Week 1

AP Physics C: Electricity and Magnetism

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

8. Electric Charges, Fields, and Gauss's Law

Starter Quiz · 课前小测

AP Physics C: Electricity and Magnetism   Name: ____________________   Score: ________

1. Two negative charges are brought near each other. They:
   - ☐ attract
   - ☐ repel
   - ☐ do nothing
   - ☐ become neutral

2. When you charge an object, what actually moves?
   - ☐ protons
   - ☐ electrons
   - ☐ the nucleus
   - ☐ new charge is created

3. The electric field at a point is the:
   - ☐ force per unit charge there
   - ☐ total charge there
   - ☐ energy per second
   - ☐ speed of a charge

4. Linear charge density λ has units of:
   - ☐ $\mathrm{C/m^3}$
   - ☐ $\mathrm{C/m^2}$
   - ☐ $\mathrm{C/m}$
   - ☐ C

5. Electric flux through a flat area is:
   - ☐ $EA\cos\theta$
   - ☐ $EA\sin\theta$
   - ☐ E/A
   - ☐ $E + A$

6. Two $+3\ \mu\mathrm{C}$ charges are 2 m apart. Find the force (in N). Use $k = 9 \times 10^9$.

   ____________________ N

7. Find the field (in N/C) 3 m from a +6 nC charge. Use $k = 9 \times 10^9$.

_______________ N/C

8. The field from a small element dq at distance r is $dE = k\,dq/$ ______.

____

9. A field $E = 100$ N/C passes straight through ($\theta = 0$) an area of 2 m^2. Find the flux ($\text{N} \cdot \text{m}^2/\text{C}$).

_______________ $\text{N} \cdot \text{m}^2/\text{C}$

10. A charge outside a closed surface adds zero net flux through it.

☐ True ☐ False

AP Physics C: Electricity and Magnetism

1. **repel**

   Like charges repel — two negatives push apart.

2. **electrons**

   Charging moves electrons; protons stay locked in the nuclei.

3. **force per unit charge there**

   $\vec{E} = \vec{F}/q$ — the force a unit positive charge would feel.

4. **C/m**

   $\lambda = dq/d\ell$ is charge per length, C/m.

5. $EA\cos\theta$

   $\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta$, with θ from the normal.

6. **0.02 N**

   $F = \frac{(9\times 10^9)(3\times 10^{-6})^2}{2^2} = \frac{9\times 10^9 \cdot 9\times 10^{-12}}{4} \approx 0.02$ N.

7. **6 N/C**

   $E = \frac{(9\times 10^9)(6\times 10^{-9})}{3^2} = \frac{54}{9} = 6$ N/C.

8. **r^2**

   Each element is a point charge: $dE = k\,dq/r^2$.

9. **200 N · m^2/C**

   $\Phi = EA\cos 0 = 100\times 2\times 1 = 200$.

10. **True**

    Outside charge sends in as much flux as it takes out — net zero.

8.1 Electric Charge and Electric Force

Name: ______________ Class: __________ Date: ____________ **Total: 11 marks**

KEY WORDS conserved 守恒 • superposition 叠加 • coulomb's law 库仑定律 • electric charge 电荷 • gravitational force 引力

Objective

Build the skills to answer exam questions on **electric charge and electric force**.

You must be able to:

- describe **electric charge** 电荷 as **quantized** in multiples of e and **conserved** 守恒
- apply **Coulomb's law** 库仑定律 $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r^2}$
- use the **superposition principle** 叠加原理 to find the net force from several charges
- compare the electrostatic force with the **gravitational force** 引力

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Coulomb's law

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = k\frac{q_1 q_2}{r^2}, \text{ with } k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \text{ N m}^2\,\text{C}^{-2}.$$

■ Superposition

The net force on a charge is the **vector sum** of the Coulomb forces from every other charge.

■ Charge is quantized and conserved

Any charge is a whole multiple of $e = 1.6 \times 10^{-19}$ C, and total charge is conserved. The electric force vastly exceeds gravity between two charged particles.

2 Practice

2.1 Two $+2.0$ μC charges are 0.30 m apart. Find the force between them. [2]

2.2 State which is stronger for two charged particles: the electrostatic force or the gravitational force. [1]

2.3 An object carries a charge of 8.0×10^{-19} C. Find the number of elementary charges. [2]

3 Exam-style questions

3.1 The Coulomb force between two point charges is proportional to [1]

- **A** r
- **B** $1/r$
- **C** $1/r^2$
- **D** r^2

3.2 To find the net force on a charge from several others, you use [1]

- **A** only the nearest charge
- **B** vector superposition of all the forces
- **C** the average of the forces
- **D** the largest force only

3.3 Charges of $+3.0$ μC sit at $x = 0$ and $x = 0.20$ m. A $+1.0$ μC charge is placed at the midpoint $x = 0.10$ m.

(a) Find the force on the middle charge from **one** of the outer charges. [2]

(b) State the net force on the middle charge, and explain why. [2]

Solutions

2.1 $F = k\dfrac{q_1 q_2}{r^2} = 8.99 \times 10^9 \times \dfrac{(2.0 \times 10^{-6})^2}{(0.30)^2} = 0.40\text{ N}.$

2.2 the electrostatic force (by far).

2.3 $N = \dfrac{8.0 \times 10^{-19}}{1.6 \times 10^{-19}} = 5.$

3.1 **C**.

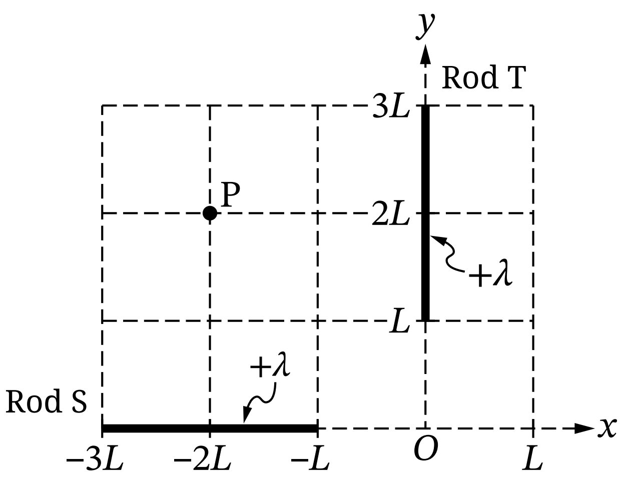
3.2 **B**.

3.3 (a) $F = 8.99 \times 10^9 \times \dfrac{(3.0 \times 10^{-6})(1.0 \times 10^{-6})}{(0.10)^2} = 2.7\text{ N}.$ (b) zero — the two outer charges push the middle charge with equal forces in opposite directions, which cancel.

Question 2: Version J

2. Thin, nonconducting, charged Rods S and T are held fixed in the xy-plane, as shown in Figure 1. Rod S is oriented along the x-axis in the region $-3L \le x \le -L$. Rod T is oriented along the y-axis in the region $L \le y \le 3L$. Each rod has uniform, positive linear charge density $+\lambda$. Point P is located at $(-2L, 2L)$.

Figure 1

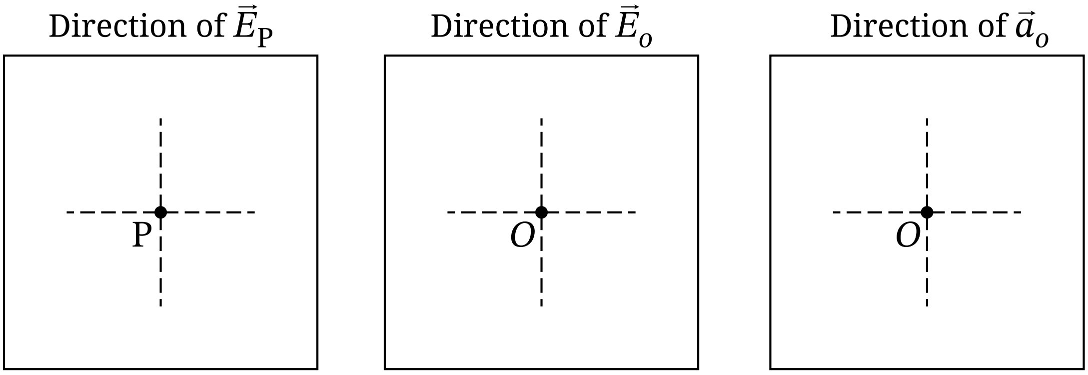


Part A

Complete the following tasks in Figure 2.

- **Draw** an arrow starting on the dot that indicates the direction of the net electric field $\vec{E}_P$ at Point P due to Rods S and T.
- **Draw** an arrow starting on the dot that indicates the direction of the net electric field $\vec{E}_O$ at the origin O due to Rods S and T.
- **Draw** an arrow starting on the dot that indicates the direction of the net acceleration $\vec{a}_O$ of a small, negatively charged sphere (not shown) immediately after the sphere is released from rest at the origin O.

Figure 2



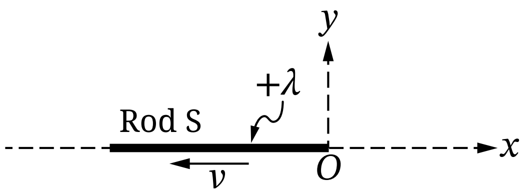
Part B

Derive an expression for the magnitude E_O of the net electric field at the origin O due to Rods S and T in terms of L, λ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

Rod T is removed. Rod S is then moved such that the right end of Rod S is at the origin O. Rod S is then moved with speed v in the $-x$-direction, as shown in Figure 3.

Figure 3



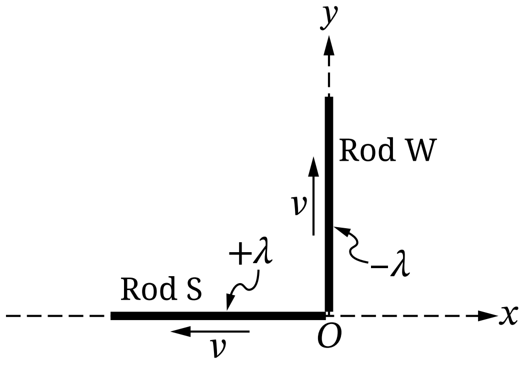
On the axes shown in Figure 4, **sketch** a graph of the electric potential V_O at the origin O due to Rod S as a function of the distance r between the origin and the right end of Rod S.

Figure 4

Part D

A new thin, nonconducting, charged Rod W has the same length as Rod S. Rod W has uniform, negative linear charge density $-\lambda$ and is oriented along the y-axis. Rods S and W are initially held fixed with one end at the origin. Starting at the same time, Rod S is moved with speed v in the $-x$-direction and Rod W is moved with speed v in the $+y$-direction, as shown in Figure 5.

Figure 5



Indicate how a sketch of V_O at the origin O due to both Rods S and W as a function of r would be different from the sketch made in part C. Briefly **justify** your answer.

Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

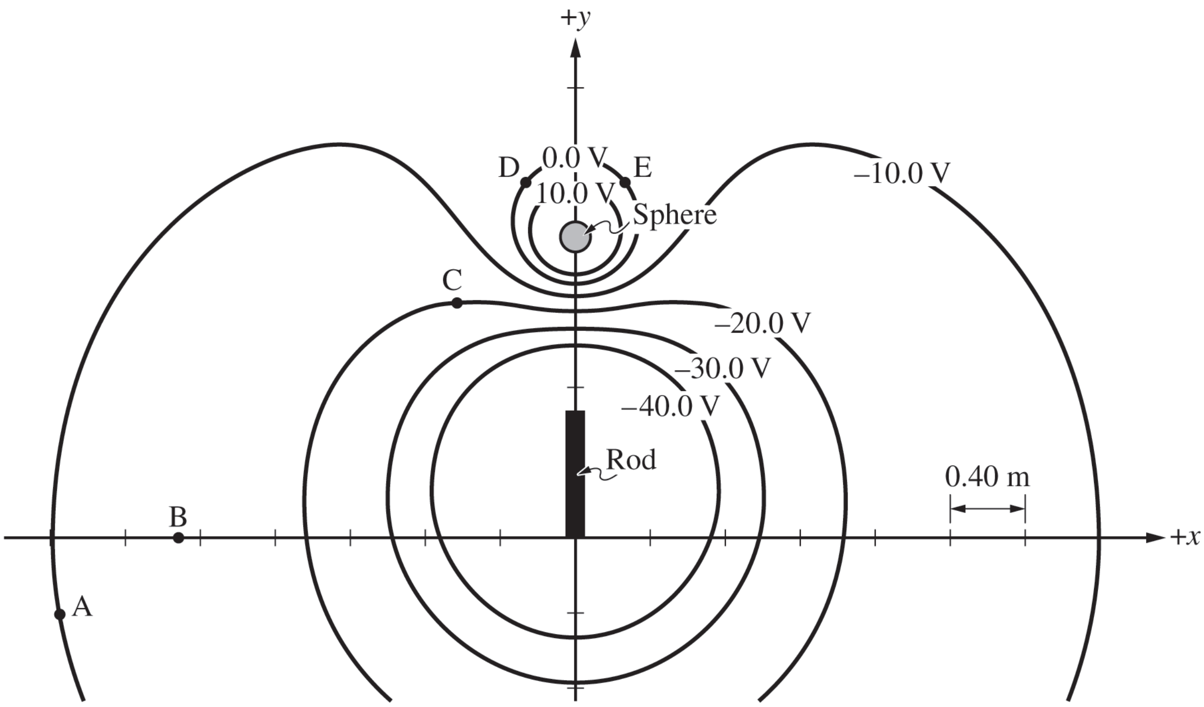


Figure 1

1. A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\text{ nC}$. The rod and sphere are held at rest on the y-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and on the y-axis are 0.40 m apart.

   (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the 0.0 V equipotential line.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is held momentarily at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE}. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a "0" in the corresponding columns, as appropriate.

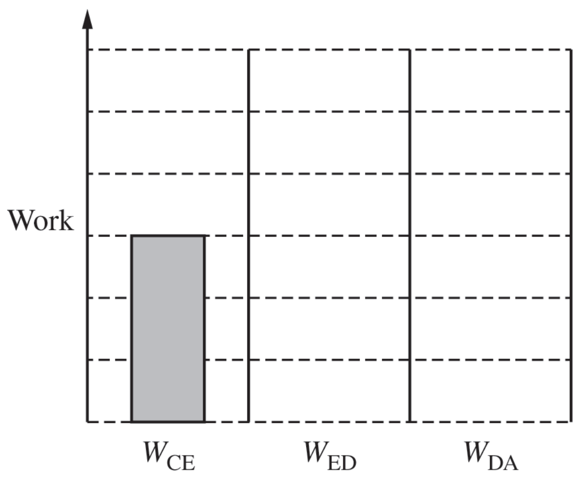


Figure 2

ii. **Calculate** the approximate magnitude of the x-component of the electric field at Position B.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position C. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

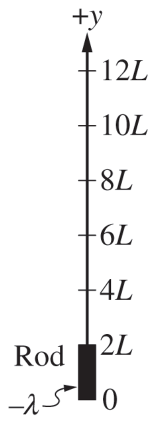


Figure 3

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the y-axis in the orientation shown in Figure 3. Position P (not shown) is located on the y-axis a distance y_P from the origin, where $y_P > 2L$.

(d) The electric potential V_P at y_P is $V_P = -k\lambda \ln\left(\dfrac{y_P}{y_P - 2L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

ii. On Figure 4, **sketch** a graph of the y-component E_y of the electric field resulting from the rod as a function of y in the region $2L < y < 12L$.

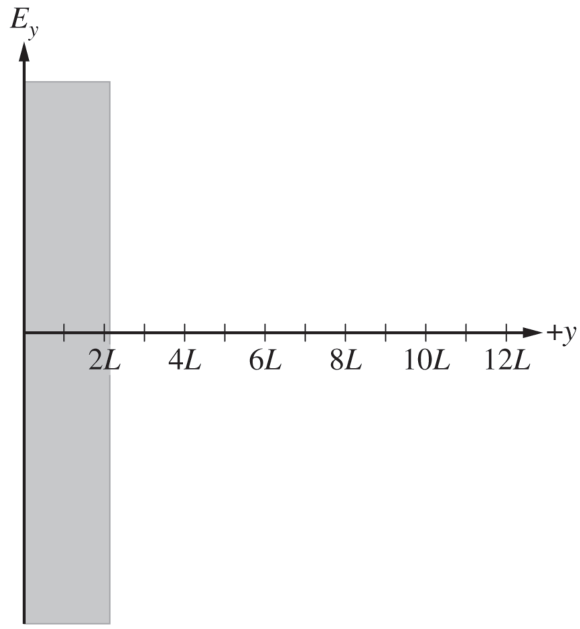


Figure 4

GO ON TO THE NEXT PAGE.

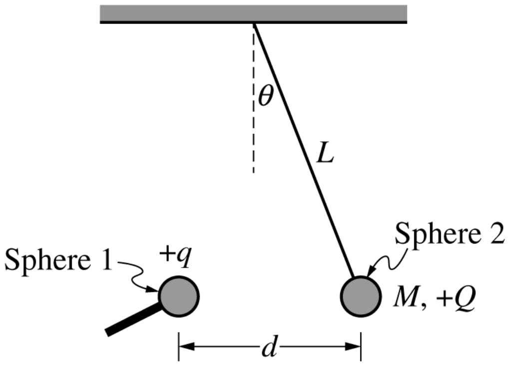
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

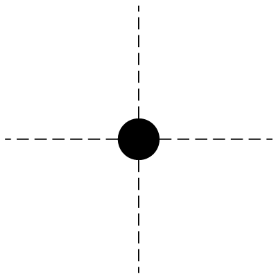


1. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere 1 is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere 2 is nonconducting with charge $+Q$ and has mass M. Sphere 2 is hung from a string of negligible mass and length L. Sphere 1 is brought near, without touching, Sphere 2, as shown. Equilibrium is established when the centers of the two spheres have the same vertical position, are a horizontal distance d apart, and the string is at an angle θ from the vertical.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(a) On the following dot that represents Sphere 2 at the position shown in the previous figure, draw and label the forces (not components) that act on Sphere 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



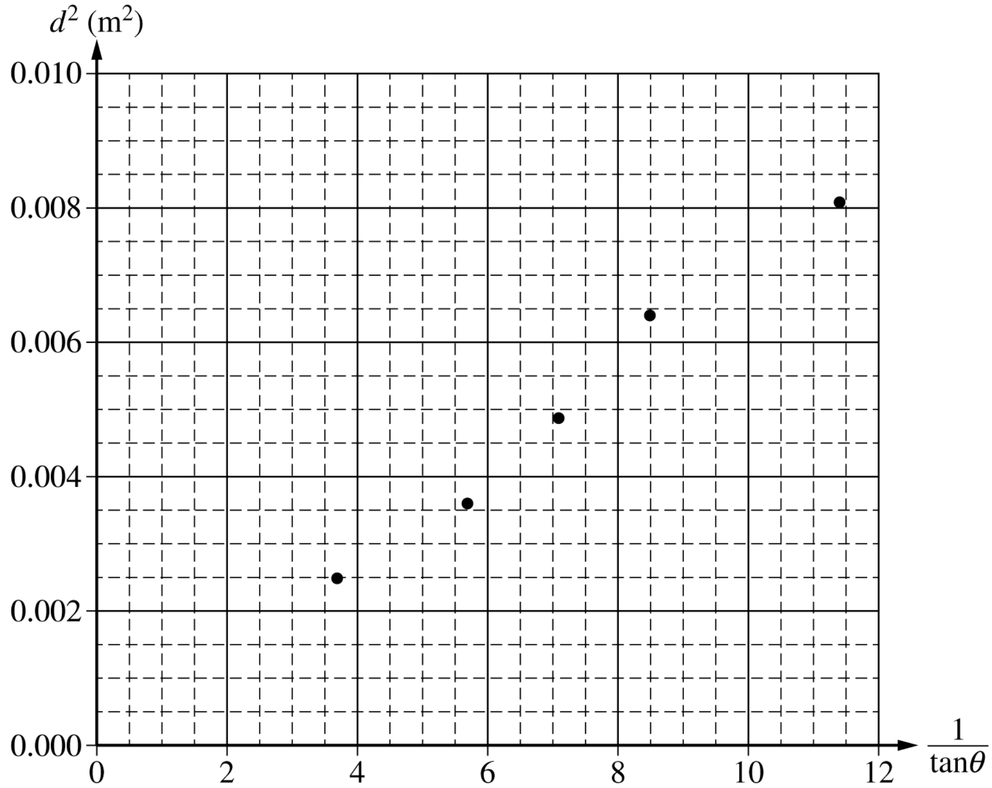
(b) Derive the relationship between the distance d and the angle θ to show that $d = \sqrt{\dfrac{Qq}{4\pi\varepsilon_0 Mg \tan\theta}}$.

(c) These values are collected in one trial: $Q = q = 6.0 \times 10^{-8}$ C, $\theta = 12°$, and $d = 0.057$ m. Calculate the expected force of tension exerted on Sphere 2 by the string.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(d) The students vary d and measure θ after equilibrium is reached. The students use the collected data to plot the following graph of d^2 vs. $\frac{1}{\tan\theta}$.



i. Draw the best-fit line for the data.

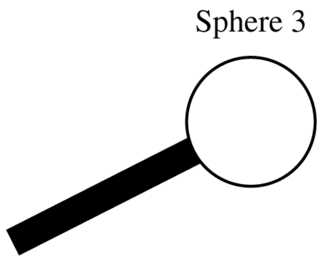
ii. Using the best-fit line, calculate an experimental value for the vacuum permittivity ε_0 when $M = 0.0050\text{ kg}$ and $Q = q = 6.0 \times 10^{-8}\text{ C}$.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(e) The students modify the experiment by replacing Sphere 1 with a conducting Sphere 3 that has the same size and charge $+q$. The experiment is repeated.

i. The circle in the following figure represents Sphere 3 when spheres 2 and 3 are at equilibrium. On the circle, draw a single “+” sign to represent the location of highest concentration of the excess positive charges.



ii. Briefly explain your reasoning for the sketch drawn in part (e)(i).

iii. In the original experiment, when the centers of the two spheres are a horizontal distance d_1 apart, the string makes an angle θ_1 from the vertical. In the modified experiment, when the centers of the two spheres are a horizontal distance d_1 apart, the string makes an angle θ_2 from the vertical.

Is θ_2 greater than, less than, or equal to θ_1 ?

_____ $\theta_2 > \theta_1$ _____ $\theta_2 < \theta_1$ _____ $\theta_2 = \theta_1$

Briefly justify your answer.

GO ON TO THE NEXT PAGE.

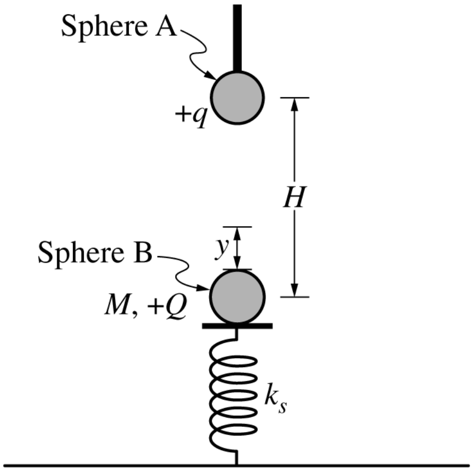
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

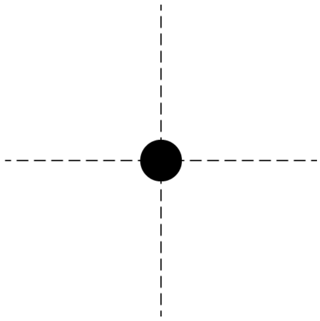
1. Students perform an experiment to determine the value of vacuum permittivity ε_0. Sphere A is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere B is nonconducting with charge $+Q$, and has mass M. Sphere B rests on an insulating platform of negligible mass that is attached to a vertical ideal spring with spring constant k_s. Sphere B and the spring are initially at rest.

Sphere A is then brought near Sphere B without touching. When the centers of the spheres are separated by a vertical distance H, the spring has been compressed a distance y, as shown in the figure. The students measure y for different values of H.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(a) On the following dot that represents Sphere B in the figure on the previous page, draw and label the forces (not components) that are exerted on Sphere B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

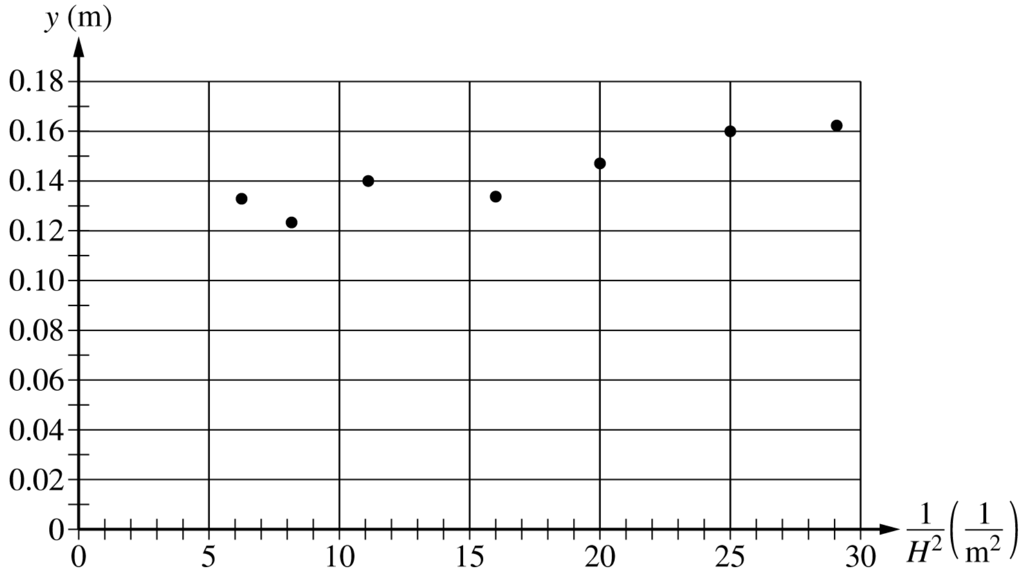


(b) Derive the relationship between y and H to show that $y = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{k_s H^2} + \frac{Mg}{k_s}$.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(c) The students plot collected data of y as a function of $\frac{1}{H^2}$, as shown in the graph.



i. Draw the best-fit line for the data.

ii. Using the best-fit line, calculate an experimental value for the vacuum permittivity ε_0 when $Q = q = 2.00 \times 10^{-6}$ C and $k_s = 25$ N/m.

iii. Using the best-fit line, calculate an experimental value for the mass of Sphere B.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(d) The students modify the experiment by replacing nonconducting Sphere B with conducting Sphere C that has the same charge $+Q$ and mass M. Sphere A is brought near Sphere C without touching, compressing the spring. Sphere C comes to rest.

