer surface** 外表面
- describe how the field just outside is **perpendicular** to the surface
- explain why a conductor is an **equipotential** 等势体

1 Worked examples

Study these first. Each one shows the method for a question type used later.

A conductor in equilibrium

- The field **inside** is zero (free charges would move until it is).
- Any **excess charge** sits on the outer surface.
- Just **outside**, the field is **perpendicular** to the surface.
- The whole conductor is a single **equipotential**.

Surface field

$E = \dfrac{\sigma}{\varepsilon_0}$ just outside. Charge crowds onto sharply curved regions (points), giving the strongest field there — the idea behind a lightning rod.

2 Practice

2.1 State the electric field inside a conductor in electrostatic equilibrium. [1]

2.2 State where excess charge sits on a solid conductor. [1]

2.3 State the direction of the field just outside a charged conductor's surface. [1]

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3 Exam-style questions

3.1 Inside a conductor in electrostatic equilibrium, the field is [1]

- **A** σ/ε_0
- **B** zero
- **C** at its maximum
- **D** kq/r^2

3.2 A conductor in electrostatic equilibrium is [1]

- **A** an equipotential
- **B** at a varying potential
- **C** an insulator
- **D** charged throughout its volume

3.3 A hollow metal sphere carries a charge $+Q$.

(a) State the field inside the metal. [1]

(b) State where the charge sits. [1]

(c) Explain why the field just outside the surface is perpendicular to it. [2]

Solutions

2.1 zero.

2.2 on its outer surface.

2.3 perpendicular to the surface.

3.1 **B**.

3.2 **A**.

3.3 (a) zero. (b) on the outer surface. (c) any component along the surface would push the surface charges until it vanished, so at equilibrium only the perpendicular component remains.

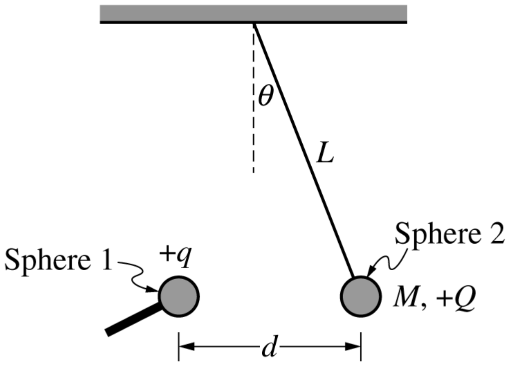
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

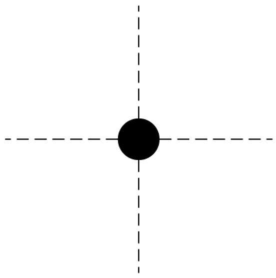


. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere 1 is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere 2 is nonconducting with charge $+Q$ and has mass M. Sphere 2 is hung from a string of negligible mass and length L. Sphere 1 is brought near, without touching, Sphere 2, as shown. Equilibrium is established when the centers of the two spheres have the same vertical position, are a horizontal distance d apart, and the string is at an angle θ from the vertical.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(a) On the following dot that represents Sphere 2 at the position shown in the previous figure, draw and label the forces (not components) that act on Sphere 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



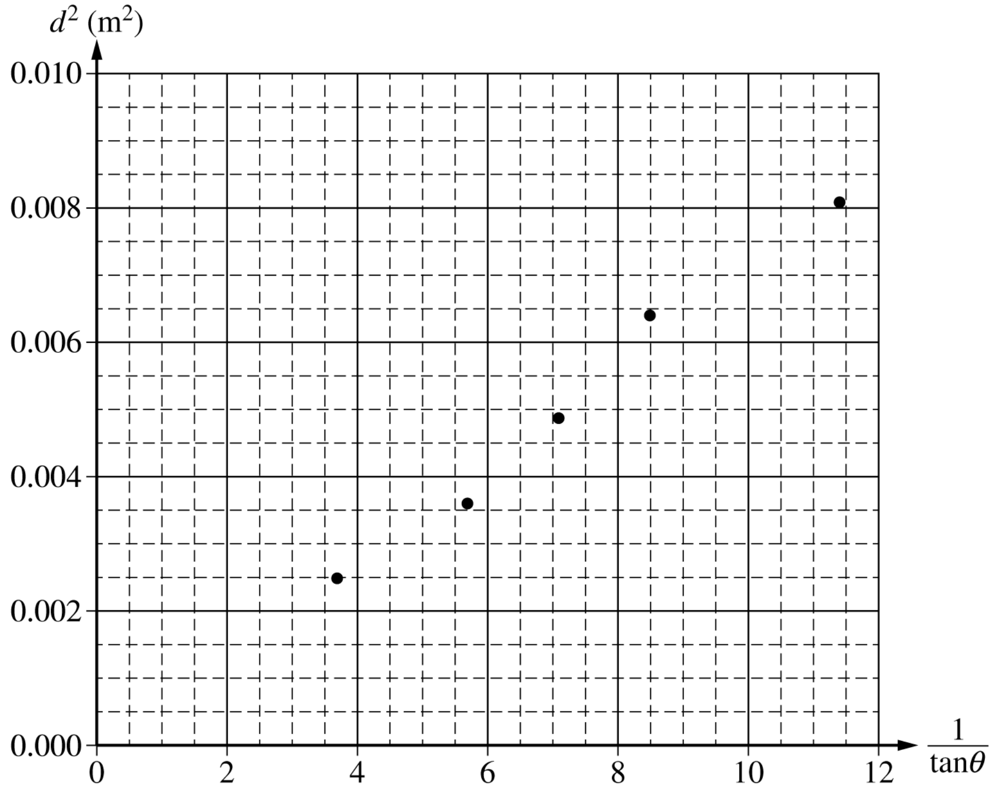
(b) Derive the relationship between the distance d and the angle θ to show that $d = \sqrt{\dfrac{Qq}{4\pi\varepsilon_0 Mg \tan\theta}}$.

(c) These values are collected in one trial: $Q = q = 6.0 \times 10^{-8}$ C, $\theta = 12°$, and $d = 0.057$ m. Calculate the expected force of tension exerted on Sphere 2 by the string.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(d) The students vary d and measure θ after equilibrium is reached. The students use the collected data to plot the following graph of d^2 vs. $\frac{1}{\tan\theta}$.



i. Draw the best-fit line for the data.

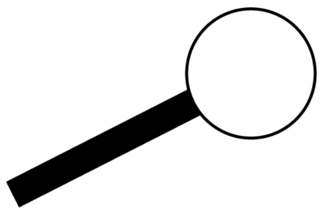
ii. Using the best-fit line, calculate an experimental value for the vacuum permittivity ε_0 when $M = 0.0050\,\text{kg}$ and $Q = q = 6.0 \times 10^{-8}\,\text{C}$.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(e) The students modify the experiment by replacing Sphere 1 with a conducting Sphere 3 that has the same size and charge $+q$. The experiment is repeated.

i. The circle in the following figure represents Sphere 3 when spheres 2 and 3 are at equilibrium. On the circle, draw a single "+" sign to represent the location of highest concentration of the excess positive charges.



ii. Briefly explain your reasoning for the sketch drawn in part (e)(i).

iii. In the original experiment, when the centers of the two spheres are a horizontal distance d_1 apart, the string makes an angle θ_1 from the vertical. In the modified experiment, when the centers of the two spheres are a horizontal distance d_1 apart, the string makes an angle θ_2 from the vertical.

Is θ_2 greater than, less than, or equal to θ_1 ?

____ $\theta_2 > \theta_1$ ____ $\theta_2 < \theta_1$ ____ $\theta_2 = \theta_1$

Briefly justify your answer.

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PHYSICS C: ELECTRICITY AND MAGNETISM

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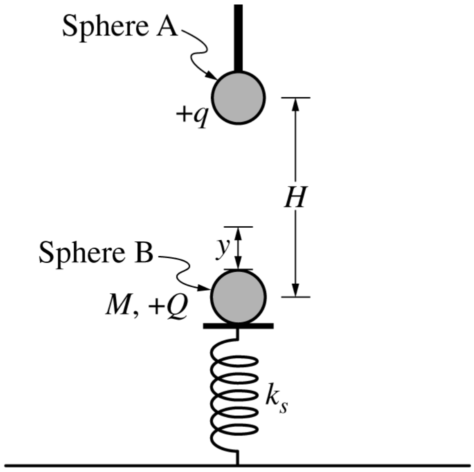

Note: Figure not drawn to scale.

1. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M. Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s. Sphere B and the spring are initially at rest.

Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.

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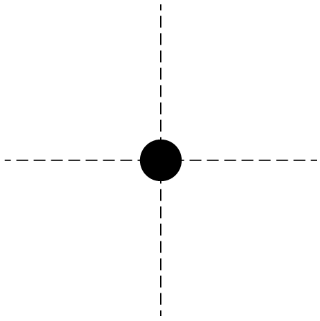
Continue your response to **QUESTION 1** on this page.

(a) On the following dot that represents Sphere B in the figure on the previous page, draw and label the forces (not components) that are exerted on Sphere B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



(b) Derive the relationship between y and H to show that $y = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{k_s H^2} + \dfrac{Mg}{k_s}$.

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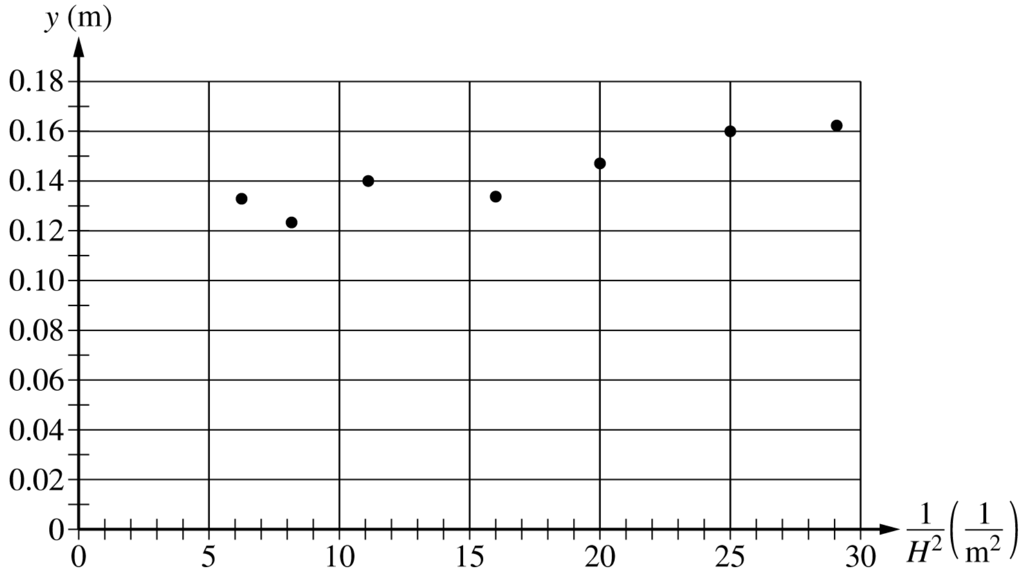
(c) The students plot collected data of y as a function of $\frac{1}{H^2}$, as shown in the graph.



i. Draw the best-fit line for the data.

ii. Using the best-fit line, calculate an experimental value for the vacuum permittivity ε_0 when $Q = q = 2.00 \times 10^{-6}$ C and $k_s = 25$ N/m.

iii. Using the best-fit line, calculate an experimental value for the mass of Sphere B.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(d) The students modify the experiment by replacing nonconducting Sphere B with conducting Sphere C that has the same charge $+Q$ and mass M. Sphere A is brought near Sphere C without touching, compressing the spring. Sphere C comes to rest.

