**Week 7**

# AP Physics C: Electricity and Magnetism

Homework bundle for this week

本周作业合集

exercises.lol

## Contents 目录

AP Physics C: Electricity and Magnetism Name: ______________ Score: ________

1. The potential energy of two point charges is proportional to:

☐ $1/r^2$

☐ $1/r$

☐ $r$

☐ $r^2$

2. Electric potential $V$ is:

☐ energy per unit charge

☐ force per unit charge

☐ charge per unit volume

☐ power per second

3. A charge is released in a field (only the electric force acts). As it speeds up, its potential energy:

☐ rises

☐ falls

☐ stays constant

☐ becomes infinite

4. Two opposite charges have a potential energy that is:

☐ positive

☐ negative

☐ zero

☐ infinite

5. Potentials from several charges add as simple scalars — no vectors needed.

☐ True ☐ False

6. A 2 C charge falls through $\Delta V = 5$ V. How much kinetic energy (in J) does it gain?

______________ J

7. Find $U$ (in J) for a +1 nC and +1 nC charge 1 m apart. Use $k = 9 \times 10^9$.

______________ J

8. Two charges give $+50$ V and $-20$ V at a point. Find the total $V$ (in V).

   ________________ V

9. The energy an electron gains crossing 1 V is one ______.

   ____________________________

10. The work done by the electric field equals the decrease in potential energy, $W = -\Delta U$.

    ☐ True ☐ False

AP Physics C: Electricity and Magnetism

1. $1/r$

   $U = kq_1q_2/r$ — energy goes as $1/r$ (force goes as $1/r^2$).

2. **energy per unit charge**

   $V = U/q$ — potential energy per unit charge, in volts.

3. **falls**

   $\Delta KE + \Delta U = 0$: as KE rises, $U$ falls.

4. **negative**

   Opposite charges attract, so $U = kq_1q_2/r < 0$ — a bound pair.

5. **True**

   $V$ is a scalar: $V = \sum kQ_i/r_i$, just add the numbers.

6. **10 J**

   $KE = q\,\Delta V = 2 \times 5 = 10$ J.

7. **9e-09 J**

   $U = \dfrac{(9\times10^9)(10^{-9})(10^{-9})}{1} = 9\times10^{-9}$ J.

8. **30 V**

   Scalars add: $50 + (-20) = 30$ V.

9. **electron-volt**

   That energy is 1 eV $= 1.6\times10^{-19}$ J.

10. **True**

    A drop in $U$ releases energy as work: $W = -\Delta U$.

# 9.1 Electric Potential Energy

Name: ______________ Class: _________ Date: ____________ **Total: 10 marks**

**KEY WORDS** work 功 • electric potential 电势 • system 系统 • electric potential energy 电势能 • conservative force

## Objective

Build the skills to answer exam questions on **electric potential energy**.

**You must be able to:**

- define **electric potential energy** 电势能 as the work to assemble a charge configuration
- calculate the potential energy of two point charges, $U = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r}$, with its sign
- find the total potential energy of a system by summing over all **pairs**
- relate the work done by the electric force to $W = -\Delta U$

## 1 Worked examples

Study these first. Each one shows the method for a question type used later.

### ■ Potential energy of a pair

$U = k\dfrac{q_1 q_2}{r}$ — **positive** for like charges, **negative** for unlike.

### ■ System of charges

Add $U$ over every **distinct pair** of charges.

### ■ Work and a conservative force

The electric force is **conservative**, so $W = -\Delta U$ and the energy depends only on position.

## 2 Practice

**2.1** Find the potential energy of a $+3.0\ \mu$C and a $-2.0\ \mu$C charge 0.40 m apart ($k = 8.99 \times 10^9$). [2]

**2.2** State the sign of the potential energy of two like charges. [1]

**2.3** The electric force does +6.0 J of work on a charge. State the change in its potential energy. [1]

## 3 Exam-style questions

**3.1** The electric potential energy of two point charges is [1]

- **A** $k\dfrac{q_1 q_2}{r^2}$
- **B** $k\dfrac{q_1 q_2}{r}$
- **C** $qE$
- **D** $\frac{1}{2}CV^2$

**3.2** The work done by the electric force equals [1]

- **A** $+\Delta U$
- **B** $-\Delta U$
- **C** $\Delta U^2$
- **D** zero always

**3.3** Three $+2.0\ \mu$C charges sit at the corners of an equilateral triangle of side 0.30 m.

(a) Find the potential energy of **one** pair. [2]

(b) State the total potential energy of the system. [2]

## Solutions

**2.1** $U = k\dfrac{q_1 q_2}{r} = 8.99 \times 10^9 \times \dfrac{(3.0 \times 10^{-6})(-2.0 \times 10^{-6})}{0.40} = -0.13$ J.

**2.2** positive.

**2.3** $\Delta U = -6.0$ J.

**3.1** **B**.

**3.2** **B**.

**3.3** (a) $U = k\dfrac{q^2}{r} = 8.99 \times 10^9 \times \dfrac{(2.0 \times 10^{-6})^2}{0.30} = 0.12$ J. (b) three identical pairs, so $U_{\text{total}} = 3 \times 0.12 = 0.36$ J.

Begin your response to **QUESTION 1** on this page.

**PHYSICS C: ELECTRICITY AND MAGNETISM**

**SECTION II**

**Time—45 minutes**

**3 Questions**

**Directions:** Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

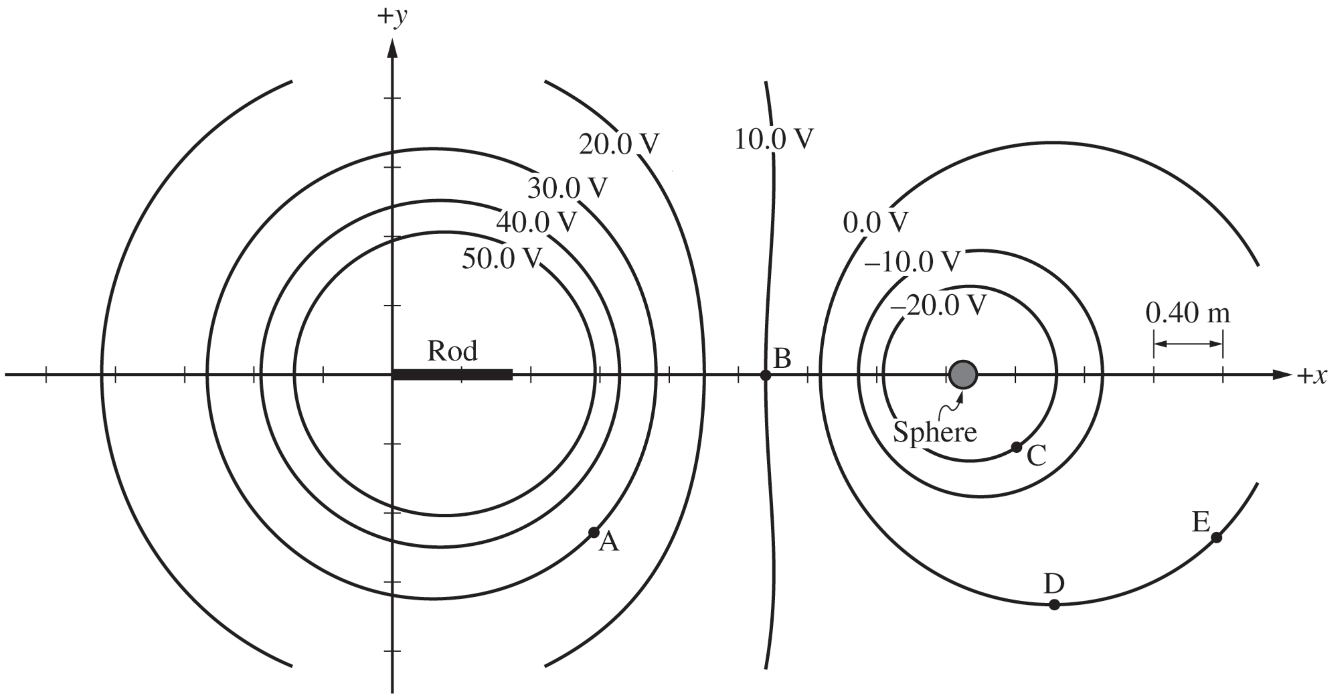


Figure 1

1. A nonconducting rod of uniform positive linear charge density is near a sphere with charge $-2.0\ \text{nC}$. The rod and sphere are held at rest on the $x$-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the $x$-axis and the $y$-axis are $0.40\ \text{m}$ apart.

(a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the $-20.0\ \text{V}$ equipotential line.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is momentarily held at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work $W_{CE}$ done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work $W_{ED}$ done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work $W_{DA}$ done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of $W_{CE}$. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a "0" in the corresponding columns, as appropriate.

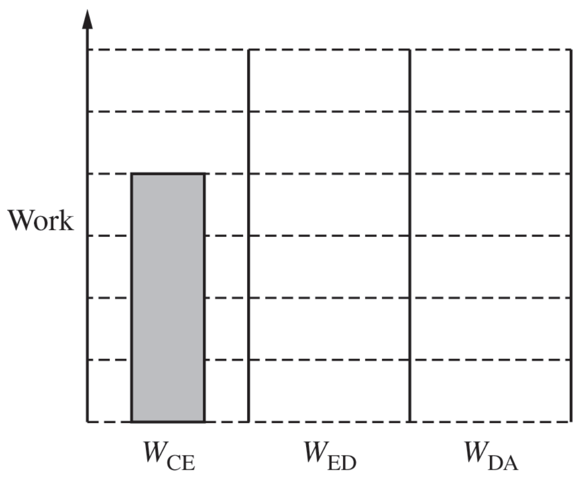


Figure 2

ii. **Calculate** the approximate magnitude of the $x$-component of the electric field at Position B.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position D. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

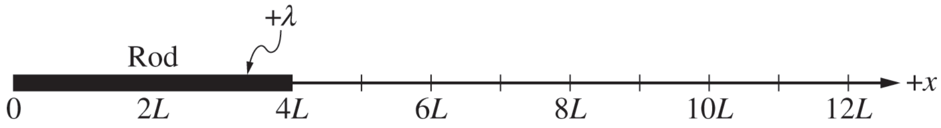


Figure 3

The sphere and the test charge are removed. The rod has length $4L$ and uniform positive linear charge density $+\lambda$. The rod is held at rest on the $x$-axis in the orientation shown in Figure 3. Position P (not shown) is located on the $x$-axis a distance $x_P$ from the origin, where $x_P > 4L$.

(d) The electric potential $V_P$ at $x_P$ is $V_P = k\lambda \ln\left(\frac{x_P}{x_P - 4L}\right)$.

i. Using integral calculus, **derive** the expression for $V_P$ provided.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

ii. On Figure 4, **sketch** a graph of the $x$-component $E_x$ of the electric field from the rod as a function of $x$ in the region $4L < x < 12L$.

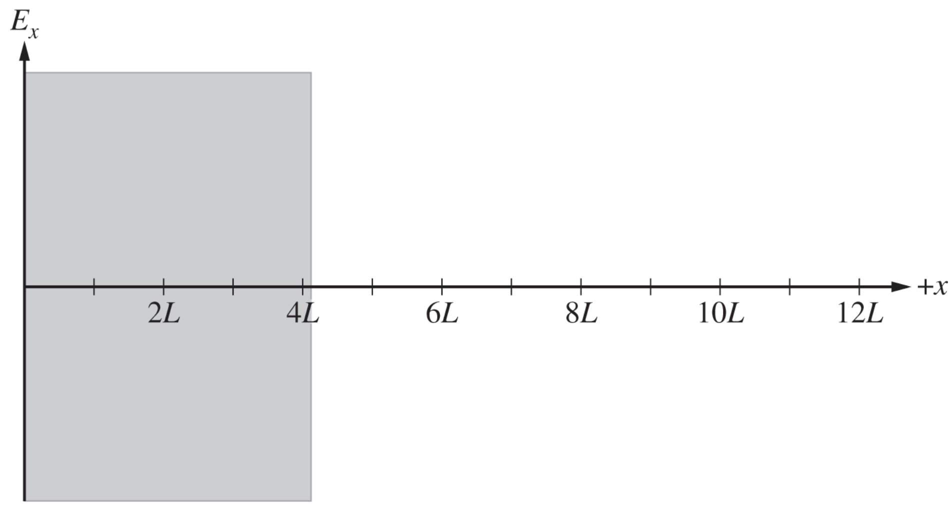


Figure 4

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Begin your response to **QUESTION 1** on this page.