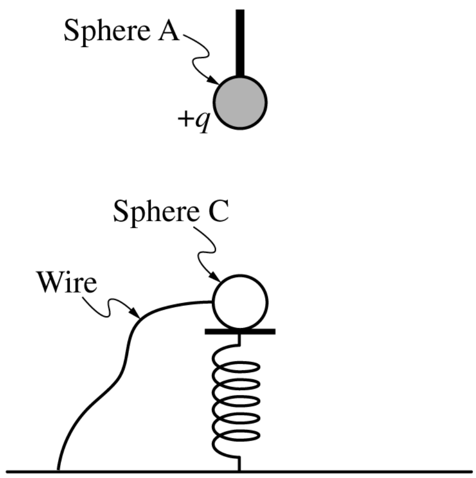
i. In the original experiment, when the centers of spheres A and B are a vertical distance H_1 apart, the spring is compressed a distance y_1. In the modified experiment, when the centers of spheres A and C are a vertical distance H_1 apart, the spring is compressed a distance y_2.

Is y_2 greater than, less than, or equal to y_1 ?

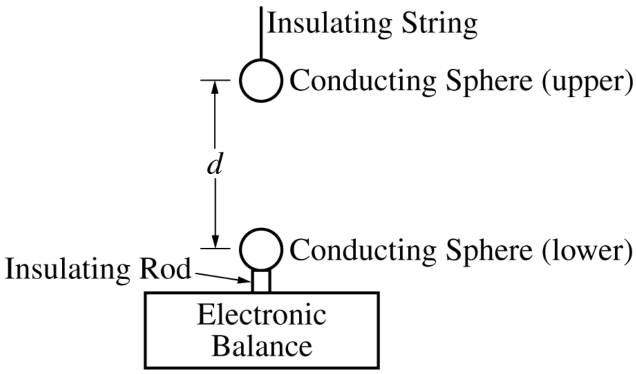
____ $y_2 > y_1$ ____ $y_2 < y_1$ ____ $y_2 = y_1$

Justify your answer.

ii. Sphere C is then grounded with a wire. On the following figure, draw an arrow indicating the direction that the platform will move immediately after being grounded. If the platform remains stationary, write "does not move."

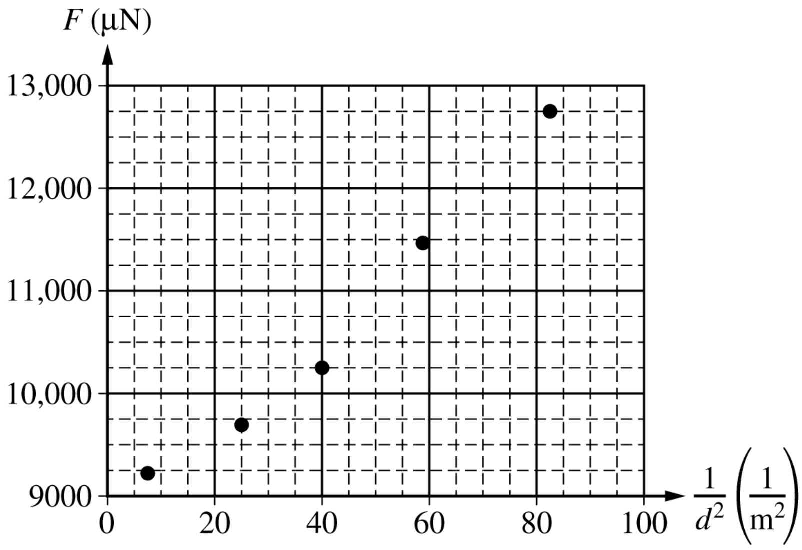


GO ON TO THE NEXT PAGE.



Students perform an experiment to study the force between two charged objects using the apparatus shown above, which contains two identical conducting spheres. The upper sphere is attached to an insulating string, which can be used to move the sphere downward. The lower sphere sits on an insulating rod, which is on an electronic balance. The electronic balance is zeroed before the lower sphere and insulating rod are in place.

For the first trial, a charge of Q is placed on each sphere and then the upper sphere is slowly moved downward. The students measure the distance d between the centers of the spheres and the magnitude F of the force that appears on the electronic balance. The recorded data are shown on the graph of F as a function of $\frac{1}{d^2}$ shown below.



(a)

i. Draw a line that represents the best fit to the points shown.

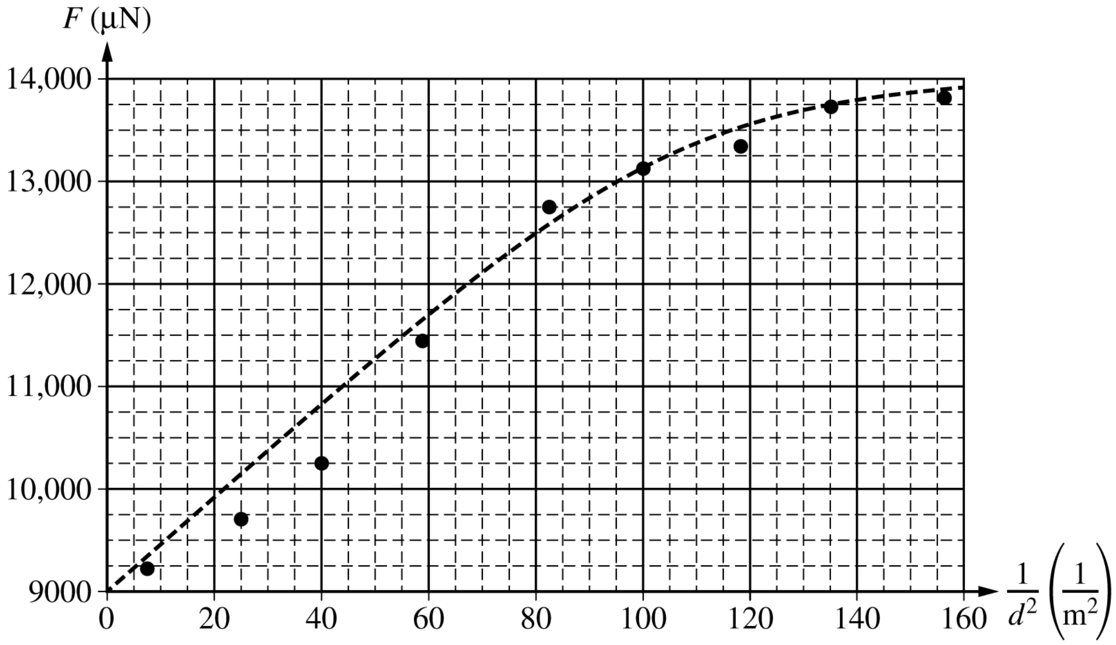
ii. Use the graph to calculate the charge Q.

iii. On the graph on the previous page, draw a circle around the data point that was taken when the distance between the centers of the spheres was the least.

iv. Determine the distance between the centers of the spheres for the data point indicated above.

v. What physical quantity does the vertical intercept represent?

Justify your answer.



The experiment is extended by collecting additional data points, which appear on the right side of the graph shown above. The new data points do not follow the linear pattern seen with the first points. The group of students tries to explain this discrepancy.

(b) One student suspects that charge is slowly leaking off the top sphere. Could this explain the discrepancy?

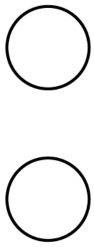
____ Yes ______ No

Justify your answer.

GO ON TO THE NEXT PAGE.

) A second student suspects that the excess charges have rearranged themselves, polarizing the spheres.

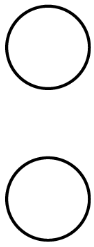
i. On the circles representing the spheres below, use a single "+" sign on each sphere to represent the locations of highest concentration of the excess positive charges.



ii. Explain how this rearrangement could be responsible for the discrepancy.

(d) A third student suggests that the experiment be modified so that the top sphere is given a negative charge that is equal in magnitude to the positive charge given to the bottom sphere.

i. On the circles representing the spheres below, use a single "+" sign on the bottom sphere to represent the location of highest concentration of the excess positive charges. Use a single "–" sign on the top sphere to represent the location of the highest concentration of the excess negative charges.



ii. For a separation distance equal to that of the data point indicated in part (a)(iii), would the magnitude of the force reading with spheres of opposite charges be greater than, less than, or equal to the magnitude of the force reading with spheres of the same charges?

_____ Greater than _____ Less than _____ Equal to

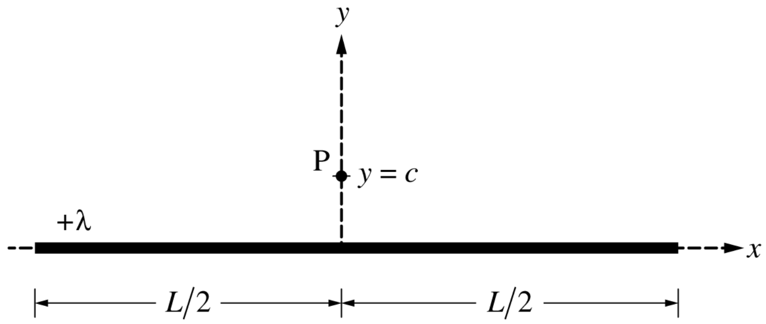
Justify your answer.

GO ON TO THE NEXT PAGE.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

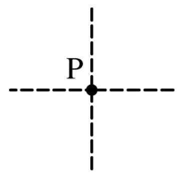


Note: Figure not drawn to scale.

1. A very long, thin, nonconducting cylinder of length L is centered on the y-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y-axis at $y = c$, where $L >> c$.

(a)

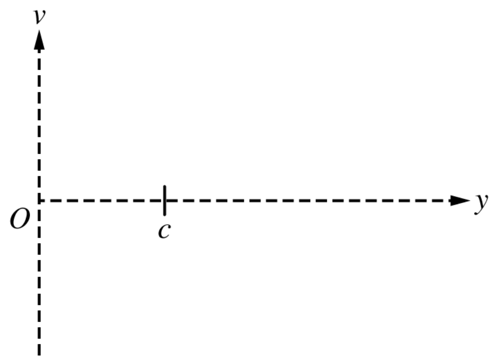
i. On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the long cylinder. The arrow should start on and point away from the dot.

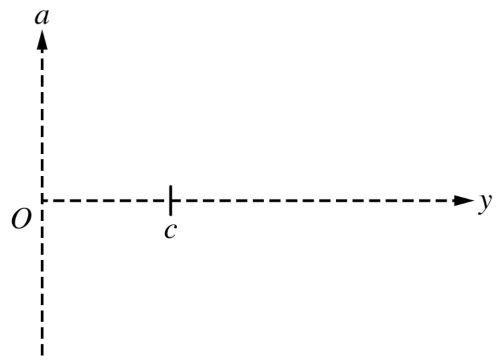


ii. Describe the shape and location of a Gaussian surface that can be used to determine the electric field at point P due to the long cylinder.

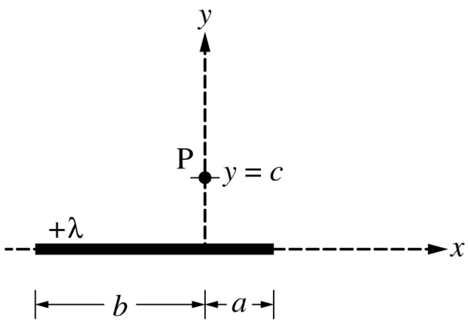
iii. Use your Gaussian surface to derive an expression for the magnitude of the electric field at point P. Express your answer in terms of λ, c, L, and physical constants, as appropriate.

(b) A proton is released from rest at point P. On the axes below, sketch the velocity v as a function of <u>position</u> y and the acceleration a as a function of <u>position</u> y for the proton.

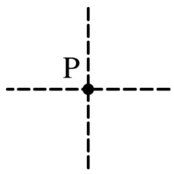




The original cylinder is now replaced with a much shorter thin, nonconducting cylinder with the same uniform linear charge density $+\lambda$, as shown in the figure below. The length of the cylinder to the right of the y-axis is a, and the length of the cylinder to the left of the y-axis is b, where $a < b$.



(c) On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the shorter cylinder. The arrow should start on and point away from the dot.

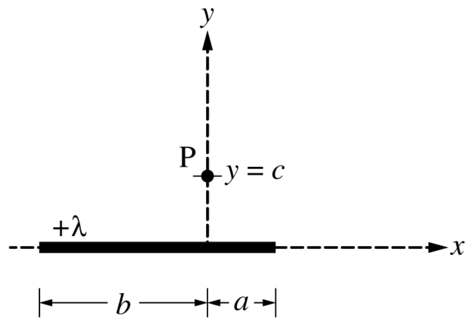


(d)

i. Is there a single Gaussian surface that can be used with Gauss's law to derive an expression for the electric field at point P?

____ Yes ____ No

ii. If your answer to part (d)(i) is yes, explain how you can use Gauss's law to derive an expression for the field at point P. If your answer to part (d)(i) is no, explain why Gauss's law cannot be applied to derive an expression for the electric field in this case.



Note: This figure is shown again for reference.

A student in class argues that using the integral shown below might be a useful approach for determining the electric field at point P.

$$E = \int \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\,dq$$

The student uses this approach and writes the following two integrals for the magnitude of the horizontal and vertical components of the electric field at point P.

Horizontal component: $$\left|E_x\right| = \frac{\lambda}{4\pi\varepsilon_0}\int_{-b}^{a}\frac{x}{\left(c^2 + x^2\right)^{3/2}}\,dx$$

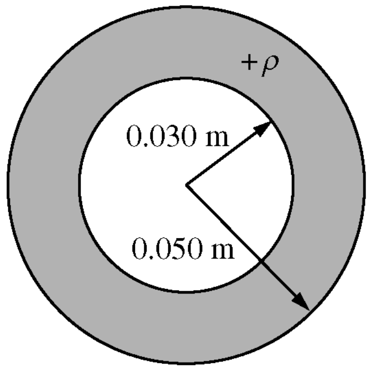
Vertical component: $$\left|E_y\right| = \frac{\lambda}{4\pi\varepsilon_0}\int_{-b}^{a}\frac{y}{\left(c^2 + x^2\right)}\,dy$$

(e)

i. One of the two expressions above is not correct. Which expression is not correct?

____ Horizontal component ____ Vertical component

ii. Identify two mistakes in the incorrect expression, and explain how to correct the mistakes.



2. A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m}: \quad \rho = 0$$

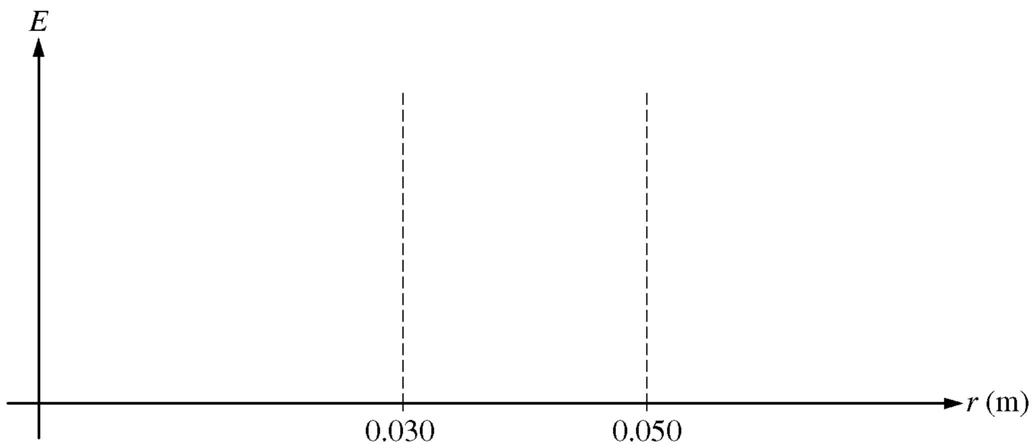
$$0.030 \text{ m} < r < 0.050 \text{ m}: \quad \rho = b/r \text{, where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

$$r > 0.050 \text{ m}: \quad \rho = 0$$

(a) Calculate the total charge of the sphere.

(b) Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.

(c) On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.



(d) Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

(e) A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.

i. Calculate the magnitude of the initial acceleration of the proton.

ii. Calculate the speed of the proton after a long time.

E&M.3.

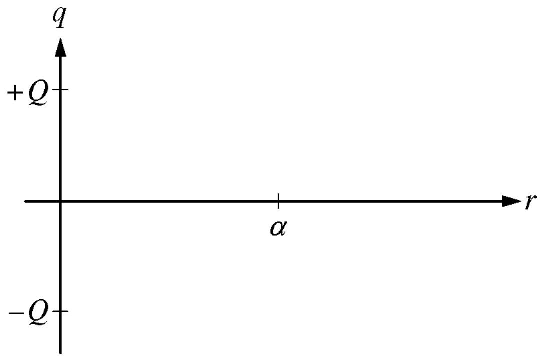
A scientist describes an electrically neutral atom with a model that consists of a nucleus that is a point particle with positive charge $+Q$ at the center of the atom and an electron volume charge density of the form

$$\rho(r) = \begin{cases} -\dfrac{\beta}{r^2} e^{-r/\alpha} & r < \alpha \\ 0 & r > \alpha \end{cases}$$

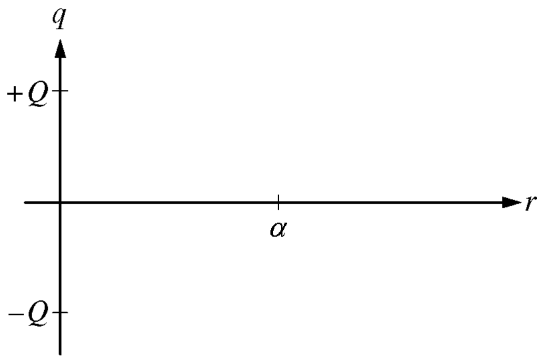
where α and β are positive constants and r is the distance from the center of the atom.

(a) On the axes below, let r stand for the radius of a Gaussian sphere. Sketch the graph for each of the following charges enclosed by the Gaussian sphere as a function of r. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

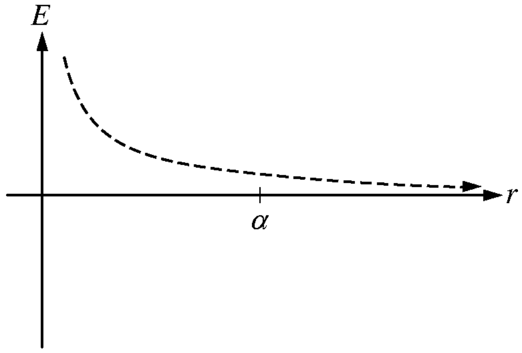
i. The nuclear charge only



ii. The electron charge only



(b) The dashed curve on the graph below represents the electric field as a function of distance r due to the positive nucleus of the atom without any electrons. The nucleus is modeled as a point particle of charge $+Q$. On the same graph, sketch the electric field as a function of distance r for the neutral atom as defined by the scientist's model, which includes the nucleus and the negative electrons surrounding it.



(c) Use Gauss's law to derive an expression for the electric field strength due to the neutral atom for the following positions in terms of $Q, \alpha, \beta, r,$ and fundamental constants, as appropriate.

i. $r > \alpha$

ii. $r < \alpha$

(d) Based on the model proposed by the scientist, what is the physical meaning of the constant α ?

STOP

END OF EXAM

8.2 Electric Charge and the Process of Charging

Name: __________ Class: __________ Date: __________ **Total: 11 marks**

KEY WORDS ground 接地 • conductor 导体 • induction 感应 • insulator 绝缘体 • conduction 接触

Objective

Build the skills to answer exam questions on **electric charge and the process of charging**.

You must be able to:

- distinguish **conductors** 导体 from **insulators** 绝缘体
- explain charging by **friction** 摩擦, **conduction** 接触, and **induction** 感应
- explain the role of **grounding** 接地 in charging
- account for charge distributions using **conservation of charge**

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Conductors and insulators

Charge moves freely in a **conductor**, but stays put in an **insulator**.

■ Three ways to charge

Friction (transfers electrons), conduction (contact shares charge), and induction (a nearby charge rearranges charge, then grounding removes some).

■ Grounding

A **ground** is a huge charge reservoir: it can supply or drain electrons, fixing the sign of an induced charge before the connection is broken.

2 Practice

2.1 State the three methods of charging an object. [2]

2.2 Explain the role of grounding when charging a conductor by induction. [2]

2.3 State where excess charge sits on an isolated solid conductor. [1]

3 Exam-style questions

3.1 Charging by induction requires [1]

- **A** direct contact only
- **B** rubbing two materials
- **C** a nearby charge and a ground connection
- **D** heating the object

3.2 Grounding a conductor allows it to [1]

- **A** gain or lose electrons
- **B** melt
- **C** become an insulator
- **D** store light

3.3 A positively charged rod is held near a neutral metal sphere; the sphere is then grounded, and the ground is removed before the rod.

(a) State the sign of the final charge on the sphere. [1]

(b) Explain the sequence of steps that produces it. [3]

Solutions

2.1 friction, conduction (contact), and induction.

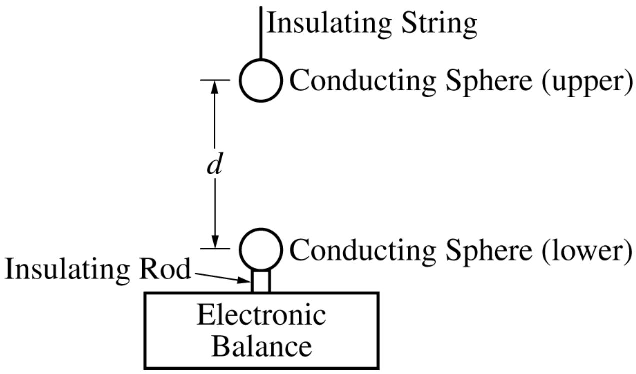
2.2 grounding lets electrons flow in or out, leaving the conductor with a net induced charge once the ground is removed.

2.3 entirely on its outer surface.

3.1 **C**.

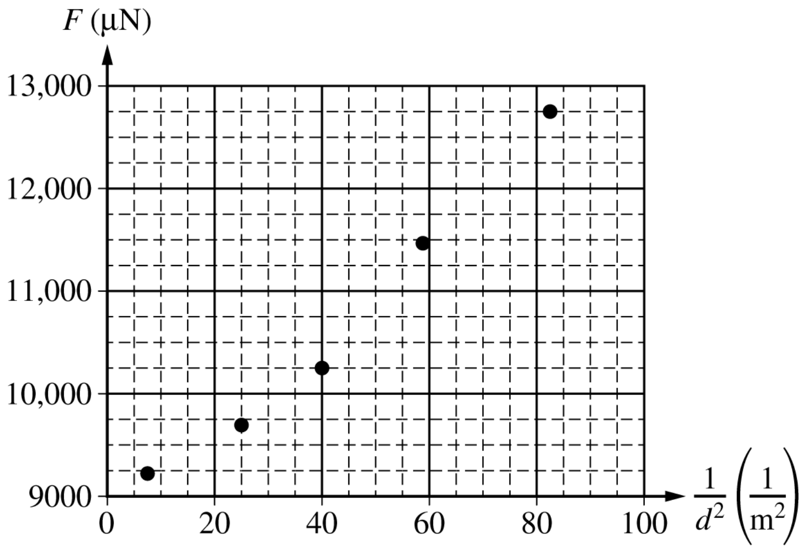
3.2 **A**.

3.3 (a) negative. (b) the positive rod attracts electrons to the near side; grounding lets more electrons flow onto the sphere; removing the ground traps this extra negative charge, then removing the rod leaves the sphere negatively charged.



Students perform an experiment to study the force between two charged objects using the apparatus shown above, which contains two identical conducting spheres. The upper sphere is attached to an insulating string, which can be used to move the sphere downward. The lower sphere sits on an insulating rod, which is on an electronic balance. The electronic balance is zeroed before the lower sphere and insulating rod are in place.

For the first trial, a charge of Q is placed on each sphere and then the upper sphere is slowly moved downward. The students measure the distance d between the centers of the spheres and the magnitude F of the force that appears on the electronic balance. The recorded data are shown on the graph of F as a function of $\frac{1}{d^2}$ shown below.



(a)

i. Draw a line that represents the best fit to the points shown.

ii. Use the graph to calculate the charge Q.

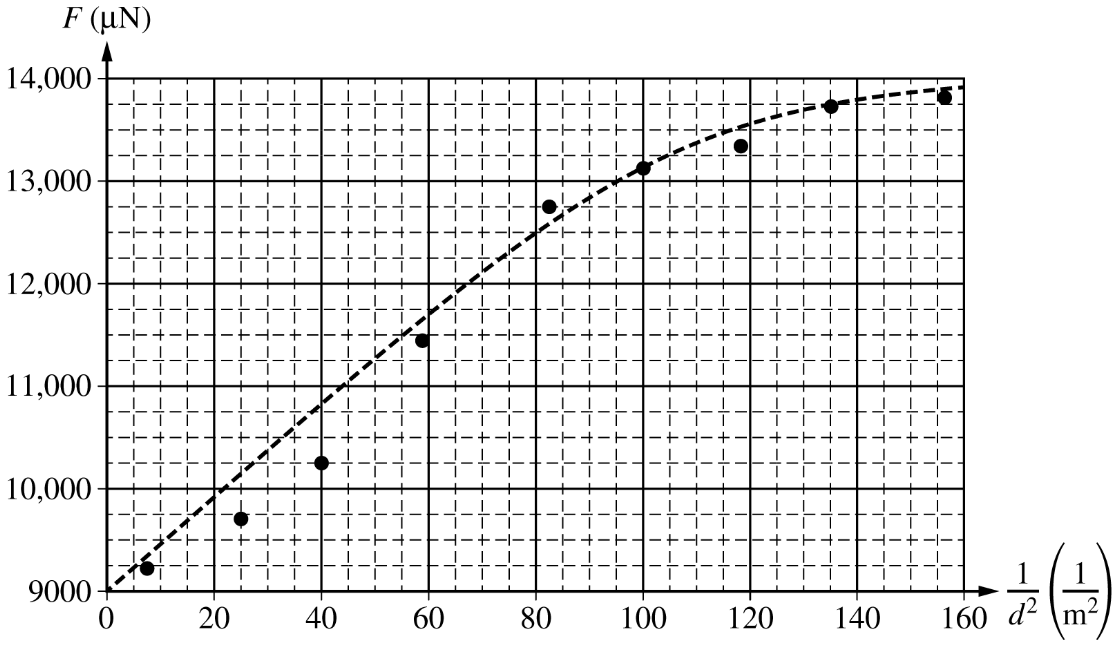
GO ON TO THE NEXT PAGE.

iii. On the graph on the previous page, draw a circle around the data point that was taken when the distance between the centers of the spheres was the least.

iv. Determine the distance between the centers of the spheres for the data point indicated above.

v. What physical quantity does the vertical intercept represent?

Justify your answer.



The experiment is extended by collecting additional data points, which appear on the right side of the graph shown above. The new data points do not follow the linear pattern seen with the first points. The group of students tries to explain this discrepancy.

(b) One student suspects that charge is slowly leaking off the top sphere. Could this explain the discrepancy?

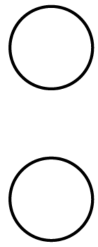
____ Yes ______ No

Justify your answer.