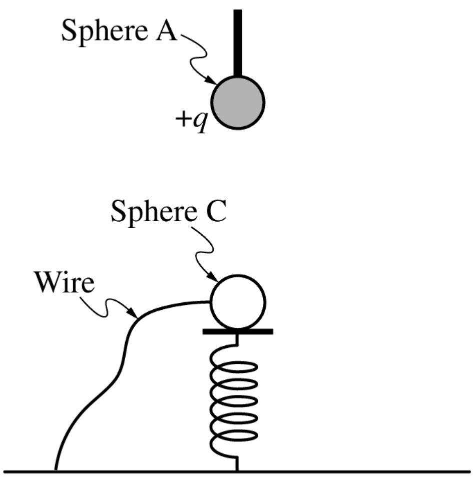
i. In the original experiment, when the centers of spheres A and B are a vertical distance H_1 apart, the spring is compressed a distance y_1. In the modified experiment, when the centers of spheres A and C are a vertical distance H_1 apart, the spring is compressed a distance y_2.

Is y_2 greater than, less than, or equal to y_1 ?

____ $y_2 > y_1$ ____ $y_2 < y_1$ ____ $y_2 = y_1$

Justify your answer.

ii. Sphere C is then grounded with a wire. On the following figure, draw an arrow indicating the direction that the platform will move immediately after being grounded. If the platform remains stationary, write "does not move."



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Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

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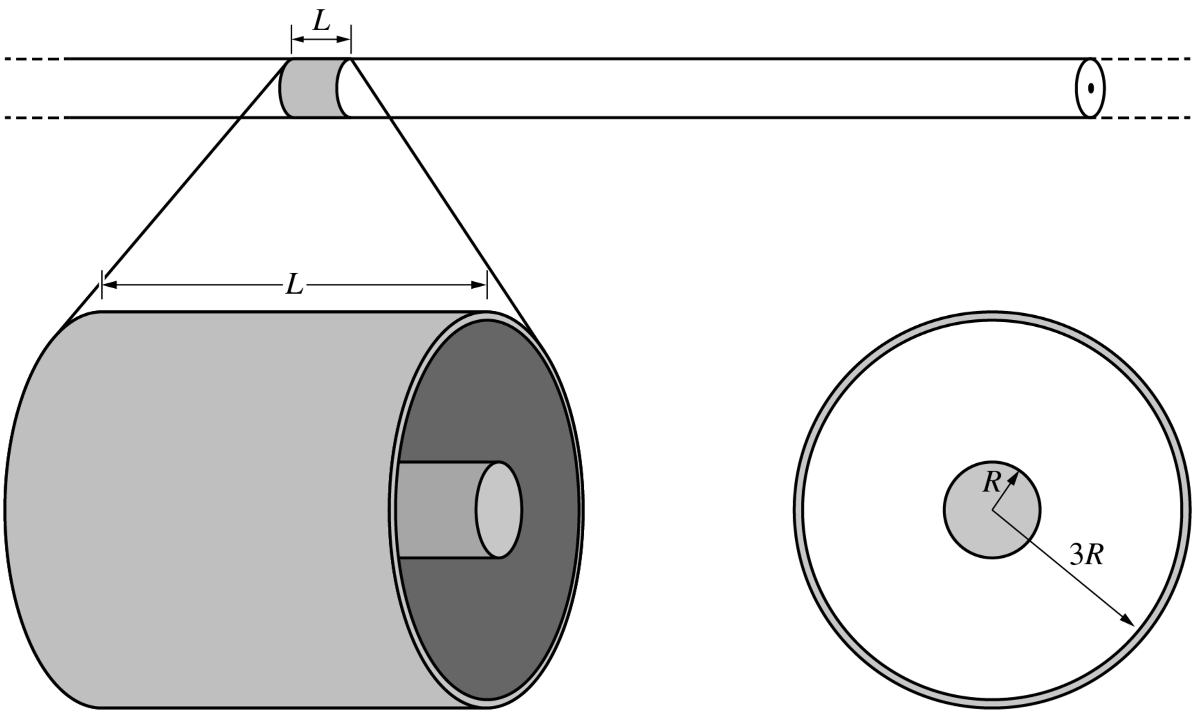

Cutout View

Cross-Section View

Note: Figures not drawn to scale.

1. A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

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Continue your response to **QUESTION 1** on this page.

(a) Determine the charge on the outer surface of the cylindrical shell within length L.

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the inner cylinder for $r < R$. Express your answers in terms of Q, R, r, L, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(c) The magnitude of the electric field at $r = R$ is $12\ \mathrm{N/C}$. Calculate the value of the electric field at $r = 2R$.

(d) Derive an expression for the absolute value of the potential difference between the surface of the nonconducting cylinder and the inner surface of the cylindrical shell.

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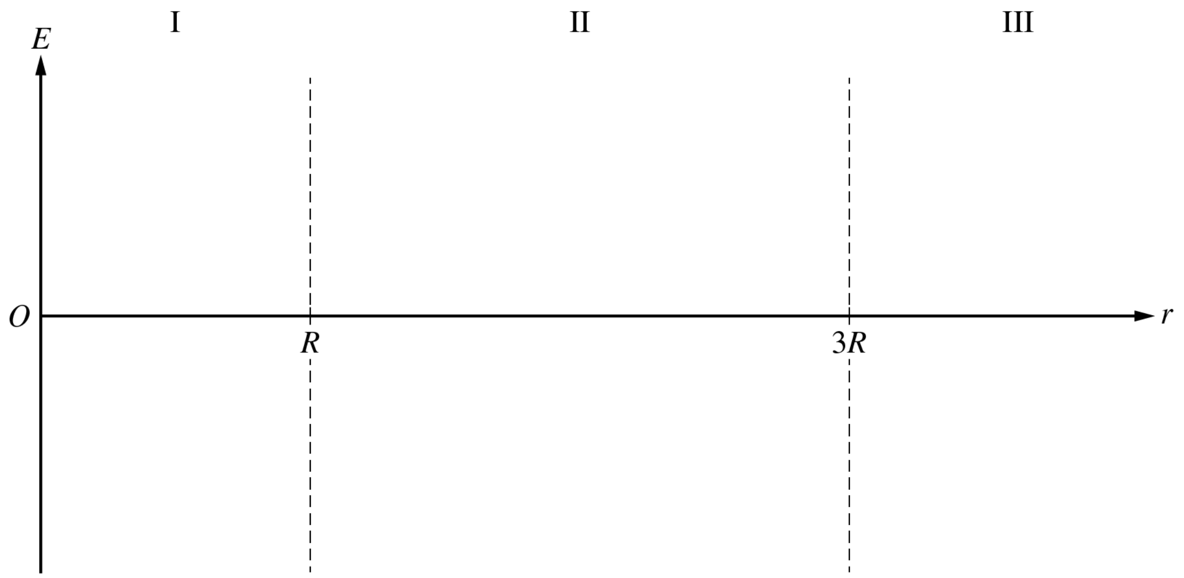
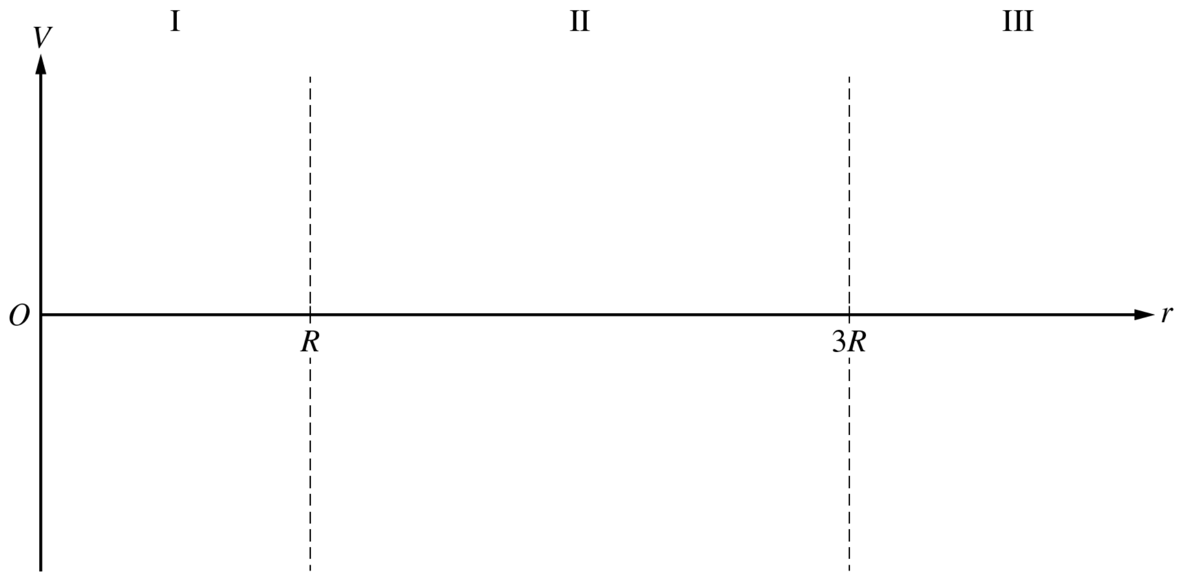
Continue your response to **QUESTION 1** on this page.

(e)

i. On the following axes that include regions I, II, and III, sketch the graph of the electric field E as a function of the distance r from the axis of the inner cylinder.



ii. On the following axes that include regions I, II, and III, sketch the graph of the electric potential V as a function of the distance r from the axis of the inner cylinder.



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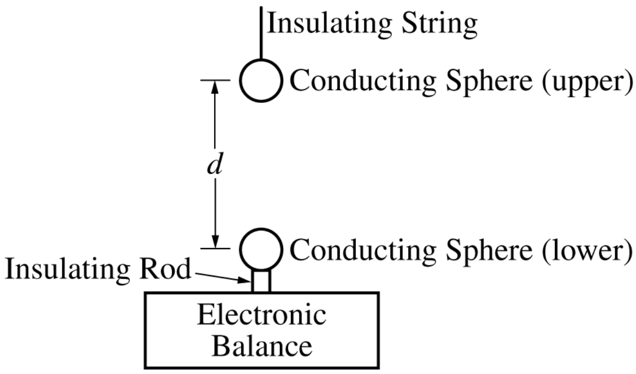
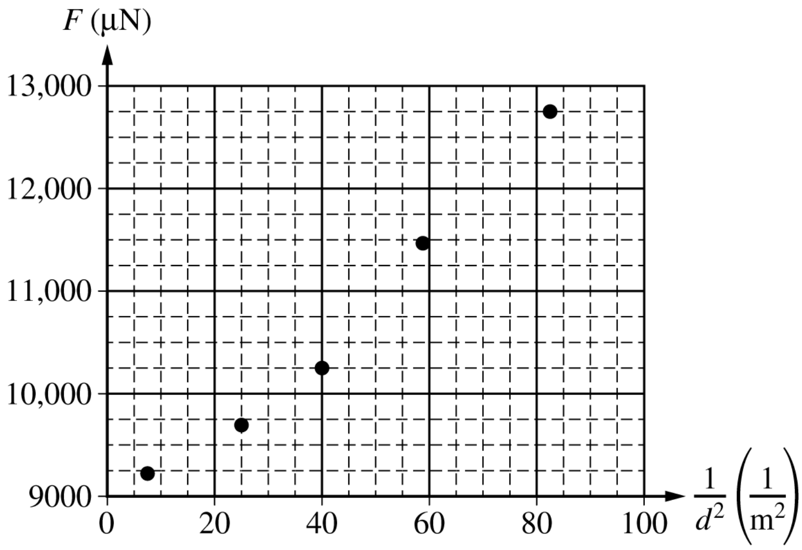


Students perform an experiment to study the force between two charged objects using the apparatus shown above, which contains two identical conducting spheres. The upper sphere is attached to an insulating string, which can be used to move the sphere downward. The lower sphere sits on an insulating rod, which is on an electronic balance. The electronic balance is zeroed before the lower sphere and insulating rod are in place.