**PHYSICS C: ELECTRICITY AND MAGNETISM**

**SECTION II**

**Time—45 minutes**

**3 Questions**

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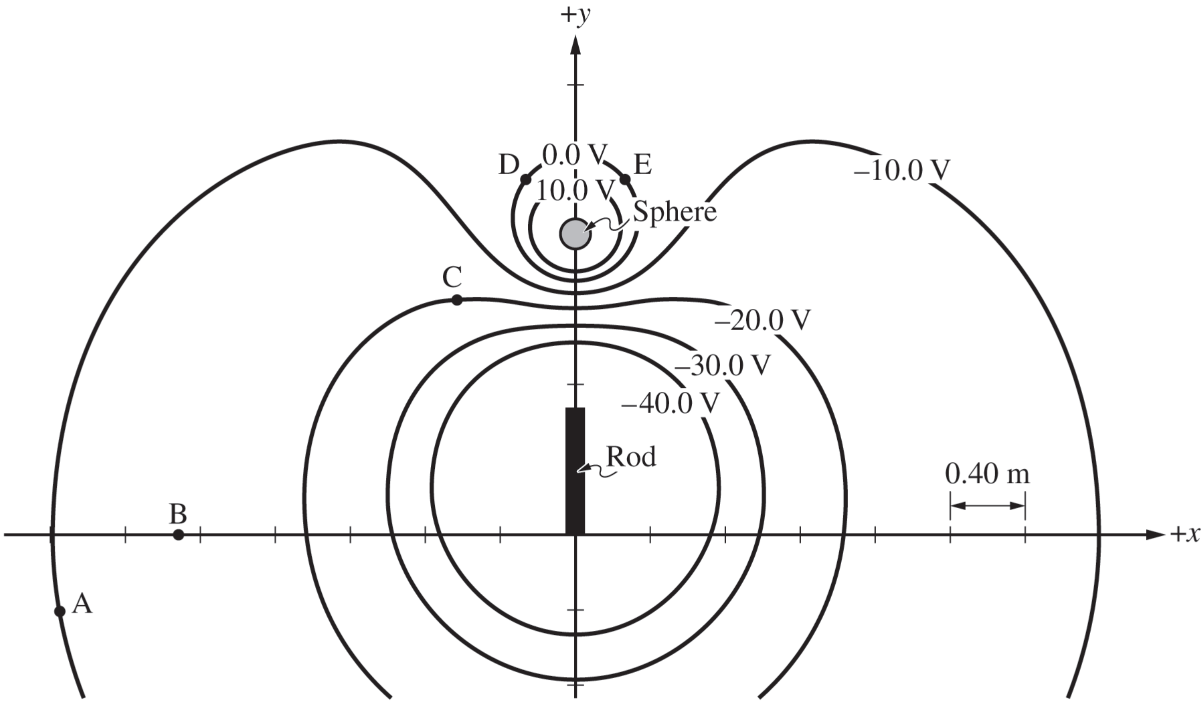


Figure 1

1. A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\ \text{nC}$. The rod and sphere are held at rest on the $y$-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the $x$-axis and on the $y$-axis are $0.40\ \text{m}$ apart.

   (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the $0.0\ \text{V}$ equipotential line.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is held momentarily at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work $W_{CE}$ done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work $W_{ED}$ done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work $W_{DA}$ done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of $W_{CE}$. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a "0" in the corresponding columns, as appropriate.

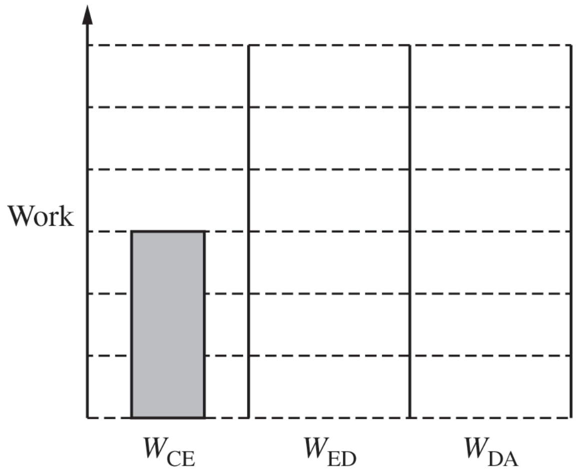


Figure 2

ii. **Calculate** the approximate magnitude of the $x$-component of the electric field at Position B.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position C. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

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Continue your response to **QUESTION 1** on this page.

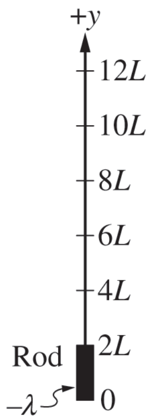


Figure 3

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the $y$-axis in the orientation shown in Figure 3. Position P (not shown) is located on the $y$-axis a distance $y_P$ from the origin, where $y_P > 2L$.

(d) The electric potential $V_P$ at $y_P$ is $V_P = -k\lambda \ln\left(\dfrac{y_P}{y_P - 2L}\right)$.

i. Using integral calculus, **derive** the expression for $V_P$ provided.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

ii. On Figure 4, **sketch** a graph of the $y$-component $E_y$ of the electric field resulting from the rod as a function of $y$ in the region $2L < y < 12L$.

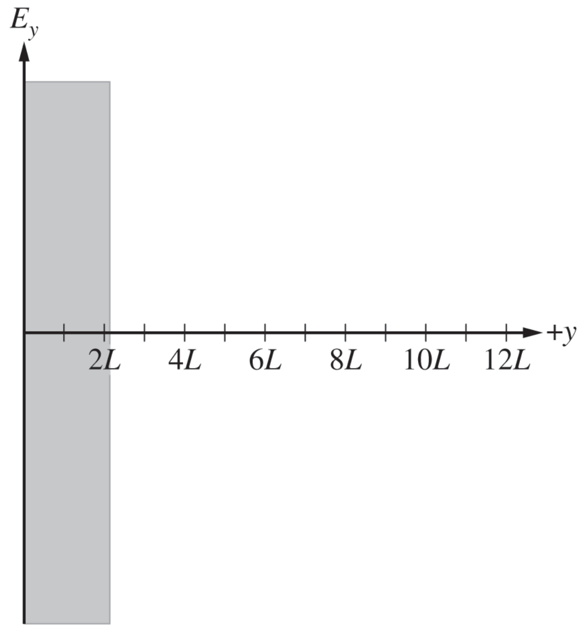


Figure 4

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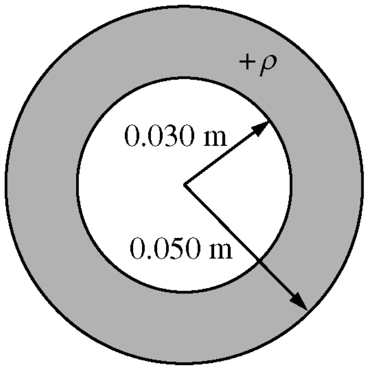



2. A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density $\rho$, as shown in the figure above. The charge density $\rho$ of the sphere is given as a function of the distance $r$ from the center of the sphere, in meters, by the following.

$$r < 0.030\text{ m}: \quad \rho = 0$$

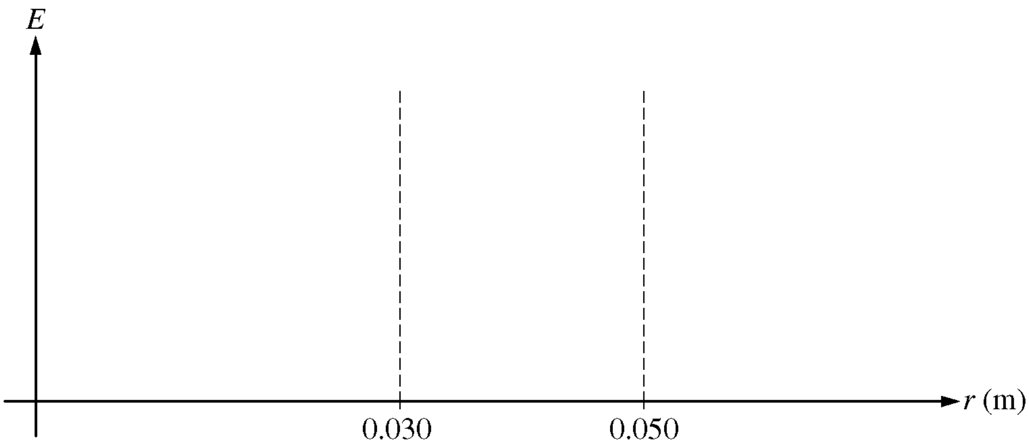
$$0.030\text{ m} < r < 0.050\text{ m}: \quad \rho = b/r\text{, where } b = 1.6\times10^{-6}\ \text{C/m}^2$$

$$r > 0.050\text{ m}: \quad \rho = 0$$

(a) Calculate the total charge of the sphere.

(b) Using Gauss's law, calculate the magnitude of the electric field $E$ at the outer surface of the sphere.

(c) On the axes below, sketch the magnitude of the electric field $E$ as a function of distance $r$ from the center of the sphere.



(d) Calculate the electric potential $V$ at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

(e) A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.

i. Calculate the magnitude of the initial acceleration of the proton.

ii. Calculate the speed of the proton after a long time.

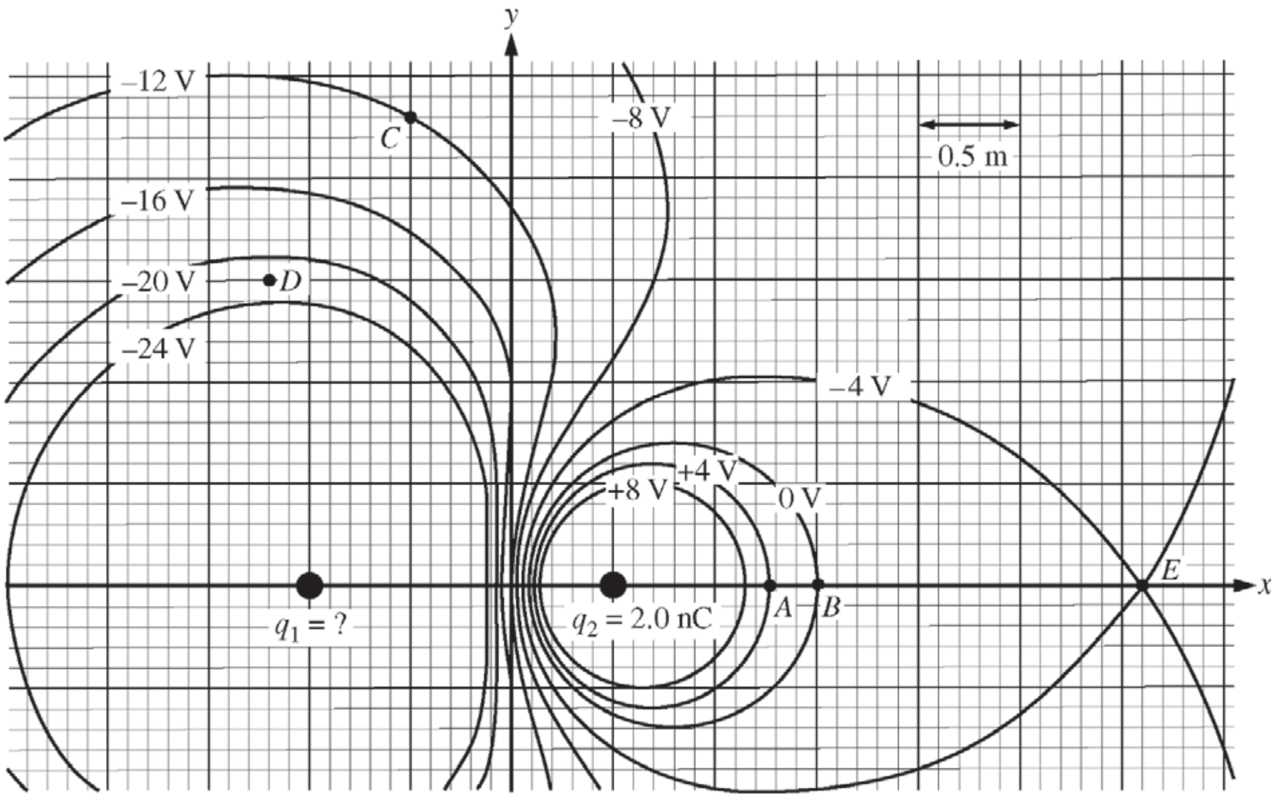
**PHYSICS C: ELECTRICITY AND MAGNETISM**

**SECTION II**

**Time—45 minutes**

**3 Questions**

**Directions:** Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.




E&M.1.

Two point charges, $q_1$ and $q_2$, are fixed in place on the $x$-axis at positions $x_1 = -1.00$ m and $x_2 = +0.50$ m, respectively. Charge $q_2$ has a value of +2.0 nC. Values of electric potential are illustrated by the given equipotentials in the diagram shown above, which is drawn to scale.

(a) Calculate the value of $q_1$.

(b) At point C on the diagram, draw a vector representing the direction of the electric field at that point.

(c) Calculate the approximate magnitude of the electric field strength at point D on the diagram.

(d) The equipotential labeled 0 V is the cross section of a nearly spherical surface. Calculate the electric flux for this surface.

(e) A proton is placed at point A and then released from rest.

i. Calculate the work done by the electric field on the proton as it moves from point *A* to point *E*.

ii. Calculate the speed of the proton when it reaches point *E*.

(f) An electron is released from rest at point B. Which of the following indicates the direction of the initial acceleration, if any, of the electron?

____ Up ____ Down

____ Left ____ Right

____ Into the page ____ Out of the page

____ The direction is undefined since the acceleration is zero.

Justify your answer.

# 9.2 Electric Potential

Name: ____________ Class: ________ Date: __________ **Total: 10 marks**

**KEY WORDS** electric potential 电势 • equipotential 等势面 • perpendicular 垂直 • scalar 标量 • field lines 电场线

## Objective

Build the skills to answer exam questions on **electric potential**.

**You must be able to:**

- define **electric potential 电势** as energy per unit charge, $V = \dfrac{U}{q}$
- calculate the potential of a point charge, $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r}$, and use superposition
- relate potential to field through $V = -\int \vec{E} \cdot d\vec{l}$ and $\vec{E} = -\nabla V$
- describe **equipotential surfaces 等势面** as perpendicular to the field

## 1 Worked examples

Study these first. Each one shows the method for a question type used later.

### ■ Electric potential

$V = \dfrac{U}{q}$, in volts. For a point charge $V = k\dfrac{q}{r}$ — a **scalar**, so potentials just add (superposition).

### ■ Field and potential

$V = -\int \vec{E} \cdot d\vec{l}$, and conversely $\vec{E} = -\nabla V$: the field points **down** the steepest potential drop.

### ■ Equipotentials

Surfaces of constant $V$ are always **perpendicular** to the field lines; no work is done moving a charge along one.