GO ON TO THE NEXT PAGE.

) A second student suspects that the excess charges have rearranged themselves, polarizing the spheres.

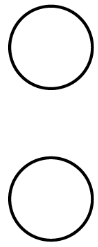
i. On the circles representing the spheres below, use a single “+” sign on each sphere to represent the locations of highest concentration of the excess positive charges.



ii. Explain how this rearrangement could be responsible for the discrepancy.

(d) A third student suggests that the experiment be modified so that the top sphere is given a negative charge that is equal in magnitude to the positive charge given to the bottom sphere.

i. On the circles representing the spheres below, use a single “+” sign on the bottom sphere to represent the location of highest concentration of the excess positive charges. Use a single “–” sign on the top sphere to represent the location of the highest concentration of the excess negative charges.



ii. For a separation distance equal to that of the data point indicated in part (a)(iii), would the magnitude of the force reading with spheres of opposite charges be greater than, less than, or equal to the magnitude of the force reading with spheres of the same charges?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

GO ON TO THE NEXT PAGE.

8.3 Electric Fields

Name: ______________ Class: __________ Date: ____________ **Total: 11 marks**

KEY WORDS electric field 电场 • test charge 检验电荷 • point charges 点电荷 • superposition 叠加

Objective

Build the skills to answer exam questions on **electric fields**.

You must be able to:

- define the **electric field** 电场 as force per unit charge, $\vec{E} = \dfrac{\vec{F}}{q}$
- calculate the field of a **point charge**, $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}$
- use **superposition** for several point charges
- find the force $\vec{F} = q\vec{E}$ on a charge in a field

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **The electric field**

$\vec{E} = \dfrac{\vec{F}}{q}$ — the force per unit **positive** test charge, in N C^{-1}.

■ **Point charge**

$E = k\dfrac{q}{r^2}$, pointing away from a positive charge and toward a negative one; add fields as vectors (superposition).

■ **Force on a charge**

$\vec{F} = q\vec{E}$. A positive charge feels a force **along** $\vec{E}$; a negative charge feels one **opposite** to $\vec{E}$.

2 Practice

2.1 Find the electric field 0.20 m from a +5.0 μC charge ($k = 8.99 \times 10^9$). [2]

2.2 A +2.0 μC charge sits in a field of 3000 N C^{-1}. Find the force on it. [2]

2.3 State the direction of the field around a negative point charge. [1]

3 Exam-style questions

3.1 The electric field is defined as [1]

- **A** qF
- **B** F/q
- **C** q/F
- **D** qr^2

3.2 The force on a charge q in a field E is [1]

- **A** q/E
- **B** qE
- **C** E/q
- **D** qE^2

3.3 A +4.0 μC charge is at the origin.

(a) Find the electric field 0.30 m from it. [2]

(b) Find the force on a −1.0 μC charge placed there, with direction. [2]

Solutions

2.1 $E = k\dfrac{q}{r^2} = 8.99 \times 10^9 \times \dfrac{5.0 \times 10^{-6}}{(0.20)^2} = 1.1 \times 10^6$ N C^{-1}.

2.2 $F = qE = (2.0 \times 10^{-6})(3000) = 6.0 \times 10^{-3}$ N.

2.3 radially inward, toward the charge.

3.1 **B**.

3.2 **B**.

3.3 (a) $E = 8.99 \times 10^9 \times \dfrac{4.0 \times 10^{-6}}{(0.30)^2} = 4.0 \times 10^5$ N C^{-1}. (b) $F = qE = (1.0 \times 10^{-6})(4.0 \times 10^5) = 0.40$ N, directed toward the $+4.0$ μC charge (attractive).

Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

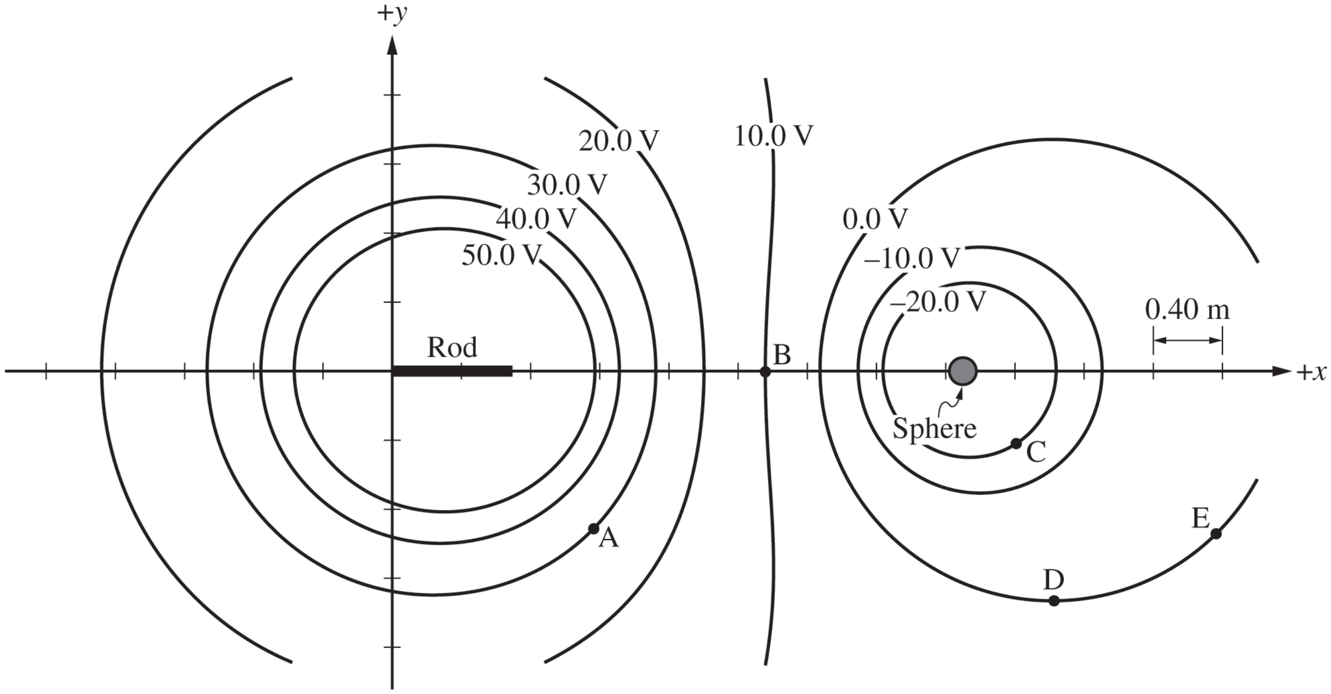


Figure 1

1. A nonconducting rod of uniform positive linear charge density is near a sphere with charge -2.0 nC. The rod and sphere are held at rest on the x-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and the y-axis are 0.40 m apart.

   (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the -20.0 V equipotential line.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is momentarily held at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE}. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a “0” in the corresponding columns, as appropriate.

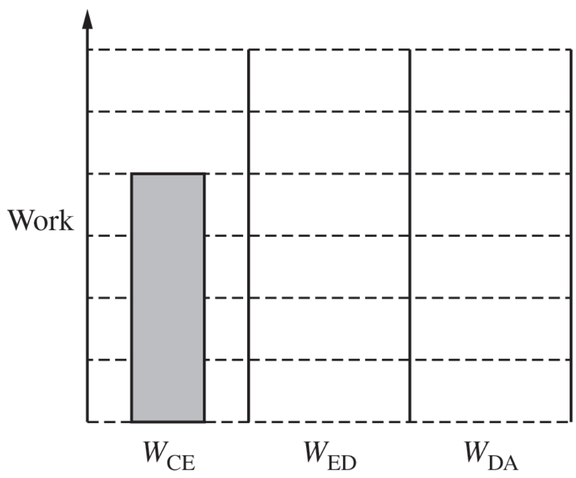


Figure 2

ii. **Calculate** the approximate magnitude of the x-component of the electric field at Position B.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position D. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

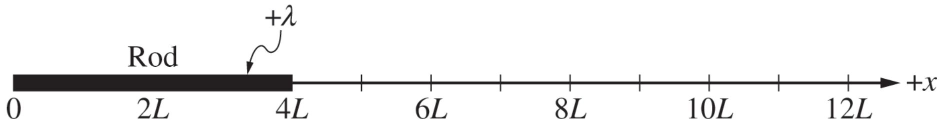


Figure 3

The sphere and the test charge are removed. The rod has length $4L$ and uniform positive linear charge density $+\lambda$. The rod is held at rest on the x-axis in the orientation shown in Figure 3. Position P (not shown) is located on the x-axis a distance x_P from the origin, where $x_P > 4L$.

(d) The electric potential V_P at x_P is $V_P = k\lambda \ln\left(\dfrac{x_P}{x_P - 4L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

ii. On Figure 4, **sketch** a graph of the x-component E_x of the electric field from the rod as a function of x in the region $4L < x < 12L$.

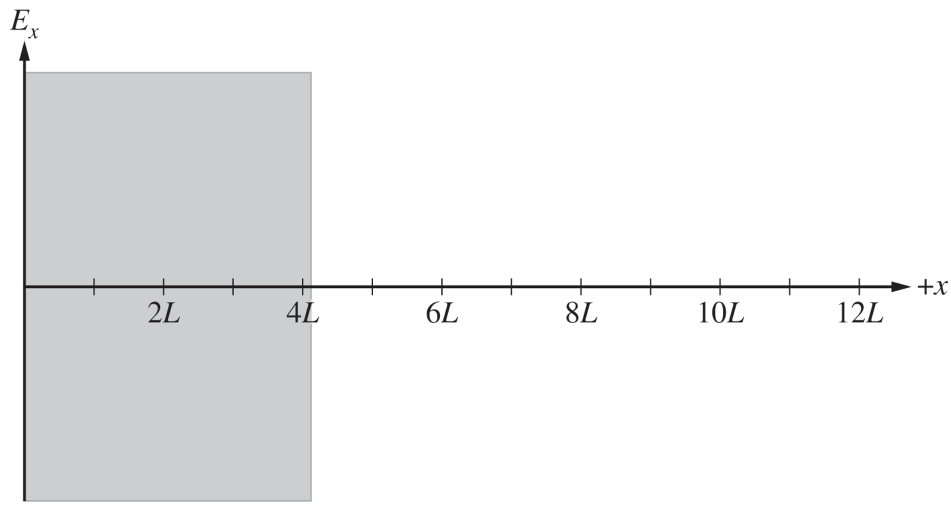


Figure 4

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

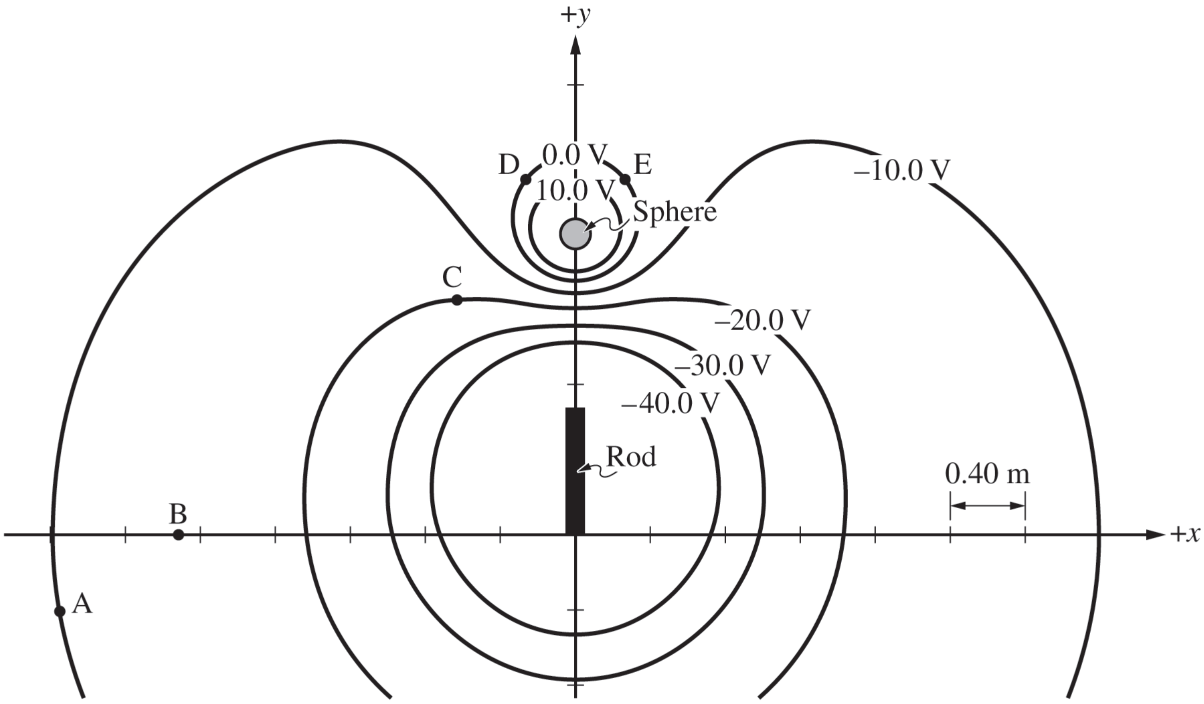


Figure 1

1. A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\ \text{nC}$. The rod and sphere are held at rest on the y-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and on the y-axis are $0.40\ \text{m}$ apart.

   (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the $0.0\ \text{V}$ equipotential line.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is held momentarily at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE}. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a "0" in the corresponding columns, as appropriate.

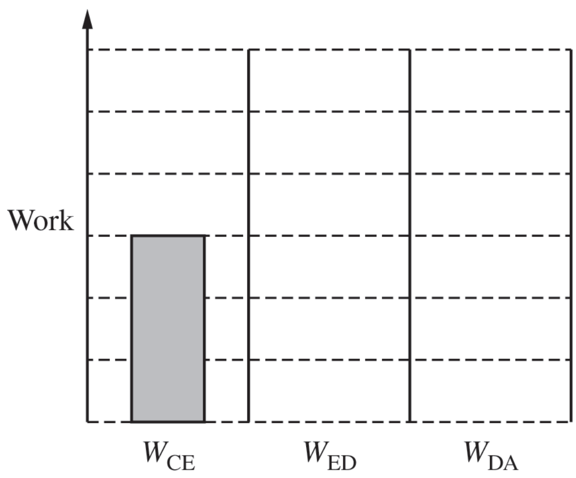


Figure 2

ii. **Calculate** the approximate magnitude of the x-component of the electric field at Position B.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position C. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

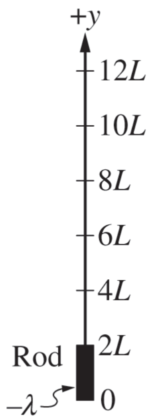


Figure 3

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the y-axis in the orientation shown in Figure 3. Position P (not shown) is located on the y-axis a distance y_P from the origin, where $y_P > 2L$.

(d) The electric potential V_P at y_P is $V_P = -k\lambda \ln\left(\dfrac{y_P}{y_P - 2L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

ii. On Figure 4, **sketch** a graph of the y-component E_y of the electric field resulting from the rod as a function of y in the region $2L < y < 12L$.

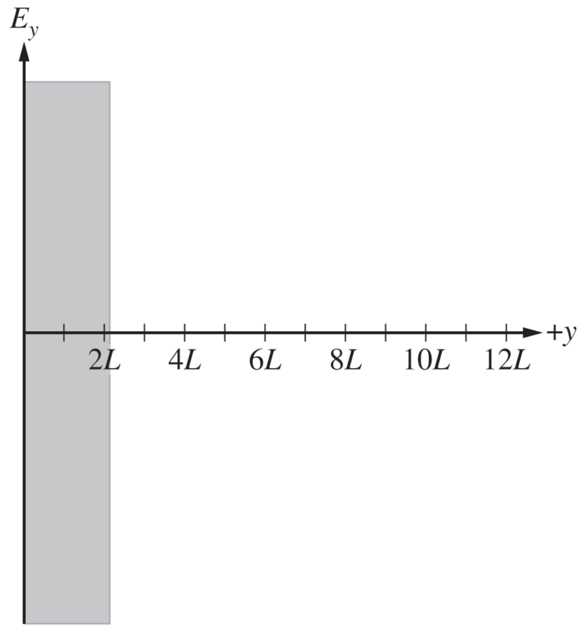


Figure 4

GO ON TO THE NEXT PAGE.

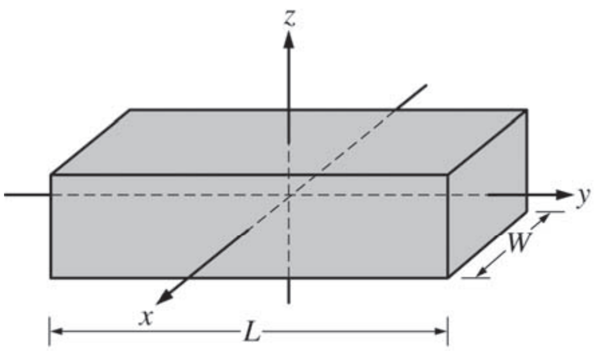
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

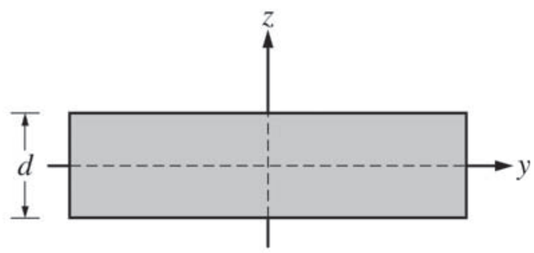
Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Perspective View

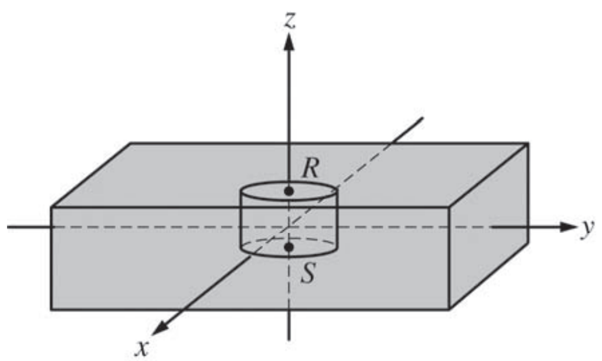


Side View

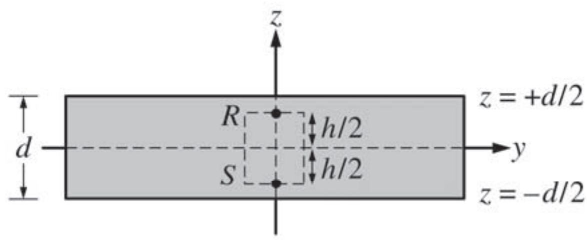
Note: Figures not drawn to scale.

1. A very large nonconducting slab with a uniform positive volume charge density ρ_0 is fixed with the origin of the xyz-axes at its center, as shown in the figure above. The thickness of the slab is d, the length is L, and the width is W, where $L >> d$ and $W >> d$. The large faces of the slab are parallel to the xy-plane.

   Consider a Gaussian cylinder with a cross-sectional area A and height h that is positioned with its axis along the z-axis, as shown in the figure below.



Perspective View



Side View

Note: Figures not drawn to scale.

(a) Draw a single vector on each of the dots below representing the direction of the electric field at the given points. If the electric field at either point is zero, write "$E = 0$" next to the point.

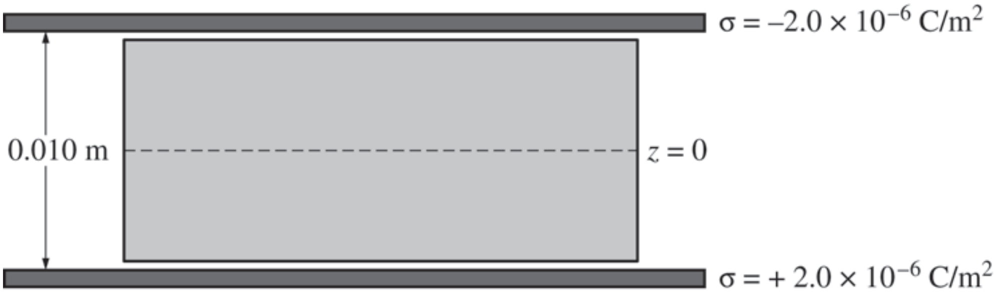
i. ii.

• R • S

b) Use Gauss's law to derive expressions for the following. Express your answers in terms of ρ_0, A, d, h, z, and physical constants, as appropriate.

i. Derive an expression for the total flux Φ through the Gaussian surface shown.

ii. Derive an expression for the magnitude of the electric field as a function of z for any position inside the slab, and show that it is equal to $E = \dfrac{\rho_0 z}{\varepsilon_0}$.



Note: Figure not drawn to scale.

The charged slab is now placed between two large metal plates separated by a distance of 0.010 m, which is approximately the thickness of the slab, but the slab does not contact either metal plate. The metal plates are charged, resulting in the surface charge densities $\sigma = \pm 2.0 \times 10^{-6}\ \mathrm{C/m^2}$, as shown in the figure above. Assume the charge distribution inside the slab remains unchanged by the presence of the charged plates and that the slab's volume charge density is $\rho_0 = 1.00 \times 10^{-3}\ \mathrm{C/m^3}$.

(c)

i. The magnitude of the electric field inside the slab is zero on the z-axis at position z_0. Which of the following correctly indicates the value for z_0?

____ $z_0 > 0$ ____ $z_0 = 0$ ____ $z_0 < 0$

Justify your answer.

ii. Calculate the value z_0.

(d) Calculate the magnitude of the electric potential difference from the center of the slab to the top of the slab.

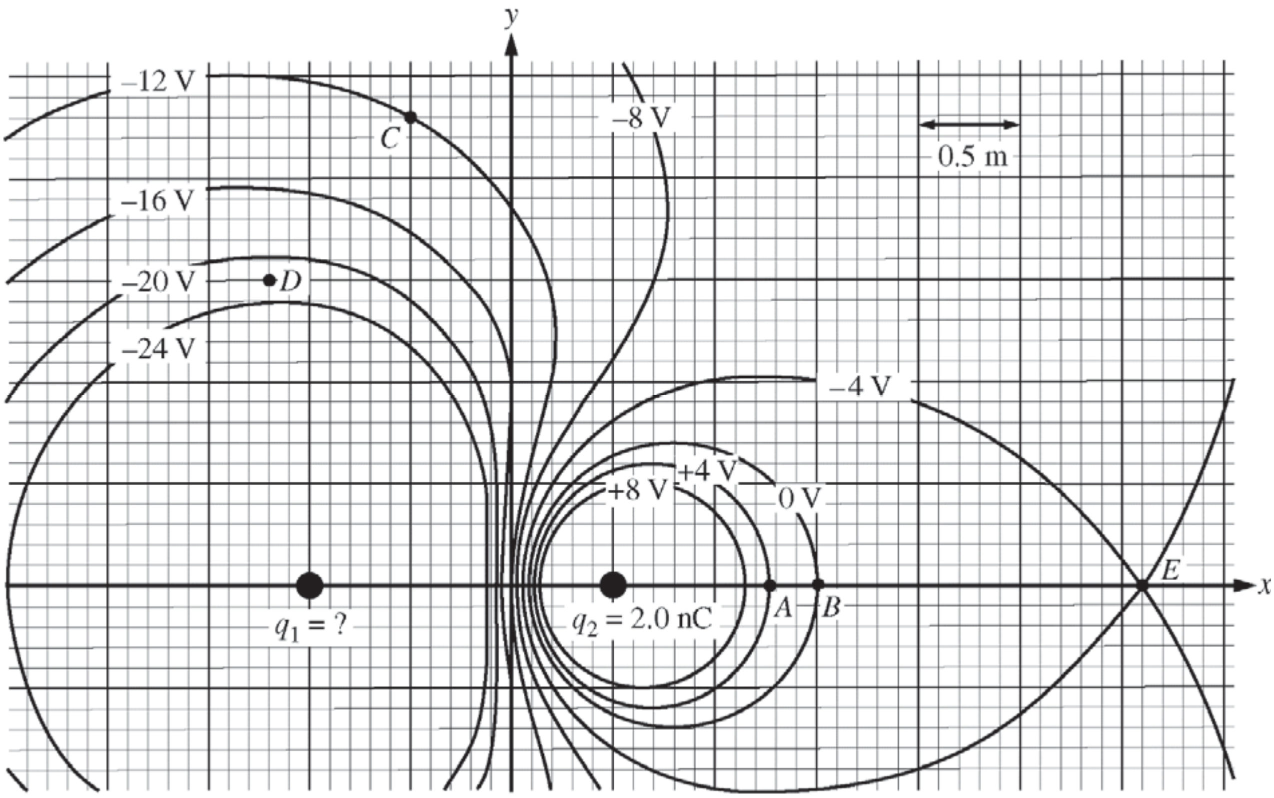
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



E&M.1.

Two point charges, q_1 and q_2, are fixed in place on the x-axis at positions $x_1 = -1.00$ m and $x_2 = +0.50$ m, respectively. Charge q_2 has a value of +2.0 nC. Values of electric potential are illustrated by the given equipotentials in the diagram shown above, which is drawn to scale.

(a) Calculate the value of q_1.

(b) At point C on the diagram, draw a vector representing the direction of the electric field at that point.

(c) Calculate the approximate magnitude of the electric field strength at point D on the diagram.

(d) The equipotential labeled 0 V is the cross section of a nearly spherical surface. Calculate the electric flux for this surface.

(e) A proton is placed at point A and then released from rest.

i. Calculate the work done by the electric field on the proton as it moves from point *A* to point *E*.

ii. Calculate the speed of the proton when it reaches point *E*.

(f) An electron is released from rest at point B. Which of the following indicates the direction of the initial acceleration, if any, of the electron?

____ Up ____ Down

____ Left ____ Right

____ Into the page ____ Out of the page

____ The direction is undefined since the acceleration is zero.

Justify your answer.

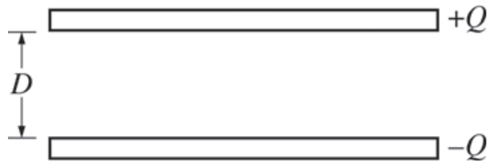
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



E&M.1.

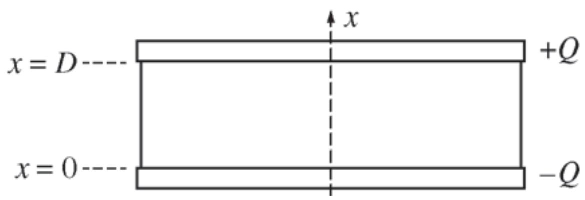
A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

(a)

i. On the figure above, draw an arrow to indicate the direction of the electric field between the plates.

ii. On the figure above, draw an appropriate Gaussian surface that will be used to derive an expression for the magnitude of the electric field E between the plates.

iii. Using Gauss's law and the Gaussian surface from part (a)-ii, derive an expression for the magnitude of the electric field E between the plates. Express your answer in terms of A, D, Q, and physical constants, as appropriate.