For the first trial, a charge of Q is placed on each sphere and then the upper sphere is slowly moved downward. The students measure the distance d between the centers of the spheres and the magnitude F of the force that appears on the electronic balance. The recorded data are shown on the graph of F as a function of $\frac{1}{d^2}$ shown below.



(a)

i. Draw a line that represents the best fit to the points shown.

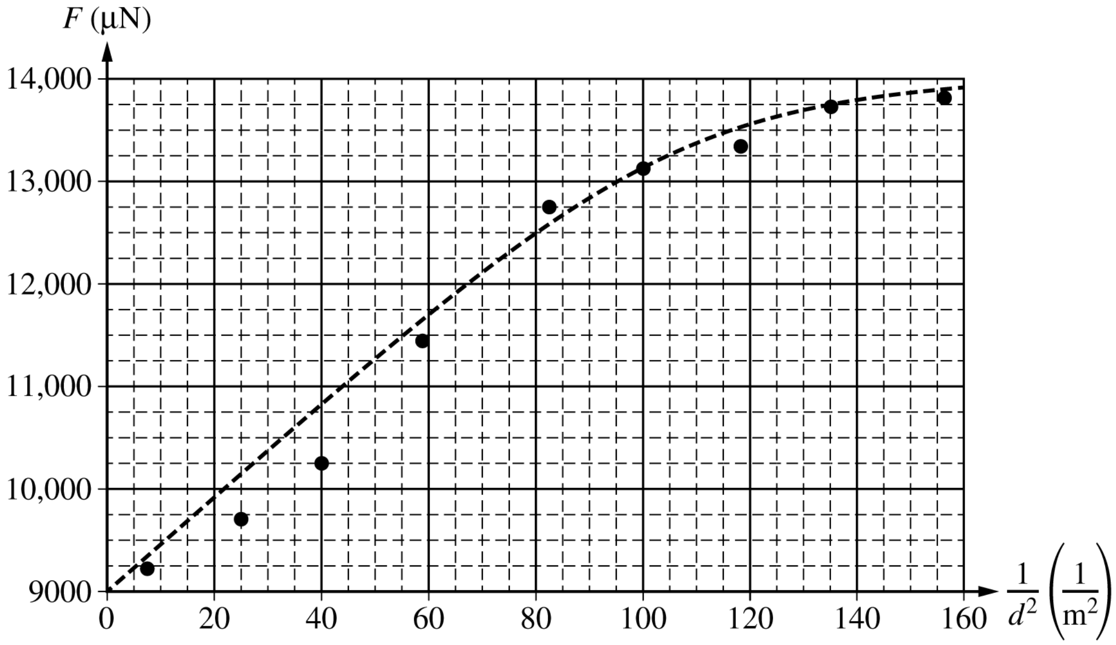
ii. Use the graph to calculate the charge Q.

iii. On the graph on the previous page, draw a circle around the data point that was taken when the distance between the centers of the spheres was the least.

iv. Determine the distance between the centers of the spheres for the data point indicated above.

v. What physical quantity does the vertical intercept represent?

Justify your answer.



The experiment is extended by collecting additional data points, which appear on the right side of the graph shown above. The new data points do not follow the linear pattern seen with the first points. The group of students tries to explain this discrepancy.

(b) One student suspects that charge is slowly leaking off the top sphere. Could this explain the discrepancy?

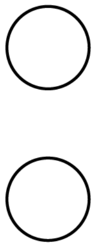
____ Yes ______ No

Justify your answer.

GO ON TO THE NEXT PAGE.

) A second student suspects that the excess charges have rearranged themselves, polarizing the spheres.

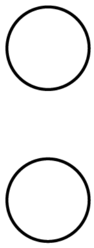
i. On the circles representing the spheres below, use a single “+” sign on each sphere to represent the locations of highest concentration of the excess positive charges.



ii. Explain how this rearrangement could be responsible for the discrepancy.

(d) A third student suggests that the experiment be modified so that the top sphere is given a negative charge that is equal in magnitude to the positive charge given to the bottom sphere.

i. On the circles representing the spheres below, use a single “+” sign on the bottom sphere to represent the location of highest concentration of the excess positive charges. Use a single “–” sign on the top sphere to represent the location of the highest concentration of the excess negative charges.



ii. For a separation distance equal to that of the data point indicated in part (a)(iii), would the magnitude of the force reading with spheres of opposite charges be greater than, less than, or equal to the magnitude of the force reading with spheres of the same charges?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

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PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

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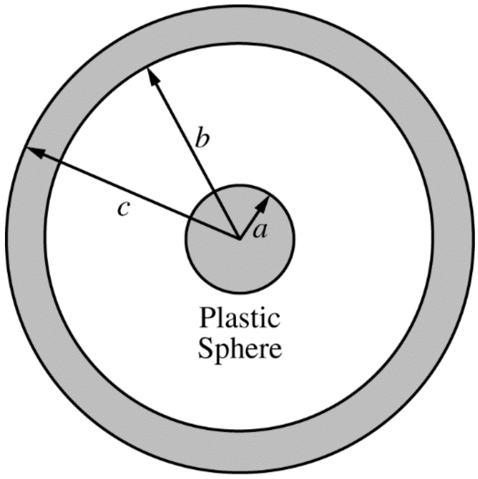


Conducting Spherical Shell

1. A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

   (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

      i. $r < a$

      ii. $a < r < b$

   (b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

      i. $r < a$

      ii. $a < r < b$

   (c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

      i. Determine the charge on the inner surface of the conducting shell.

      Justify your answer.

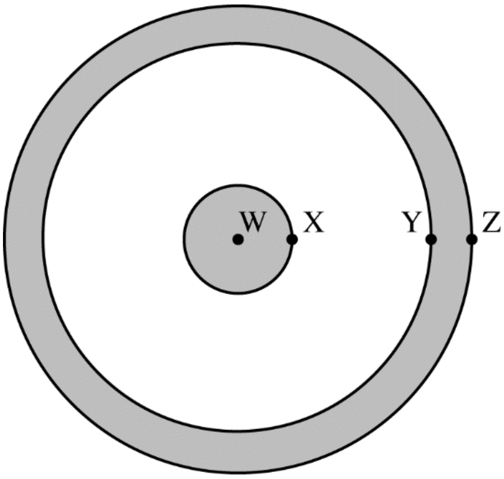
      ii. Determine the charge on the outer surface of the conducting shell.

(d)

i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.

ii. The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

____ W ____ X ____ Y ____ Z



10.2 Redistribution of Charge Between Conductors

Name: ______________ Class: __________ Date: ____________ **Total: 9 marks**

KEY WORDS potential 电势 • work 功 • conductor 导体

Objective

Build the skills to answer exam questions on **redistribution of charge between conductors**.

You must be able to:

- describe how charge **redistributes** when two conductors touch
- explain that connected conductors reach a common **potential** 电势
- use **conservation of charge** to find the final charge on each
- analyse charge sharing between two spheres

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Contact reaches a common potential

When two conductors are joined, charge flows until both are at the **same potential**.

■ Conservation of charge

The total charge before equals the total after — it is only shared, never lost.

■ Two spheres

Joined spheres share charge so that $V = k\dfrac{Q}{r}$ is equal; a larger sphere ends up with **more** charge. Two **identical** spheres split the total equally.

2 Practice

2.1 A metal sphere with $+8.0\ \mu\text{C}$ touches an identical neutral sphere, then they separate. Find the charge on each. [2]

2.2 State what becomes equal on two conductors joined by a wire at equilibrium. [1]

2.3 State the conservation law used in charge sharing. [1]

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3 Exam-style questions

3.1 Two conductors connected by a wire reach a common [1]

- **A** charge
- **B** potential
- **C** mass
- **D** field

3.2 When two identical charged spheres touch, the total charge is [1]

- **A** doubled
- **B** conserved and shared equally
- **C** reduced to zero
- **D** lost to the surroundings

3.3 A sphere carrying $+12\ \mu C$ briefly touches an identical neutral sphere.

(a) Find the charge on each sphere after they separate. [2]

(b) State the principle you used. [1]

Solutions

2.1 they are identical, so the charge splits equally: $\frac{8.0}{2} = 4.0\ \mu\text{C}$ each.

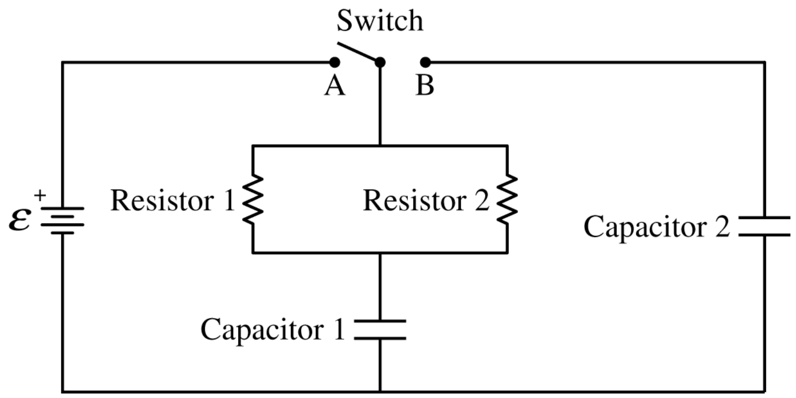
2.2 their electric potential.

2.3 conservation of charge.

3.1 **B**.

3.2 **B**.

3.3 (a) $\frac{12}{2} = 6.0\ \mu\text{C}$ on each. (b) conservation of charge (with equal sharing between identical spheres).



. The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

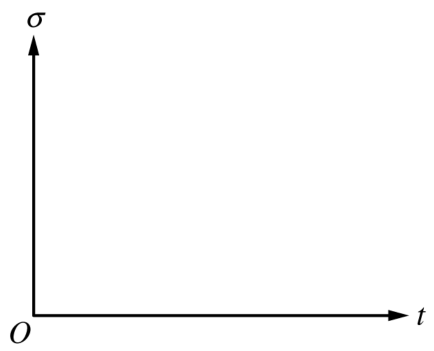
At time $t = 0$, the switch is closed to Position A.

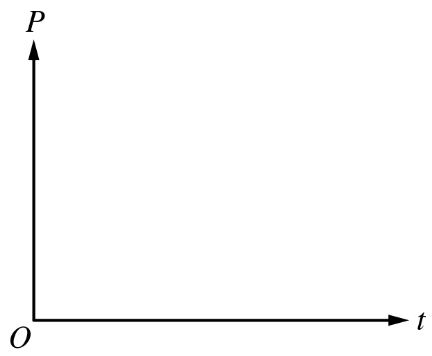
(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(b) On the axes shown, sketch graphs of the surface charge density σ on the positive plate of Capacitor 1 and the total power P dissipated by the resistors as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.





A long time after the switch is closed to Position A, the charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

(c) At time t_1, the switch is closed to Position B.

i. Immediately after time t_1, is the direction of the current in the switch directed toward the left, directed toward the right, or is there no current? Briefly justify your answer.

ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

iii. Derive an expression for the total energy E_R dissipated by resistors 1 and 2 from immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.

With the switch still closed to Position B, the parallel plates of Capacitor 2 are moved so that the separation distance increases by a factor of 2.

(d) Determine the ratio $\dfrac{U_2}{U_1}$ of the energy U_2 stored in Capacitor 2 to the energy U_1 stored in Capacitor 1 a long time after the plates of Capacitor 2 have been moved. Briefly justify your answer.

GO ON TO THE NEXT PAGE.