## 2 Practice

**2.1** Find the potential 0.30 m from a +6.0 $\mu$C charge ($k = 8.99 \times 10^9$). [2]

**2.2** State the relationship between equipotential surfaces and field lines. [1]

**2.3** State whether electric potential is a scalar or a vector. [1]

## 3 Exam-style questions

**3.1** The electric potential of a point charge is [1]

- **A** $k\frac{q}{r^2}$
- **B** $k\frac{q}{r}$
- **C** $k\frac{q^2}{r}$
- **D** $qE$

**3.2** Equipotential surfaces are always _________ to the electric field. [1]

- **A** parallel
- **B** perpendicular
- **C** at 45°
- **D** unrelated

**3.3** Two +4.0 $\mu$C charges are 0.40 m apart.

(a) Find the potential at the midpoint between them. [3]

(b) State why the two potentials can simply be added. [1]

## Solutions

**2.1** $V = k\dfrac{q}{r} = 8.99 \times 10^9 \times \dfrac{6.0 \times 10^{-6}}{0.30} = 1.8 \times 10^5$ V.

**2.2** they are perpendicular to each other.

**2.3** a scalar.

**3.1** **B**.

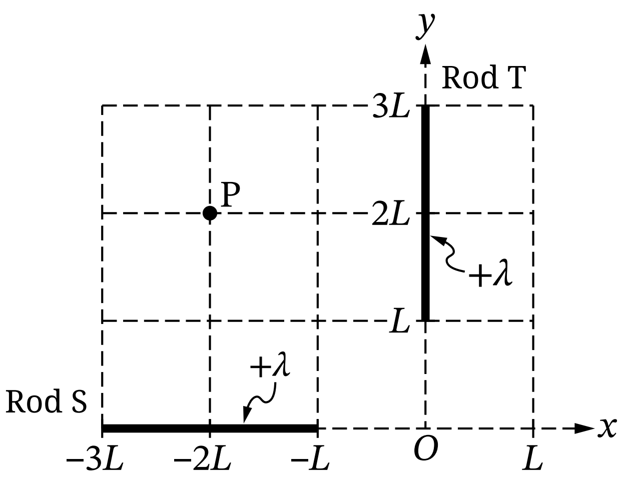
**3.2** **B**.

**3.3** (a) each charge is 0.20 m from the midpoint: $V = 2 \times k\dfrac{q}{r} = 2 \times 8.99 \times 10^9 \times \dfrac{4.0 \times 10^{-6}}{0.20} = 3.6 \times 10^5$ V. (b) potential is a scalar, so contributions add directly (no direction).

## Question 2: Version J

**2.** Thin, nonconducting, charged Rods S and T are held fixed in the $xy$-plane, as shown in Figure 1. Rod S is oriented along the $x$-axis in the region $-3L \leq x \leq -L$. Rod T is oriented along the $y$-axis in the region $L \leq y \leq 3L$. Each rod has uniform, positive linear charge density $+\lambda$. Point P is located at $(-2L, 2L)$.

**Figure 1**

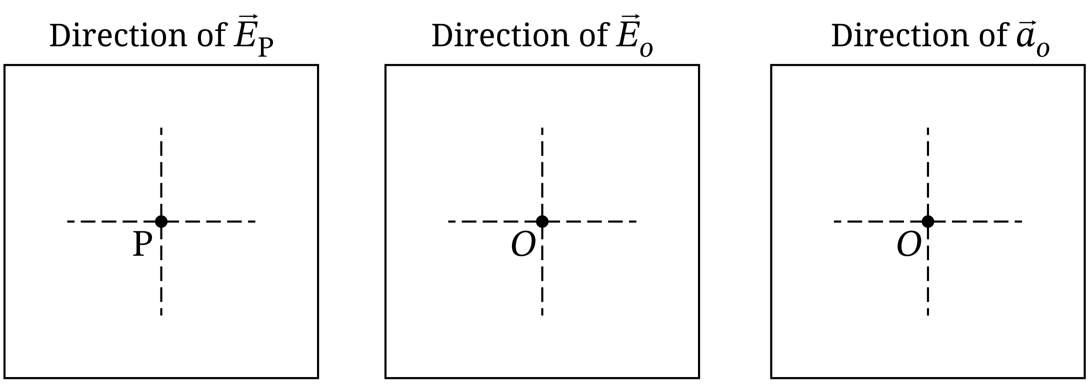



### Part A

Complete the following tasks in Figure 2.

- **Draw** an arrow starting on the dot that indicates the direction of the net electric field $\vec{E}_P$ at Point P due to Rods S and T.
- **Draw** an arrow starting on the dot that indicates the direction of the net electric field $\vec{E}_O$ at the origin $O$ due to Rods S and T.
- **Draw** an arrow starting on the dot that indicates the direction of the net acceleration $\vec{a}_O$ of a small, negatively charged sphere (not shown) immediately after the sphere is released from rest at the origin $O$.

**Figure 2**



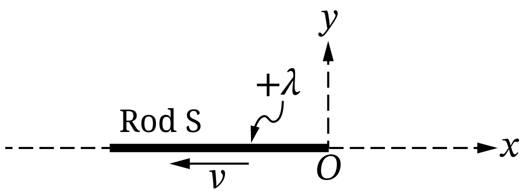
**Part B**

**Derive** an expression for the magnitude $E_O$ of the net electric field at the origin $O$ due to Rods S and T in terms of $L$, $\lambda$, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

**Part C**

Rod T is removed. Rod S is then moved such that the right end of Rod S is at the origin $O$. Rod S is then moved with speed $v$ in the $-x$-direction, as shown in Figure 3.

**Figure 3**



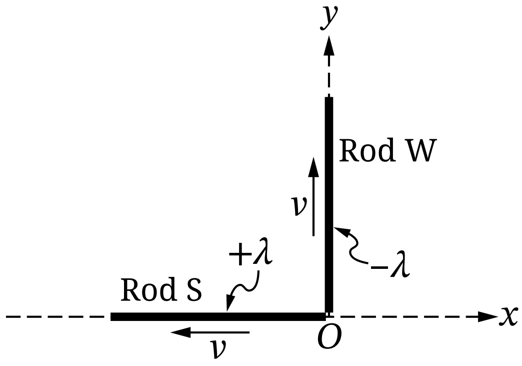

On the axes shown in Figure 4, **sketch** a graph of the electric potential $V_O$ at the origin $O$ due to Rod S as a function of the distance $r$ between the origin and the right end of Rod S.

**Figure 4**

**Part D**

A new thin, nonconducting, charged Rod W has the same length as Rod S. Rod W has uniform, negative linear charge density $-\lambda$ and is oriented along the $y$-axis. Rods S and W are initially held fixed with one end at the origin. Starting at the same time, Rod S is moved with speed $v$ in the $-x$-direction and Rod W is moved with speed $v$ in the $+y$-direction, as shown in Figure 5.

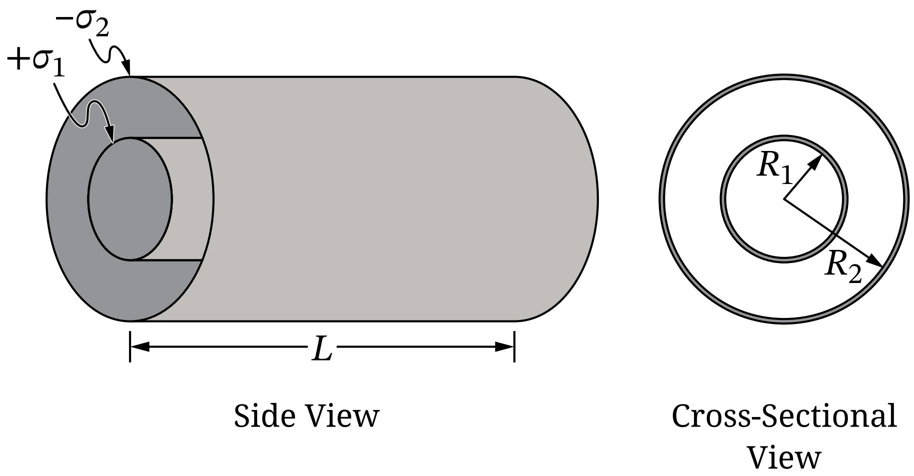
**Figure 5**




**Indicate** how a sketch of $V_O$ at the origin $O$ due to both Rods S and W as a function of $r$ would be different from the sketch made in part C. Briefly **justify** your answer.

**Question 1: Version J**

**1.** An isolated, air-filled, charged capacitor consists of two conducting, coaxial, cylindrical shells that each have length $L$. The inner shell has radius $R_1$ and the outer shell has radius $R_2$, as shown in Figure 1, where $R_1 < R_2 \ll L$. The surface charge densities (amounts of charge per unit area) of the inner and outer shells are $+\sigma_1$ and $-\sigma_2$, respectively. The absolute values of the total charges on the shells are equal.




Note: Figures not drawn to scale.

Figure 1

**A.**

i. Using Gauss's law, **derive** an expression for the magnitude $E$ of the electric field as a function of the radial distance $r$ from the center of the capacitor for the region $R_1 < r < R_2$. Express your answer in terms of $R_1$, $\sigma_1$, $r$, and physical constants, as appropriate.

ii. **Derive** an expression for the absolute value $|\Delta V|$ of the potential difference between the outer and inner shells in terms of $R_1$, $R_2$, $\sigma_1$, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

iii. On the axes shown in Figure 2, **sketch** a graph of $E$ as a function of $r$ from $r = 0$ to a position that is outside the outer shell.

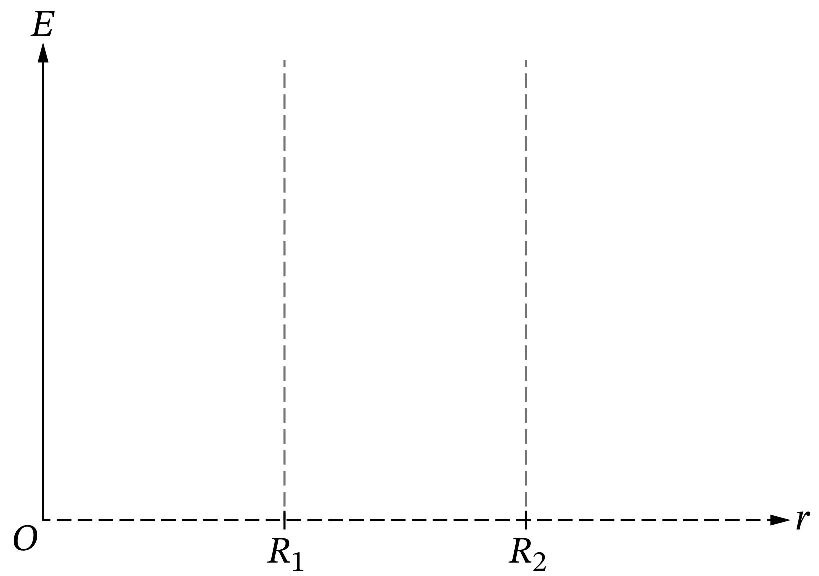
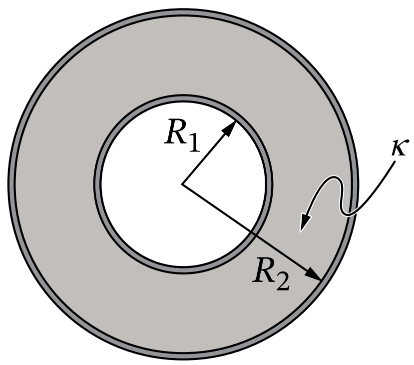



Figure 2

**B.** A material of dielectric constant $\kappa$ is inserted into the isolated, charged capacitor such that the material fills the region $R_1 < r < R_2$, as shown in Figure 3.




Cross-Sectional View

Figure 3

**Derive** an expression for the capacitance $C$ of the capacitor with the material inserted in terms of $L$, $R_1$, $R_2$, $\kappa$, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Begin your response to **QUESTION 1** on this page.

**PHYSICS C: ELECTRICITY AND MAGNETISM**

**SECTION II**

**Time—45 minutes**

**3 Questions**

**Directions:** Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

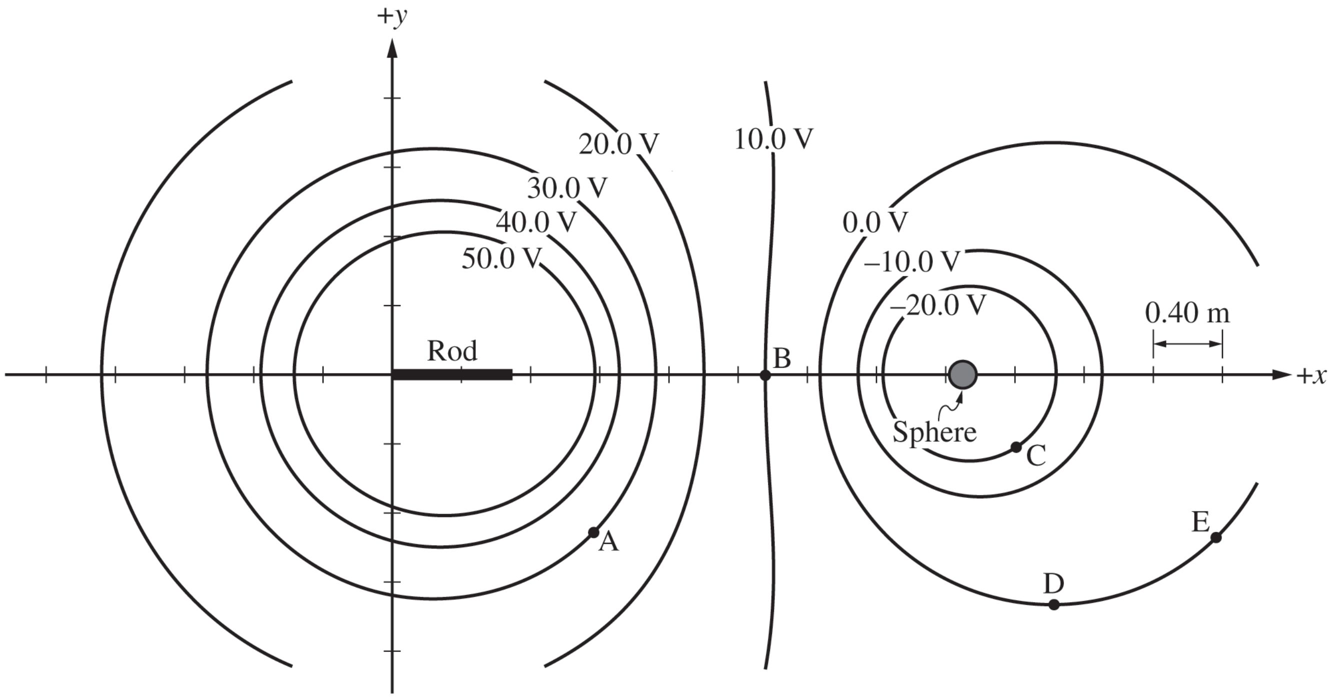



Figure 1

1. A nonconducting rod of uniform positive linear charge density is near a sphere with charge $-2.0\ \text{nC}$. The rod and sphere are held at rest on the $x$-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the $x$-axis and the $y$-axis are $0.40\ \text{m}$ apart.