The space between the plates is now filled with a dielectric material that is engineered so that its dielectric constant varies with the distance from the bottom plate to the top plate, defined by the x-axis indicated in the diagram above. As a result, the electric field between the plates is given by $\vec{E} = -\dfrac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\hat{i}$, where κ_0 is a positive constant. Express all algebraic answers to the remaining parts in terms of A, D, Q, κ_0, x, and physical constants, as appropriate.

(b) Determine an expression for the dielectric constant κ as a function of x.

(c)

i. Write, but do NOT solve, an equation that could be used to determine the potential difference V between the plates of the capacitor.

ii. Using the equation from part (c)-i, derive an expression for the potential difference $V_D - V_0$, where V_D is the potential of the top plate and V_0 is the potential of the bottom plate.

(d) Determine the capacitance of the capacitor.

(e) The energy stored in the capacitor that has a varying dielectric is U_V. A second capacitor that has a constant dielectric of value κ_0 is also given a charge Q. The energy stored in the second capacitor is U_C. How do the values of U_V and U_C compare?

____ $U_V < U_C$ ____ $U_V > U_C$ ____ $U_V = U_C$

Justify your answer.

E&M.3.

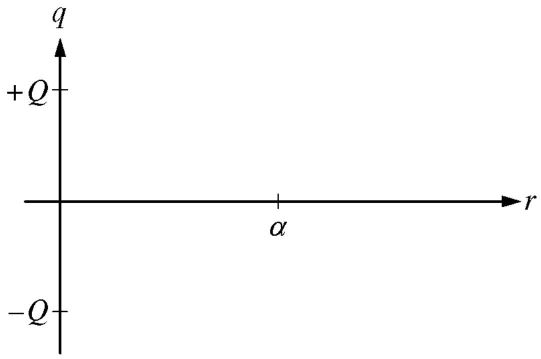
A scientist describes an electrically neutral atom with a model that consists of a nucleus that is a point particle with positive charge $+Q$ at the center of the atom and an electron volume charge density of the form

$$\rho(r) = \begin{cases} -\dfrac{\beta}{r^2} e^{-r/\alpha} & r < \alpha \\ 0 & r > \alpha \end{cases}$$

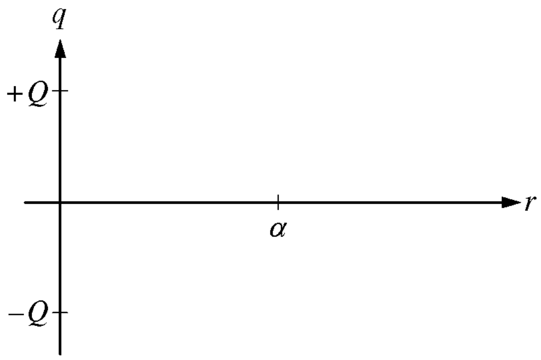
where α and β are positive constants and r is the distance from the center of the atom.

(a) On the axes below, let r stand for the radius of a Gaussian sphere. Sketch the graph for each of the following charges enclosed by the Gaussian sphere as a function of r. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

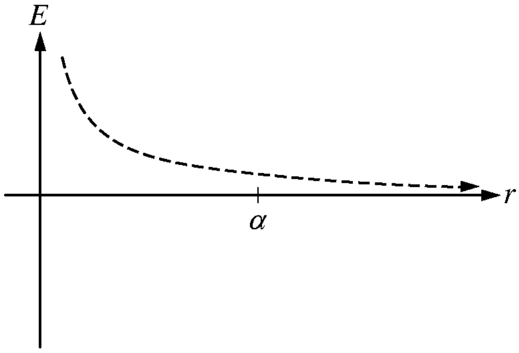
i. The <u>nuclear charge</u> only



ii. The <u>electron charge</u> only



(b) The dashed curve on the graph below represents the electric field as a function of distance r due to the positive nucleus of the atom without any electrons. The nucleus is modeled as a point particle of charge $+Q$. On the same graph, sketch the electric field as a function of distance r for the neutral atom as defined by the scientist's model, which includes the nucleus and the negative electrons surrounding it.



(c) Use Gauss's law to derive an expression for the electric field strength due to the neutral atom for the following positions in terms of $Q, \alpha, \beta, r,$ and fundamental constants, as appropriate.

i. $r > \alpha$

ii. $r < \alpha$

(d) Based on the model proposed by the scientist, what is the physical meaning of the constant α ?

STOP

END OF EXAM

AP PHYSICS C: ELECTRICITY AND MAGNETISM • EXERCISE SHEET

8.4 Electric Fields of Charge Distributions

Name: ______________ Class: __________ Date: ____________ **Total: 8 marks**

KEY WORDS linear charge density 线电荷密度 • volume charge density 体电荷密度 • surface charge density 面电荷 • symmetry 对称性 • continuous charge distribution 连续电荷分布

Objective

Build the skills to answer exam questions on **electric fields of charge distributions**.

You must be able to:

- treat a **continuous charge distribution 连续电荷分布** as many elements dq
- relate dq to **linear**, **surface**, and **volume charge density** (λ, σ, ρ)
- set up an **integral** for the field of a symmetric distribution
- use **symmetry 对称性** to see which field components cancel

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ From point charge to integral

Split the charge into elements dq, each giving $dE = \dfrac{1}{4\pi\varepsilon_0}\dfrac{dq}{r^2}$, then $\vec{E} = \displaystyle\int d\vec{E}$.

■ Charge densities

$\lambda = \dfrac{dq}{dl}$ (per length), $\sigma = \dfrac{dq}{dA}$ (per area), $\rho = \dfrac{dq}{dV}$ (per volume).

■ Symmetry and the far field

By symmetry, components perpendicular to the axis cancel; only the axial part survives. Far away, any finite distribution looks like a **point charge** ($E \to kQ/r^2$).

2 Practice

2.1 Write the charge dq on a length dl of a rod with linear charge density λ. [1]

2.2 State the SI units of surface charge density σ. [1]

2.3 State what the field of any finite charge distribution looks like from very far away. [1]

3 Exam-style questions

3.1 The linear charge density λ is defined as [1]

- **A** charge per unit volume
- **B** charge per unit area
- **C** charge per unit length
- **D** charge $\times$ length

3.2 To find the field of a continuous distribution you [1]

- **A** add two point charges
- **B** integrate $d\vec{E}$ over all the charge elements dq
- **C** use $V = IR$
- **D** ignore symmetry

3.3 A ring of radius a carries total charge Q. On its axis at distance x the field is $E = \dfrac{kQx}{(x^2 + a^2)^{3/2}}$.

(a) State the field at the centre of the ring ($x = 0$). [1]

(b) State what E approaches when $x \gg a$, and explain. [2]

Solutions

2.1 $dq = \lambda\, dl$.

2.2 C m^{-2}.

2.3 it approaches the field of a point charge.

3.1 **C**.

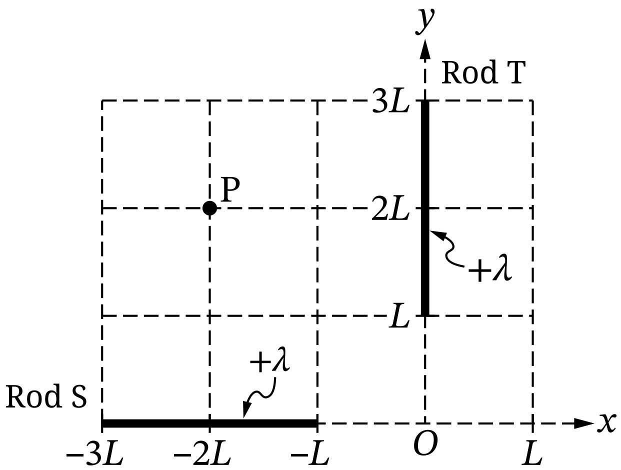
3.2 **B**.

3.3 (a) $E = 0$ (the contributions cancel by symmetry). (b) $E \to \dfrac{kQ}{x^2}$ — far away the ring looks like a point charge Q.

Question 2: Version J

2. Thin, nonconducting, charged Rods S and T are held fixed in the xy-plane, as shown in Figure 1. Rod S is oriented along the x-axis in the region $-3L \leq x \leq -L$. Rod T is oriented along the y-axis in the region $L \leq y \leq 3L$. Each rod has uniform, positive linear charge density $+\lambda$. Point P is located at $(-2L, 2L)$.

Figure 1



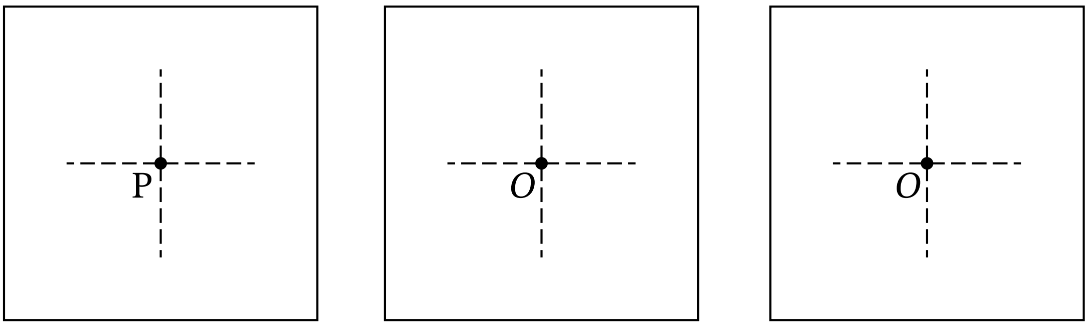
Part A

Complete the following tasks in Figure 2.

- **Draw** an arrow starting on the dot that indicates the direction of the net electric field $\vec{E}_P$ at Point P due to Rods S and T.
- **Draw** an arrow starting on the dot that indicates the direction of the net electric field $\vec{E}_O$ at the origin O due to Rods S and T.
- **Draw** an arrow starting on the dot that indicates the direction of the net acceleration $\vec{a}_O$ of a small, negatively charged sphere (not shown) immediately after the sphere is released from rest at the origin O.

Figure 2

Direction of $\vec{E}_P$ | Direction of $\vec{E}_o$ | Direction of $\vec{a}_o$



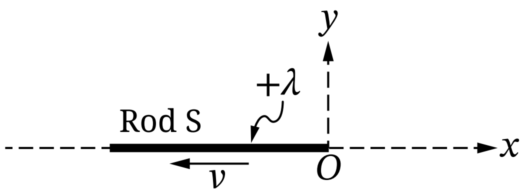
Part B

Derive an expression for the magnitude E_O of the net electric field at the origin O due to Rods S and T in terms of L, λ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

Rod T is removed. Rod S is then moved such that the right end of Rod S is at the origin O. Rod S is then moved with speed v in the $-x$-direction, as shown in Figure 3.

Figure 3



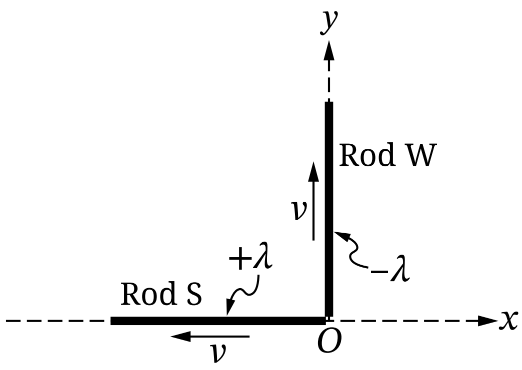
On the axes shown in Figure 4, **sketch** a graph of the electric potential V_O at the origin O due to Rod S as a function of the distance r between the origin and the right end of Rod S.

Figure 4

Part D

A new thin, nonconducting, charged Rod W has the same length as Rod S. Rod W has uniform, negative linear charge density $-\lambda$ and is oriented along the y-axis. Rods S and W are initially held fixed with one end at the origin. Starting at the same time, Rod S is moved with speed v in the $-x$-direction and Rod W is moved with speed v in the $+y$-direction, as shown in Figure 5.

Figure 5



Indicate how a sketch of V_O at the origin O due to both Rods S and W as a function of r would be different from the sketch made in part C. Briefly **justify** your answer.

Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

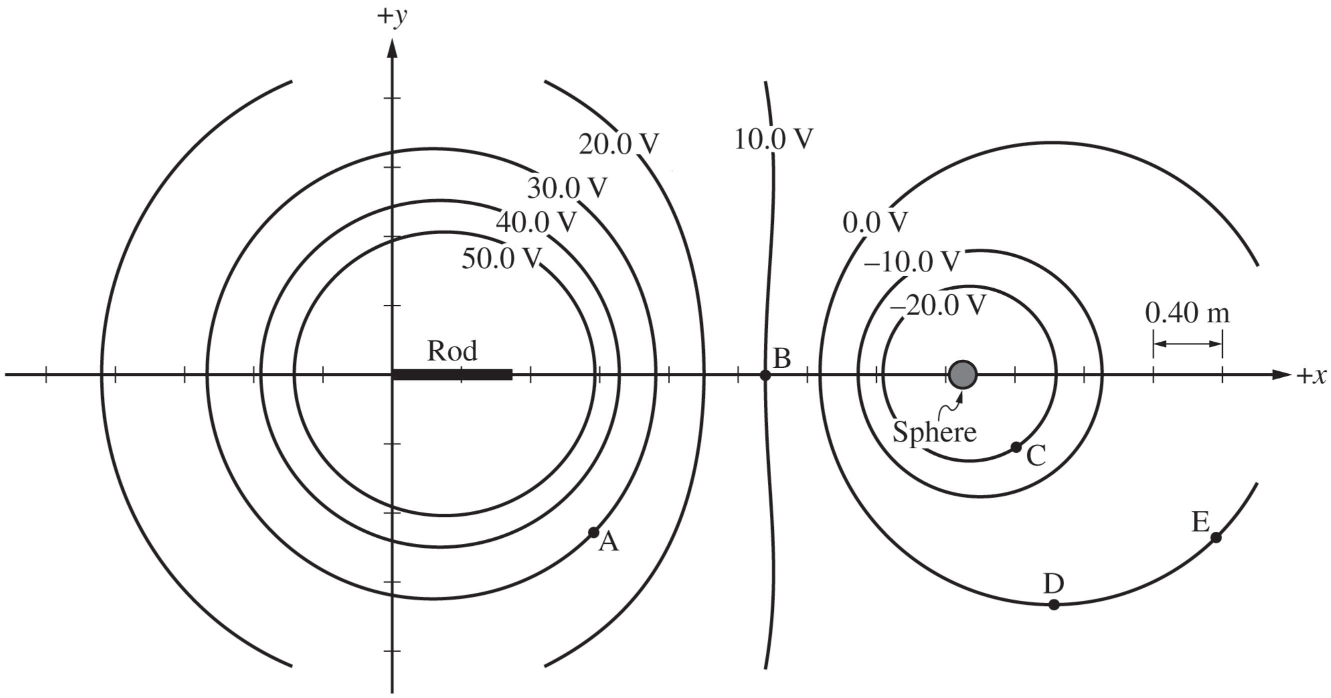


Figure 1

1. A nonconducting rod of uniform positive linear charge density is near a sphere with charge -2.0 nC. The rod and sphere are held at rest on the x-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and the y-axis are 0.40 m apart.

   (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the -20.0 V equipotential line.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is momentarily held at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE}. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a “0” in the corresponding columns, as appropriate.

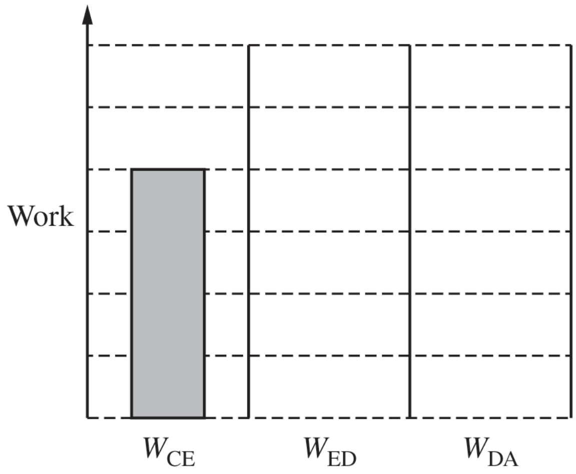


Figure 2

ii. **Calculate** the approximate magnitude of the x-component of the electric field at Position B.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position D. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

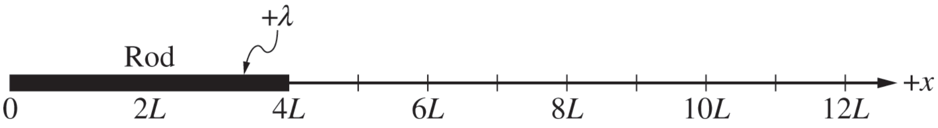


Figure 3

The sphere and the test charge are removed. The rod has length $4L$ and uniform positive linear charge density $+\lambda$. The rod is held at rest on the x-axis in the orientation shown in Figure 3. Position P (not shown) is located on the x-axis a distance x_P from the origin, where $x_P > 4L$.

(d) The electric potential V_P at x_P is $V_P = k\lambda \ln\left(\dfrac{x_P}{x_P - 4L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

ii. On Figure 4, **sketch** a graph of the x-component E_x of the electric field from the rod as a function of x in the region $4L < x < 12L$.

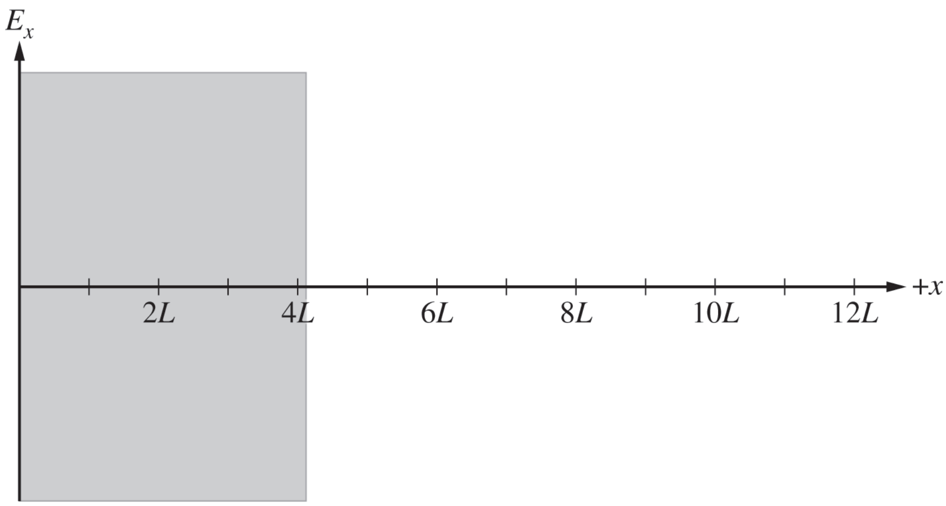


Figure 4

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

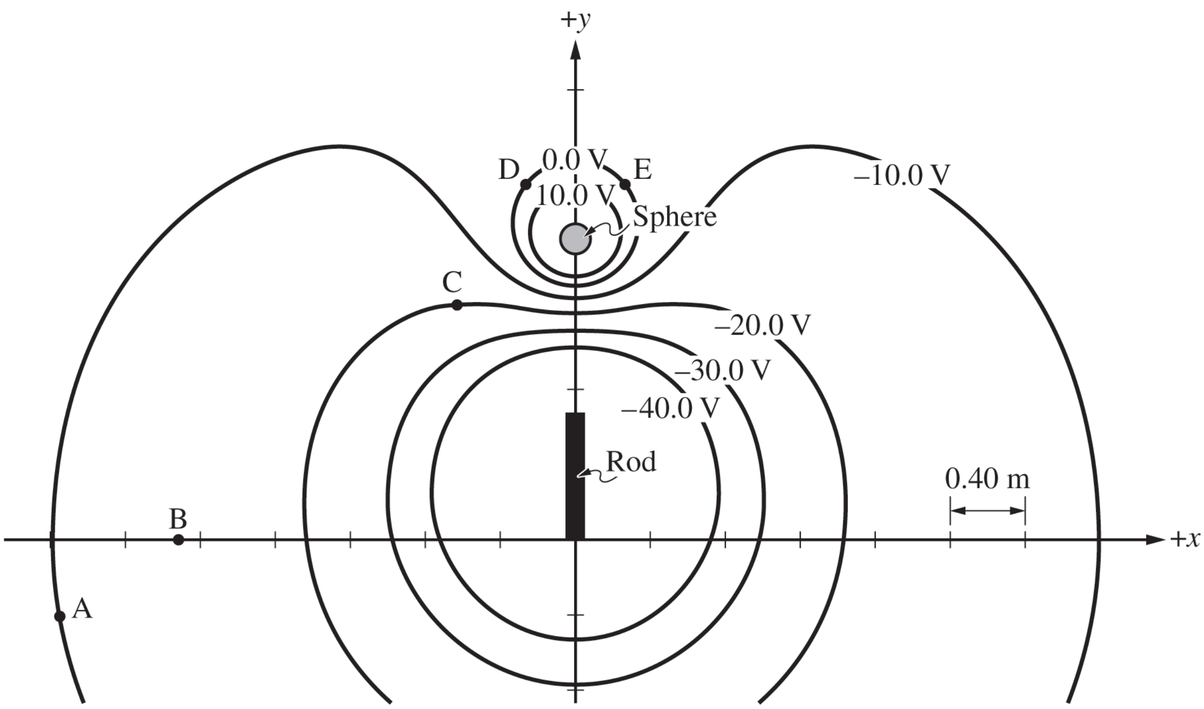


Figure 1

1. A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\,\text{nC}$. The rod and sphere are held at rest on the y-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and on the y-axis are $0.40\,\text{m}$ apart.

   (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the $0.0\,\text{V}$ equipotential line.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is held momentarily at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE}. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a "0" in the corresponding columns, as appropriate.

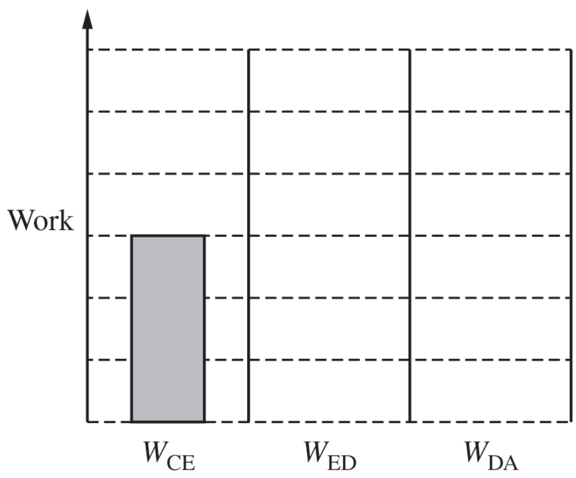


Figure 2

ii. **Calculate** the approximate magnitude of the x-component of the electric field at Position B.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position C. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

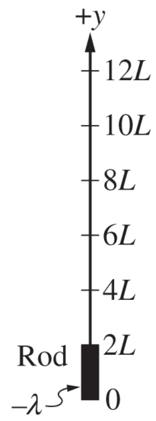


Figure 3

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the y-axis in the orientation shown in Figure 3. Position P (not shown) is located on the y-axis a distance y_P from the origin, where $y_P > 2L$.

(d) The electric potential V_P at y_P is $V_P = -k\lambda \ln\left(\dfrac{y_P}{y_P - 2L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

ii. On Figure 4, **sketch** a graph of the y-component E_y of the electric field resulting from the rod as a function of y in the region $2L < y < 12L$.

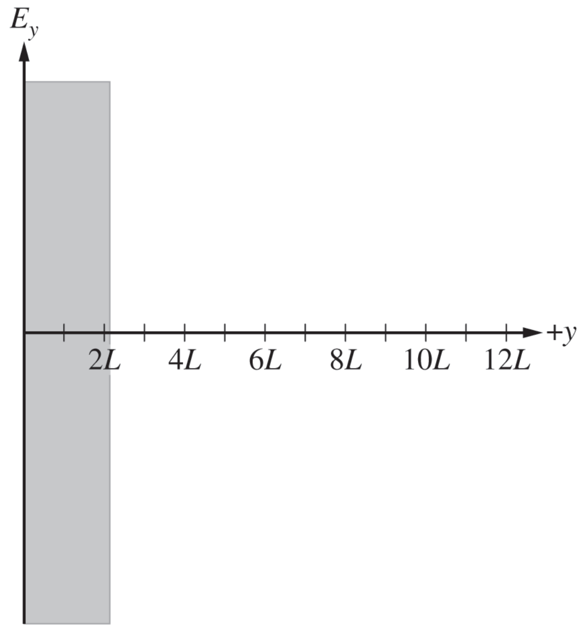


Figure 4

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 1** on this page.

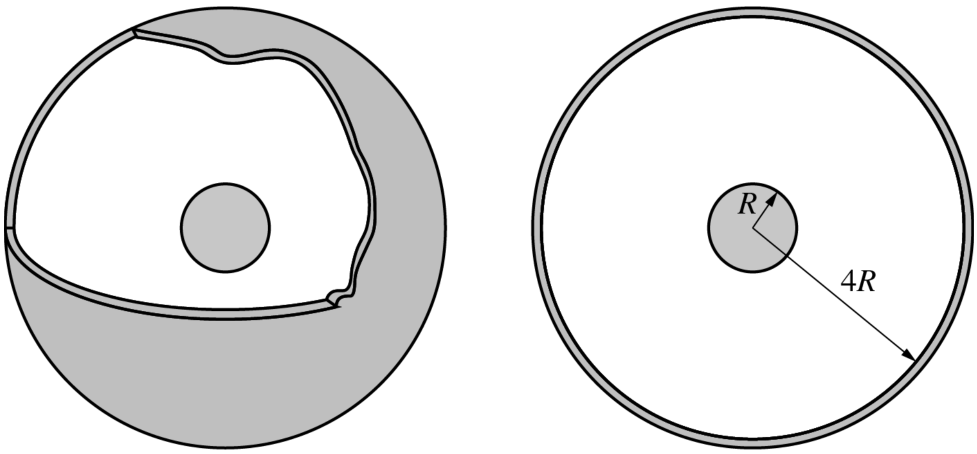
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Cutout View

Cross-Section View

Note: Figures not drawn to scale.

1. A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.

(a) Determine the charge on the outer surface of the shell.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the sphere for $r < R$. Express your answer in terms of Q, R, r, and physical constants, as appropriate.

(c) The magnitude of the electric field at $r = R$ is $8\,\text{N}/\text{C}$. Calculate the value of the electric field at $r = 2R$.

