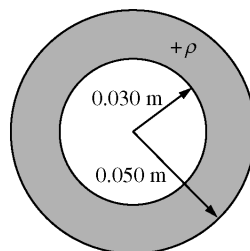


Question 2

15 points



A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ , as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m: } \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m: } \rho = b/r, \text{ where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

$$r > 0.050 \text{ m: } \rho = 0$$

- (a) LO CNV-2.D.a, SP 6.B, 6.C
3 points

Calculate the total charge of the sphere.

For indicating the need to integrate the expression for charge density to determine the total charge on the sphere		1 point
$Q = \int \rho dV$		
For proper substitutions into the integration		1 point
$Q = \int \frac{(1.6 \times 10^{-6})}{r} (4\pi r^2) dr$		
For using the proper limits of integration		1 point
$Q = 4\pi (1.6 \times 10^{-6}) \int_{r=0.03}^{r=0.05} r dr = 4\pi (1.6 \times 10^{-6}) \left[\frac{r^2}{2} \right]_{r=0.03}^{r=0.05}$		
$Q = (2\pi)(1.6 \times 10^{-6})(0.05^2 - 0.03^2) = 1.61 \times 10^{-8} \text{ C}$		

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Question 2 (continued)

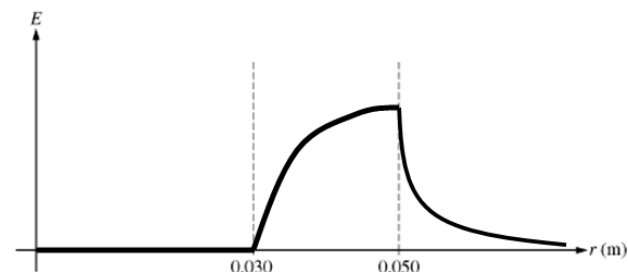
- (b) LO CNV-2.D.a, SP 6.C
3 points

Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.

For correctly evaluating the surface integral in Gauss's law		1 point
$\oint E \cdot dA = E(4\pi r^2)$		
For correctly substituting the answer from part (a) and correct radius into above equation		1 point
$\frac{Q_{enc}}{\epsilon_0} = E(4\pi r^2) \therefore E = \frac{Q_{enc}}{4\pi\epsilon_0 r^2} = \frac{(1.61 \times 10^{-8} \text{ C})}{(4\pi)(8.85 \times 10^{-12})(0.05)^2}$		
For an answer consistent with part (a) with correct units		1 point
$E = 5.79 \times 10^4 \text{ N/C}$		

- (c) LO CNV-2.C, SP 3.C
3 points

On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.



For clearly showing a graph with a value of $E = 0$ for $r < 0.030 \text{ m}$		1 point
For a continuous graph that starts at zero, is concave down, and increases in value from $r = 0.030$ to $r = 0.050$		1 point
For a continuous graph that decreases asymptotically toward the horizontal axis for $r > 0.050 \text{ m}$		1 point

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Question 2 (continued)

- (d) LO CNV-1.G.a, SP 6.B, 6.C
2 points

Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

For substituting the total charge from part (a) into a correct expression for electric potential	1 point
For substituting $r = 0.05$ m into a correct expression for electric potential	1 point
$V_R = \frac{Q_{\text{tot}}}{4\pi\epsilon_0 r} = \frac{(9 \times 10^9)(1.61 \times 10^{-8} \text{ C})}{(0.05 \text{ m})} = 2900 \text{ V}$	
<i>Alternate Solution</i>	<i>Alternate Points</i>
For substituting the total charge from part (a) into an integration for electric potential	1 point
$\Delta V_R = V_R - V_\infty = V_R = -\int E dr = -\int \frac{Q_{\text{enc}}}{4\pi\epsilon_0 r^2} dr = -\int \frac{(9 \times 10^9)(1.61 \times 10^{-8} \text{ C})}{r^2} dr$	
For integrating with correct limits of integration	1 point
$V_R = -\int_{r=\infty}^{r=0.05 \text{ m}} \frac{145}{r^2} dr = -145 \left[-\frac{1}{r} \right]_{r=\infty}^{r=0.05 \text{ m}} = 145 \left(\frac{1}{(0.05 \text{ m})} - \frac{1}{\infty} \right) = 2900 \text{ V}$	

A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.

- (e) i. LO ACT-1.D, SP 6.B, 6.C
2 points

Calculate the magnitude of the initial acceleration of the proton.

For using a correct expression of Newton's second law in terms of the electric field	1 point
$F = ma \therefore qE = ma \therefore a = \frac{qE}{m}$	
For correctly substituting into equation above	1 point
$a = \frac{qE}{m} = \frac{(1.6 \times 10^{-19} \text{ C})(5.79 \times 10^4 \text{ N/C})}{1.67 \times 10^{-27} \text{ kg}} = 5.55 \times 10^{12} \text{ m/s}^2$	

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Question 2 (continued)

- (e) continued
ii. LO CNV-1.E, SP 6.B, 6.C
2 points

Calculate the speed of the proton after a long time.

For a correct expression of kinetic energy in terms of the electric potential difference	1 point
$-q\Delta V = \frac{1}{2}mv^2$	
For correctly substituting into equation above	1 point
$v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{-2(1.6 \times 10^{-19} \text{ C})(0 - 2900 \text{ V})}{(1.67 \times 10^{-27} \text{ kg})}} = 7.45 \times 10^5 \text{ m/s}$	

Learning Objectives

ACT-1.D: Determine the motion of a charged object of specified charge and mass under the influence of an electrostatic force.

CNV-1.E: Calculate the work done or changes in kinetic energy (or changes in speed) of a charged particle when it is moved through some known potential difference.

CNV-1.G.a: Use the general relationship between electric field and electric potential to calculate the relationships between the magnitude of electric field or the potential difference as a function of position.

CNV-2.C: State and use Gauss's law in integral form to derive unknown electric fields for planar, spherical, or cylindrically symmetrical charge distributions.

CNV-2.D.a: Using appropriate mathematics (which may involve calculus), calculate the total charge contained in lines, surfaces, or volumes when given a linear-charge density, a surface-charge density, or a volume-charge density of the charge configuration.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.

6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.

6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.