With the capacitors still charged as in part (d), the switch is now closed to Position A.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.

i. Derive an expression for the current I_0 from the battery immediately after the switch is closed to Position A.

ii. Determine the current I_∞ from the battery a long time after the switch is closed to Position A.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

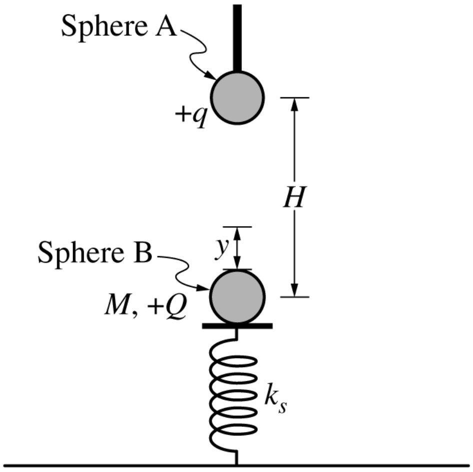
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

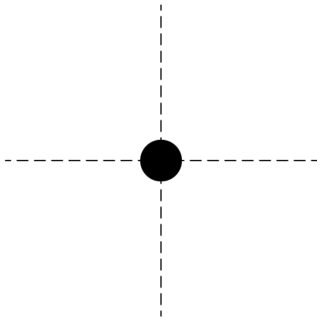
1. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M. Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s. Sphere B and the spring are initially at rest.

Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(a) On the following dot that represents Sphere B in the figure on the previous page, draw and label the forces (not components) that are exerted on Sphere B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



(b) Derive the relationship between y and H to show that $y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{k_s H^2} + \frac{Mg}{k_s}$.

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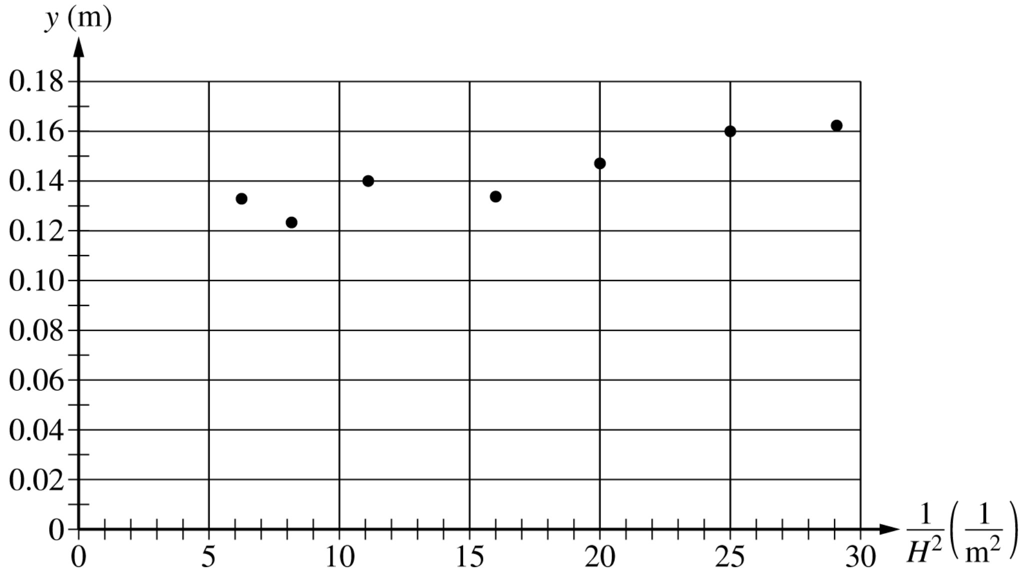
Continue your response to **QUESTION 1** on this page.

(c) The students plot collected data of y as a function of $\frac{1}{H^2}$, as shown in the graph.



i. Draw the best-fit line for the data.

ii. Using the best-fit line, calculate an experimental value for the vacuum permittivity ε_0 when $Q = q = 2.00 \times 10^{-6}$ C and $k_s = 25$ N/m.

iii. Using the best-fit line, calculate an experimental value for the mass of Sphere B.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(d) The students modify the experiment by replacing nonconducting Sphere B with conducting Sphere C that has the same charge $+Q$ and mass M. Sphere A is brought near Sphere C without touching, compressing the spring. Sphere C comes to rest.

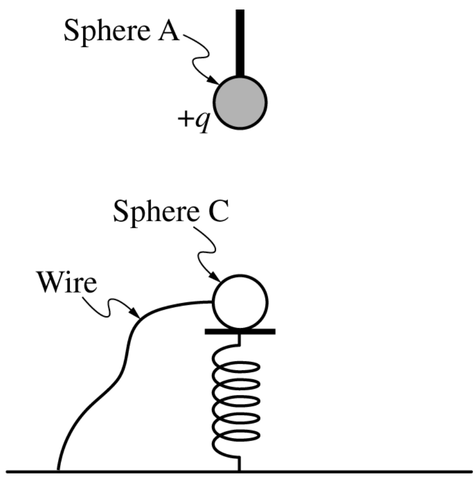
i. In the original experiment, when the centers of spheres A and B are a vertical distance H_1 apart, the spring is compressed a distance y_1. In the modified experiment, when the centers of spheres A and C are a vertical distance H_1 apart, the spring is compressed a distance y_2.

Is y_2 greater than, less than, or equal to y_1 ?

____ $y_2 > y_1$ ____ $y_2 < y_1$ ____ $y_2 = y_1$

Justify your answer.

ii. Sphere C is then grounded with a wire. On the following figure, draw an arrow indicating the direction that the platform will move immediately after being grounded. If the platform remains stationary, write "does not move."



GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

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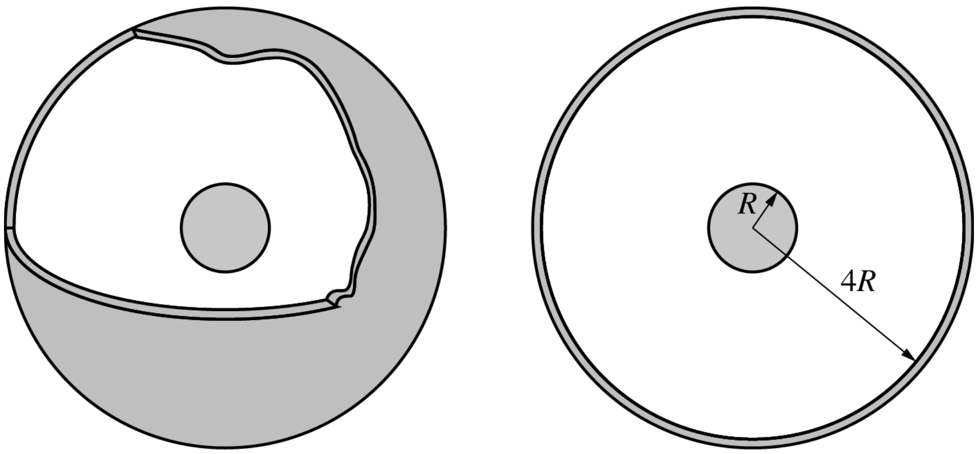


Cutout View   Cross-Section View

Note: Figures not drawn to scale.

. A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.

(a) Determine the charge on the outer surface of the shell.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the sphere for $r < R$. Express your answer in terms of Q, R, r, and physical constants, as appropriate.

(c) The magnitude of the electric field at $r = R$ is $8\,\mathrm{N/C}$. Calculate the value of the electric field at $r = 2R$.

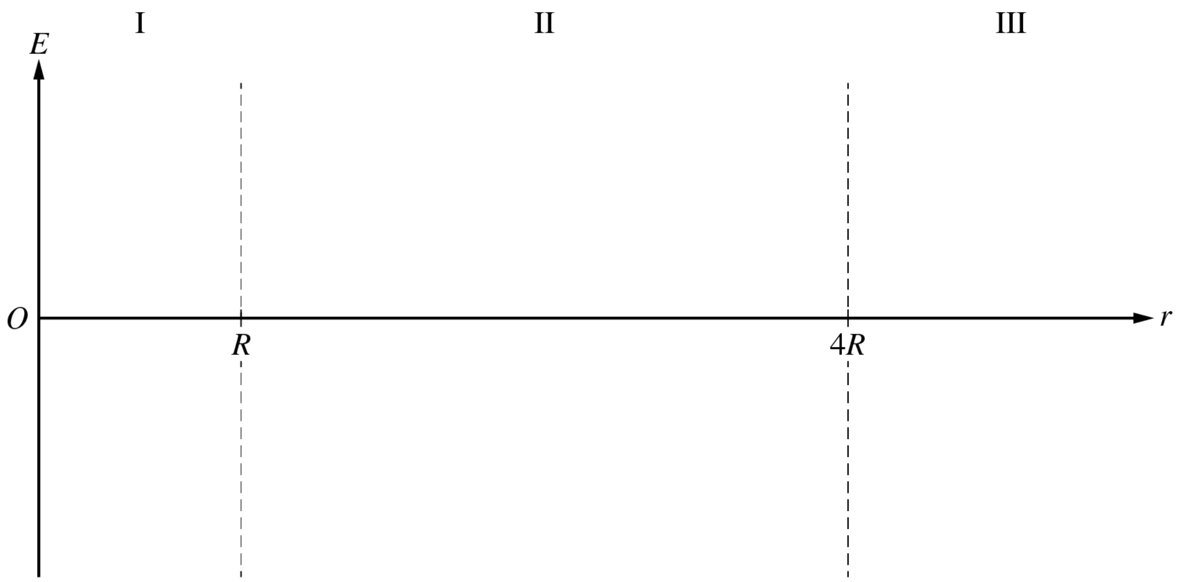
(d) Derive an expression for the absolute value of the potential difference between the outer surface of the sphere and the inner surface of the shell. Express your answer in terms of Q, R, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

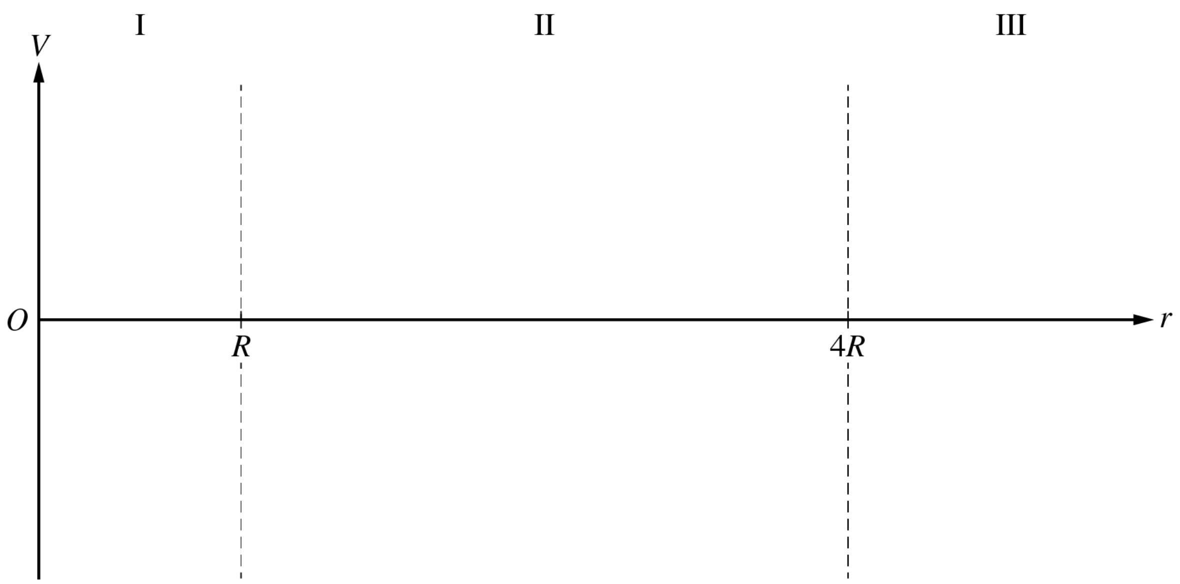
Continue your response to **QUESTION 1** on this page.