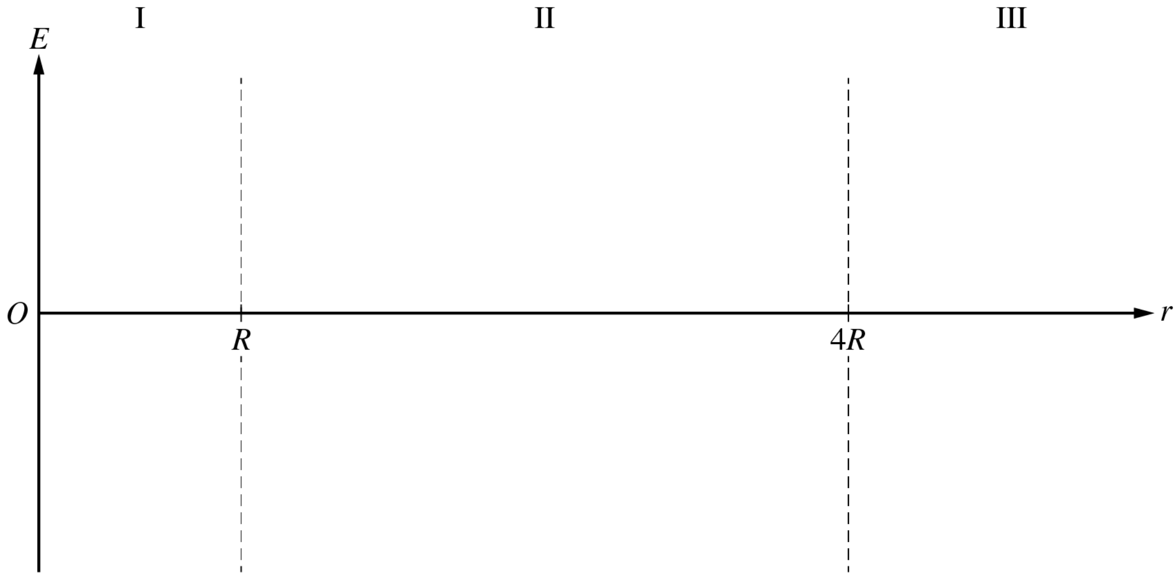
(d) Derive an expression for the absolute value of the potential difference between the outer surface of the sphere and the inner surface of the shell. Express your answer in terms of Q, R, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

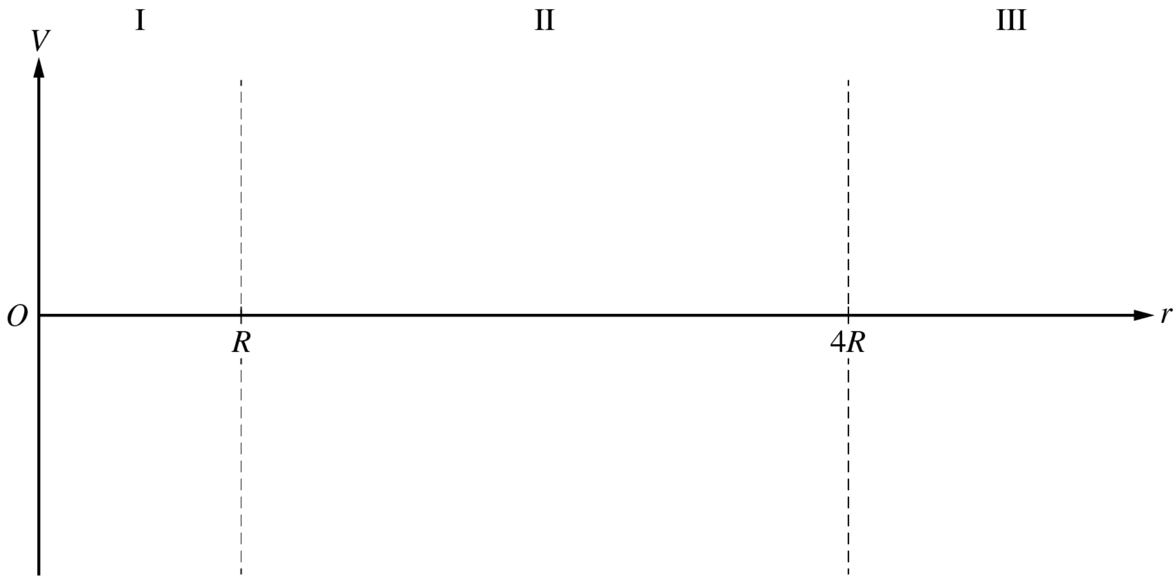
Continue your response to **QUESTION 1** on this page.

(e)

i. On the following axes that include regions I, II, and III, sketch a graph of the electric field E as a function of the distance r from the center of the sphere.



ii. On the following axes that include regions I, II, and III, sketch a graph of the electric potential V as a function of the distance r from the center of the sphere.



GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 1** on this page.

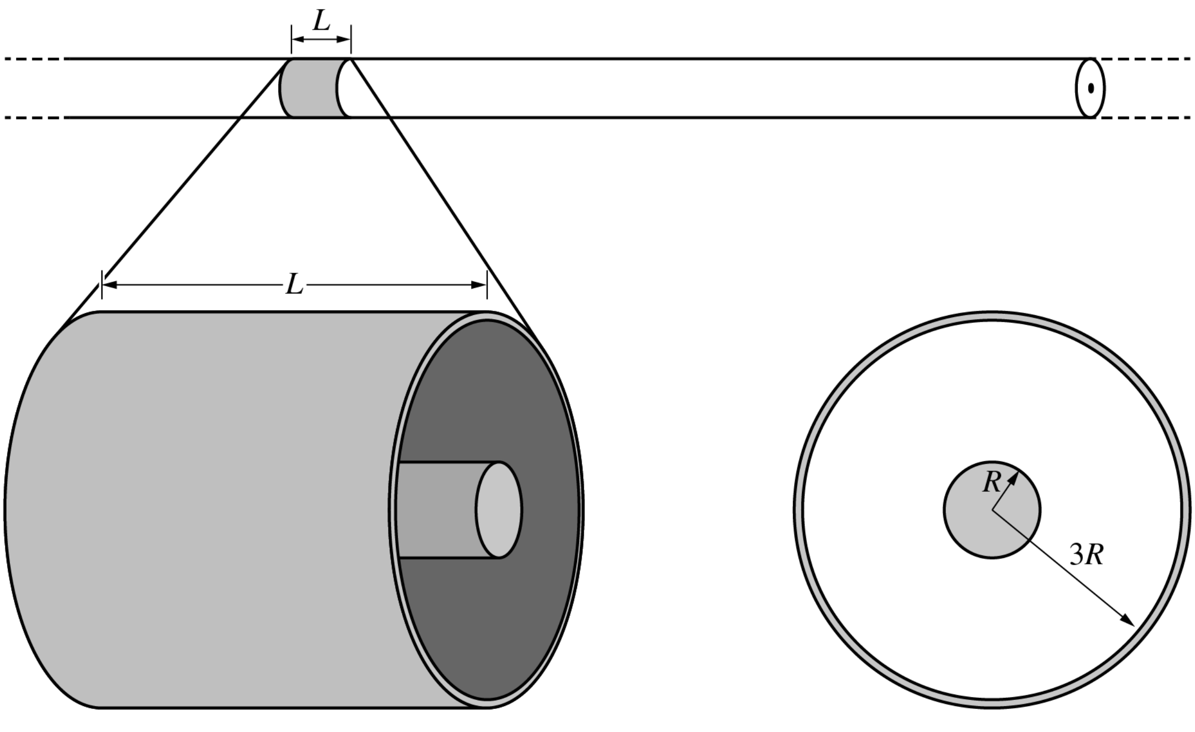
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Cutout View

Cross-Section View

<u>Note:</u> Figures not drawn to scale.

1. A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(a) Determine the charge on the outer surface of the cylindrical shell within length L.

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the inner cylinder for $r < R$. Express your answers in terms of Q, R, r, L, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(c) The magnitude of the electric field at $r = R$ is $12\,\mathrm{N/C}$. Calculate the value of the electric field at $r = 2R$.

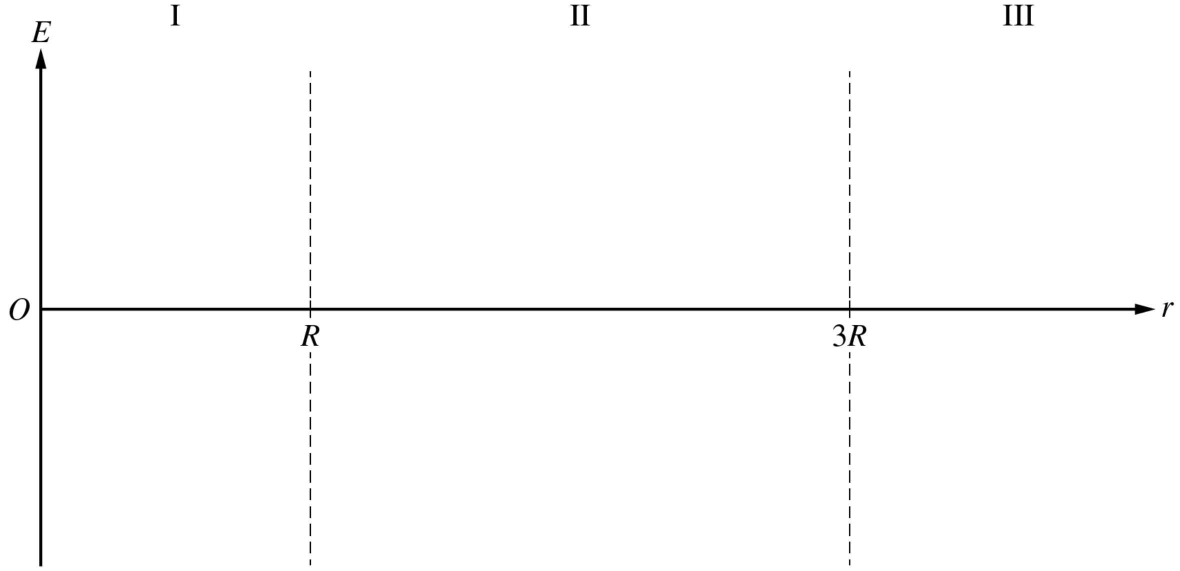
(d) Derive an expression for the absolute value of the potential difference between the surface of the nonconducting cylinder and the inner surface of the cylindrical shell.

GO ON TO THE NEXT PAGE.

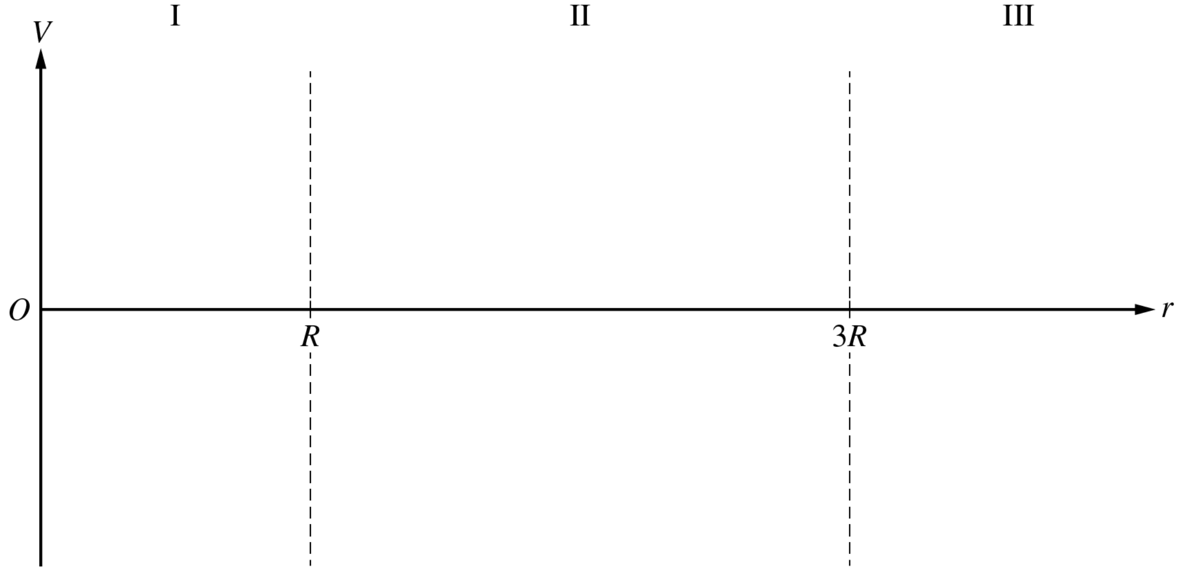
Continue your response to **QUESTION 1** on this page.

(e)

i. On the following axes that include regions I, II, and III, sketch the graph of the electric field E as a function of the distance r from the axis of the inner cylinder.



ii. On the following axes that include regions I, II, and III, sketch the graph of the electric potential V as a function of the distance r from the axis of the inner cylinder.

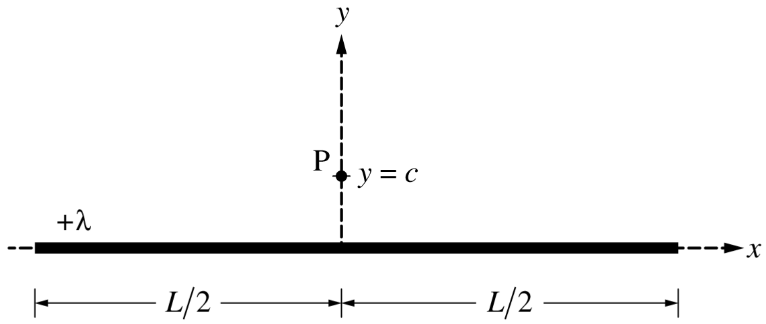


GO ON TO THE NEXT PAGE.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

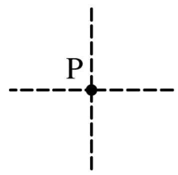


Note: Figure not drawn to scale.

1. A very long, thin, nonconducting cylinder of length L is centered on the y-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y-axis at $y = c$, where $L >> c$.

(a)

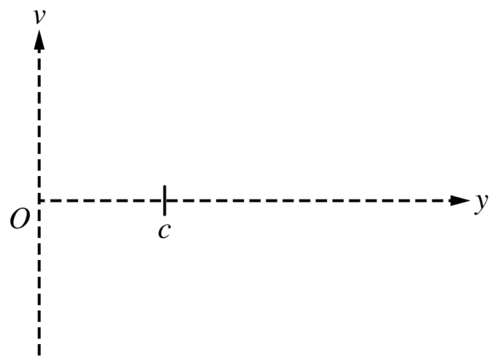
i. On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the long cylinder. The arrow should start on and point away from the dot.

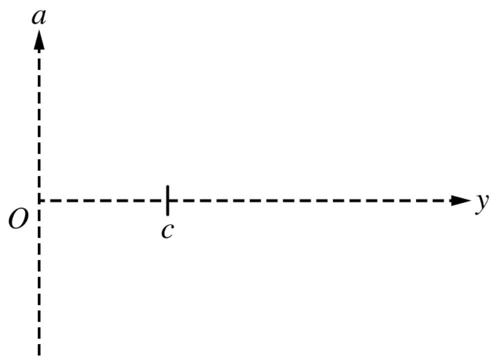


ii. Describe the shape and location of a Gaussian surface that can be used to determine the electric field at point P due to the long cylinder.

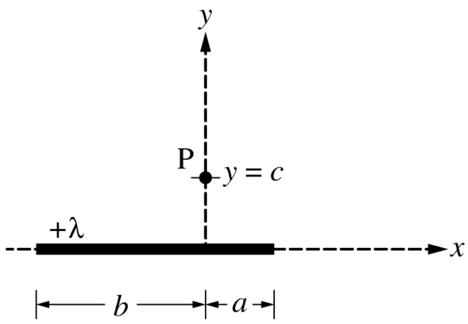
iii. Use your Gaussian surface to derive an expression for the magnitude of the electric field at point P. Express your answer in terms of λ, c, L, and physical constants, as appropriate.

(b) A proton is released from rest at point P. On the axes below, sketch the velocity v as a function of <u>position</u> y and the acceleration a as a function of <u>position</u> y for the proton.

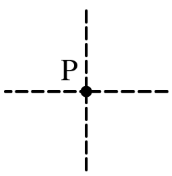




The original cylinder is now replaced with a much shorter thin, nonconducting cylinder with the same uniform linear charge density $+\lambda$, as shown in the figure below. The length of the cylinder to the right of the y-axis is a, and the length of the cylinder to the left of the y-axis is b, where $a < b$.



(c) On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the shorter cylinder. The arrow should start on and point away from the dot.

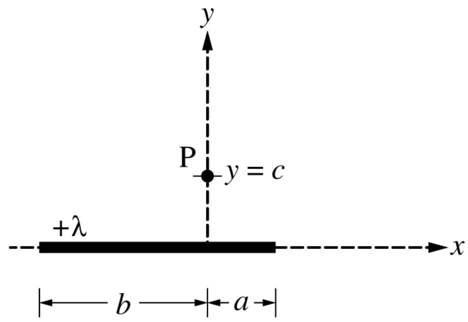


(d)

i. Is there a single Gaussian surface that can be used with Gauss's law to derive an expression for the electric field at point P?

____ Yes ____ No

ii. If your answer to part (d)(i) is yes, explain how you can use Gauss's law to derive an expression for the field at point P. If your answer to part (d)(i) is no, explain why Gauss's law cannot be applied to derive an expression for the electric field in this case.



Note: This figure is shown again for reference.

A student in class argues that using the integral shown below might be a useful approach for determining the electric field at point P.

$$E = \int \frac{1}{4\pi\varepsilon_0} \frac{1}{r^2}\, dq$$

The student uses this approach and writes the following two integrals for the magnitude of the horizontal and vertical components of the electric field at point P.

Horizontal component: $$|E_x| = \frac{\lambda}{4\pi\varepsilon_0} \int_{-b}^{a} \frac{x}{\left(c^2 + x^2\right)^{3/2}}\, dx$$

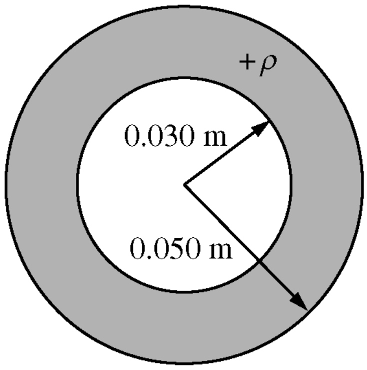
Vertical component: $$|E_y| = \frac{\lambda}{4\pi\varepsilon_0} \int_{-b}^{a} \frac{y}{\left(c^2 + x^2\right)}\, dy$$

(e)

i. One of the two expressions above is not correct. Which expression is not correct?

____ Horizontal component ____ Vertical component

ii. Identify two mistakes in the incorrect expression, and explain how to correct the mistakes.



2. A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030\text{ m:}\quad \rho = 0$$

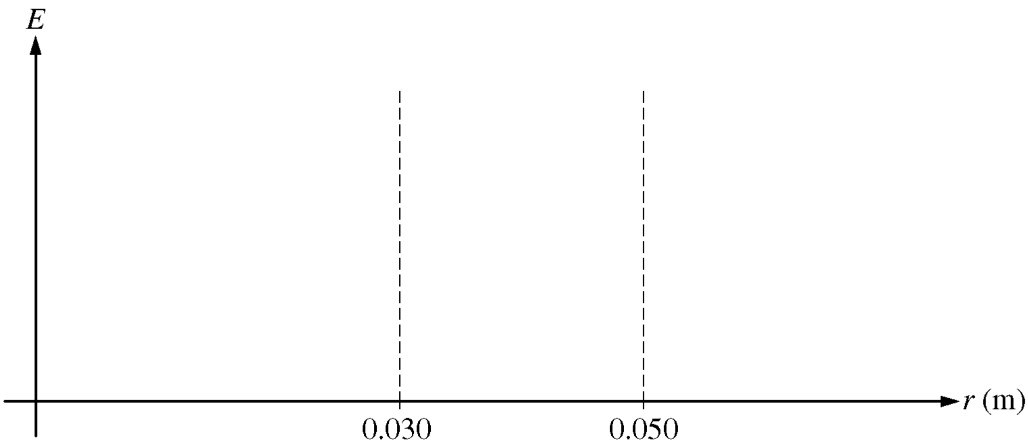
$$0.030\text{ m} < r < 0.050\text{ m:}\quad \rho = b/r\text{, where } b = 1.6 \times 10^{-6}\ \mathrm{C/m^2}$$

$$r > 0.050\text{ m:}\quad \rho = 0$$

(a) Calculate the total charge of the sphere.

(b) Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.

(c) On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.



(d) Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

(e) A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.

i. Calculate the magnitude of the initial acceleration of the proton.

ii. Calculate the speed of the proton after a long time.

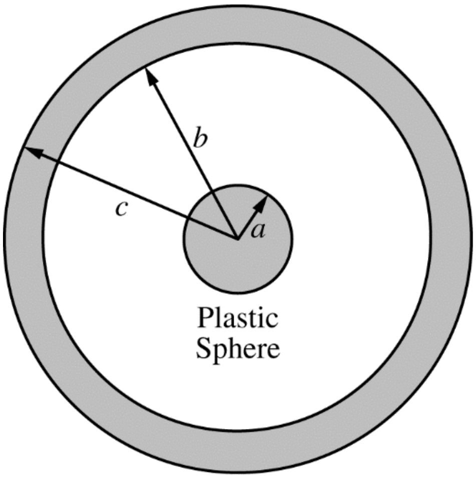
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Conducting Spherical Shell

1. A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

   (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

   i. $r < a$

   ii. $a < r < b$

   (b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

   i. $r < a$

   ii. $a < r < b$

   (c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

   i. Determine the charge on the inner surface of the conducting shell.

   Justify your answer.

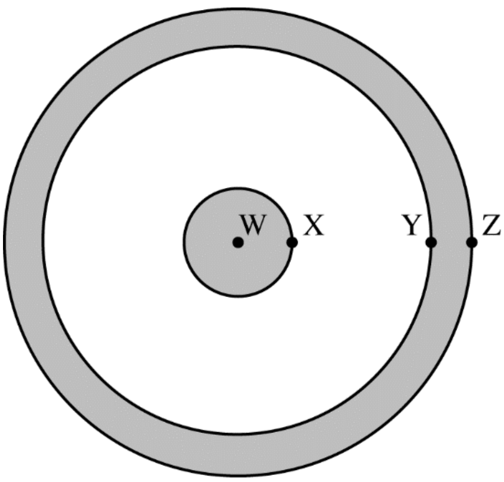
   ii. Determine the charge on the outer surface of the conducting shell.

(d)

i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.

ii. The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

____ W ____ X ____ Y ____ Z



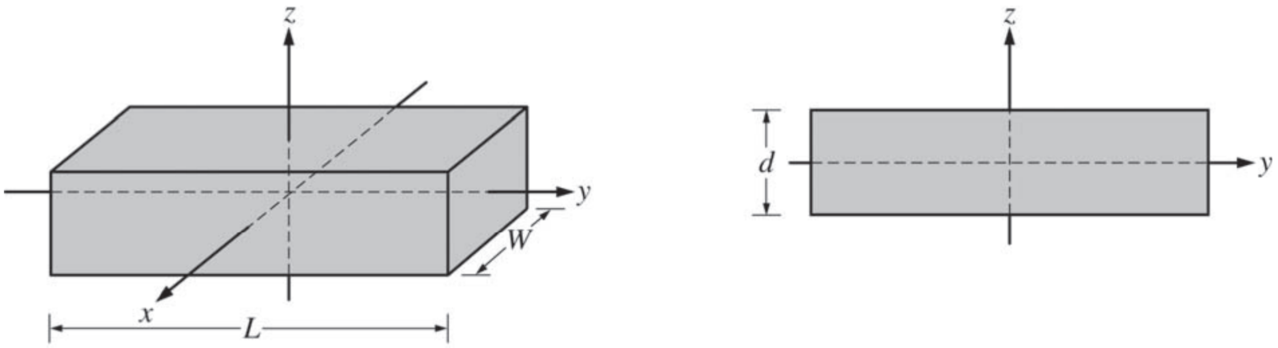
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

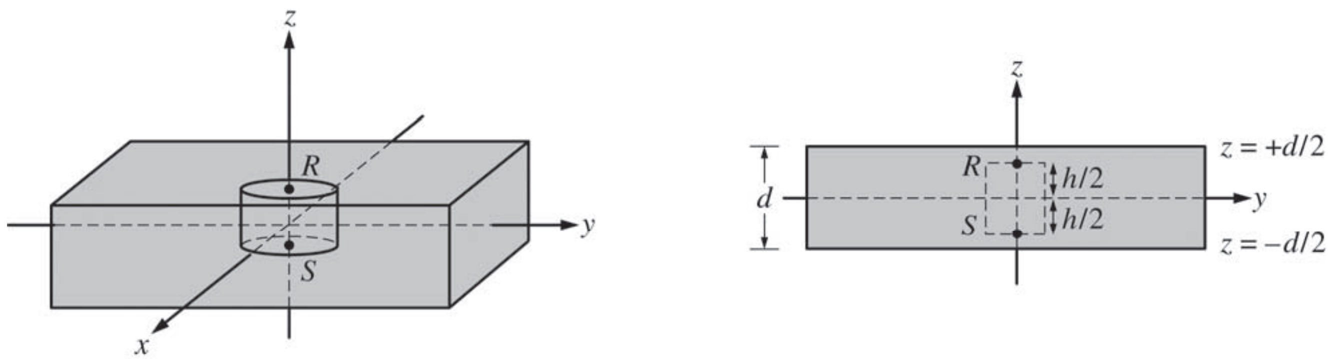


Perspective View Side View

Note: Figures not drawn to scale.

1. A very large nonconducting slab with a uniform positive volume charge density ρ_0 is fixed with the origin of the *xyz*-axes at its center, as shown in the figure above. The thickness of the slab is d, the length is L, and the width is W, where $L >> d$ and $W >> d$. The large faces of the slab are parallel to the *xy*-plane.

   Consider a Gaussian cylinder with a cross-sectional area A and height h that is positioned with its axis along the *z*-axis, as shown in the figure below.



Perspective View Side View

Note: Figures not drawn to scale.

(a) Draw a single vector on each of the dots below representing the direction of the electric field at the given points. If the electric field at either point is zero, write "$E = 0$" next to the point.

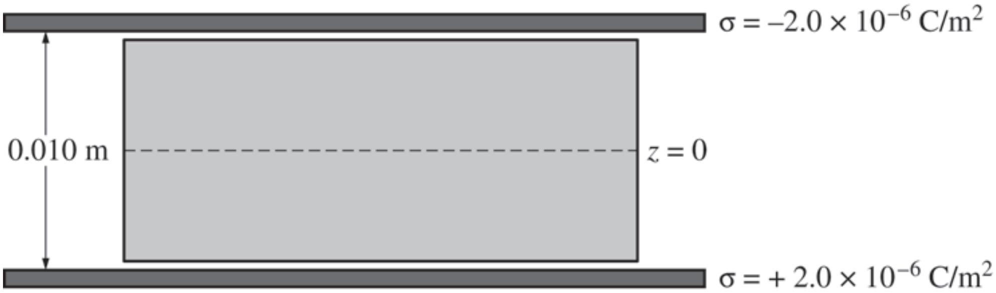
i. ii.