   (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the $-20.0\ \text{V}$ equipotential line.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is momentarily held at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work $W_{CE}$ done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work $W_{ED}$ done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work $W_{DA}$ done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of $W_{CE}$. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a "0" in the corresponding columns, as appropriate.

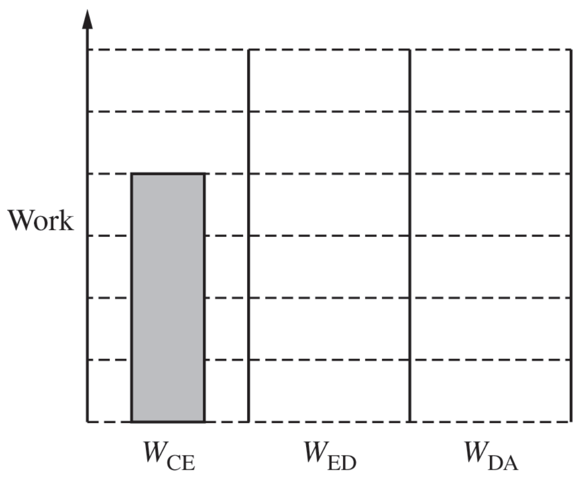



Figure 2

ii. **Calculate** the approximate magnitude of the $x$-component of the electric field at Position B.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position D. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

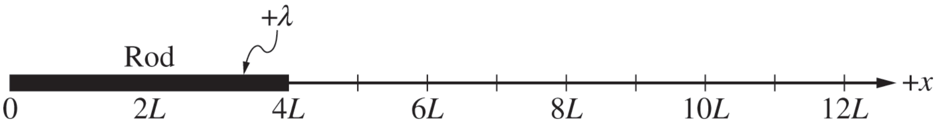



Figure 3

The sphere and the test charge are removed. The rod has length $4L$ and uniform positive linear charge density $+\lambda$. The rod is held at rest on the $x$-axis in the orientation shown in Figure 3. Position P (not shown) is located on the $x$-axis a distance $x_P$ from the origin, where $x_P > 4L$.

(d) The electric potential $V_P$ at $x_P$ is $V_P = k\lambda \ln\left(\dfrac{x_P}{x_P - 4L}\right)$.

i. Using integral calculus, **derive** the expression for $V_P$ provided.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

ii. On Figure 4, **sketch** a graph of the $x$-component $E_x$ of the electric field from the rod as a function of $x$ in the region $4L < x < 12L$.

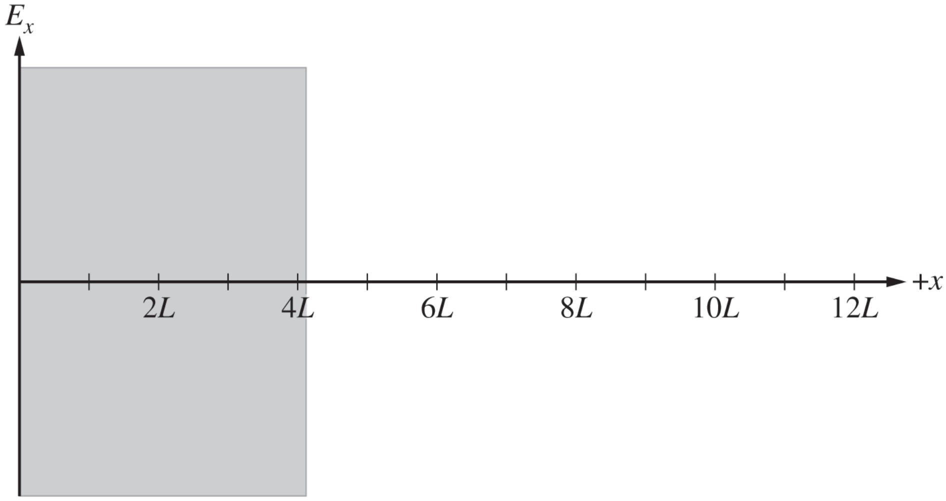



Figure 4

**GO ON TO THE NEXT PAGE.**

Begin your response to **QUESTION 1** on this page.

**PHYSICS C: ELECTRICITY AND MAGNETISM**

**SECTION II**

**Time—45 minutes**

**3 Questions**

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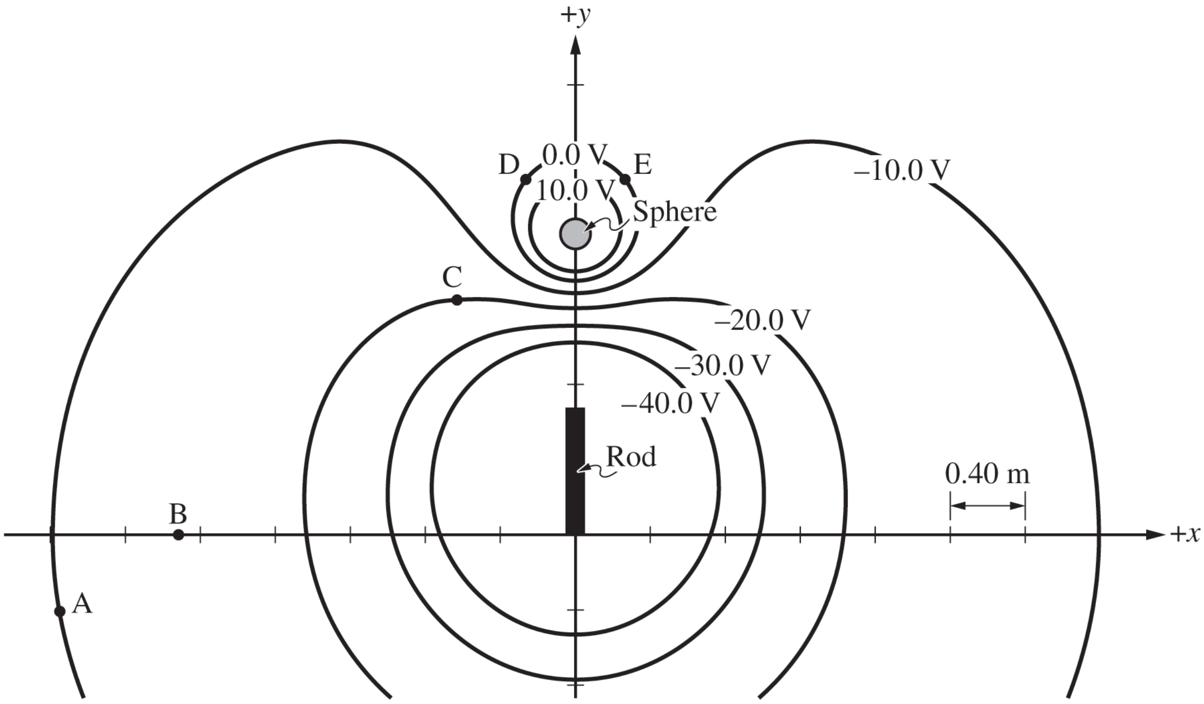


Figure 1

1. A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\ \text{nC}$. The rod and sphere are held at rest on the $y$-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the $x$-axis and on the $y$-axis are $0.40\ \text{m}$ apart.

   (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the $0.0\ \text{V}$ equipotential line.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

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(b) The bar shown in Figure 2 represents the absolute value of the work $W_{CE}$ done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work $W_{ED}$ done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work $W_{DA}$ done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of $W_{CE}$. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a “0” in the corresponding columns, as appropriate.

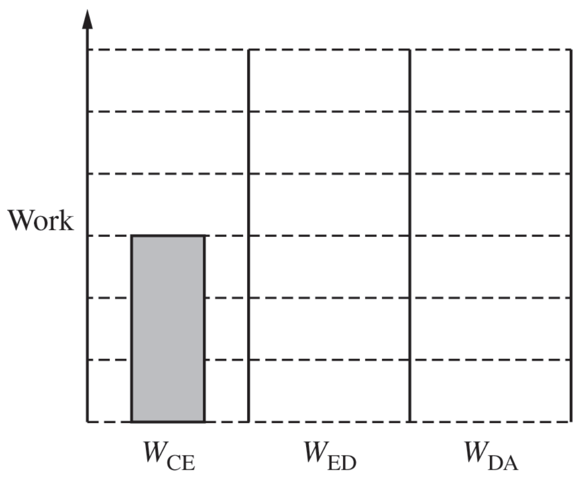


Figure 2

ii. **Calculate** the approximate magnitude of the $x$-component of the electric field at Position B.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position C. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

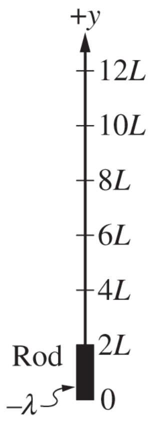


Figure 3

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the $y$-axis in the orientation shown in Figure 3. Position P (not shown) is located on the $y$-axis a distance $y_P$ from the origin, where $y_P > 2L$.

(d) The electric potential $V_P$ at $y_P$ is $V_P = -k\lambda \ln\left(\dfrac{y_P}{y_P - 2L}\right)$.

i. Using integral calculus, **derive** the expression for $V_P$ provided.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

ii. On Figure 4, **sketch** a graph of the $y$-component $E_y$ of the electric field resulting from the rod as a function of $y$ in the region $2L < y < 12L$.

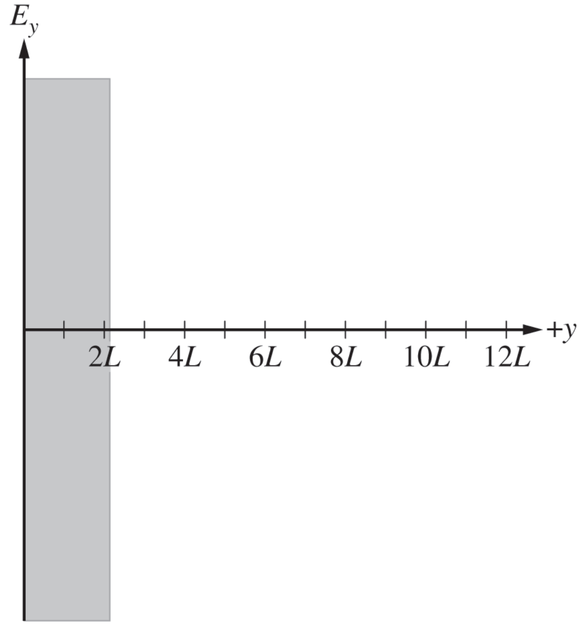


Figure 4

**GO ON TO THE NEXT PAGE.**

Begin your response to **QUESTION 1** on this page.

## PHYSICS C: ELECTRICITY AND MAGNETISM

## SECTION II

**Time—45 minutes**

**3 Questions**

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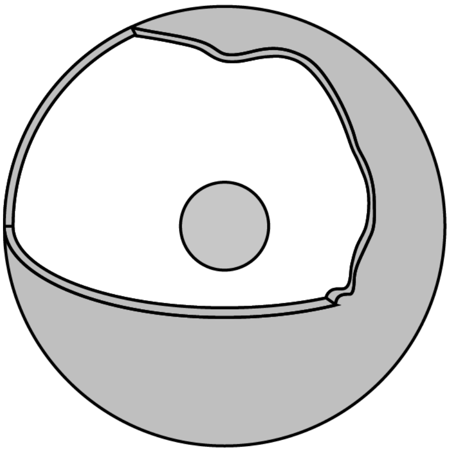
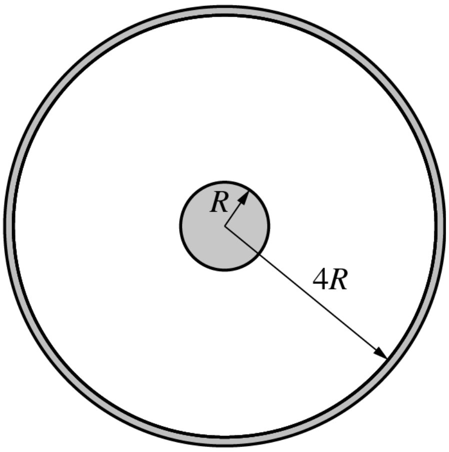


Cutout View




Cross-Section View

Note: Figures not drawn to scale.