(e)

i. On the following axes that include regions I, II, and III, sketch a graph of the electric field E as a function of the distance r from the center of the sphere.



ii. On the following axes that include regions I, II, and III, sketch a graph of the electric potential V as a function of the distance r from the center of the sphere.



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10.3 Capacitors

Name: ______________ Class: __________ Date: ____________ **Total: 11 marks**

KEY WORDS capacitor 电容器 • capacitance 电容 • series 串联 • parallel-plate capacitor 平行板电容器 • series and parallel 串联和并联

Objective

Build the skills to answer exam questions on **capacitors**.

You must be able to:

- define **capacitance 电容** as $C = \dfrac{Q}{V}$
- use the parallel-plate result $C = \dfrac{\varepsilon_0 A}{d}$
- calculate the **energy stored**, $U = \frac{1}{2}CV^2 = \frac{1}{2}\dfrac{Q^2}{C}$
- combine capacitors in **series and parallel 串联和并联**

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Capacitance

$C = \dfrac{Q}{V}$ (farads). For parallel plates, $C = \dfrac{\varepsilon_0 A}{d}$ — larger area or smaller gap gives more capacitance.

■ Energy stored

$U = \frac{1}{2}CV^2 = \frac{1}{2}\dfrac{Q^2}{C}$, stored in the field (energy density $u = \frac{1}{2}\varepsilon_0 E^2$).

■ Combining capacitors

- **Parallel:** $C = C_1 + C_2 + \dots$
- **Series:** $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dots$ (less than the smallest).

2 Practice

2.1 A parallel-plate capacitor has area 0.020 m^2 and separation 1.0 mm ($\varepsilon_0 = 8.85 \times 10^{-12}$). Find its capacitance. [2]

2.2 Two 4.0 μF capacitors are connected in parallel. Find the total capacitance. [1]

2.3 Find the energy stored in a 10 μF capacitor at 12 V. [2]

3 Exam-style questions

3.1 The capacitance of a parallel-plate capacitor is [1]

- **A** $\dfrac{\varepsilon_0 d}{A}$
- **B** $\dfrac{\varepsilon_0 A}{d}$
- **C** $\varepsilon_0 A d$
- **D** $\dfrac{A}{\varepsilon_0 d}$

3.2 Two capacitors in series have a total capacitance that is [1]

- **A** the sum of the two
- **B** less than either one
- **C** the larger of the two
- **D** zero

3.3 A 5.0 μF capacitor is charged to 100 V.

(a) Find the charge stored. [2]

(b) Find the energy stored. [2]

Solutions

2.1 $C = \dfrac{\varepsilon_0 A}{d} = \dfrac{(8.85 \times 10^{-12})(0.020)}{1.0 \times 10^{-3}} = 1.8 \times 10^{-10}$ F.

2.2 $C = 4.0 + 4.0 = 8.0\ \mu$F.

2.3 $U = \frac{1}{2}CV^2 = \frac{1}{2}(10 \times 10^{-6})(12)^2 = 7.2 \times 10^{-4}$ J.

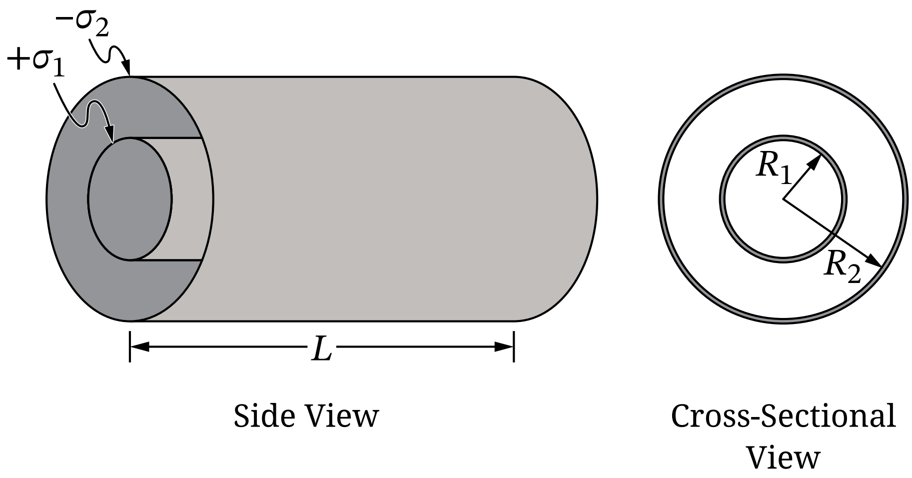
3.1 **B**.

3.2 **B**.

3.3 (a) $Q = CV = (5.0 \times 10^{-6})(100) = 5.0 \times 10^{-4}$ C. (b) $U = \frac{1}{2}CV^2 = \frac{1}{2}(5.0 \times 10^{-6})(100)^2 = 0.025$ J.

Question 1: Version J

1. An isolated, air-filled, charged capacitor consists of two conducting, coaxial, cylindrical shells that each have length L. The inner shell has radius R_1 and the outer shell has radius R_2, as shown in Figure 1, where $R_1 < R_2 \ll L$. The surface charge densities (amounts of charge per unit area) of the inner and outer shells are $+\sigma_1$ and $-\sigma_2$, respectively. The absolute values of the total charges on the shells are equal.



Note: Figures not drawn to scale.

Figure 1

A.

i. Using Gauss's law, **derive** an expression for the magnitude E of the electric field as a function of the radial distance r from the center of the capacitor for the region $R_1 < r < R_2$. Express your answer in terms of R_1, σ_1, r, and physical constants, as appropriate.

ii. **Derive** an expression for the absolute value $|\Delta V|$ of the potential difference between the outer and inner shells in terms of R_1, R_2, σ_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

iii. On the axes shown in Figure 2, **sketch** a graph of E as a function of r from $r = 0$ to a position that is outside the outer shell.

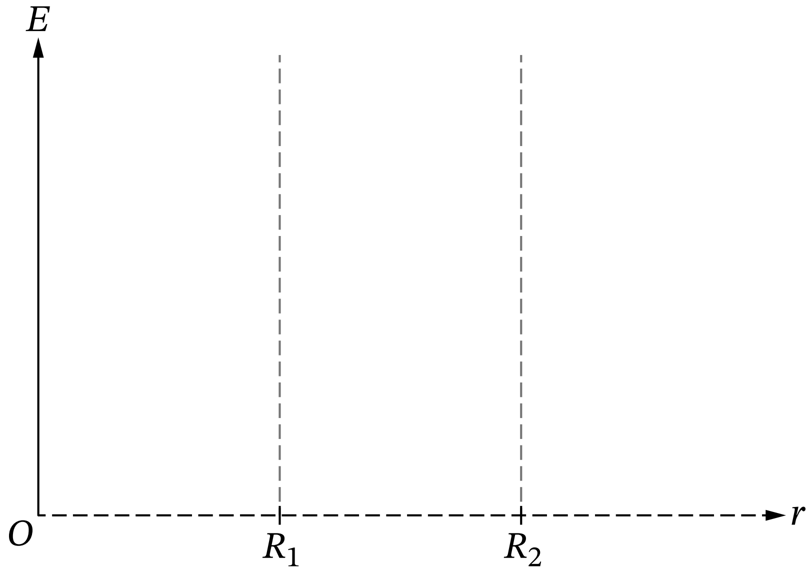
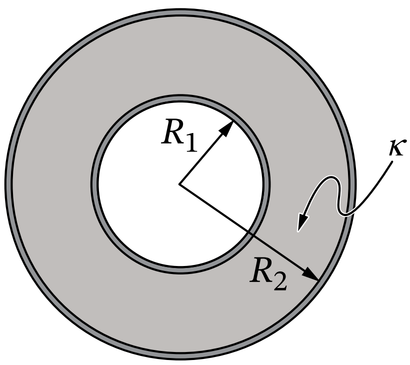


Figure 2

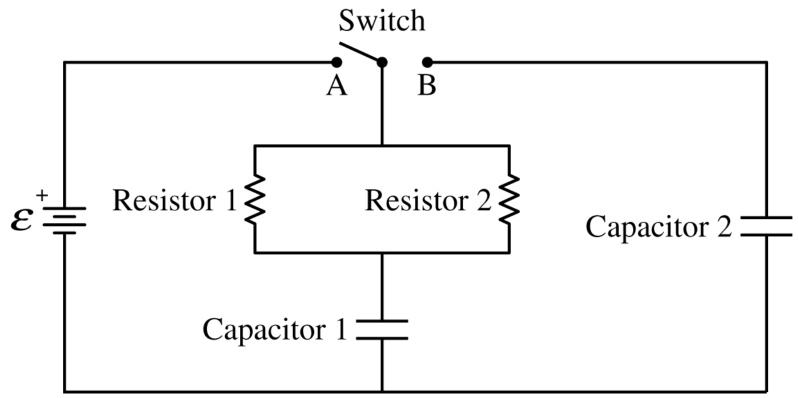
B. A material of dielectric constant κ is inserted into the isolated, charged capacitor such that the material fills the region $R_1 < r < R_2$, as shown in Figure 3.



Cross-Sectional View

Figure 3

Derive an expression for the capacitance C of the capacitor with the material inserted in terms of L, R_1, R_2, κ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.



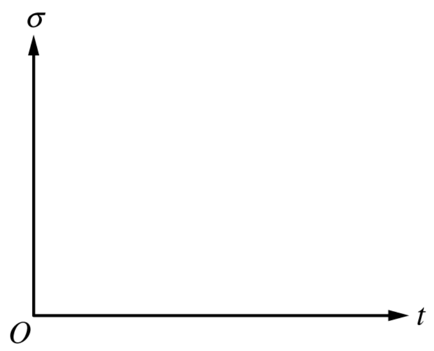
. The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

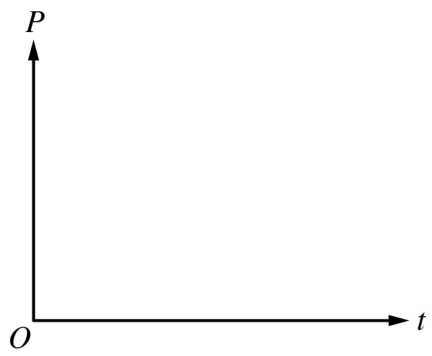
At time $t = 0$, the switch is closed to Position A.

(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

(b) On the axes shown, sketch graphs of the surface charge density σ on the positive plate of Capacitor 1 and the total power P dissipated by the resistors as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.





A long time after the switch is closed to Position A, the charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

(c) At time t_1, the switch is closed to Position B.

i. Immediately after time t_1, is the direction of the current in the switch directed toward the left, directed toward the right, or is there no current? Briefly justify your answer.

ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

iii. Derive an expression for the total energy E_R dissipated by resistors 1 and 2 from immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.

With the switch still closed to Position B, the parallel plates of Capacitor 2 are moved so that the separation distance increases by a factor of 2.

(d) Determine the ratio $\frac{U_2}{U_1}$ of the energy U_2 stored in Capacitor 2 to the energy U_1 stored in Capacitor 1 a long time after the plates of Capacitor 2 have been moved. Briefly justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

With the capacitors still charged as in part (d), the switch is now closed to Position A.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.

i. Derive an expression for the current I_0 from the battery immediately after the switch is closed to Position A.

ii. Determine the current I_∞ from the battery a long time after the switch is closed to Position A.