• R • S

b) Use Gauss's law to derive expressions for the following. Express your answers in terms of ρ_0, A, d, h, z, and physical constants, as appropriate.

i. Derive an expression for the total flux Φ through the Gaussian surface shown.

ii. Derive an expression for the magnitude of the electric field as a function of z for any position inside the slab, and show that it is equal to $E = \dfrac{\rho_0 z}{\varepsilon_0}$.



Note: Figure not drawn to scale.

The charged slab is now placed between two large metal plates separated by a distance of 0.010 m, which is approximately the thickness of the slab, but the slab does not contact either metal plate. The metal plates are charged, resulting in the surface charge densities $\sigma = \pm 2.0 \times 10^{-6}\ \mathrm{C/m^2}$, as shown in the figure above. Assume the charge distribution inside the slab remains unchanged by the presence of the charged plates and that the slab's volume charge density is $\rho_0 = 1.00 \times 10^{-3}\ \mathrm{C/m^3}$.

(c)

i. The magnitude of the electric field inside the slab is zero on the z-axis at position z_0. Which of the following correctly indicates the value for z_0?

____ $z_0 > 0$ ____ $z_0 = 0$ ____ $z_0 < 0$

Justify your answer.

ii. Calculate the value z_0.

(d) Calculate the magnitude of the electric potential difference from the center of the slab to the top of the slab.

E&M.3.

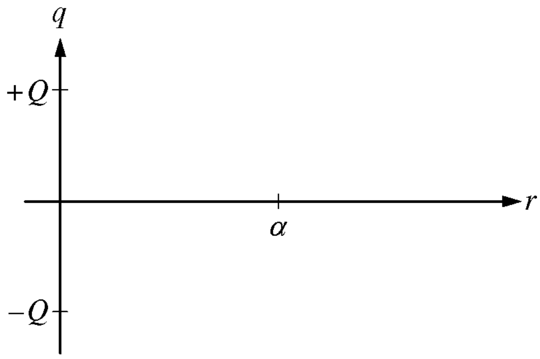
A scientist describes an electrically neutral atom with a model that consists of a nucleus that is a point particle with positive charge $+Q$ at the center of the atom and an electron volume charge density of the form

$$\rho(r) = \begin{cases} -\dfrac{\beta}{r^2} e^{-r/\alpha} & r < \alpha \\ 0 & r > \alpha \end{cases}$$

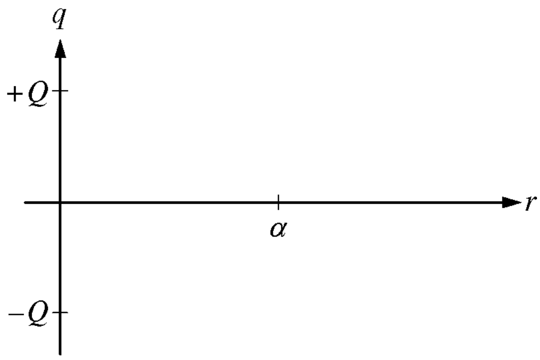
where α and β are positive constants and r is the distance from the center of the atom.

(a) On the axes below, let r stand for the radius of a Gaussian sphere. Sketch the graph for each of the following charges enclosed by the Gaussian sphere as a function of r. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

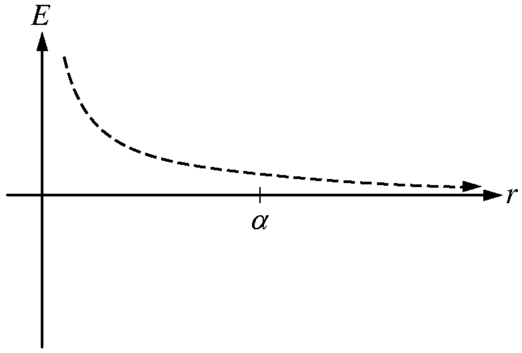
i. The nuclear charge only



ii. The electron charge only



(b) The dashed curve on the graph below represents the electric field as a function of distance r due to the positive nucleus of the atom without any electrons. The nucleus is modeled as a point particle of charge $+Q$. On the same graph, sketch the electric field as a function of distance r for the neutral atom as defined by the scientist's model, which includes the nucleus and the negative electrons surrounding it.



(c) Use Gauss's law to derive an expression for the electric field strength due to the neutral atom for the following positions in terms of $Q, \alpha, \beta, r,$ and fundamental constants, as appropriate.

i. $r > \alpha$

ii. $r < \alpha$

(d) Based on the model proposed by the scientist, what is the physical meaning of the constant α ?

STOP

END OF EXAM

8.5 Electric Flux

Name: ____________ Class: ________ Date: __________ **Total: 10 marks**

KEY WORDS electric flux 电通量 • gaussian surface 高斯面 • gauss's law 高斯定律 • area vector 面积矢量 • uniform 均匀

Objective

Build the skills to answer exam questions on **electric flux**.

You must be able to:

- define **electric flux 电通量** through a surface as $\Phi_E = \int \vec{E} \cdot d\vec{A}$
- relate flux to the component of $\vec{E}$ perpendicular to the surface
- calculate the flux through flat surfaces in a uniform field
- see how a convenient **Gaussian surface 高斯面** simplifies a flux calculation

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Electric flux

$\Phi_E = \int \vec{E} \cdot d\vec{A}$. For a **uniform** field through a **flat** surface this is $\Phi_E = EA\cos\theta$, where θ is the angle between $\vec{E}$ and the area vector (the normal).

■ Only the perpendicular part counts

Flux is greatest when the surface **faces** the field ($\theta = 0$) and zero when the field lies **along** the surface ($\theta = 90°$).

■ Closed surface

The **net** flux out of a closed surface depends on how much field enters versus leaves — the basis of Gauss's law.

2 Practice

2.1 Find the flux through a 0.20 m^2 flat surface placed face-on to a uniform 500 N C^{-1} field. [2]

2.2 State when the flux through a flat surface is zero, relative to the field. [1]

2.3 State what the net flux through a closed surface depends on. [1]

3 Exam-style questions

3.1 The electric flux through a flat surface in a uniform field is [1]

- **A** EA
- **B** $EA\cos\theta$
- **C** E/A
- **D** EA^2

3.2 The flux through a surface is zero when the field is [1]

- **A** perpendicular to the surface
- **B** parallel to the surface (grazing it)
- **C** very strong
- **D** uniform

3.3 A uniform field of 300 N C^{-1} passes through a 0.40 m^2 square.

(a) Find the flux when the square faces the field. [2]

(b) Find the flux when the square is tilted so its normal is $60°$ from the field. [2]

Solutions

2.1 $\Phi_E = EA = 500 \times 0.20 = 100$ N m^2 C^{-1}.

2.2 when the field is parallel to the surface (perpendicular to its normal).

2.3 the net charge enclosed by the surface.

3.1 **B**.

3.2 **B**.

3.3 (a) $\Phi_E = EA = 300 \times 0.40 = 120$ N m^2 C^{-1}. (b) $\Phi_E = EA\cos 60° = 120 \times 0.5 = 60$ N m^2 C^{-1}.

Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

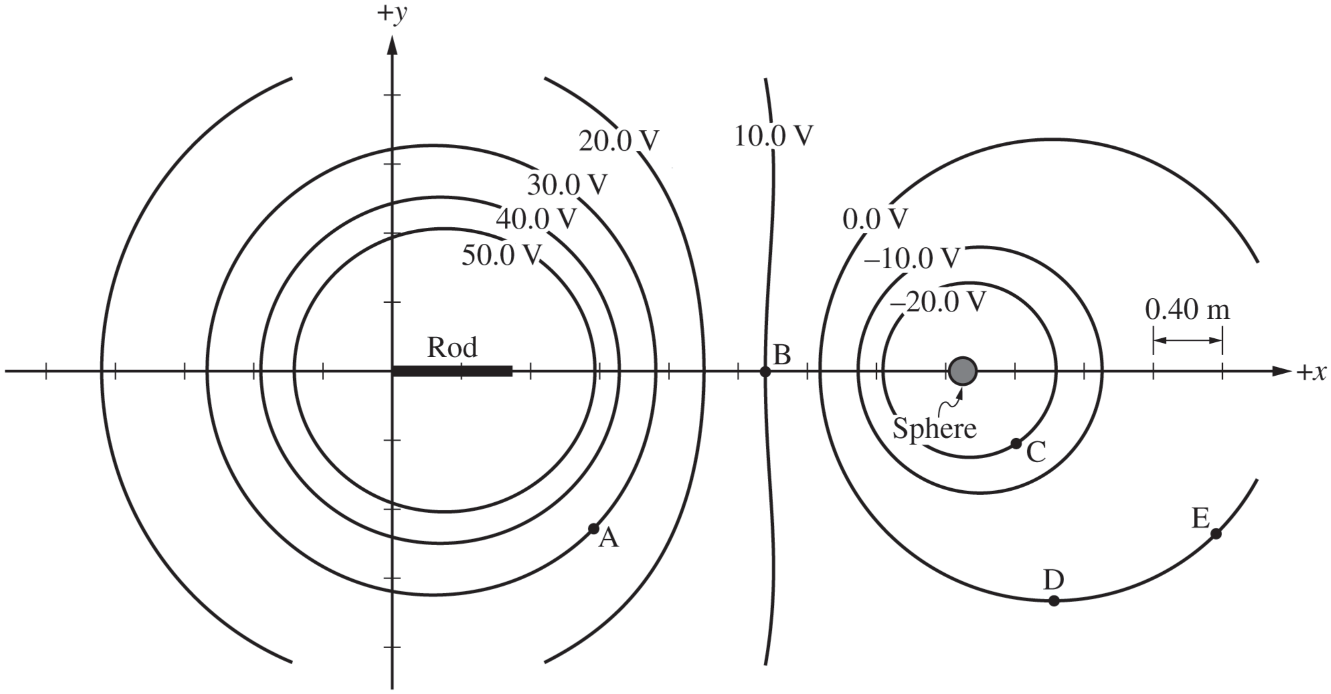


Figure 1

1. A nonconducting rod of uniform positive linear charge density is near a sphere with charge -2.0 nC. The rod and sphere are held at rest on the x-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and the y-axis are 0.40 m apart.

   (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the -20.0 V equipotential line.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is momentarily held at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE}. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a "0" in the corresponding columns, as appropriate.

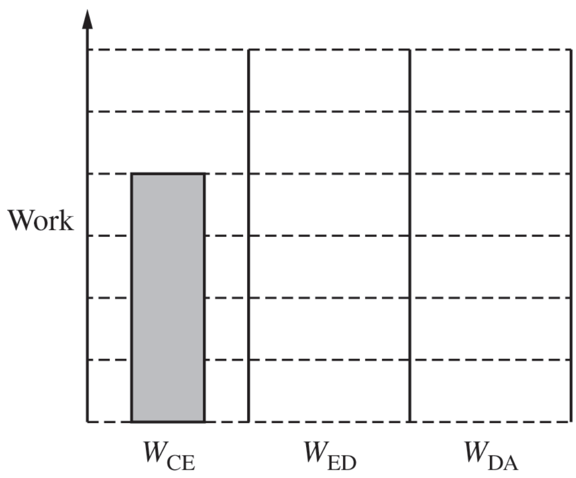


Figure 2

ii. **Calculate** the approximate magnitude of the x-component of the electric field at Position B.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position D. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

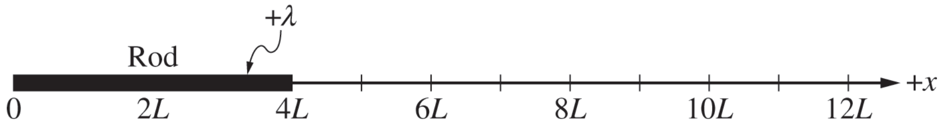


Figure 3

The sphere and the test charge are removed. The rod has length $4L$ and uniform positive linear charge density $+\lambda$. The rod is held at rest on the x-axis in the orientation shown in Figure 3. Position P (not shown) is located on the x-axis a distance x_P from the origin, where $x_P > 4L$.

(d) The electric potential V_P at x_P is $V_P = k\lambda \ln\left(\dfrac{x_P}{x_P - 4L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

ii. On Figure 4, **sketch** a graph of the x-component E_x of the electric field from the rod as a function of x in the region $4L < x < 12L$.

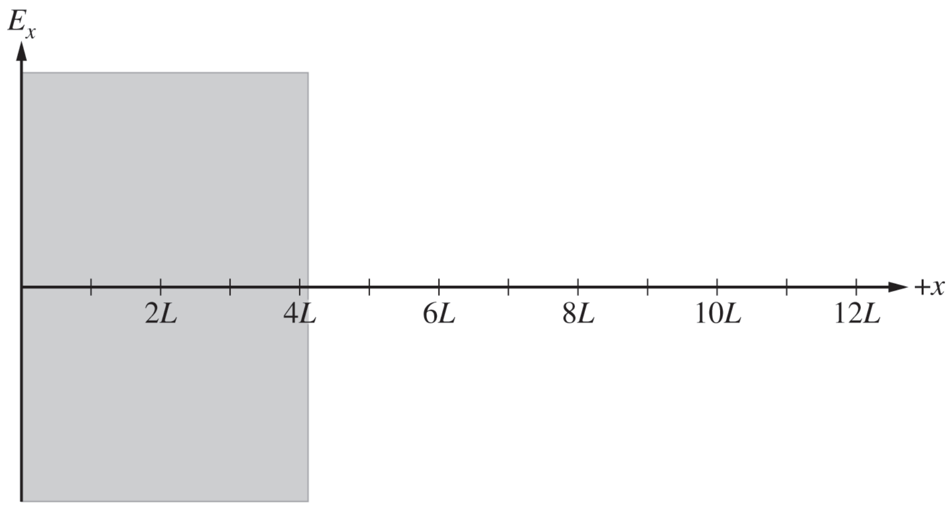


Figure 4

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

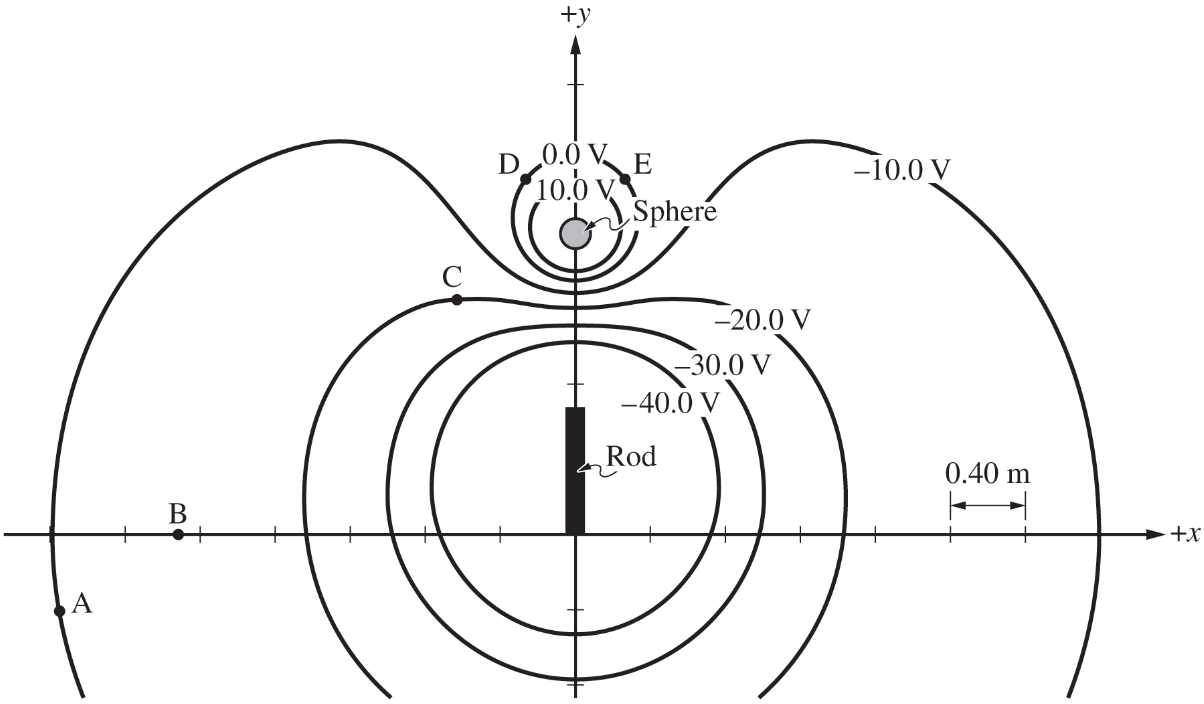


Figure 1

1. A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\ \text{nC}$. The rod and sphere are held at rest on the y-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and on the y-axis are $0.40\ \text{m}$ apart.

   (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the $0.0\ \text{V}$ equipotential line.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is held momentarily at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE}. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a "0" in the corresponding columns, as appropriate.

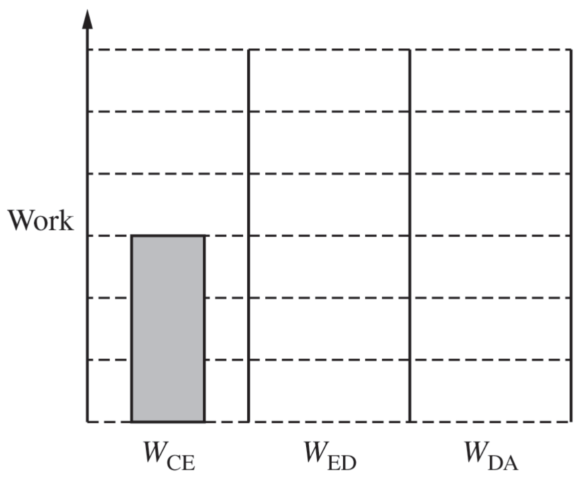


Figure 2

ii. **Calculate** the approximate magnitude of the x-component of the electric field at Position B.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position C. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

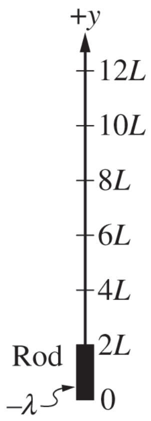


Figure 3

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the y-axis in the orientation shown in Figure 3. Position P (not shown) is located on the y-axis a distance y_P from the origin, where $y_P > 2L$.

(d) The electric potential V_P at y_P is $V_P = -k\lambda \ln\left(\dfrac{y_P}{y_P - 2L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

ii. On Figure 4, **sketch** a graph of the y-component E_y of the electric field resulting from the rod as a function of y in the region $2L < y < 12L$.

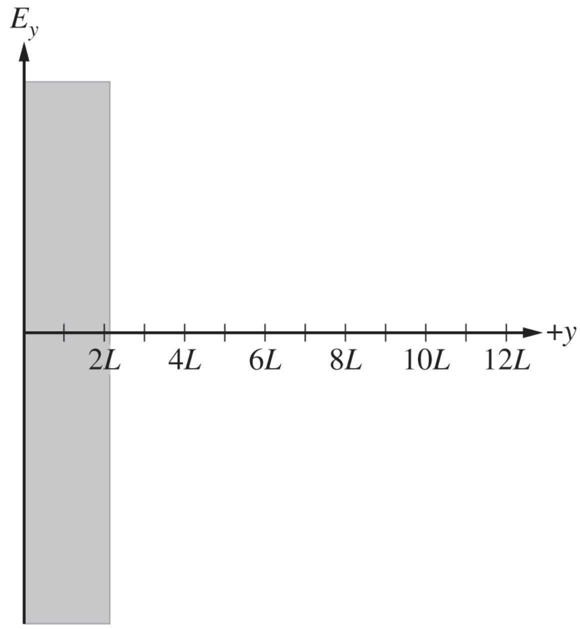


Figure 4

GO ON TO THE NEXT PAGE.

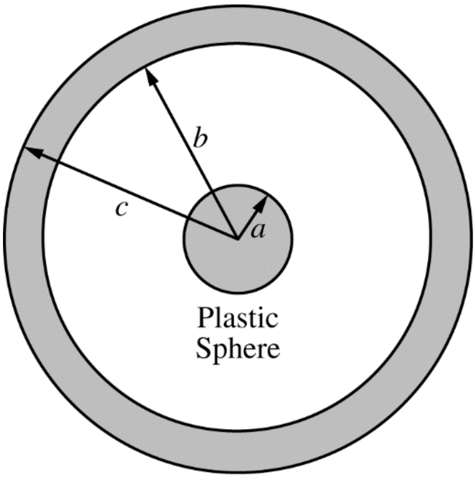
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Conducting Spherical Shell

1. A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

   (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

   i. $r < a$

   ii. $a < r < b$

   (b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

   i. $r < a$

   ii. $a < r < b$

   (c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

   i. Determine the charge on the inner surface of the conducting shell.

   Justify your answer.

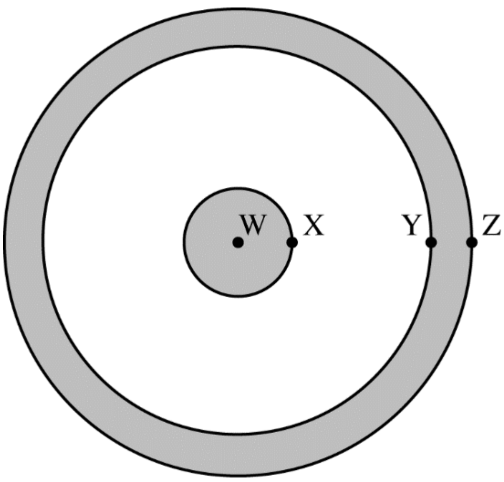
   ii. Determine the charge on the outer surface of the conducting shell.

(d)

i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.

ii. The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

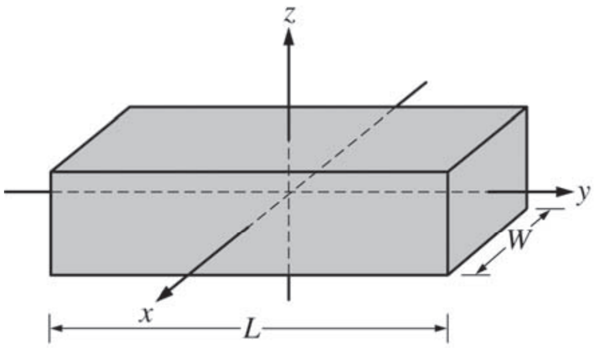
____ W ____ X ____ Y ____ Z



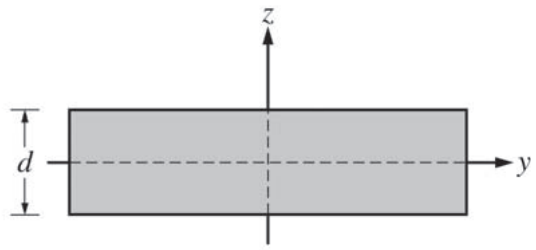
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Perspective View

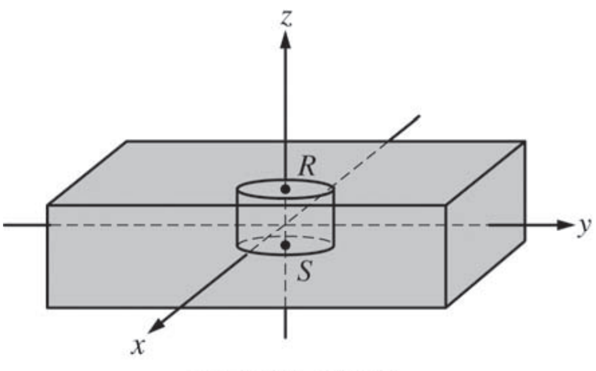


Side View

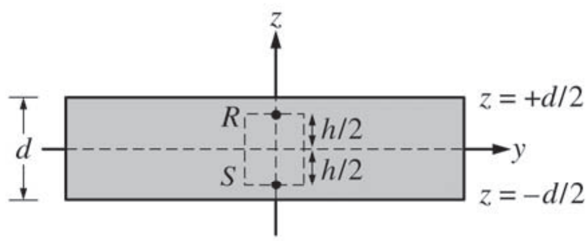
Note: Figures not drawn to scale.

1. A very large nonconducting slab with a uniform positive volume charge density ρ_0 is fixed with the origin of the xyz-axes at its center, as shown in the figure above. The thickness of the slab is d, the length is L, and the width is W, where $L >> d$ and $W >> d$. The large faces of the slab are parallel to the xy-plane.

   Consider a Gaussian cylinder with a cross-sectional area A and height h that is positioned with its axis along the z-axis, as shown in the figure below.



Perspective View



Side View

Note: Figures not drawn to scale.

(a) Draw a single vector on each of the dots below representing the direction of the electric field at the given points. If the electric field at either point is zero, write "$E = 0$" next to the point.

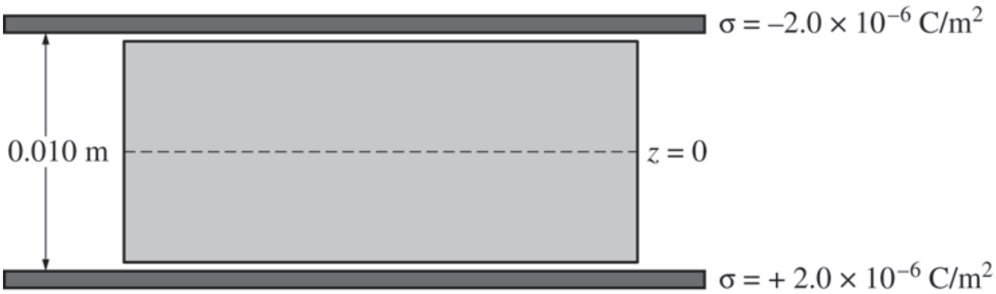
i. • R

ii. • S

b) Use Gauss's law to derive expressions for the following. Express your answers in terms of ρ_0, A, d, h, z, and physical constants, as appropriate.

i. Derive an expression for the total flux Φ through the Gaussian surface shown.

ii. Derive an expression for the magnitude of the electric field as a function of z for any position inside the slab, and show that it is equal to $E = \dfrac{\rho_0 z}{\varepsilon_0}$.



Note: Figure not drawn to scale.

The charged slab is now placed between two large metal plates separated by a distance of 0.010 m, which is approximately the thickness of the slab, but the slab does not contact either metal plate. The metal plates are charged, resulting in the surface charge densities $\sigma = \pm 2.0 \times 10^{-6}\ \mathrm{C/m^2}$, as shown in the figure above. Assume the charge distribution inside the slab remains unchanged by the presence of the charged plates and that the slab's volume charge density is $\rho_0 = 1.00 \times 10^{-3}\ \mathrm{C/m^3}$.

(c)

i. The magnitude of the electric field inside the slab is zero on the z-axis at position z_0. Which of the following correctly indicates the value for z_0?

____ $z_0 > 0$ ____ $z_0 = 0$ ____ $z_0 < 0$

Justify your answer.

ii. Calculate the value z_0.

(d) Calculate the magnitude of the electric potential difference from the center of the slab to the top of the slab.