1. A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are $R$ and $4R$, respectively, as shown in the cross-section view.

(a) Determine the charge on the outer surface of the shell.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

(b) Using Gauss's law, derive an expression for the electric field a distance $r$ from the center of the sphere for $r < R$. Express your answer in terms of $Q$, $R$, $r$, and physical constants, as appropriate.

(c) The magnitude of the electric field at $r = R$ is $8\,\mathrm{N/C}$. Calculate the value of the electric field at $r = 2R$.

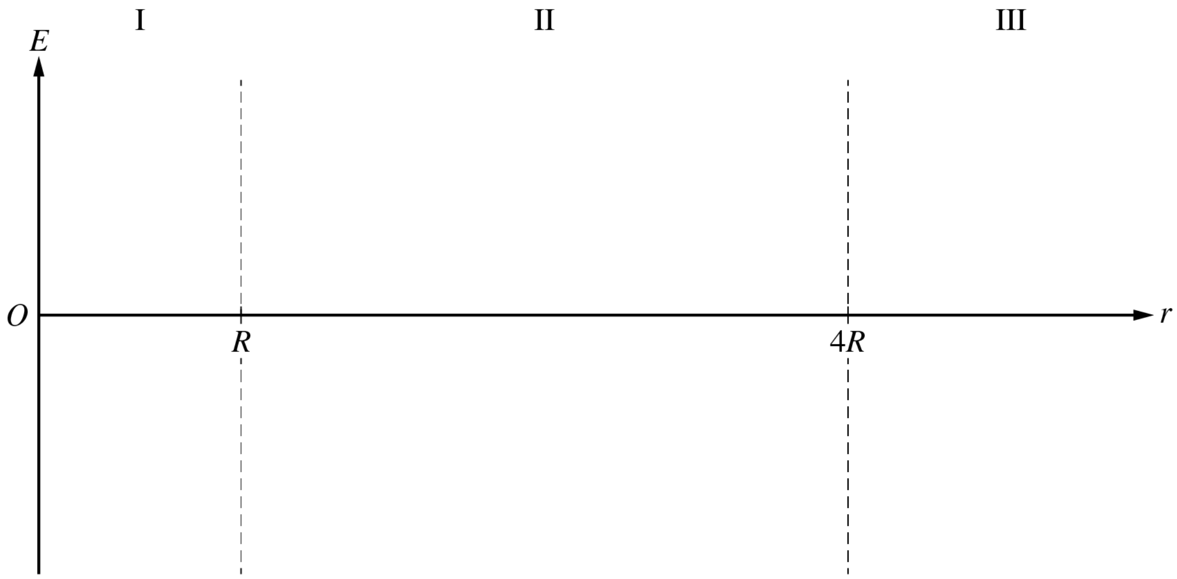
(d) Derive an expression for the absolute value of the potential difference between the outer surface of the sphere and the inner surface of the shell. Express your answer in terms of $Q$, $R$, and physical constants, as appropriate.

**GO ON TO THE NEXT PAGE.**

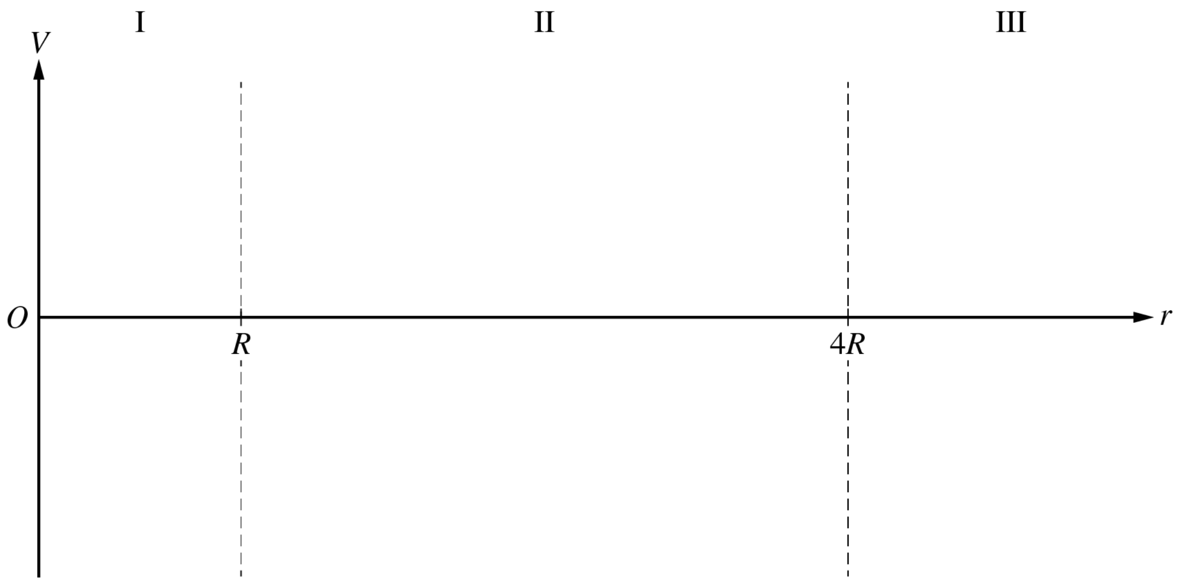
Continue your response to **QUESTION 1** on this page.

(e)

i. On the following axes that include regions I, II, and III, sketch a graph of the electric field $E$ as a function of the distance $r$ from the center of the sphere.




ii. On the following axes that include regions I, II, and III, sketch a graph of the electric potential $V$ as a function of the distance $r$ from the center of the sphere.




**GO ON TO THE NEXT PAGE.**

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**PHYSICS C: ELECTRICITY AND MAGNETISM**

**SECTION II**

**Time—45 minutes**

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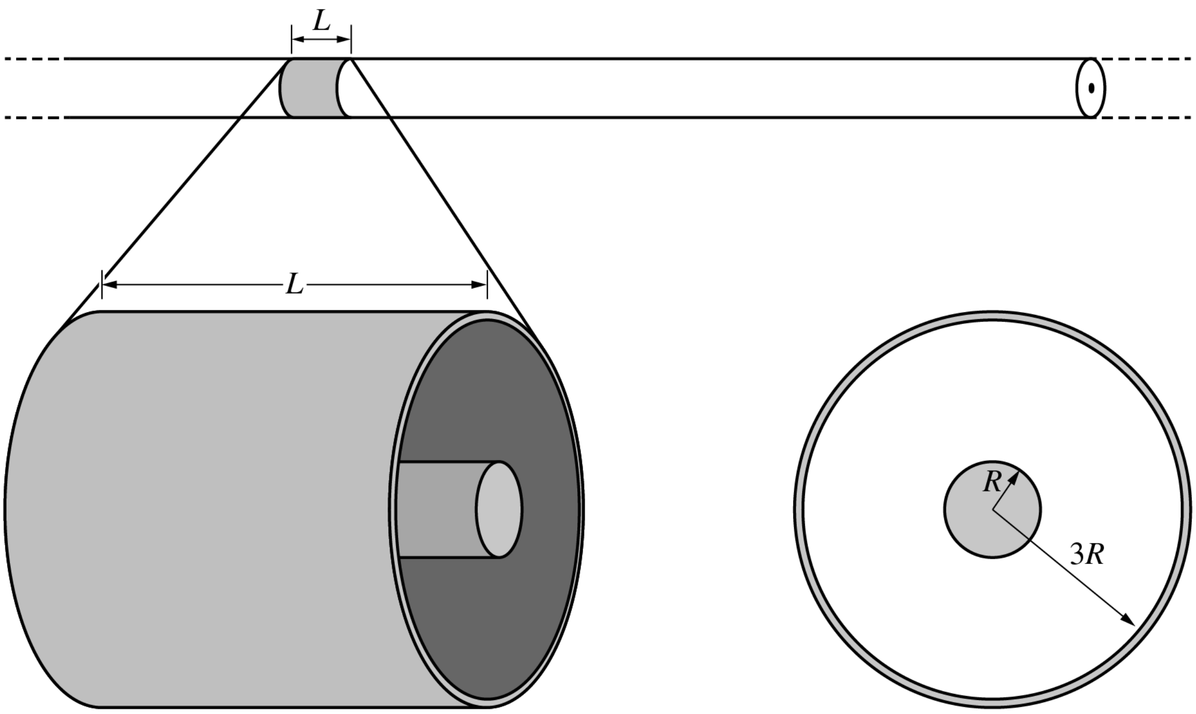



Cutout View

Cross-Section View

Note: Figures not drawn to scale.

1. A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length $L$ of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length $L$ of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are $R$ and $3R$, respectively, as shown in the cross-section view.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

(a) Determine the charge on the outer surface of the cylindrical shell within length $L$.

(b) Using Gauss's law, derive an expression for the electric field a distance $r$ from the center of the inner cylinder for $r < R$. Express your answers in terms of $Q$, $R$, $r$, $L$, and physical constants, as appropriate.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

(c) The magnitude of the electric field at $r = R$ is $12\ \text{N/C}$. Calculate the value of the electric field at $r = 2R$.

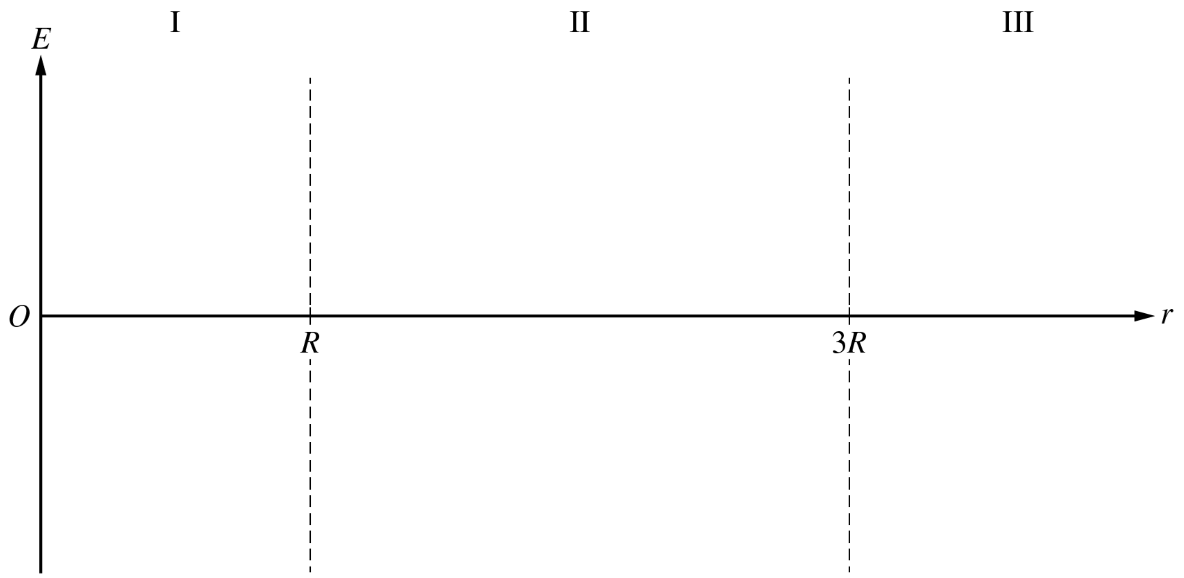
(d) Derive an expression for the absolute value of the potential difference between the surface of the nonconducting cylinder and the inner surface of the cylindrical shell.

**GO ON TO THE NEXT PAGE.**

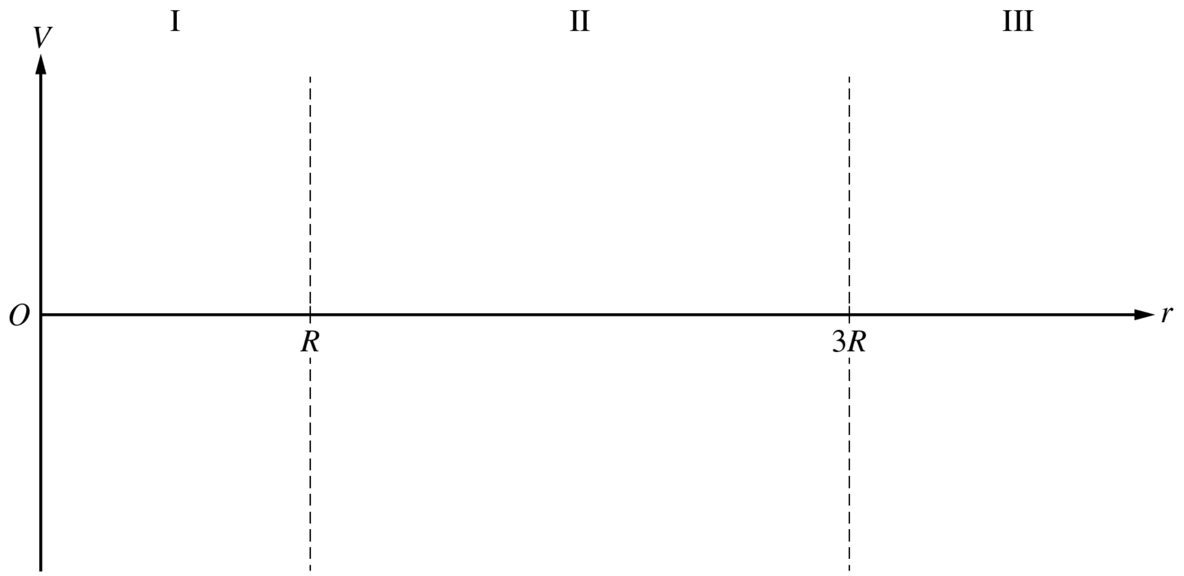
Continue your response to **QUESTION 1** on this page.

