

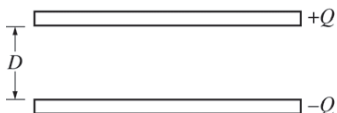
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

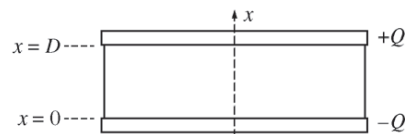


E&M.1.

A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D . The plates of the capacitor are each given a charge of magnitude Q , as shown in the figure above. Ignore edge effects.

(a)

- On the figure above, draw an arrow to indicate the direction of the electric field between the plates.
- On the figure above, draw an appropriate Gaussian surface that will be used to derive an expression for the magnitude of the electric field E between the plates.
- Using Gauss's law and the Gaussian surface from part (a)-ii, derive an expression for the magnitude of the electric field E between the plates. Express your answer in terms of A , D , Q , and physical constants, as appropriate.



The space between the plates is now filled with a dielectric material that is engineered so that its dielectric constant varies with the distance from the bottom plate to the top plate, defined by the x -axis indicated in the diagram above. As a result, the electric field between the plates is given by $\vec{E} = -\frac{Q}{\epsilon_0 \kappa_0 e^{-x/D} A} \hat{i}$, where κ_0 is a

positive constant. Express all algebraic answers to the remaining parts in terms of A , D , Q , κ_0 , x , and physical constants, as appropriate.

- (b) Determine an expression for the dielectric constant κ as a function of x .

(c)

- Write, but do NOT solve, an equation that could be used to determine the potential difference V between the plates of the capacitor.
- Using the equation from part (c)-i, derive an expression for the potential difference $V_D - V_0$, where V_D is the potential of the top plate and V_0 is the potential of the bottom plate.

- (d) Determine the capacitance of the capacitor.

- (e) The energy stored in the capacitor that has a varying dielectric is U_V . A second capacitor that has a constant dielectric of value κ_0 is also given a charge Q . The energy stored in the second capacitor is U_C . How do the values of U_V and U_C compare?

_____ $U_V < U_C$ _____ $U_V > U_C$ _____ $U_V = U_C$

Justify your answer.