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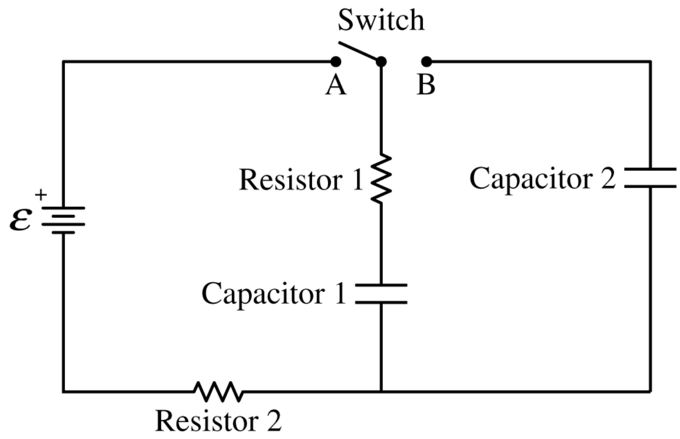
STOP

END OF EXAM



. The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

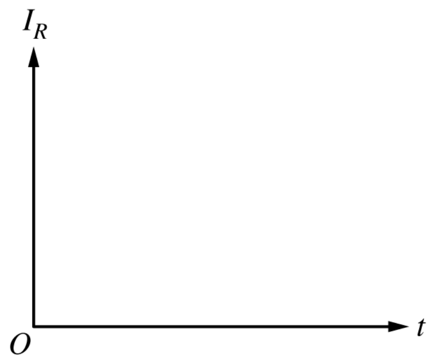
At time $t = 0$, the switch is closed to Position A.

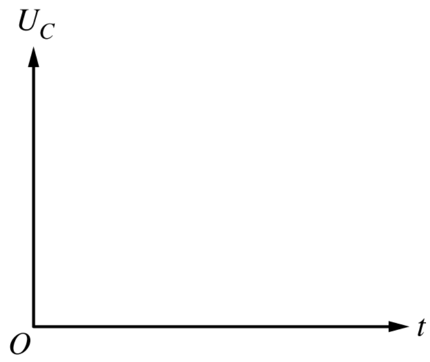
(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(b) On the axes shown, sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.





A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

(c) At time t_1, the switch is closed to Position B.

i. Immediately after t_1, is the direction of the current in Resistor 1 directed up, directed down, or is there no current? Briefly justify your answer.

ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

iii. Derive an expression for the total energy dissipated by Resistor 1 immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.

With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

(d) Determine the charge on the positive plate of Capacitor 2 a long time after the dielectric has been inserted. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

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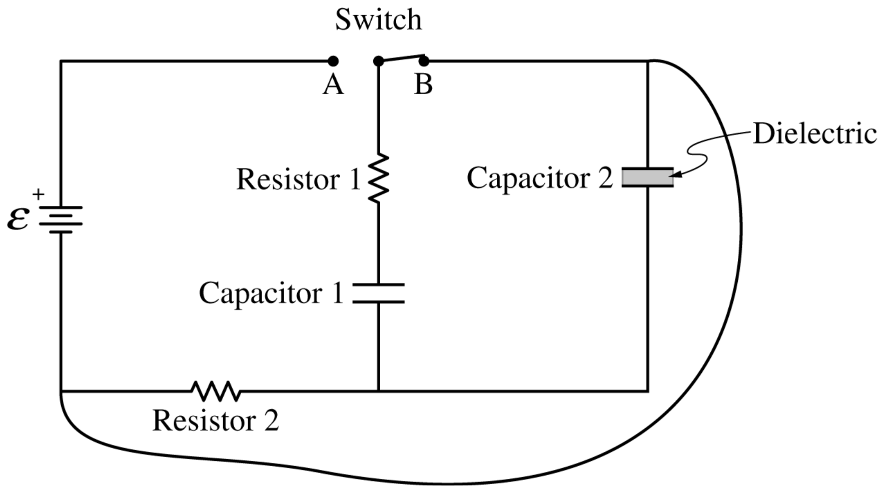
Continue your response to **QUESTION 3** on this page.



With the switch still closed to Position B, a wire of negligible resistance is connected between two corners of the circuit, as shown.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.

i. Derive an expression for the current in Resistor 2 immediately after the wire is connected to the circuit.

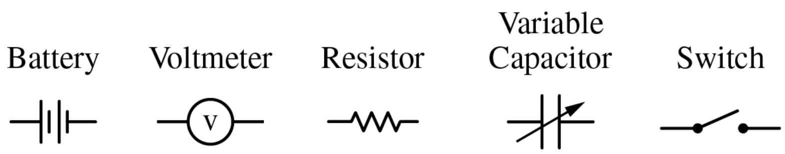
ii. Determine the current in Resistor 2 a long time after the wire is connected to the circuit.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

. The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.



(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the potential difference across the capacitor as it discharges through the resistor.

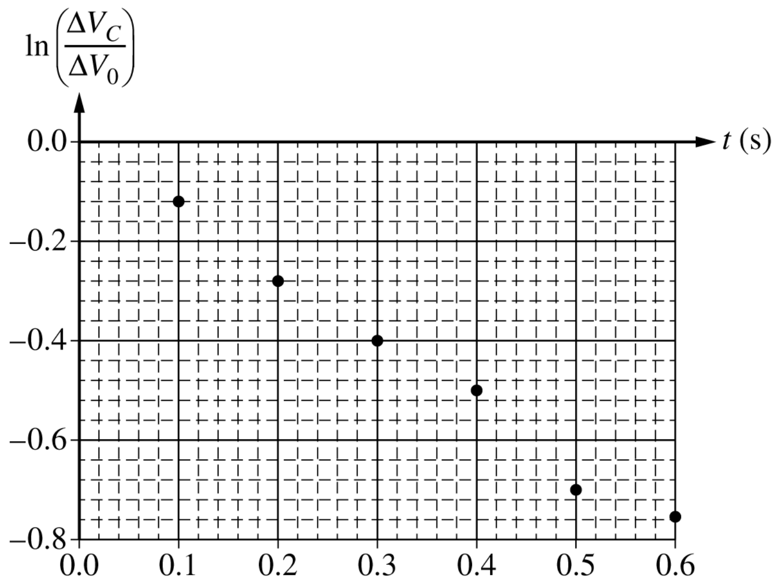
The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

(b) Show that the potential difference ΔV_C across the capacitor as a function of time t is $\Delta V_C(t) = \Delta V_0 e^{-\frac{t}{RC}}$ as the capacitor discharges.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(c) The experiment is performed using a resistor of $R = 150\,\text{k}\Omega$. Data for the potential difference ΔV_C across the capacitor as a function of t are recorded and a plot of $\ln\left(\dfrac{\Delta V_C}{\Delta V_0}\right)$ as a function of t is created on the graph below.



i. Draw the best-fit line for the data.

ii. Using the best-fit line, calculate a value for the unknown capacitance C.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(d) The capacitor is adjusted so that the surface area of the plates is increased, and the experiment is repeated. Would the slope of the best-fit line in the second experiment be more steep, less steep, or unchanged compared to the slope of the best-fit line in part (c)?

___ More steep ___ Less steep ___ Unchanged

Briefly justify your answer.

(e) The ideal battery is then replaced with a non-ideal battery with internal resistance r, and the experiment is repeated.

i. Would the slope of the graph in this final experiment change compared to the graph in part (c)?

___ Yes ___ No

Briefly justify your answer.

ii. Would the vertical intercept of the graph in this final experiment change compared to the graph in part (c)?

___ Yes ___ No

Briefly justify your answer.

GO ON TO THE NEXT PAGE.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

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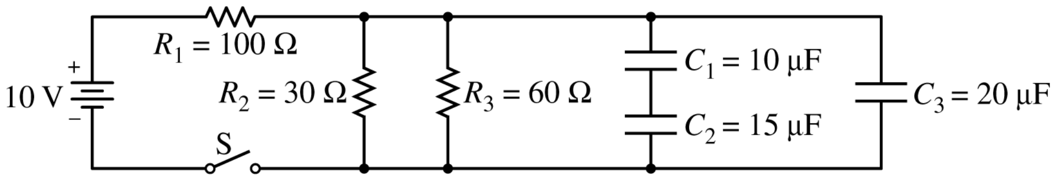


1. The circuit shown above is composed of an ideal 10 V battery, three resistors and three capacitors with the values shown, and an open switch S. The capacitors are initially uncharged. Switch S is now closed.

(a) Calculate the current through R_1 immediately after switch S is closed.

Switch S has been closed for a long time, and the circuit has reached a steady state.

(b) Calculate the potential difference across R_1.

(c)
i. Calculate the charge stored on the positive plate of capacitor C_2.

ii. Is the charge stored on capacitor C_3 greater than, less than, or equal to the charge stored on capacitor C_2 ?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

GO ON TO THE NEXT PAGE.

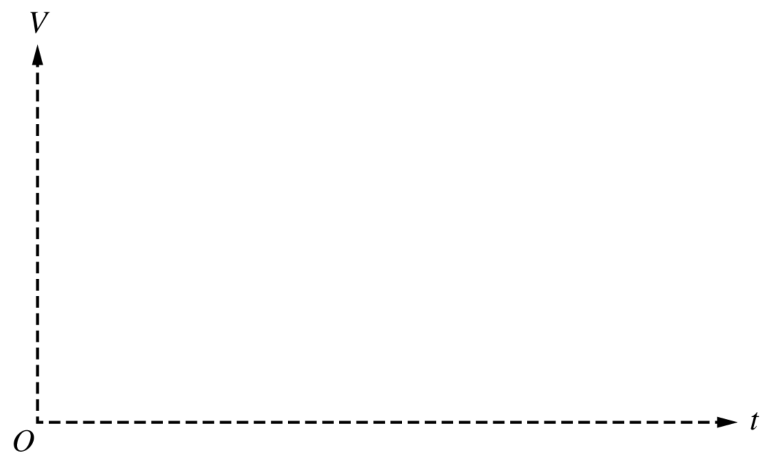
Switch S is then opened.

(d)

i. Determine the current through R_1 immediately after the switch is opened.

ii. Calculate the current through R_2 immediately after the switch is opened.

(e) On the axes below, sketch a graph of the potential difference V across capacitor C_2 as a function of time t if switch S is opened at time $t = 0$. Label the maximum value.



Capacitor C_3 is replaced by two $10\,\mu\text{F}$ capacitors connected in series, switch S is closed, and the circuit reaches equilibrium. Switch S is then opened at time $t = 0$.

(f) For $t > 0$, would the sketch of a graph of the new voltage across C_2 as a function of time be above, below, or the same as the sketch for part (e) ?

_____ Above _____ Below _____ The same

Justify your answer.

PHYSICS C: ELECTRICITY AND MAGNETISM

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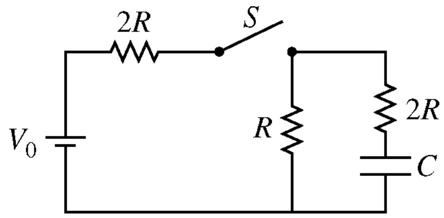


1. The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

   (a) Derive an expression for the steady-state current supplied by the battery.

   (b) Derive an expression for the charge on the capacitor.

   (c) Derive an expression for the energy stored in the capacitor.

   Now the switch is opened at time $t = 0$.

   (d) Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time t after the switch is opened.

(e)

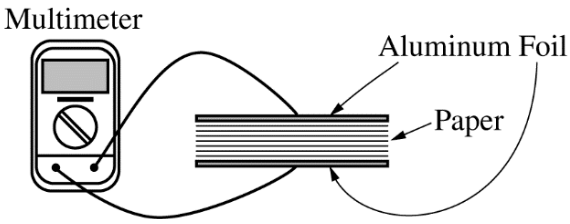
i. Calculate the current in resistor R immediately after the switch is opened.

ii. On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.

(f) Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c) ?

____ Greater than ____ Less than ____ Equal to

Justify your answer.