(e)

i. On the following axes that include regions I, II, and III, sketch the graph of the electric field $E$ as a function of the distance $r$ from the axis of the inner cylinder.




ii. On the following axes that include regions I, II, and III, sketch the graph of the electric potential $V$ as a function of the distance $r$ from the axis of the inner cylinder.

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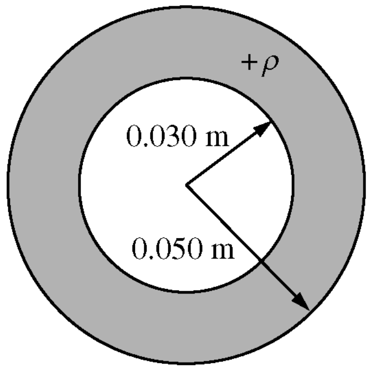



2. A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density $\rho$, as shown in the figure above. The charge density $\rho$ of the sphere is given as a function of the distance $r$ from the center of the sphere, in meters, by the following.

$$r < 0.030\text{ m:}\quad \rho = 0$$

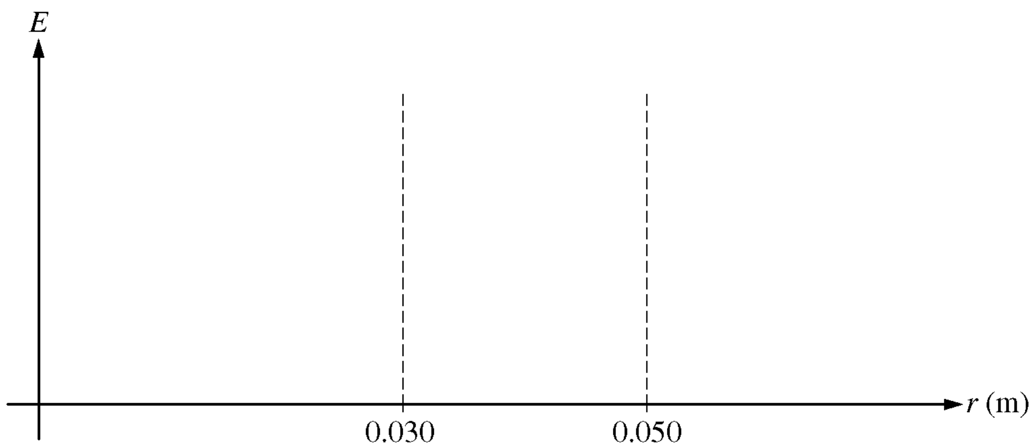
$$0.030\text{ m} < r < 0.050\text{ m:}\quad \rho = b/r\text{, where } b = 1.6 \times 10^{-6}\ \mathrm{C/m^2}$$

$$r > 0.050\text{ m:}\quad \rho = 0$$

(a) Calculate the total charge of the sphere.

(b) Using Gauss's law, calculate the magnitude of the electric field $E$ at the outer surface of the sphere.

(c) On the axes below, sketch the magnitude of the electric field $E$ as a function of distance $r$ from the center of the sphere.



(d) Calculate the electric potential $V$ at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

(e) A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.

i. Calculate the magnitude of the initial acceleration of the proton.

ii. Calculate the speed of the proton after a long time.

## PHYSICS C: ELECTRICITY AND MAGNETISM

### SECTION II

**Time—45 minutes**

**3 Questions**

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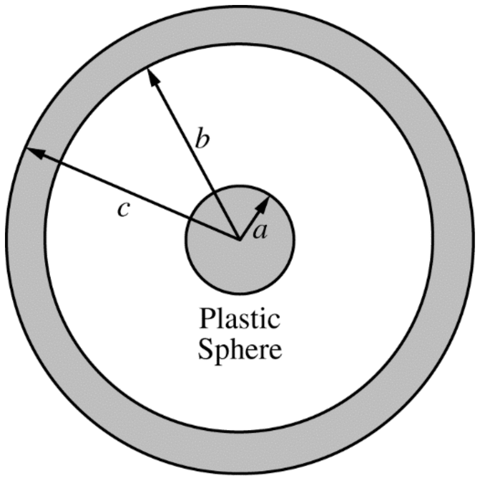



Conducting Spherical Shell

1. A solid plastic sphere of radius *a* and a conducting spherical shell of inner radius *b* and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where $\beta$ is a positive constant and *r* is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of $\beta$, *r*, *a*, and physical constants, as appropriate.

   (a) Consider a Gaussian sphere of radius *r* concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

      i. $r < a$

      ii. $a < r < b$

   (b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

      i. $r < a$

      ii. $a < r < b$

   (c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

      i. Determine the charge on the inner surface of the conducting shell.

      Justify your answer.

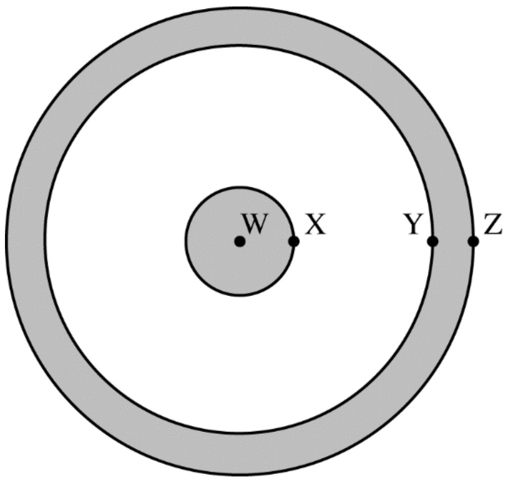
      ii. Determine the charge on the outer surface of the conducting shell.

(d)

i. On the axes below, sketch the electric field $E$ as a function of distance $r$ from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.


ii. The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

____ W ____ X ____ Y ____ Z



# PHYSICS C: ELECTRICITY AND MAGNETISM

## SECTION II

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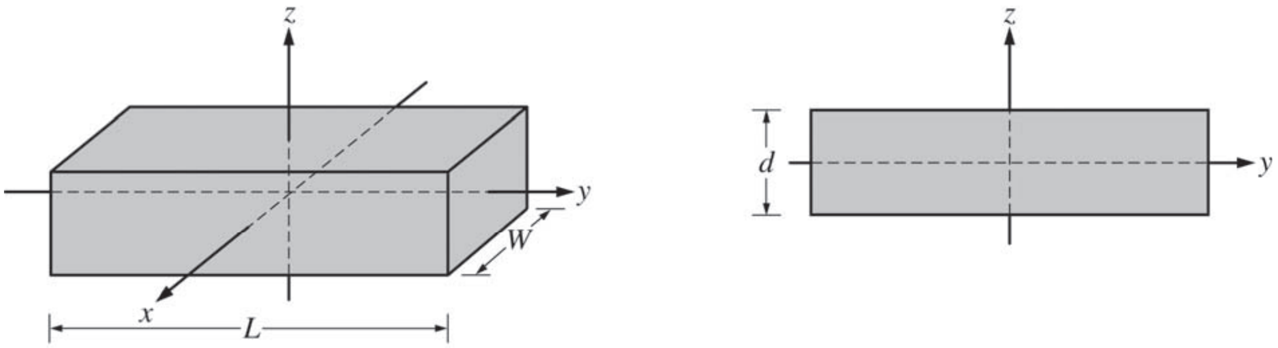
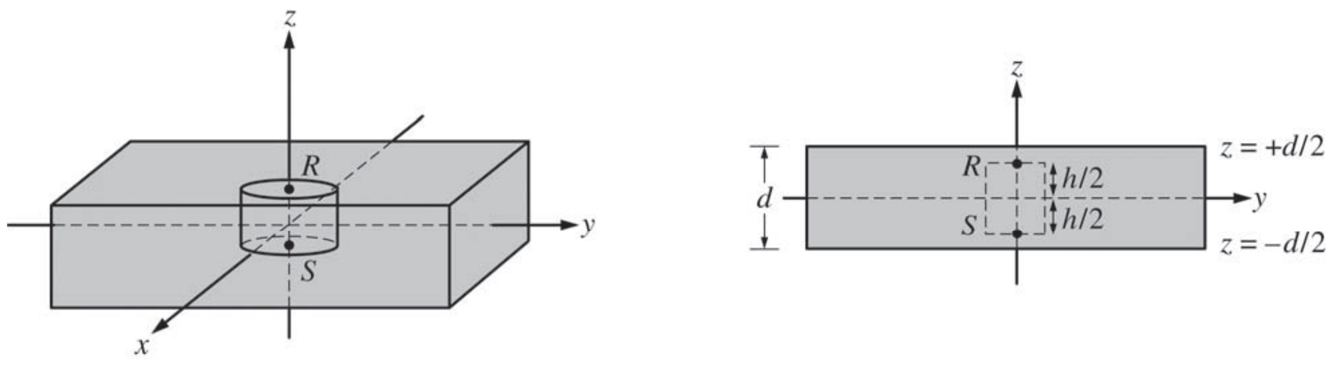



Perspective View  Side View

Note: Figures not drawn to scale.

1. A very large nonconducting slab with a uniform positive volume charge density $\rho_0$ is fixed with the origin of the *xyz*-axes at its center, as shown in the figure above. The thickness of the slab is $d$, the length is $L$, and the width is $W$, where $L >> d$ and $W >> d$. The large faces of the slab are parallel to the *xy*-plane.

   Consider a Gaussian cylinder with a cross-sectional area $A$ and height $h$ that is positioned with its axis along the *z*-axis, as shown in the figure below.




Perspective View  Side View

Note: Figures not drawn to scale.

(a) Draw a single vector on each of the dots below representing the direction of the electric field at the given points. If the electric field at either point is zero, write "$E = 0$" next to the point.

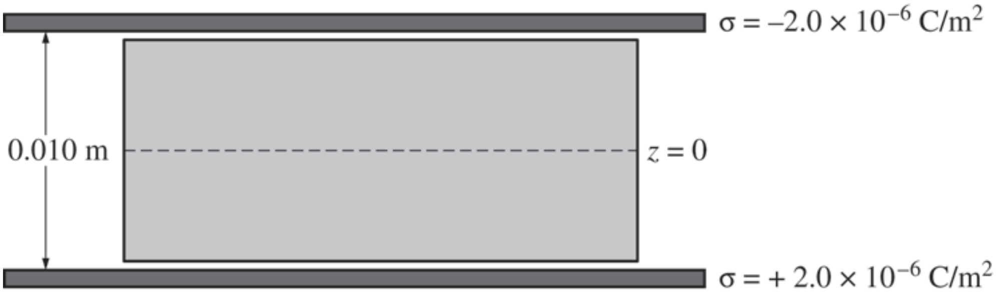
i. ii.

• $R$ • $S$

b) Use Gauss's law to derive expressions for the following. Express your answers in terms of $\rho_0$, $A$, $d$, $h$, $z$, and physical constants, as appropriate.

i. Derive an expression for the total flux $\Phi$ through the Gaussian surface shown.

ii. Derive an expression for the magnitude of the electric field as a function of $z$ for any position inside the slab, and show that it is equal to $E = \dfrac{\rho_0 z}{\varepsilon_0}$.




Note: Figure not drawn to scale.

The charged slab is now placed between two large metal plates separated by a distance of 0.010 m, which is approximately the thickness of the slab, but the slab does not contact either metal plate. The metal plates are charged, resulting in the surface charge densities $\sigma = \pm 2.0 \times 10^{-6}\ \text{C/m}^2$, as shown in the figure above. Assume the charge distribution inside the slab remains unchanged by the presence of the charged plates and that the slab's volume charge density is $\rho_0 = 1.00 \times 10^{-3}\ \text{C/m}^3$.

(c)

i. The magnitude of the electric field inside the slab is zero on the $z$-axis at position $z_0$. Which of the following correctly indicates the value for $z_0$?

____ $z_0 > 0$ ____ $z_0 = 0$ ____ $z_0 < 0$

Justify your answer.

ii. Calculate the value $z_0$.

(d) Calculate the magnitude of the electric potential difference from the center of the slab to the top of the slab.

# PHYSICS C: ELECTRICITY AND MAGNETISM

## SECTION II

**Time—45 minutes**

**3 Questions**

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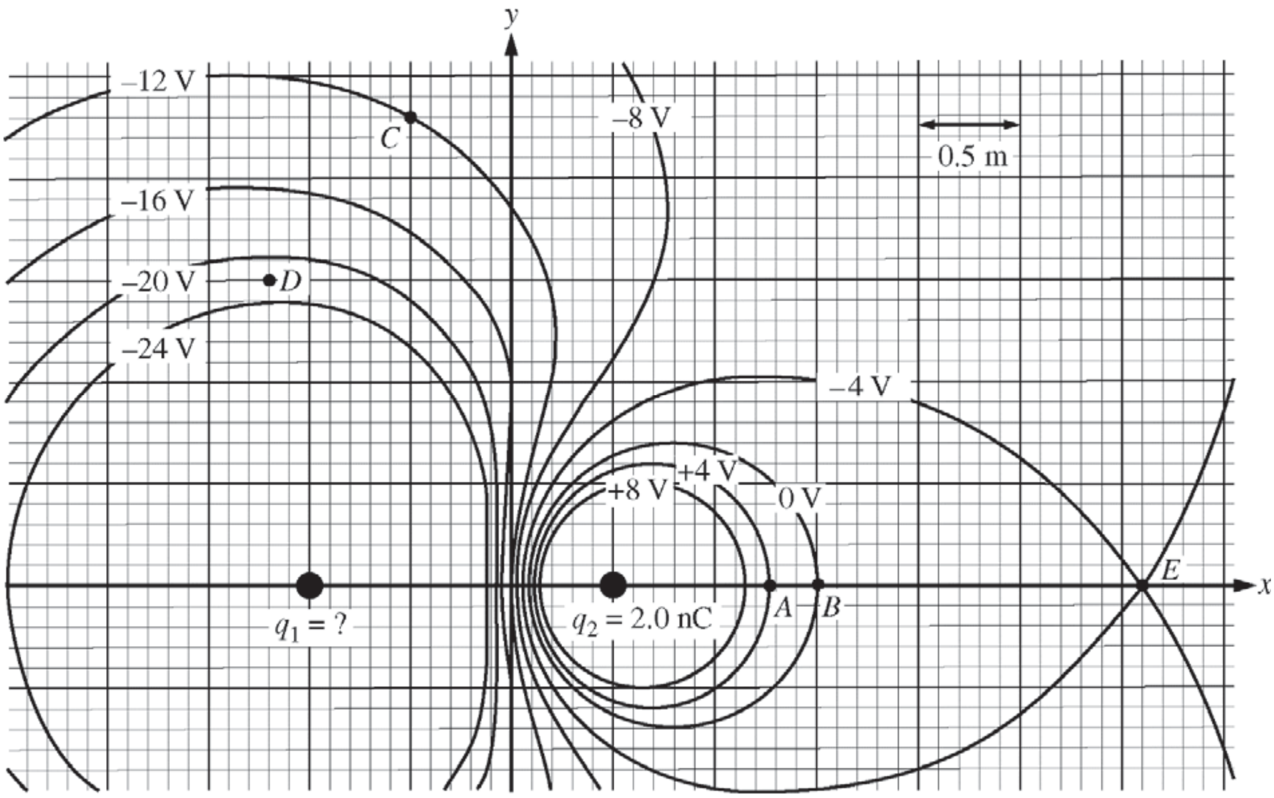


E&M.1.