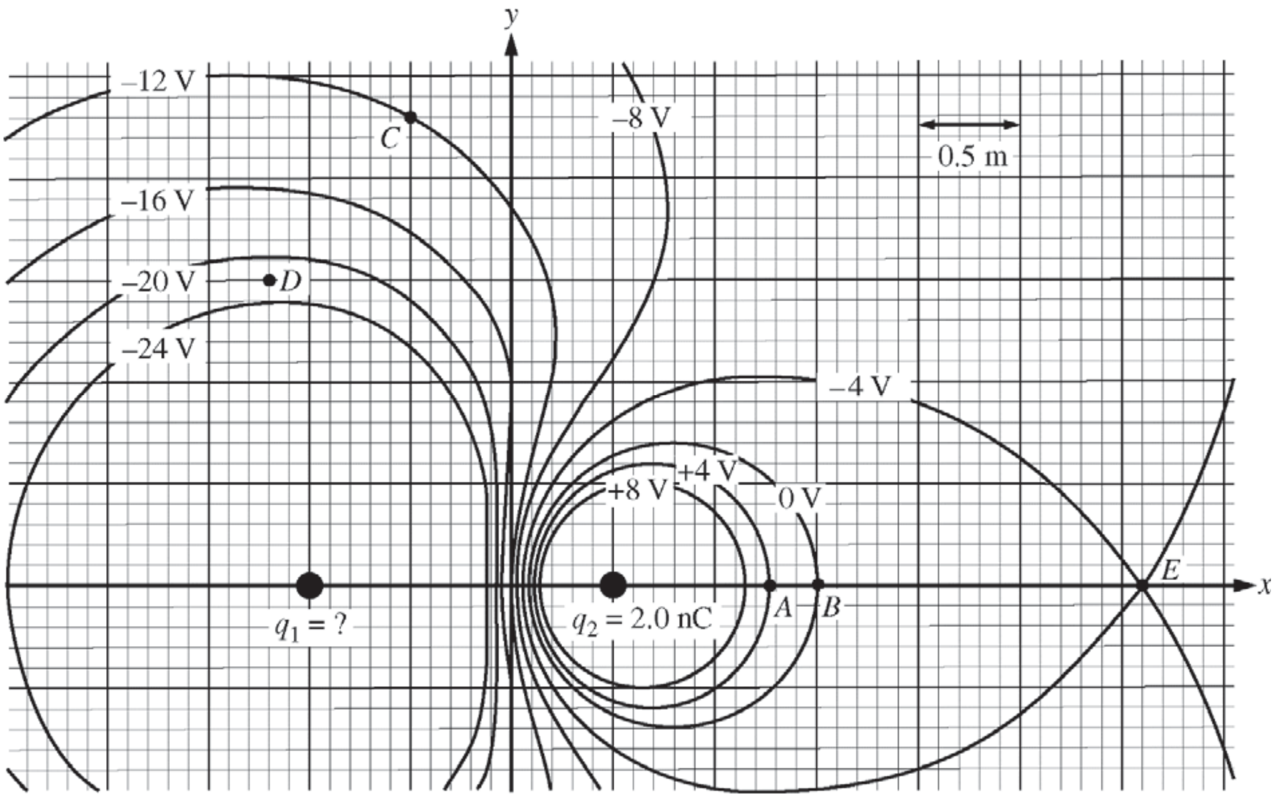
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



E&M.1.

Two point charges, q_1 and q_2, are fixed in place on the x-axis at positions $x_1 = -1.00$ m and $x_2 = +0.50$ m, respectively. Charge q_2 has a value of +2.0 nC. Values of electric potential are illustrated by the given equipotentials in the diagram shown above, which is drawn to scale.

(a) Calculate the value of q_1.

(b) At point C on the diagram, draw a vector representing the direction of the electric field at that point.

(c) Calculate the approximate magnitude of the electric field strength at point D on the diagram.

(d) The equipotential labeled 0 V is the cross section of a nearly spherical surface. Calculate the electric flux for this surface.

(e) A proton is placed at point A and then released from rest.

i. Calculate the work done by the electric field on the proton as it moves from point *A* to point *E*.

ii. Calculate the speed of the proton when it reaches point *E*.

(f) An electron is released from rest at point B. Which of the following indicates the direction of the initial acceleration, if any, of the electron?

____ Up ____ Down

____ Left ____ Right

____ Into the page ____ Out of the page

____ The direction is undefined since the acceleration is zero.

Justify your answer.

8.6 Gauss's Law

Name: ______________ Class: __________ Date: ____________ **Total: 10 marks**

KEY WORDS gauss's law 高斯定律 • gaussian surface 高斯面 • electric field 电场 • electric flux 电通量 • conductor 导体

Objective

Build the skills to answer exam questions on **Gauss's law**.

You must be able to:

- state **Gauss's law** 高斯定律, $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\varepsilon_0}$
- choose a **Gaussian surface** 高斯面 whose symmetry makes $\vec{E}$ constant and perpendicular
- apply Gauss's law to spherical, cylindrical, and planar distributions
- explain why $\vec{E} = 0$ inside a conductor in electrostatic equilibrium

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Gauss's law

$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\varepsilon_0}$: the net flux through a closed surface equals the enclosed charge divided by ε_0.

■ Choose the symmetry

Pick a Gaussian surface (sphere, cylinder, box) on which E is constant and either perpendicular or parallel to the surface, so the integral becomes $E \times A$.

■ A point charge

For a sphere of radius r around charge q: $E(4\pi r^2) = \frac{q}{\varepsilon_0}$, giving $E = \frac{kq}{r^2}$. Inside a conductor at equilibrium, no charge is enclosed, so $E = 0$.

2 Practice

2.1 State Gauss's law. [1]

2.2 State the electric field inside a conductor in electrostatic equilibrium. [1]

2.3 A charge of 5.0×10^{-9} C is enclosed by a closed surface ($\varepsilon_0 = 8.85 \times 10^{-12}$). Find the flux through it. [2]

3 Exam-style questions

3.1 Gauss's law relates the flux through a closed surface to the [1]

- **A** total field everywhere
- **B** charge enclosed
- **C** surface area
- **D** current

3.2 Inside a charged conducting sphere in equilibrium, the electric field is [1]

- **A** kq/r^2
- **B** zero
- **C** at its maximum
- **D** infinite

3.3 A point charge $+8.0 \times 10^{-9}$ C sits at the centre of a spherical surface of radius 0.20 m ($\varepsilon_0 = 8.85 \times 10^{-12}$, $k = 8.99 \times 10^9$).

(a) Find the flux through the sphere. [2]

(b) Find the field at the surface. [2]

Solutions

2.1 the net electric flux through a closed surface equals the enclosed charge divided by ε_0.

2.2 zero.

2.3 $\Phi_E = \dfrac{Q_{\text{enc}}}{\varepsilon_0} = \dfrac{5.0 \times 10^{-9}}{8.85 \times 10^{-12}} = 565\ \text{N m}^2\,\text{C}^{-1}$.

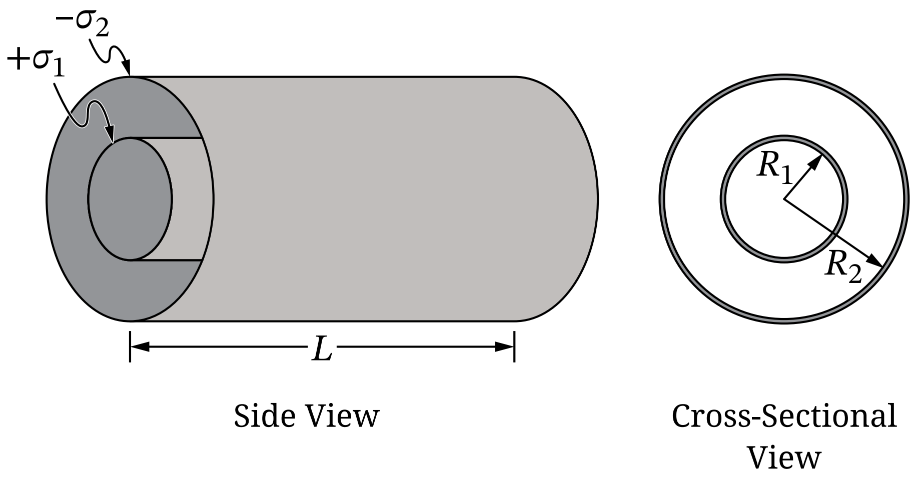
3.1 **B**.

3.2 **B**.

3.3 (a) $\Phi_E = \dfrac{Q}{\varepsilon_0} = \dfrac{8.0 \times 10^{-9}}{8.85 \times 10^{-12}} = 904\ \text{N m}^2\,\text{C}^{-1}$. (b) $E = \dfrac{kq}{r^2} = \dfrac{8.99 \times 10^9 \times 8.0 \times 10^{-9}}{(0.20)^2} =$ $1.8 \times 10^3\ \text{N C}^{-1}$.

Question 1: Version J

1. An isolated, air-filled, charged capacitor consists of two conducting, coaxial, cylindrical shells that each have length L. The inner shell has radius R_1 and the outer shell has radius R_2, as shown in Figure 1, where $R_1 < R_2 \ll L$. The surface charge densities (amounts of charge per unit area) of the inner and outer shells are $+\sigma_1$ and $-\sigma_2$, respectively. The absolute values of the total charges on the shells are equal.



Note: Figures not drawn to scale.

Figure 1

A.

i. Using Gauss's law, **derive** an expression for the magnitude E of the electric field as a function of the radial distance r from the center of the capacitor for the region $R_1 < r < R_2$. Express your answer in terms of R_1, σ_1, r, and physical constants, as appropriate.

ii. **Derive** an expression for the absolute value $|\Delta V|$ of the potential difference between the outer and inner shells in terms of R_1, R_2, σ_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

iii. On the axes shown in Figure 2, **sketch** a graph of E as a function of r from $r = 0$ to a position that is outside the outer shell.

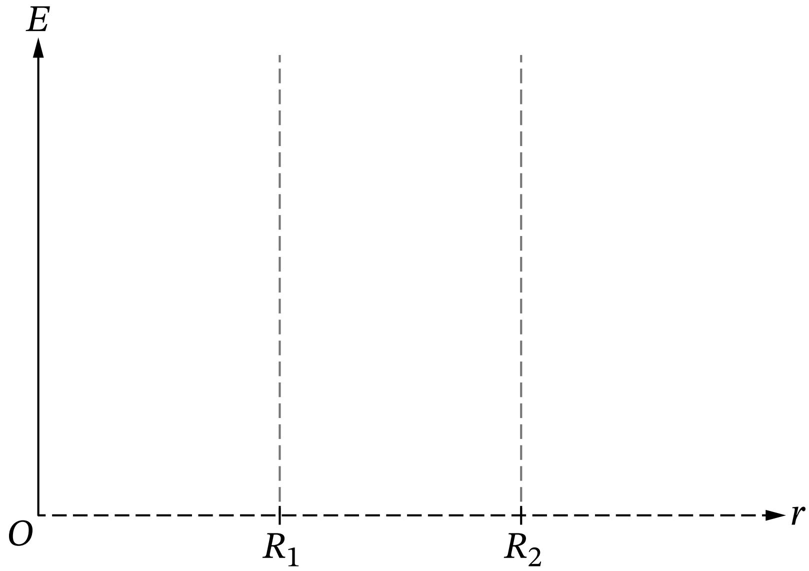
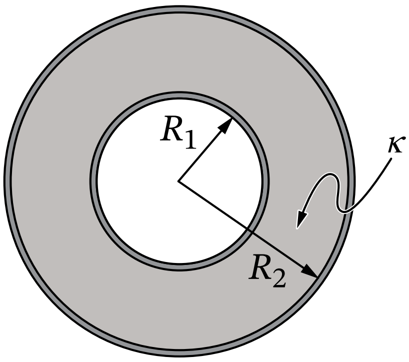


Figure 2

B. A material of dielectric constant κ is inserted into the isolated, charged capacitor such that the material fills the region $R_1 < r < R_2$, as shown in Figure 3.



Cross-Sectional View

Figure 3

Derive an expression for the capacitance C of the capacitor with the material inserted in terms of L, R_1, R_2, κ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

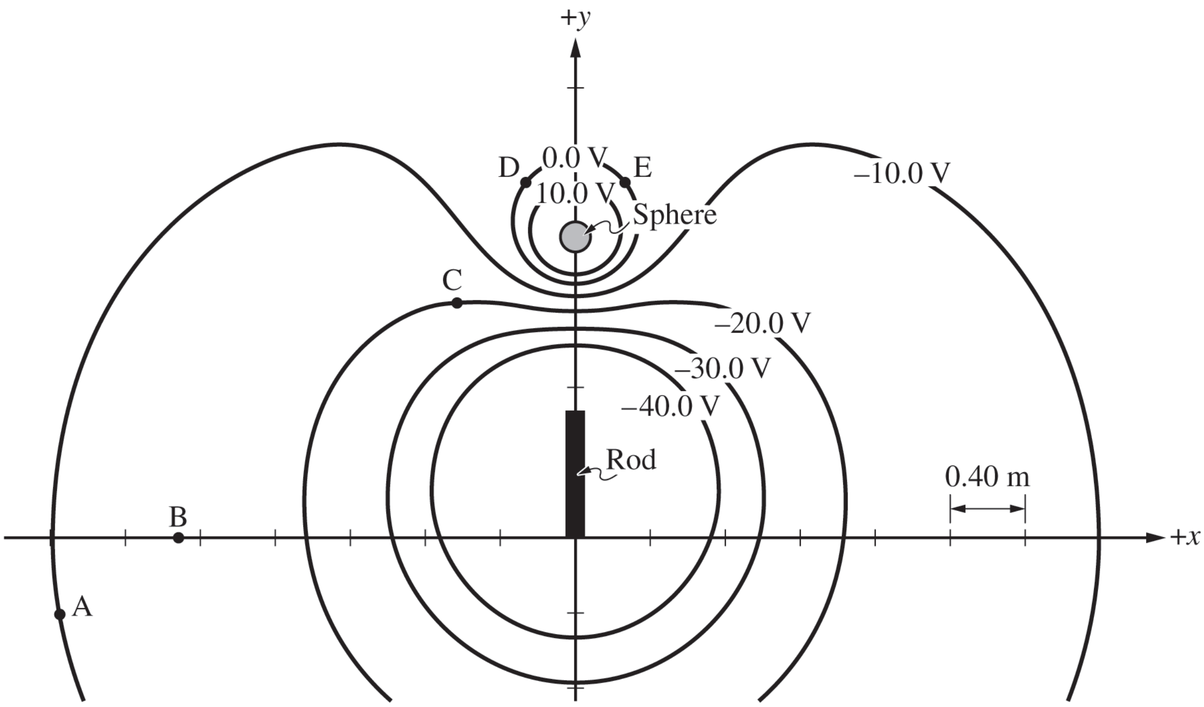


Figure 1

1. A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\text{ nC}$. The rod and sphere are held at rest on the y-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and on the y-axis are 0.40 m apart.

   (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the 0.0 V equipotential line.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is held momentarily at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE}. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a “0” in the corresponding columns, as appropriate.

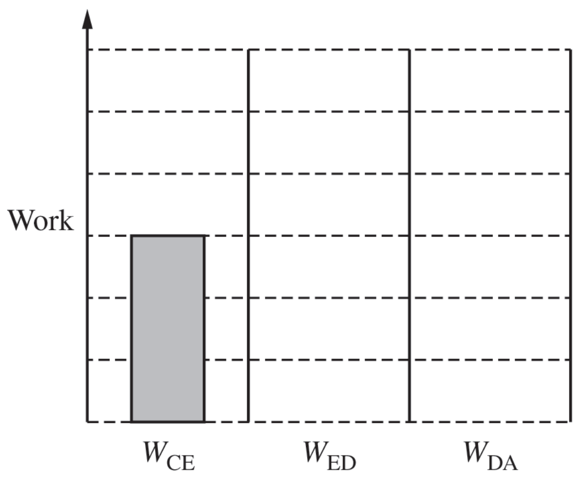


Figure 2

ii. **Calculate** the approximate magnitude of the x-component of the electric field at Position B.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position C. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

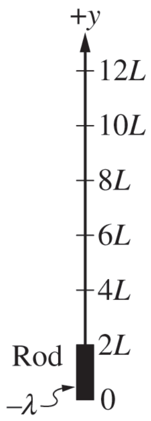


Figure 3

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the y-axis in the orientation shown in Figure 3. Position P (not shown) is located on the y-axis a distance y_P from the origin, where $y_P > 2L$.

(d) The electric potential V_P at y_P is $V_P = -k\lambda \ln\left(\dfrac{y_P}{y_P - 2L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

ii. On Figure 4, **sketch** a graph of the y-component E_y of the electric field resulting from the rod as a function of y in the region $2L < y < 12L$.

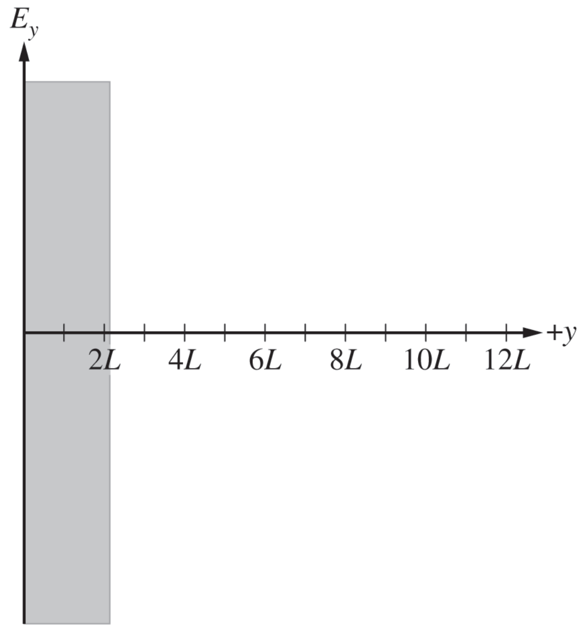


Figure 4

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 1** on this page.

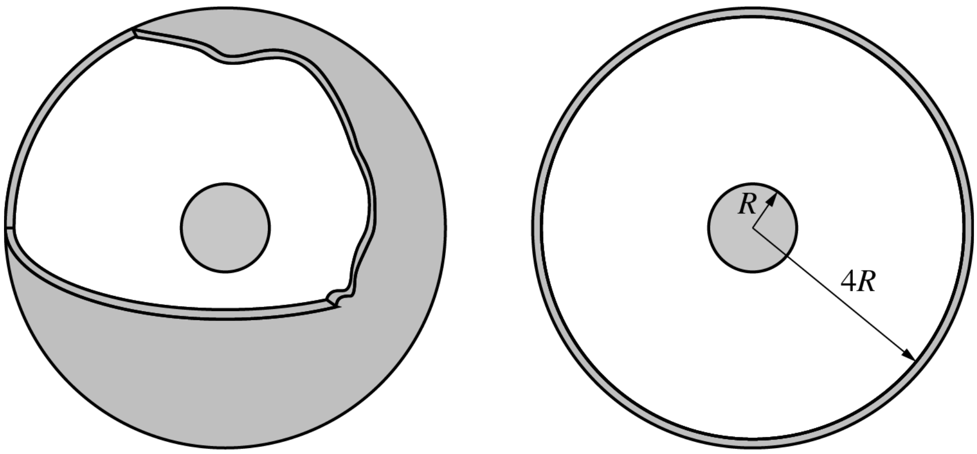
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

irections: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, hich are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this ooklet in the spaces provided after each part.



Cutout View

Cross-Section View

Note: Figures not drawn to scale.

. A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.

(a) Determine the charge on the outer surface of the shell.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the sphere for $r < R$. Express your answer in terms of Q, R, r, and physical constants, as appropriate.

(c) The magnitude of the electric field at $r = R$ is $8\,\mathrm{N/C}$. Calculate the value of the electric field at $r = 2R$.

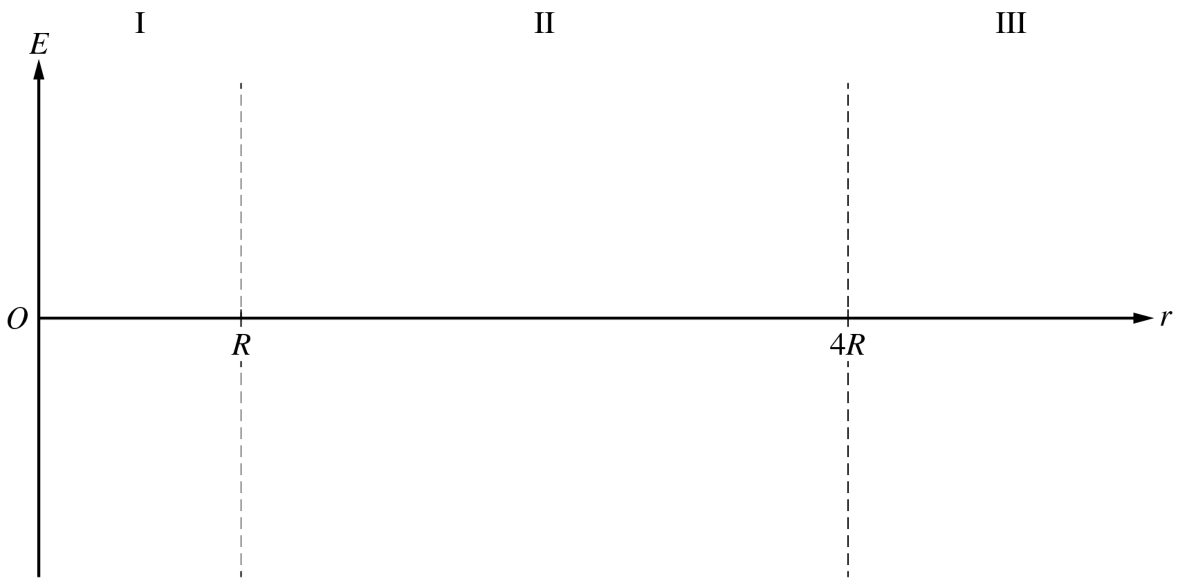
(d) Derive an expression for the absolute value of the potential difference between the outer surface of the sphere and the inner surface of the shell. Express your answer in terms of Q, R, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

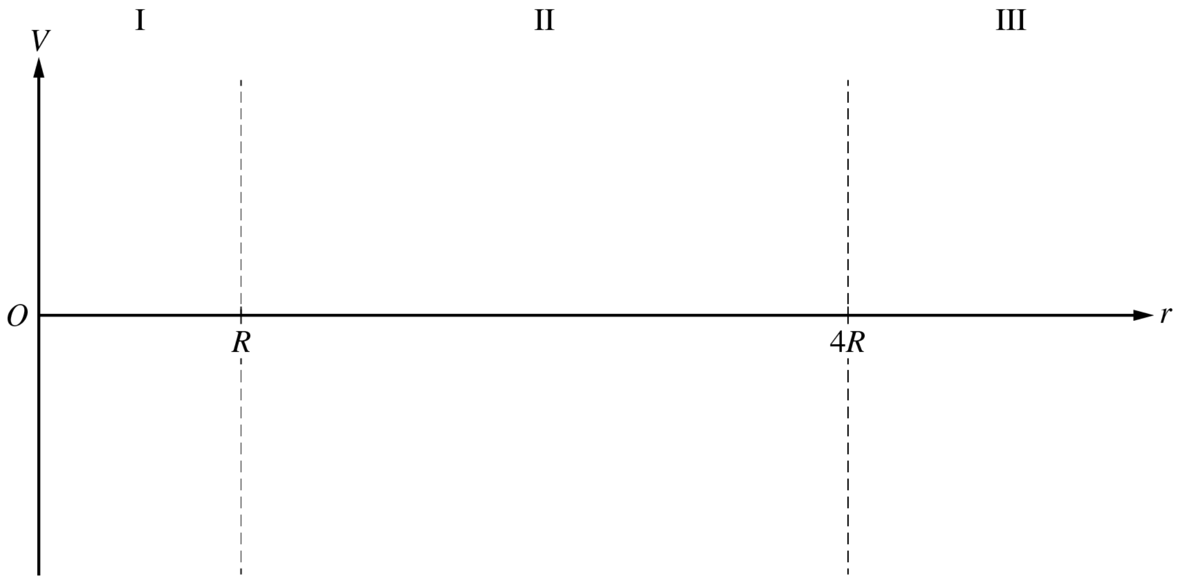
Continue your response to **QUESTION 1** on this page.

(e)

i. On the following axes that include regions I, II, and III, sketch a graph of the electric field E as a function of the distance r from the center of the sphere.



ii. On the following axes that include regions I, II, and III, sketch a graph of the electric potential V as a function of the distance r from the center of the sphere.



GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 1** on this page.

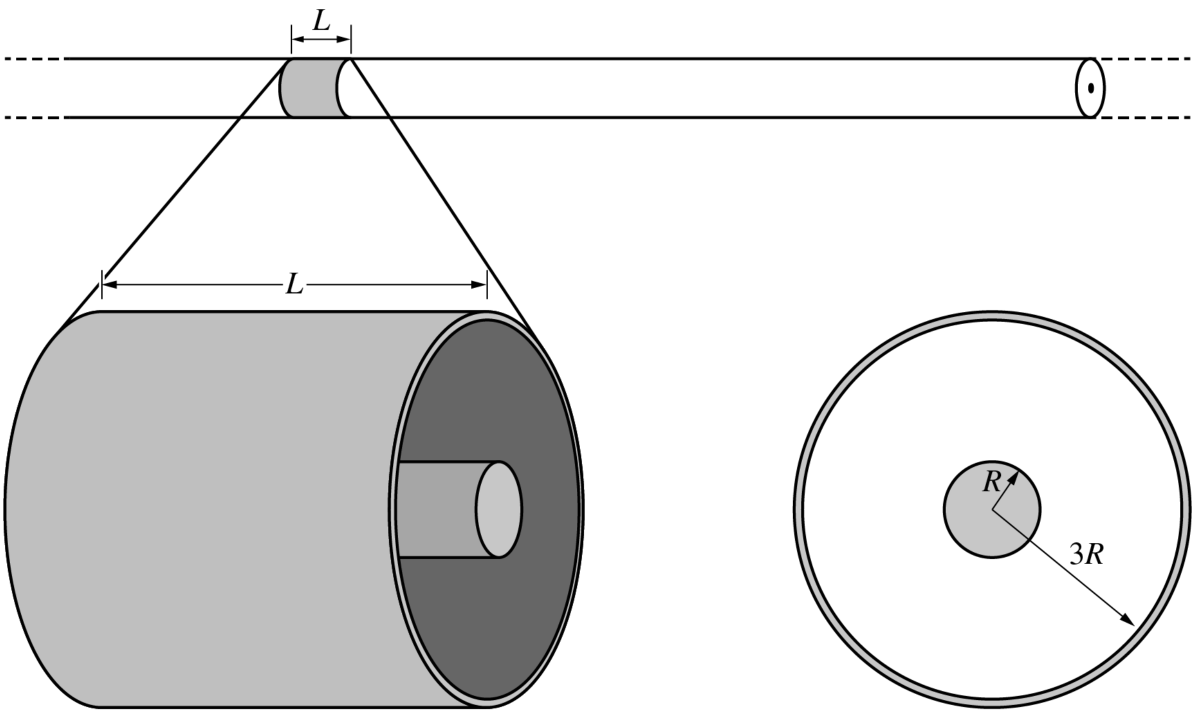
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Cutout View　　　　Cross-Section View

Note: Figures not drawn to scale.