2. An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \text{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)		
500	0.045	6.5×10^{-11}		
400	0.036	7.4×10^{-11}		
300	0.027	8.9×10^{-11}		
200	0.018	11.9×10^{-11}		
100	0.010	21.0×10^{-11}		

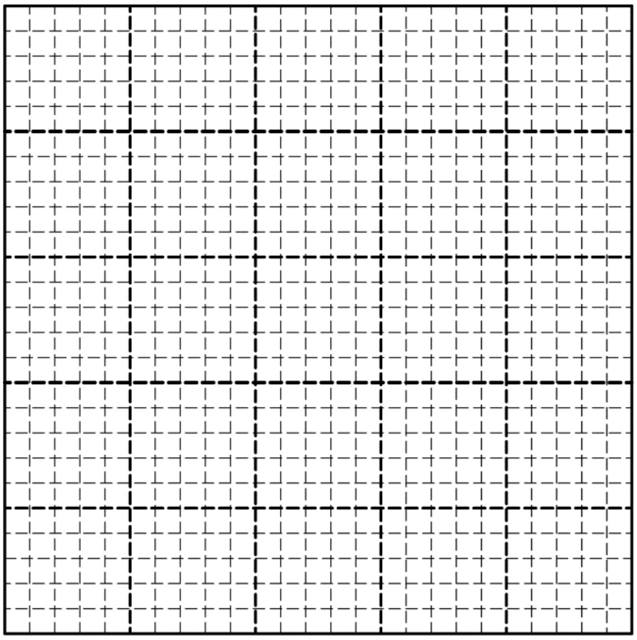
(a) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: ____________

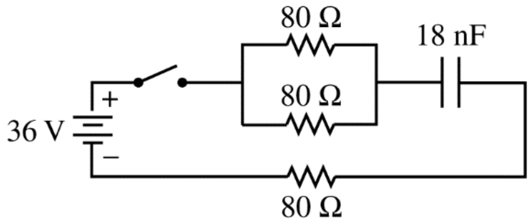
Horizontal axis: ____________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

(b) Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(c) Using the straight line, calculate a dielectric constant for the paper.



The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical $80\ \Omega$ resistors, and an open switch, as shown in the figure above.

(d) Calculate the current in the battery immediately after the switch is closed.

(e) Determine the time constant for this circuit.

(f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

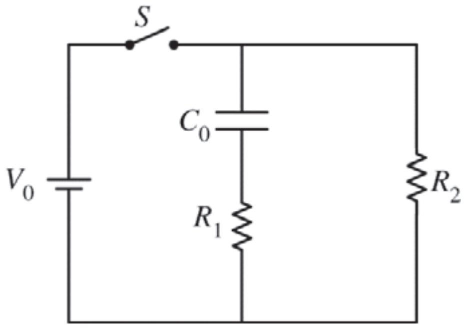
____ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

ii. Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

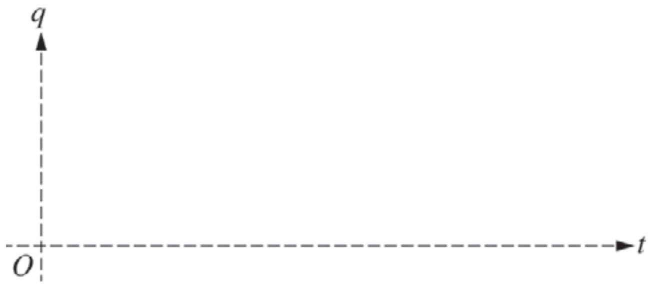
____ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

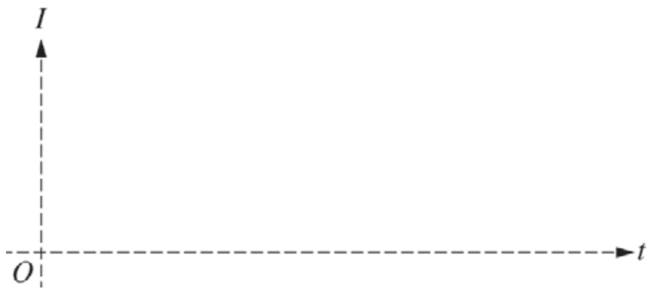


2. In the circuit above, an ideal battery of voltage V_0 is connected to a capacitor with capacitance C_0 and resistors with resistances R_1 and R_2, with $R_1 > R_2$. The switch S is open, and the capacitor is initially uncharged.

(a) The switch is closed at time $t = 0$. On the axes below, sketch the charge q on the capacitor as a function of time t. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



(b) On the axes below, sketch the current I through each resistor as a function of time t. Clearly label the two curves as I_1 and I_2, the currents through resistors R_1 and R_2, respectively. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



The circuit is constructed using an ideal 1.5 V battery, an 80 μF capacitor, and resistors $R_1 = 150\ \Omega$ and $R_2 = 100\ \Omega$. The switch is closed, allowing the capacitor to fully charge. The switch is then opened, allowing the capacitor to discharge.

(c) The time it takes to charge the capacitor to 50% of its maximum charge is Δt_C. The time it takes for the capacitor to discharge to 50% of its maximum charge is Δt_D. Which of the following correctly relates the two time intervals?

____ $\Delta t_C > \Delta t_D$ ____ $\Delta t_C = \Delta t_D$ ____ $\Delta t_C < \Delta t_D$

Justify your answer.

(d)

i. Calculate the current through resistor R_2 immediately after the switch is opened.

ii. Is the current through resistor R_2 increasing, decreasing, or constant immediately after the switch is opened?

____ Increasing ____ Decreasing ____ Constant

Justify your answer.

(e)

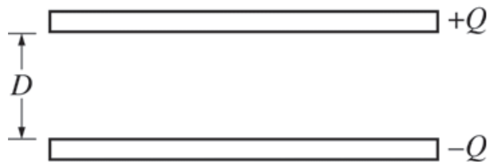
i. Calculate the energy stored in the capacitor immediately after the switch is opened.

ii. Calculate the energy dissipated by resistor R_1 as the capacitor completely discharges.

PHYSICS C: ELECTRICITY AND MAGNETISM
SECTION II
Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



E&M.1.

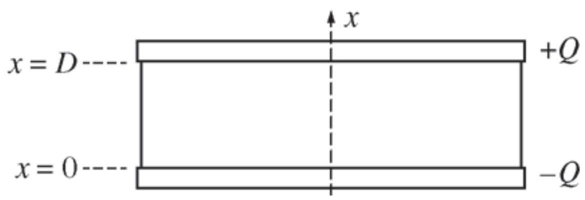
A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

(a)

i. On the figure above, draw an arrow to indicate the direction of the electric field between the plates.

ii. On the figure above, draw an appropriate Gaussian surface that will be used to derive an expression for the magnitude of the electric field E between the plates.

iii. Using Gauss's law and the Gaussian surface from part (a)-ii, derive an expression for the magnitude of the electric field E between the plates. Express your answer in terms of A, D, Q, and physical constants, as appropriate.



The space between the plates is now filled with a dielectric material that is engineered so that its dielectric constant varies with the distance from the bottom plate to the top plate, defined by the x-axis indicated in the diagram above. As a result, the electric field between the plates is given by $\vec{E} = -\dfrac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\hat{i}$, where κ_0 is a positive constant. Express all algebraic answers to the remaining parts in terms of A, D, Q, κ_0, x, and physical constants, as appropriate.

(b) Determine an expression for the dielectric constant κ as a function of x.

(c)

i. Write, but do NOT solve, an equation that could be used to determine the potential difference V between the plates of the capacitor.

ii. Using the equation from part (c)-i, derive an expression for the potential difference $V_D - V_0$, where V_D is the potential of the top plate and V_0 is the potential of the bottom plate.

(d) Determine the capacitance of the capacitor.

(e) The energy stored in the capacitor that has a varying dielectric is U_V. A second capacitor that has a constant dielectric of value κ_0 is also given a charge Q. The energy stored in the second capacitor is U_C. How do the values of U_V and U_C compare?

____ $U_V < U_C$ ____ $U_V > U_C$ ____ $U_V = U_C$

Justify your answer.

10.4 Dielectrics

Name: ______________ Class: __________ Date: ____________ **Total: 9 marks**

KEY WORDS dielectric 电介质 • dielectric constant 介电常数 • polarized 极化 • capacitor 电容器 • capacitance 电容

Objective

Build the skills to answer exam questions on **dielectrics**.

You must be able to:

- describe a **dielectric 电介质** as an insulator that becomes **polarized** in a field
- explain how the **dielectric constant 介电常数** κ increases capacitance, $C = \kappa C_0$
- describe how a dielectric reduces the field for a fixed charge
- compare a capacitor with a dielectric at **constant charge** versus **constant voltage**

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ What a dielectric does

A dielectric is an insulator that **polarizes** in a field; its aligned charges partly cancel the applied field.

■ Effect on capacitance

Inserting a dielectric raises the capacitance to $C = \kappa C_0$ ($\kappa > 1$).

■ Constant charge vs constant voltage

- **Isolated (constant Q):** the field and voltage **drop**, so $U = \frac{1}{2}Q^2/C$ falls.
- **Connected to a battery (constant V):** more charge flows on, so Q and $U = \frac{1}{2}CV^2$ **rise**.

2 Practice

2.1 State what happens to a dielectric placed in an electric field. [1]

2.2 A capacitor $C_0 = 2.0\ \mu\text{F}$ has a dielectric of $\kappa = 3.0$ inserted. Find the new capacitance. [2]

2.3 State what a dielectric does to the field between the plates at fixed charge. [1]

3 Exam-style questions

3.1 Inserting a dielectric ($\kappa > 1$) makes the capacitance [1]

- **A** smaller
- **B** larger
- **C** zero
- **D** unchanged

3.2 A dielectric is an [1]

- **A** conductor
- **B** insulator that polarizes
- **C** source of charge
- **D** magnet

3.3 A 4.0 μF capacitor is charged to 50 V, then **disconnected**; a dielectric of $\kappa = 2.0$ is inserted.

(a) Find the new capacitance. [1]

(b) State what happens to the voltage, and why. [2]

Solutions

2.1 it becomes polarized (its charges shift to partly oppose the field).

2.2 $C = \kappa C_0 = 3.0 \times 2.0 = 6.0\ \mu\text{F}$.

2.3 it reduces the field.

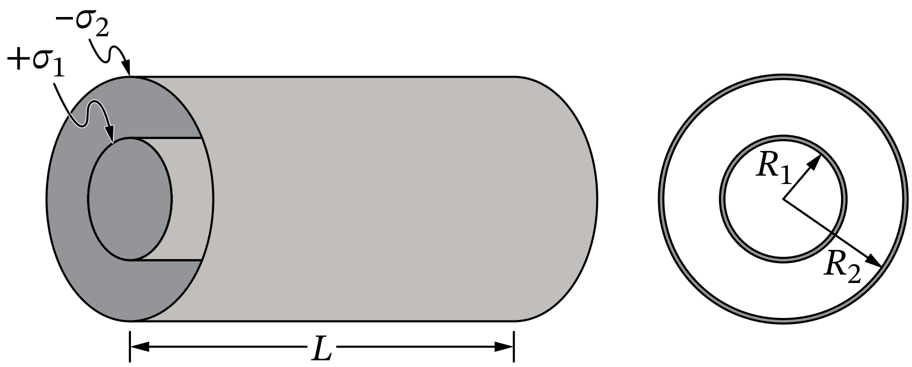
3.1 **B**.

3.2 **B**.

3.3 (a) $C = \kappa C_0 = 2.0 \times 4.0 = 8.0\ \mu\text{F}$. (b) the voltage halves to 25 V — the charge is fixed and $V = Q/C$, so doubling C halves V.

Question 1: Version J

1. An isolated, air-filled, charged capacitor consists of two conducting, coaxial, cylindrical shells that each have length L. The inner shell has radius R_1 and the outer shell has radius R_2, as shown in Figure 1, where $R_1 < R_2 \ll L$. The surface charge densities (amounts of charge per unit area) of the inner and outer shells are $+\sigma_1$ and $-\sigma_2$, respectively. The absolute values of the total charges on the shells are equal.