Two point charges, $q_1$ and $q_2$, are fixed in place on the $x$-axis at positions $x_1 = -1.00$ m and $x_2 = +0.50$ m, respectively. Charge $q_2$ has a value of +2.0 nC. Values of electric potential are illustrated by the given equipotentials in the diagram shown above, which is drawn to scale.

(a) Calculate the value of $q_1$.

(b) At point C on the diagram, draw a vector representing the direction of the electric field at that point.

(c) Calculate the approximate magnitude of the electric field strength at point D on the diagram.

(d) The equipotential labeled 0 V is the cross section of a nearly spherical surface. Calculate the electric flux for this surface.

(e) A proton is placed at point A and then released from rest.

i. Calculate the work done by the electric field on the proton as it moves from point *A* to point *E*.

ii. Calculate the speed of the proton when it reaches point *E*.

(f) An electron is released from rest at point B. Which of the following indicates the direction of the initial acceleration, if any, of the electron?

____ Up ____ Down

____ Left ____ Right

____ Into the page ____ Out of the page

____ The direction is undefined since the acceleration is zero.

Justify your answer.

## PHYSICS C: ELECTRICITY AND MAGNETISM

### SECTION II
**Time—45 minutes**

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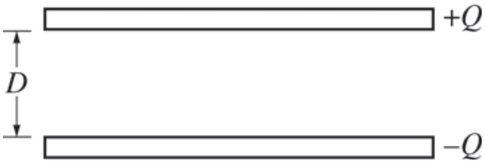



E&M.1.

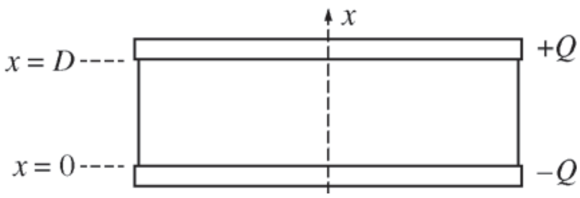
A parallel-plate capacitor is constructed of two parallel metal plates, each with area $A$ and separated by a distance $D$. The plates of the capacitor are each given a charge of magnitude $Q$, as shown in the figure above. Ignore edge effects.

(a)

i. On the figure above, draw an arrow to indicate the direction of the electric field between the plates.

ii. On the figure above, draw an appropriate Gaussian surface that will be used to derive an expression for the magnitude of the electric field $E$ between the plates.

iii. Using Gauss's law and the Gaussian surface from part (a)-ii, derive an expression for the magnitude of the electric field $E$ between the plates. Express your answer in terms of $A$, $D$, $Q$, and physical constants, as appropriate.




The space between the plates is now filled with a dielectric material that is engineered so that its dielectric constant varies with the distance from the bottom plate to the top plate, defined by the x-axis indicated in the diagram above. As a result, the electric field between the plates is given by $\vec{E} = -\dfrac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\hat{i}$, where $\kappa_0$ is a positive constant. Express all algebraic answers to the remaining parts in terms of $A$, $D$, $Q$, $\kappa_0$, $x$, and physical constants, as appropriate.

(b) Determine an expression for the dielectric constant $\kappa$ as a function of $x$.

(c)

i. Write, but do NOT solve, an equation that could be used to determine the potential difference $V$ between the plates of the capacitor.

ii. Using the equation from part (c)-i, derive an expression for the potential difference $V_D - V_0$, where $V_D$ is the potential of the top plate and $V_0$ is the potential of the bottom plate.

(d) Determine the capacitance of the capacitor.

(e) The energy stored in the capacitor that has a varying dielectric is $U_V$. A second capacitor that has a constant dielectric of value $\kappa_0$ is also given a charge $Q$. The energy stored in the second capacitor is $U_C$. How do the values of $U_V$ and $U_C$ compare?

____ $U_V < U_C$ ____ $U_V > U_C$ ____ $U_V = U_C$

Justify your answer.

# 9.3 Conservation of Electric Energy

Name: ______________ Class: __________ Date: ____________ **Total: 11 marks**

**KEY WORDS** electron volt 电子伏特 • conservation of energy 能量守恒 • work 功 • potential difference 电势差 • kinetic energy 动能

## Objective

Build the skills to answer exam questions on **conservation of electric energy**.

**You must be able to:**

- relate the change in a charge's **kinetic energy** to $W = q\,\Delta V$
- apply **conservation of energy 能量守恒** to a charge moving through a potential difference
- define and use the **electron volt 电子伏特** (eV)
- find the final **speed** of a charge accelerated through a potential difference

## 1 Worked examples

Study these first. Each one shows the method for a question type used later.

### ■ Energy from a potential difference

$\Delta KE = q\,\Delta V$: the field does work $q\,\Delta V$ on a charge that falls through $\Delta V$.

### ■ The electron volt

1 eV is the energy an electron gains through 1 V: 1 eV $= 1.6 \times 10^{-19}$ J.

### ■ Final speed

Set $\frac{1}{2}mv^2 = q\,\Delta V$ and solve for $v$.

## 2 Practice

**2.1** An electron moves through a potential difference of 50 V. State the energy it gains, in eV. [1]

**2.2** Convert 3.0 eV to joules ($e = 1.6 \times 10^{-19}$ C). [2]

**2.3** A +2.0 $\mu$C charge moves through 300 V. Find the energy it gains, in joules. [2]

## 3 Exam-style questions

**3.1** One electron volt equals [1]

- **A** 1 J
- **B** $1.6 \times 10^{-19}$ J
- **C** 1 V
- **D** 1 C

**3.2** A charge accelerated through a potential difference $\Delta V$ gains kinetic energy [1]

- **A** $q/\Delta V$
- **B** $q\,\Delta V$
- **C** $\Delta V/q$
- **D** $\frac{1}{2}q\,\Delta V^2$

**3.3** A proton ($m = 1.67 \times 10^{-27}$ kg, $q = 1.6 \times 10^{-19}$ C) accelerates from rest through 500 V.

(a) Find its kinetic energy in joules. [2]

(b) Find its final speed. [2]

## Solutions

**2.1** 50 eV.

**2.2** $3.0 \times 1.6 \times 10^{-19} = 4.8 \times 10^{-19}$ J.

**2.3** $\Delta KE = q\,\Delta V = (2.0 \times 10^{-6})(300) = 6.0 \times 10^{-4}$ J.

**3.1** **B**.

**3.2** **B**.

**3.3** (a) $\Delta KE = q\,\Delta V = (1.6 \times 10^{-19})(500) = 8.0 \times 10^{-17}$ J. (b) $\frac{1}{2}mv^2 = 8.0 \times 10^{-17} \Rightarrow$ $v = \sqrt{\dfrac{2(8.0 \times 10^{-17})}{1.67 \times 10^{-27}}} = 3.1 \times 10^5$ m s$^{-1}$.

Begin your response to **QUESTION 1** on this page.

**PHYSICS C: ELECTRICITY AND MAGNETISM**

**SECTION II**

**Time—45 minutes**

**3 Questions**

**Directions:** Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

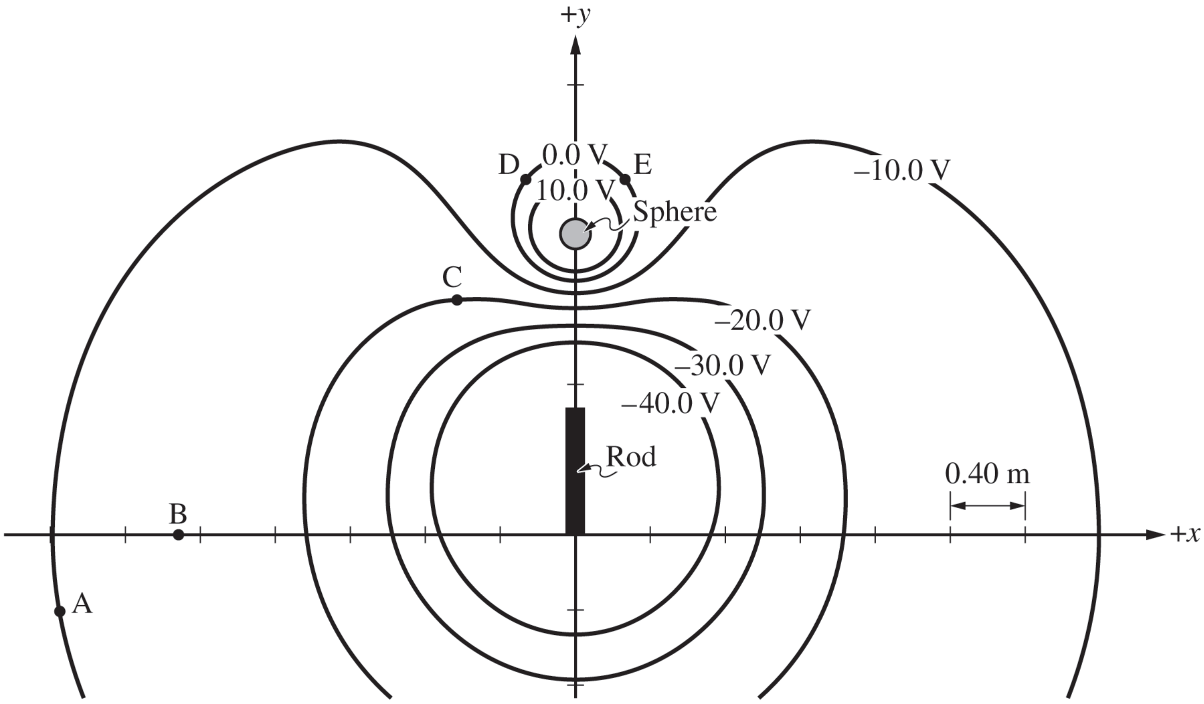



Figure 1

1. A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\text{ nC}$. The rod and sphere are held at rest on the $y$-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the $x$-axis and on the $y$-axis are $0.40\text{ m}$ apart.

   (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the $0.0\text{ V}$ equipotential line.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is held momentarily at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work $W_{CE}$ done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work $W_{ED}$ done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work $W_{DA}$ done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of $W_{CE}$. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a “0” in the corresponding columns, as appropriate.

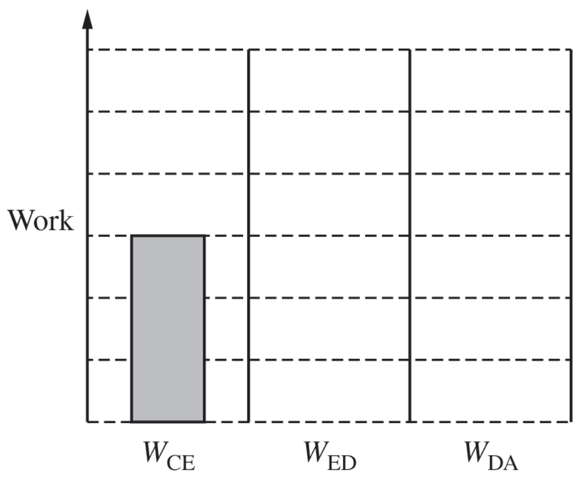



Figure 2

ii. **Calculate** the approximate magnitude of the $x$-component of the electric field at Position B.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position C. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

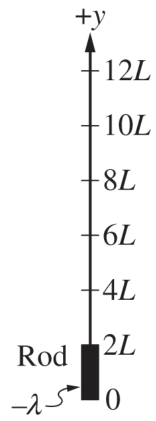



Figure 3

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the $y$-axis in the orientation shown in Figure 3. Position P (not shown) is located on the $y$-axis a distance $y_P$ from the origin, where $y_P > 2L$.

(d) The electric potential $V_P$ at $y_P$ is $V_P = -k\lambda \ln\left(\dfrac{y_P}{y_P - 2L}\right)$.

i. Using integral calculus, **derive** the expression for $V_P$ provided.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

ii. On Figure 4, **sketch** a graph of the $y$-component $E_y$ of the electric field resulting from the rod as a function of $y$ in the region $2L < y < 12L$.

