Question 2

15 points

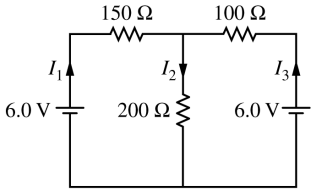

Figure 1

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(a)

i. LO CNV-6.F.b, SP 6.A
3 points

Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

For an equation representing the sum of the currents at one of the junctions		1 point
$I_1 - I_2 + I_3 = 0$		
For an equation representing the sum of the potential differences around one of the loops		1 point
For an equation representing the sum of the potential differences around a loop different from the above loop		1 point
$6 - 150I_1 - 200I_2 = 0$		
$6 - 100I_3 - 200I_2 = 0$		
$6 - 150I_1 + 100I_3 - 6 = 0$		
Note: Full credit is earned for two correct loop equations using loop currents.		

ii. LO CNV-6.F.b, SP 6.C
2 points

Calculate the current in the $200\ \Omega$ resistor.

For combining the equations from part (a)(i)		1 point
$I_1 - I_2 + I_3 = 0$ $-I_1 - 1.33I_2 = -.04 \ \therefore\ -4.33I_2 = -0.10$ $-I_3 - 2I_2 = -.06$		
Note: Credit is earned if students indicate they used a calculator function to solve the system of equations.		
For a correct answer with correct units		1 point
$-4.33I_2 = -0.10 \therefore I_2 = 0.023$ A		

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Question 2 (continued)

(a) continued

iii. LO CNV-5.A.a, SP 6.C
1 point

Calculate the power dissipated by the $200\ \Omega$ resistor.

For using a correct equation to calculate the power in the $200\ \Omega$ resistor		1 point
$P = I^2R = (0.023\text{ A})^2(200\ \Omega) \therefore P = 0.107$ W		

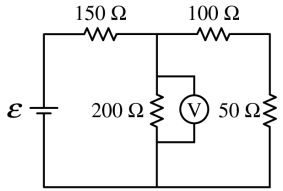

Figure 2

The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

(b) LO CNV-6.C.a, SP 6.B, 6.C
2 points

Calculate the current through the $50\ \Omega$ resistor.

For correctly calculating the equivalent resistance of the branch with the $50\ \Omega$ resistor		1 point
$R = 100\ \Omega + 50\ \Omega = 150\ \Omega$		
For using the correct potential difference in Ohm's law to calculate the current in the $50\ \Omega$ resistor		1 point
$I = \frac{V}{R} = \frac{(4.4\text{ V})}{(100\ \Omega + 50\ \Omega)} \therefore I = 0.029$ A		

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Question 2 (continued)

(c) LO CNV-6.C.a, SP 6.A, 6.C
3 points

Calculate the voltage $\mathcal{E}$ of the battery.

For using a correct equation to determine the current through the 150 Ω resistor		1 point
For correctly substituting the current from part (b) as I_3		1 point
$I_1 = I_2 + I_3 = \frac{(4.4\text{ V})}{(200\ \Omega)} + 0.029\text{ A} = 0.051\text{ A}$		
For using a correct equation to determine the emf of the battery		1 point
$\mathcal{E} = I_1 R_1 + V_2 = (0.051\text{ A})(150\ \Omega) + 4.4\text{ V} = 12.1\text{ V}$		
Alternate Third Point		
For calculating the equivalent resistance of the circuit and substituting this resistance into a correct equation to determine the emf of the battery		*1 point*
$R_T = 150\ \Omega + \frac{1}{\frac{1}{200\ \Omega} + \frac{1}{150\ \Omega}} = 236\ \Omega$		
$\mathcal{E} = I_1 R_T = (0.051\text{ A})(236\ \Omega) = 12.0\text{ V}$		
Alternate Solution		*Alternate Points*
For using a correct equation to determine the equivalent resistance of the parallel resistors		*1 point*
$\frac{1}{R_P} = \frac{1}{200\ \Omega} + \frac{1}{150\ \Omega} \therefore R_P = 86\ \Omega$		
For correctly substituting the given potential difference and the calculated equivalent resistance to determine the total current of the circuit		*1 point*
$I_T = \frac{(4.4\text{ V})}{(86\ \Omega)} = 0.051\text{ A}$		
For calculating the equivalent resistance of the circuit and substituting into a correct equation to determine the emf of the battery		*1 point*
$R_T = 150\ \Omega + 86\ \Omega = 236\ \Omega$		
$\mathcal{E} = I_1 R_T = (0.051\text{ A})(236\ \Omega) = 12.0\text{ V}$		

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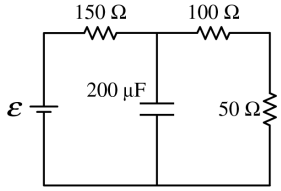
Question 2 (continued)

(d)

i. LO CNV-7.B.a, SP 6.A, 6.C
2 points

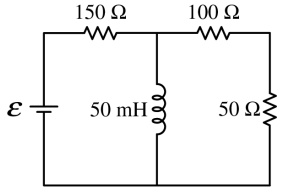
The 200 Ω resistor in the circuit in Figure 2 is replaced with a 200 μF capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the 50 Ω resistor.



For substituting the voltage consistent with part (c) into Ohm's law		1 point
For correctly calculating the equivalent resistance of the circuit		1 point
$I = \frac{\mathcal{E}}{R_{tot}} = \frac{(12.1\text{ V})}{(150\ \Omega + 100\ \Omega + 50\ \Omega)} = 40.3\text{ mA}$		

ii. LO CNV-10.C.a, SP 7.A, 7.C
2 points

The 200 Ω resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the 50 Ω resistor greater than, less than, or equal to the current calculated in part (b)?



____ Greater than ____ Less than ____ Equal to

Justify your answer.

For correctly selecting "Less than" with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because steady state is reached, the inductor will act as a short circuit. So all the current will pass through the inductor and no current will pass through the 50 Ω resistor.		

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Question 2 (continued)

Learning Objectives

CNV-5.A.a: Derive expressions that relate current, voltage, and resistance to the rate at which heat is produced in a resistor.
CNV-6.C.a: Calculate voltage, current, and power dissipation for any resistor in a circuit containing a network of known resistors with a single battery or energy source.
CNV-6.F.b: Set up simultaneous equations to calculate at least two unknowns (currents or resistance values) in a multi-loop circuit.
CNV-7.B.a: Calculate the potential difference across a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.
CNV-10.C.a: Calculate initial transient currents and final steady-state currents through any part of a series or parallel circuit containing an inductor and one or more resistors.

Science Practices

6.A: Extract quantities from narratives or mathematical relationships to solve problems.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.