Side View

Cross-Sectional View

<u>Note</u>: Figures not drawn to scale.

Figure 1

A.

i. Using Gauss's law, **derive** an expression for the magnitude E of the electric field as a function of the radial distance r from the center of the capacitor for the region $R_1 < r < R_2$. Express your answer in terms of R_1, σ_1, r, and physical constants, as appropriate.

ii. **Derive** an expression for the absolute value $|\Delta V|$ of the potential difference between the outer and inner shells in terms of R_1, R_2, σ_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

iii. On the axes shown in Figure 2, **sketch** a graph of E as a function of r from $r = 0$ to a position that is outside the outer shell.

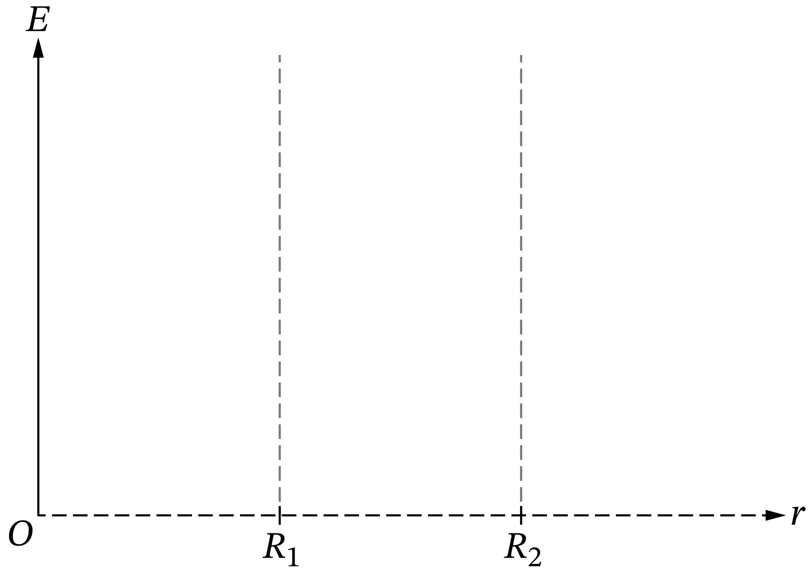
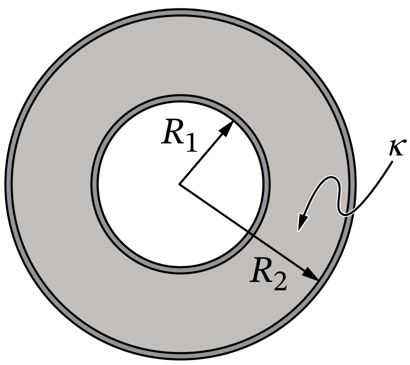


Figure 2

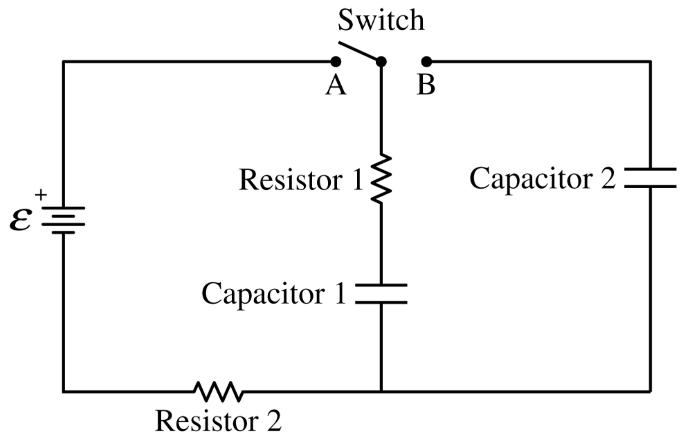
B. A material of dielectric constant κ is inserted into the isolated, charged capacitor such that the material fills the region $R_1 < r < R_2$, as shown in Figure 3.



Cross-Sectional View

Figure 3

Derive an expression for the capacitance C of the capacitor with the material inserted in terms of L, R_1, R_2, κ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.



. The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

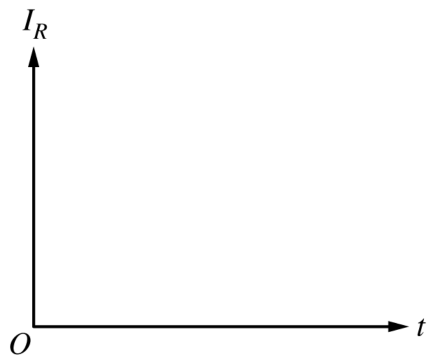
At time $t = 0$, the switch is closed to Position A.

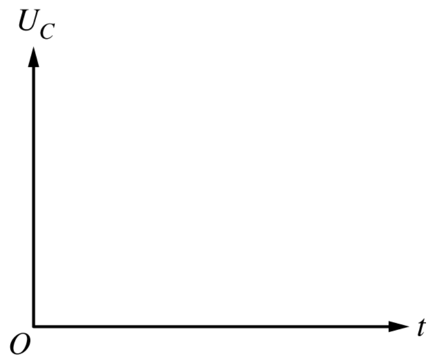
(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(b) On the axes shown, sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.





A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

(c) At time t_1, the switch is closed to Position B.

i. Immediately after t_1, is the direction of the current in Resistor 1 directed up, directed down, or is there no current? Briefly justify your answer.

ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

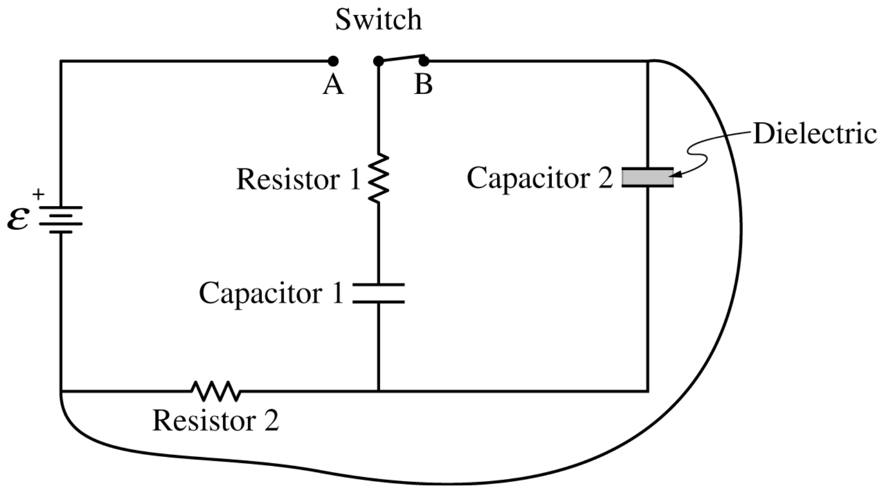
iii. Derive an expression for the total energy dissipated by Resistor 1 immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.

With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

(d) Determine the charge on the positive plate of Capacitor 2 a long time after the dielectric has been inserted. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.



With the switch still closed to Position B, a wire of negligible resistance is connected between two corners of the circuit, as shown.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.

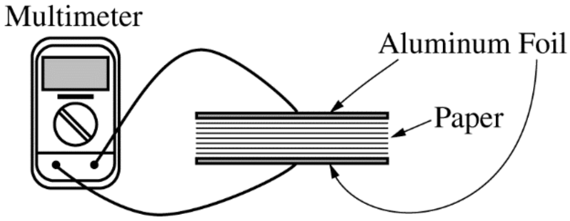
i. Derive an expression for the current in Resistor 2 immediately after the wire is connected to the circuit.

ii. Determine the current in Resistor 2 a long time after the wire is connected to the circuit.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM



2. An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \text{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

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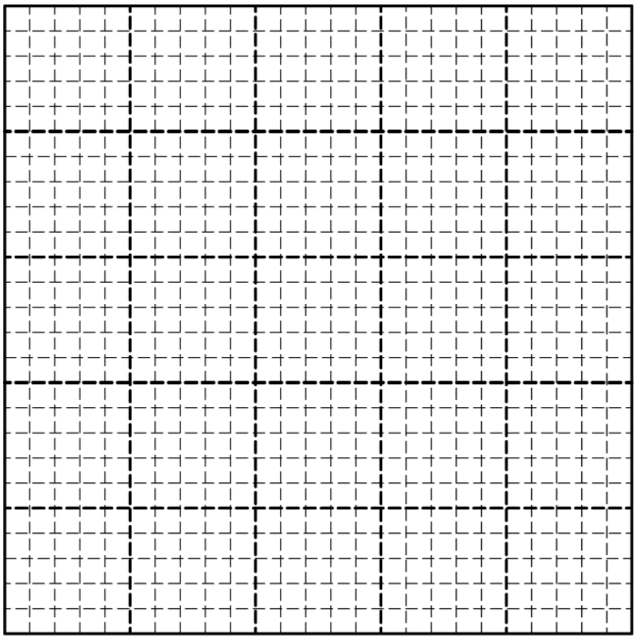
(a) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: ____________

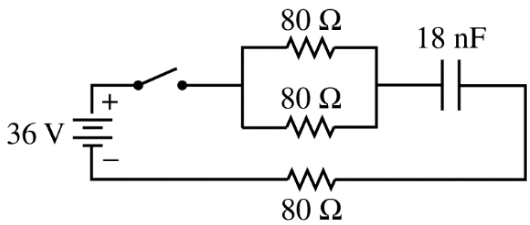
Horizontal axis: ____________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

(b) Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(c) Using the straight line, calculate a dielectric constant for the paper.



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Explain experimentally what could account for this.

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Explain experimentally what could account for this.

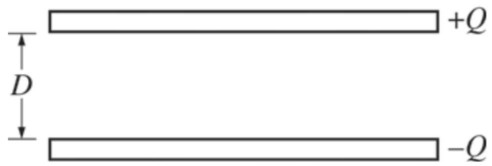
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



E&M.1.

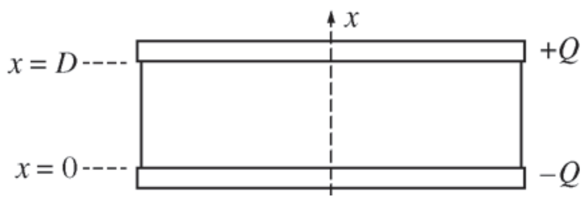
A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

(a)

i. On the figure above, draw an arrow to indicate the direction of the electric field between the plates.

ii. On the figure above, draw an appropriate Gaussian surface that will be used to derive an expression for the magnitude of the electric field E between the plates.

iii. Using Gauss's law and the Gaussian surface from part (a)-ii, derive an expression for the magnitude of the electric field E between the plates. Express your answer in terms of A, D, Q, and physical constants, as appropriate.



The space between the plates is now filled with a dielectric material that is engineered so that its dielectric constant varies with the distance from the bottom plate to the top plate, defined by the x-axis indicated in the diagram above. As a result, the electric field between the plates is given by $\vec{E} = -\dfrac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\hat{i}$, where κ_0 is a positive constant. Express all algebraic answers to the remaining parts in terms of A, D, Q, κ_0, x, and physical constants, as appropriate.

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ii. Using the equation from part (c)-i, derive an expression for the potential difference $V_D - V_0$, where V_D is the potential of the top plate and V_0 is the potential of the bottom plate.

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(e) The energy stored in the capacitor that has a varying dielectric is U_V. A second capacitor that has a constant dielectric of value κ_0 is also given a charge Q. The energy stored in the second capacitor is U_C. How do the values of U_V and U_C compare?

____ $U_V < U_C$ ____ $U_V > U_C$ ____ $U_V = U_C$

Justify your answer.