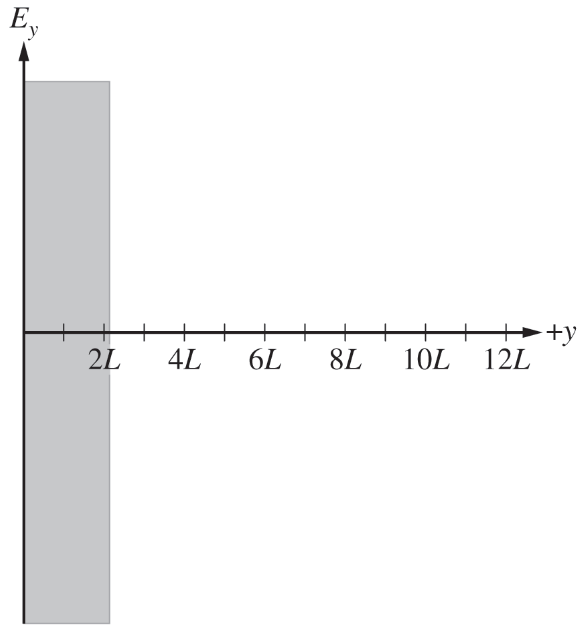



Figure 4

**GO ON TO THE NEXT PAGE.**

# PHYSICS C: ELECTRICITY AND MAGNETISM

## SECTION II
**Time—45 minutes**

**3 Questions**

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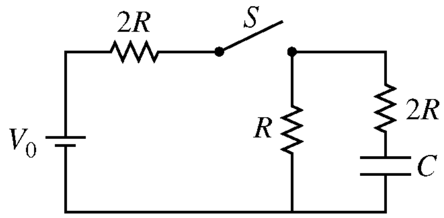



1. The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage $V_0$, a capacitor of capacitance $C$, and a switch $S$. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of $V_0$, $R$, $C$, and fundamental constants, as appropriate.

   (a) Derive an expression for the steady-state current supplied by the battery.

   (b) Derive an expression for the charge on the capacitor.

   (c) Derive an expression for the energy stored in the capacitor.

   Now the switch is opened at time $t = 0$.

   (d) Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time $t$ after the switch is opened.

(e)

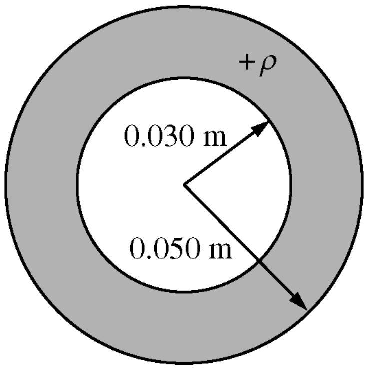
i. Calculate the current in resistor R immediately after the switch is opened.

ii. On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.


(f) Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c) ?

____ Greater than ____ Less than ____ Equal to

Justify your answer.




2. A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density $\rho$, as shown in the figure above. The charge density $\rho$ of the sphere is given as a function of the distance $r$ from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m}: \ \rho = 0$$

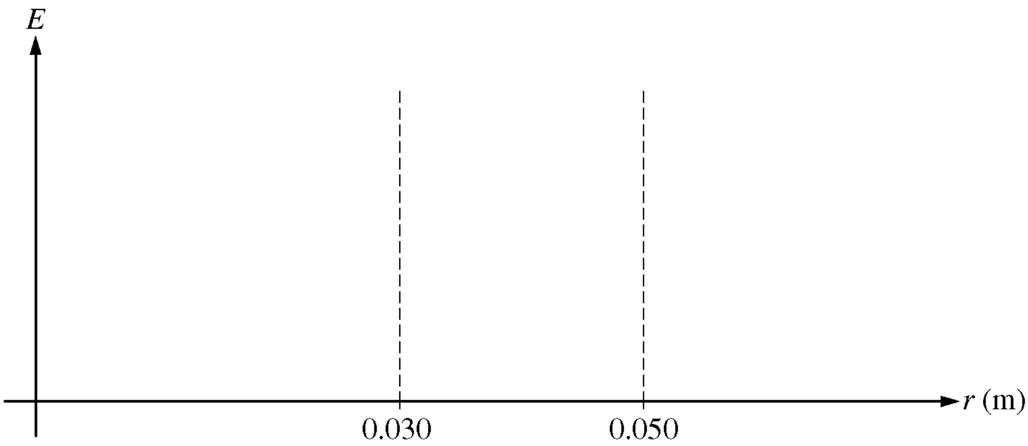
$$0.030 \text{ m} < r < 0.050 \text{ m}: \ \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

$$r > 0.050 \text{ m}: \ \rho = 0$$

(a) Calculate the total charge of the sphere.

(b) Using Gauss's law, calculate the magnitude of the electric field $E$ at the outer surface of the sphere.

(c) On the axes below, sketch the magnitude of the electric field $E$ as a function of distance $r$ from the center of the sphere.

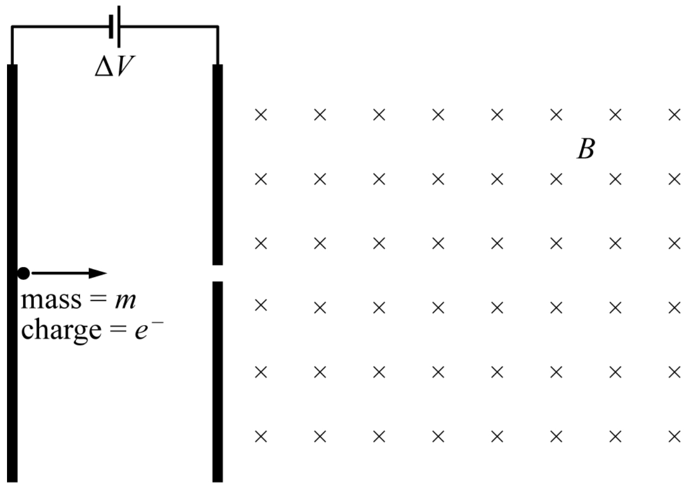


(d) Calculate the electric potential $V$ at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

(e) A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.

i. Calculate the magnitude of the initial acceleration of the proton.

ii. Calculate the speed of the proton after a long time.




3. Two plates are set up with a potential difference $V$ between them. A small sphere of mass $m$ and charge $-e$ is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude $B$ directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of $V$, $m$, $e$, $B$, and fundamental constants, as appropriate.

(a)

i. What is the initial direction of the force on the sphere as it enters the magnetic field?

____ Into the page ____ Out of the page

____ Toward the top of the page ____ Toward the bottom of the page

ii. Describe the path taken by the sphere after it enters the magnetic field.

(b) Derive an expression for the speed of the sphere as it passes through the small opening.

(c) Derive an expression for the radius of the path taken by the sphere as it moves through the magnetic field.

An experiment is performed in which a beam of electrons is accelerated across the space between the plates and passes through the small opening. After passing through the opening, the electrons travel in a semicircular path and strike the right-hand plate. The potential difference between the plates is varied in regular increments, as shown in the table below. For each potential difference, the magnetic field is varied in order to cause the beam to strike the right-hand plate at a distance of 0.020 m from the opening.

| Potential difference (V) | 60 | 70 | 100 | 110 | 120 | 140 |
|---|---|---|---|---|---|---|
| Magnetic field $\left(\text{T} \times 10^{-3}\right)$ | 2.62 | 2.78 | 3.39 | 3.54 | 3.78 | 3.99 |
| | | | | | | |
| | | | | | | |

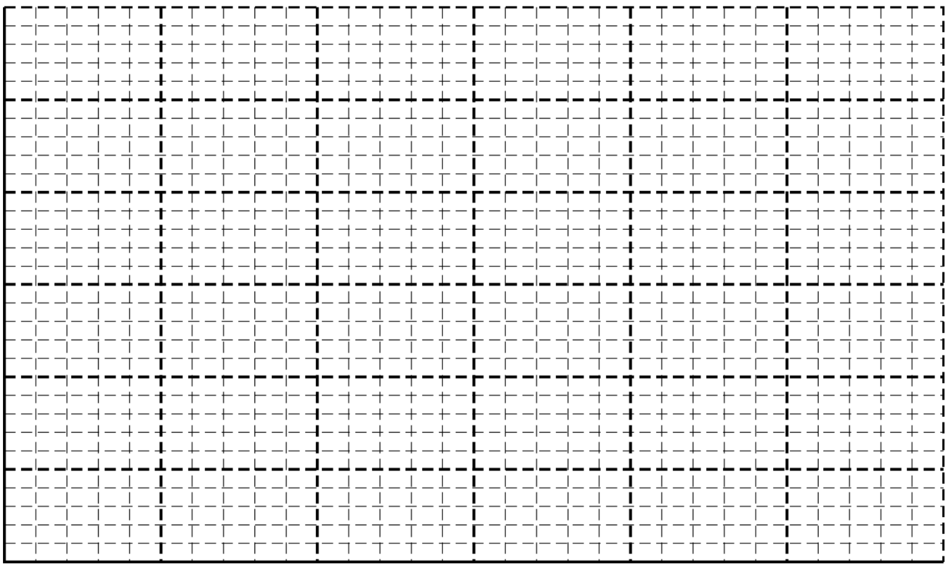
(d) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the mass-to-charge ratio of an electron.

Vertical axis: ____________

Horizontal axis: ____________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

(e) On the graph below, plot the relationship determined in part (d). Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



(f) Using the straight line from part (e), determine the mass-to-charge ratio of an electron.

**STOP**

**END OF EXAM**

# PHYSICS C: ELECTRICITY AND MAGNETISM

## SECTION II

**Time—45 minutes**

**3 Questions**

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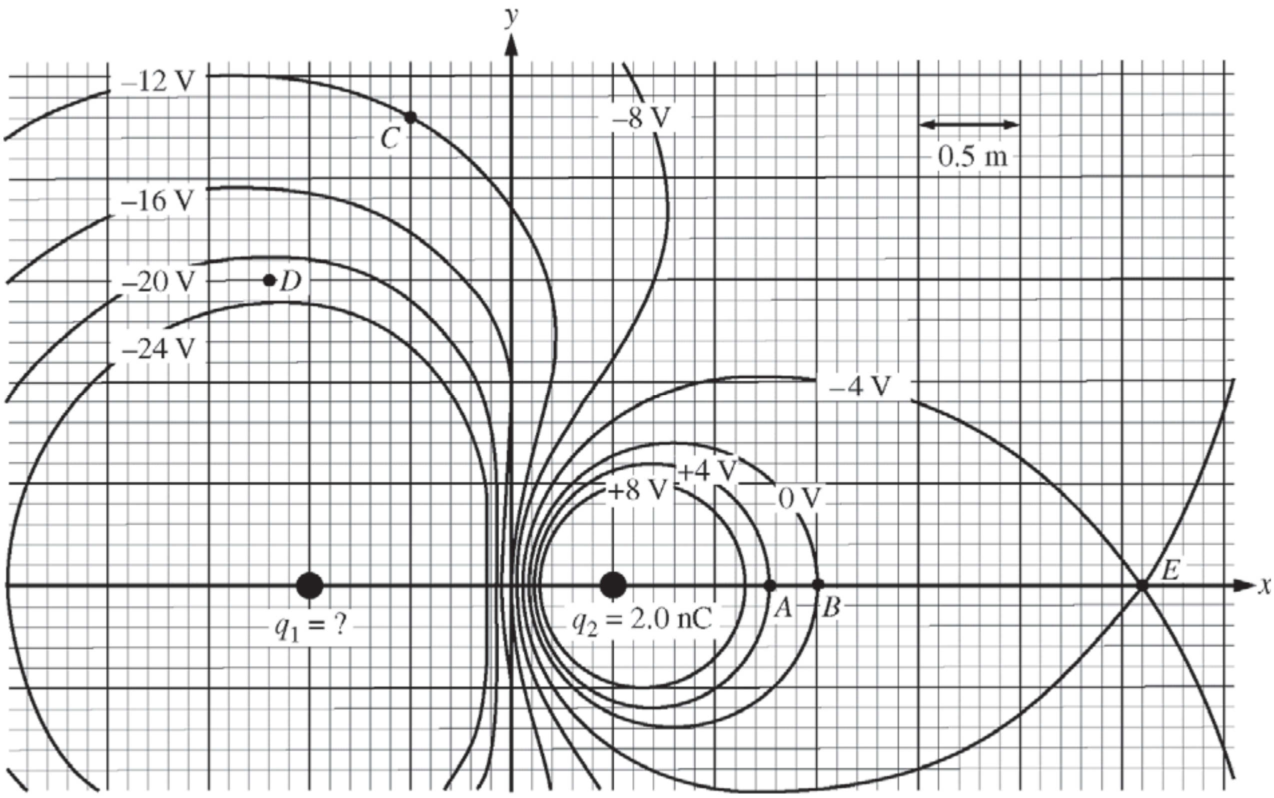


E&M.1.

Two point charges, $q_1$ and $q_2$, are fixed in place on the $x$-axis at positions $x_1 = -1.00$ m and $x_2 = +0.50$ m, respectively. Charge $q_2$ has a value of +2.0 nC. Values of electric potential are illustrated by the given equipotentials in the diagram shown above, which is drawn to scale.

(a) Calculate the value of $q_1$.

(b) At point C on the diagram, draw a vector representing the direction of the electric field at that point.

(c) Calculate the approximate magnitude of the electric field strength at point D on the diagram.

(d) The equipotential labeled 0 V is the cross section of a nearly spherical surface. Calculate the electric flux for this surface.

(e) A proton is placed at point A and then released from rest.

i. Calculate the work done by the electric field on the proton as it moves from point *A* to point *E*.

ii. Calculate the speed of the proton when it reaches point *E*.

(f) An electron is released from rest at point B. Which of the following indicates the direction of the initial acceleration, if any, of the electron?

____ Up ____ Down

____ Left ____ Right

____ Into the page ____ Out of the page

____ The direction is undefined since the acceleration is zero.

Justify your answer.