1. A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(a) Determine the charge on the outer surface of the cylindrical shell within length L.

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the inner cylinder for $r < R$. Express your answers in terms of Q, R, r, L, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(c) The magnitude of the electric field at $r = R$ is $12\ \mathrm{N/C}$. Calculate the value of the electric field at $r = 2R$.

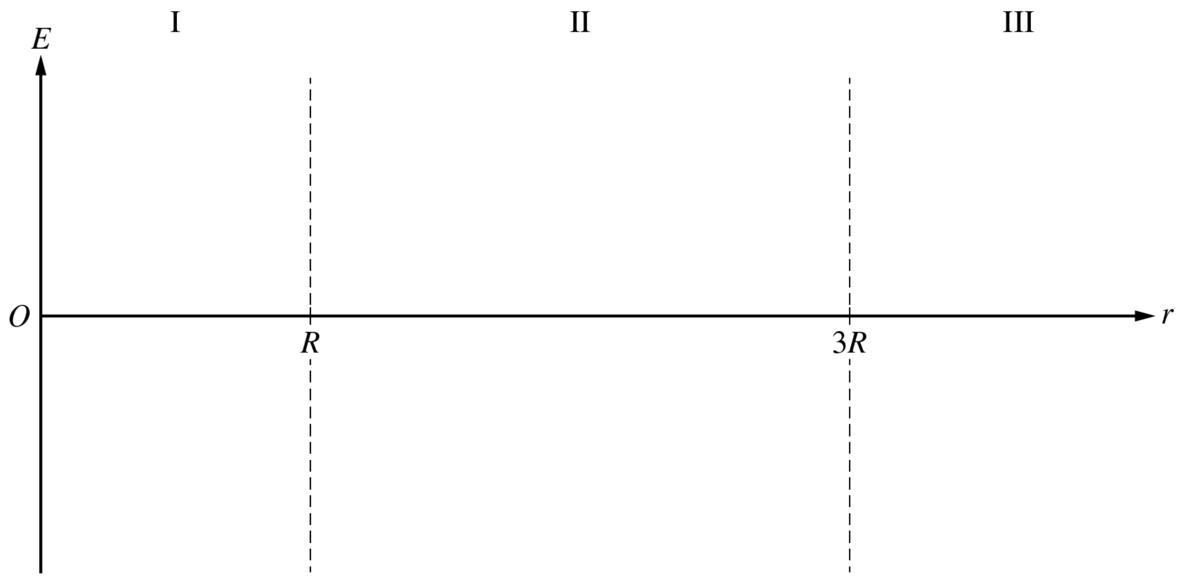
(d) Derive an expression for the absolute value of the potential difference between the surface of the nonconducting cylinder and the inner surface of the cylindrical shell.

GO ON TO THE NEXT PAGE.

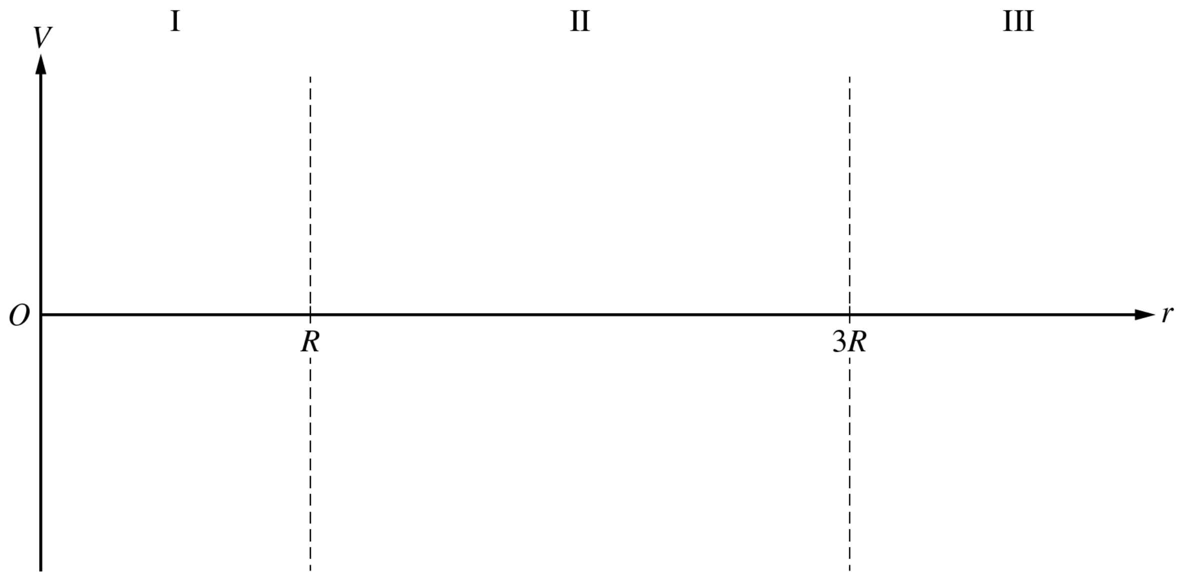
Continue your response to **QUESTION 1** on this page.

(e)

i. On the following axes that include regions I, II, and III, sketch the graph of the electric field E as a function of the distance r from the axis of the inner cylinder.



ii. On the following axes that include regions I, II, and III, sketch the graph of the electric potential V as a function of the distance r from the axis of the inner cylinder.



GO ON TO THE NEXT PAGE.

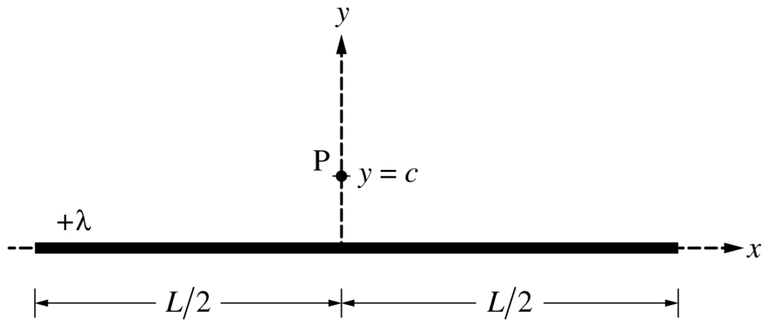
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

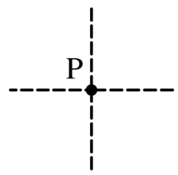


Note: Figure not drawn to scale.

1. A very long, thin, nonconducting cylinder of length L is centered on the *y*-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the *y*-axis at $y = c$, where $L >> c$.

(a)

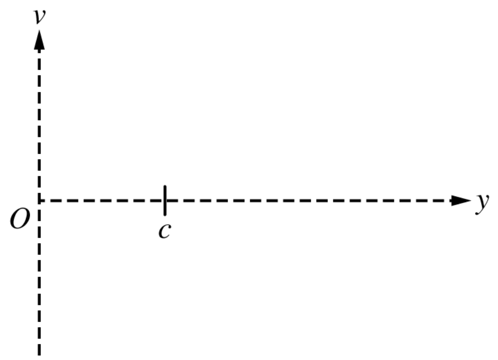
i. On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the long cylinder. The arrow should start on and point away from the dot.

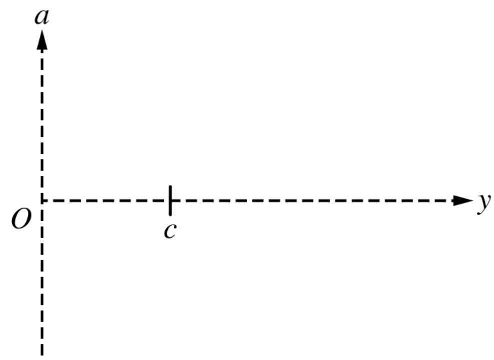


ii. Describe the shape and location of a Gaussian surface that can be used to determine the electric field at point P due to the long cylinder.

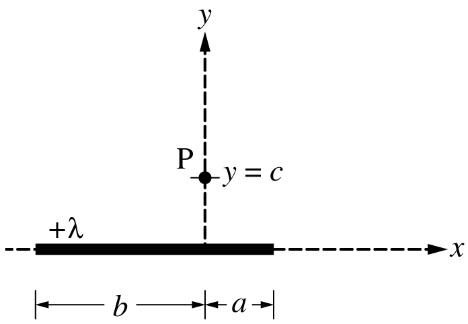
iii. Use your Gaussian surface to derive an expression for the magnitude of the electric field at point P. Express your answer in terms of λ, c, L, and physical constants, as appropriate.

(b) A proton is released from rest at point P. On the axes below, sketch the velocity v as a function of position y and the acceleration a as a function of position y for the proton.

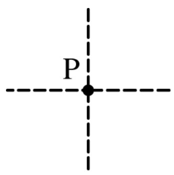




The original cylinder is now replaced with a much shorter thin, nonconducting cylinder with the same uniform linear charge density $+\lambda$, as shown in the figure below. The length of the cylinder to the right of the y-axis is a, and the length of the cylinder to the left of the y-axis is b, where $a < b$.



(c) On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the shorter cylinder. The arrow should start on and point away from the dot.

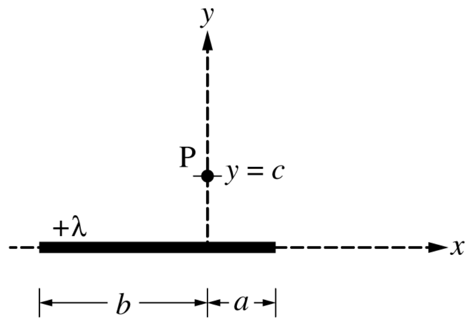


(d)

i. Is there a single Gaussian surface that can be used with Gauss's law to derive an expression for the electric field at point P?

____ Yes ____ No

ii. If your answer to part (d)(i) is yes, explain how you can use Gauss's law to derive an expression for the field at point P. If your answer to part (d)(i) is no, explain why Gauss's law cannot be applied to derive an expression for the electric field in this case.



Note: This figure is shown again for reference.

A student in class argues that using the integral shown below might be a useful approach for determining the electric field at point P.

$$E = \int \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\,dq$$

The student uses this approach and writes the following two integrals for the magnitude of the horizontal and vertical components of the electric field at point P.

Horizontal component: $$\left|E_x\right| = \frac{\lambda}{4\pi\varepsilon_0}\int_{-b}^{a}\frac{x}{\left(c^2 + x^2\right)^{3/2}}\,dx$$

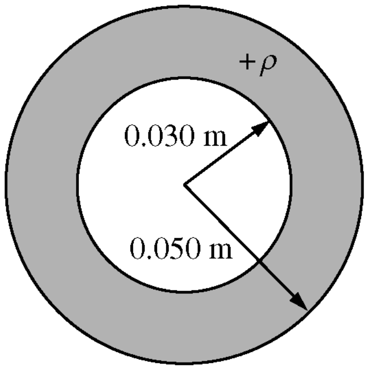
Vertical component: $$\left|E_y\right| = \frac{\lambda}{4\pi\varepsilon_0}\int_{-b}^{a}\frac{y}{\left(c^2 + x^2\right)}\,dy$$

(e)

i. One of the two expressions above is not correct. Which expression is not correct?

____ Horizontal component ____ Vertical component

ii. Identify two mistakes in the incorrect expression, and explain how to correct the mistakes.



2. A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m}: \quad \rho = 0$$

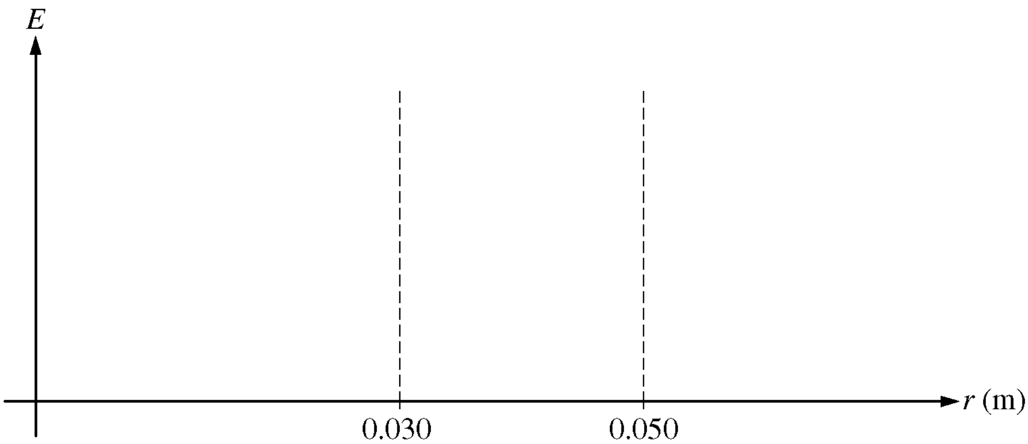
$$0.030 \text{ m} < r < 0.050 \text{ m}: \quad \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

$$r > 0.050 \text{ m}: \quad \rho = 0$$

(a) Calculate the total charge of the sphere.

(b) Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.

(c) On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.



(d) Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

(e) A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.

i. Calculate the magnitude of the initial acceleration of the proton.

ii. Calculate the speed of the proton after a long time.

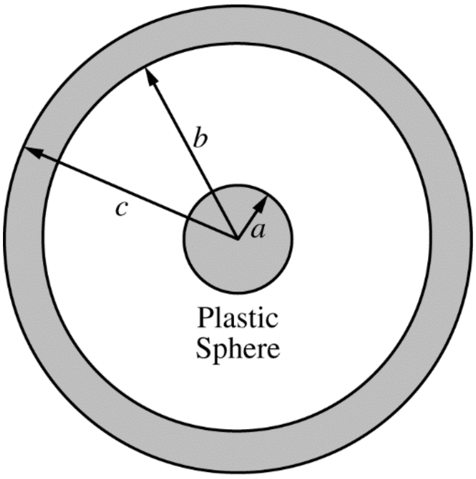
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Conducting Spherical Shell

1. A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

   (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

   i. $r < a$

   ii. $a < r < b$

   (b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

   i. $r < a$

   ii. $a < r < b$

   (c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

   i. Determine the charge on the inner surface of the conducting shell.

   Justify your answer.

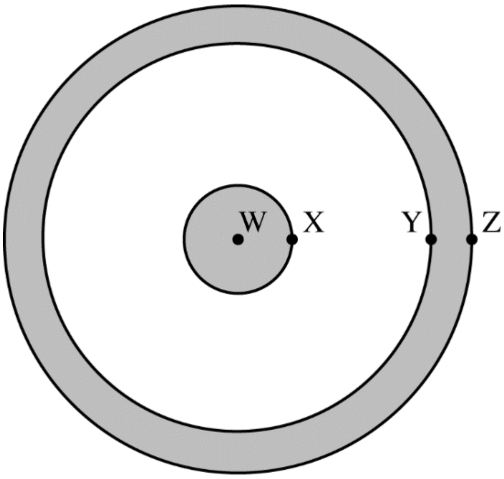
   ii. Determine the charge on the outer surface of the conducting shell.

(d)

i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.

ii. The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

____ W ____ X ____ Y ____ Z



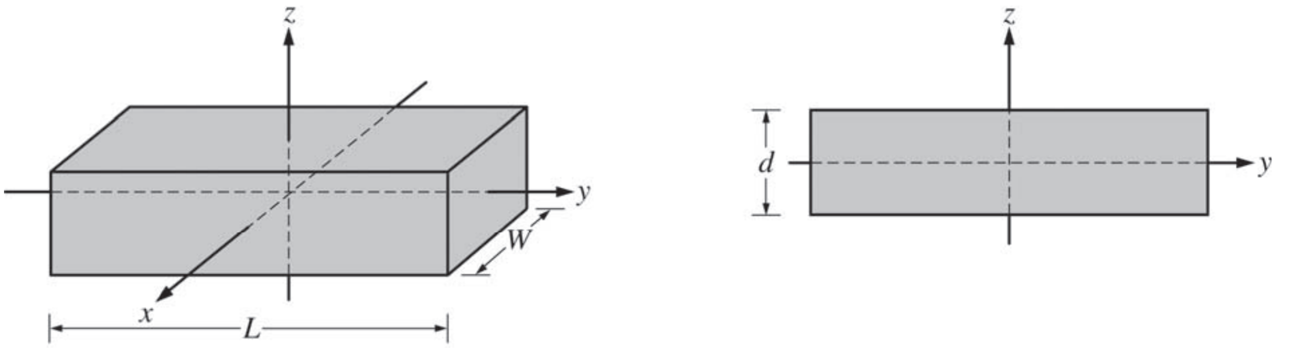
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

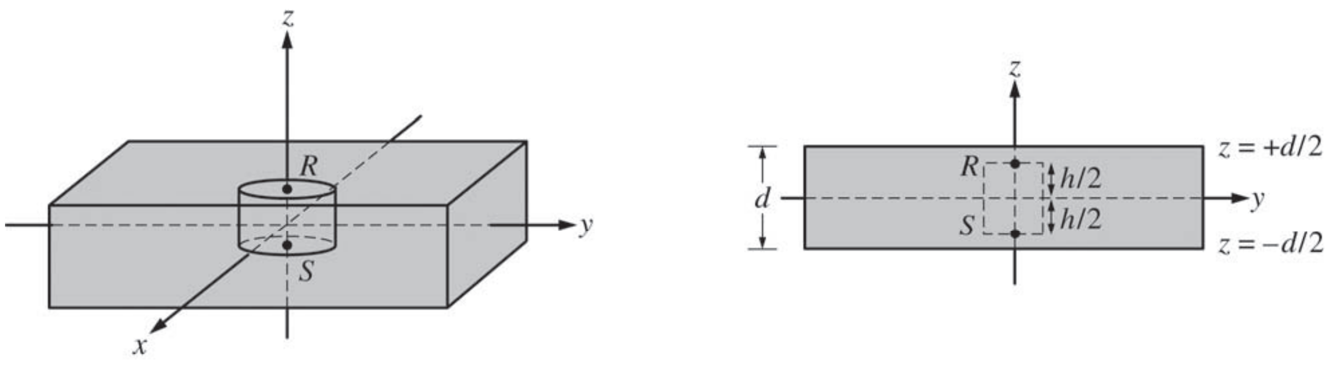


Perspective View Side View

Note: Figures not drawn to scale.

1. A very large nonconducting slab with a uniform positive volume charge density ρ_0 is fixed with the origin of the *xyz*-axes at its center, as shown in the figure above. The thickness of the slab is d, the length is L, and the width is W, where $L >> d$ and $W >> d$. The large faces of the slab are parallel to the *xy*-plane.

   Consider a Gaussian cylinder with a cross-sectional area A and height h that is positioned with its axis along the *z*-axis, as shown in the figure below.



Perspective View Side View

Note: Figures not drawn to scale.

(a) Draw a single vector on each of the dots below representing the direction of the electric field at the given points. If the electric field at either point is zero, write "$E = 0$" next to the point.

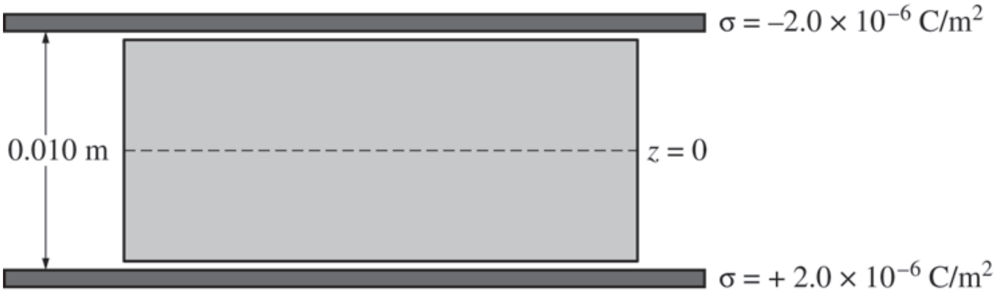
i. • R

ii. • S

b) Use Gauss's law to derive expressions for the following. Express your answers in terms of ρ_0, A, d, h, z, and physical constants, as appropriate.

i. Derive an expression for the total flux Φ through the Gaussian surface shown.

ii. Derive an expression for the magnitude of the electric field as a function of z for any position inside the slab, and show that it is equal to $E = \dfrac{\rho_0 z}{\varepsilon_0}$.



Note: Figure not drawn to scale.

The charged slab is now placed between two large metal plates separated by a distance of 0.010 m, which is approximately the thickness of the slab, but the slab does not contact either metal plate. The metal plates are charged, resulting in the surface charge densities $\sigma = \pm 2.0 \times 10^{-6}\ \mathrm{C/m^2}$, as shown in the figure above. Assume the charge distribution inside the slab remains unchanged by the presence of the charged plates and that the slab's volume charge density is $\rho_0 = 1.00 \times 10^{-3}\ \mathrm{C/m^3}$.

(c)

i. The magnitude of the electric field inside the slab is zero on the z-axis at position z_0. Which of the following correctly indicates the value for z_0?

____ $z_0 > 0$ ____ $z_0 = 0$ ____ $z_0 < 0$

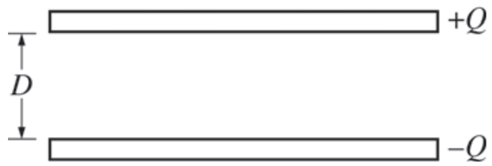
Justify your answer.

ii. Calculate the value z_0.

(d) Calculate the magnitude of the electric potential difference from the center of the slab to the top of the slab.

PHYSICS C: ELECTRICITY AND MAGNETISM
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



E&M.1.

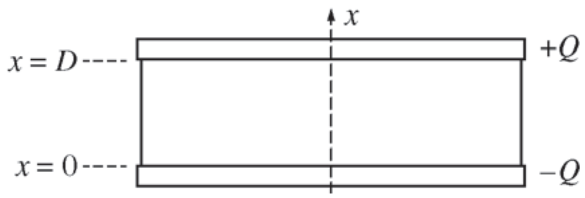
A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

(a)

i. On the figure above, draw an arrow to indicate the direction of the electric field between the plates.

ii. On the figure above, draw an appropriate Gaussian surface that will be used to derive an expression for the magnitude of the electric field E between the plates.

iii. Using Gauss's law and the Gaussian surface from part (a)-ii, derive an expression for the magnitude of the electric field E between the plates. Express your answer in terms of A, D, Q, and physical constants, as appropriate.



The space between the plates is now filled with a dielectric material that is engineered so that its dielectric constant varies with the distance from the bottom plate to the top plate, defined by the x-axis indicated in the diagram above. As a result, the electric field between the plates is given by $\vec{E} = -\dfrac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\hat{i}$, where κ_0 is a positive constant. Express all algebraic answers to the remaining parts in terms of A, D, Q, κ_0, x, and physical constants, as appropriate.

(b) Determine an expression for the dielectric constant κ as a function of x.

(c)

i. Write, but do NOT solve, an equation that could be used to determine the potential difference V between the plates of the capacitor.

ii. Using the equation from part (c)-i, derive an expression for the potential difference $V_D - V_0$, where V_D is the potential of the top plate and V_0 is the potential of the bottom plate.

(d) Determine the capacitance of the capacitor.

(e) The energy stored in the capacitor that has a varying dielectric is U_V. A second capacitor that has a constant dielectric of value κ_0 is also given a charge Q. The energy stored in the second capacitor is U_C. How do the values of U_V and U_C compare?

____ $U_V < U_C$ ____ $U_V > U_C$ ____ $U_V = U_C$

Justify your answer.

E&M.3.

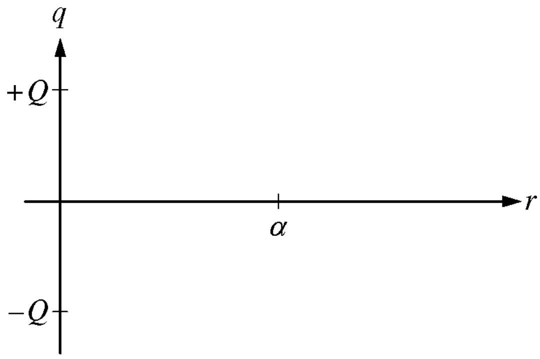
A scientist describes an electrically neutral atom with a model that consists of a nucleus that is a point particle with positive charge $+Q$ at the center of the atom and an electron volume charge density of the form

$$\rho(r) = \begin{cases} -\dfrac{\beta}{r^2} e^{-r/\alpha} & r < \alpha \\ 0 & r > \alpha \end{cases}$$

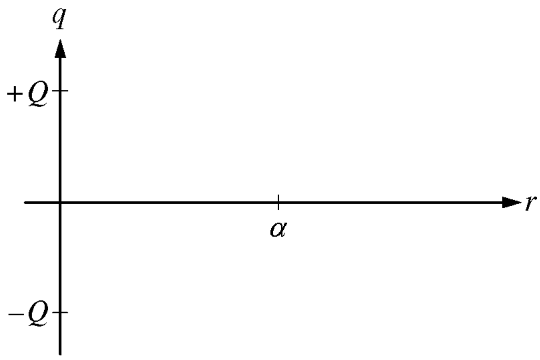
where α and β are positive constants and r is the distance from the center of the atom.

(a) On the axes below, let r stand for the radius of a Gaussian sphere. Sketch the graph for each of the following charges enclosed by the Gaussian sphere as a function of r. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

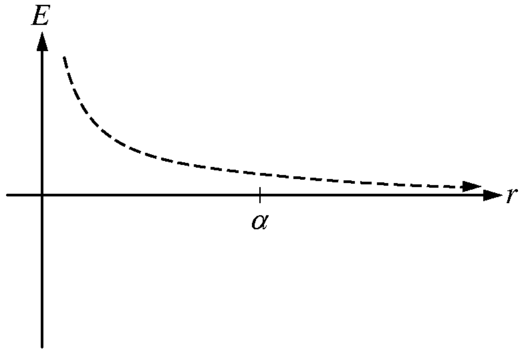
i. The nuclear charge only



ii. The electron charge only



(b) The dashed curve on the graph below represents the electric field as a function of distance r due to the positive nucleus of the atom without any electrons. The nucleus is modeled as a point particle of charge $+Q$. On the same graph, sketch the electric field as a function of distance r for the neutral atom as defined by the scientist's model, which includes the nucleus and the negative electrons surrounding it.



(c) Use Gauss's law to derive an expression for the electric field strength due to the neutral atom for the following positions in terms of $Q, \alpha, \beta, r,$ and fundamental constants, as appropriate.

i. $r > \alpha$

ii. $r < \alpha$

(d) Based on the model proposed by the scientist, what is the physical meaning of the constant α ?

STOP

END OF EXAM