

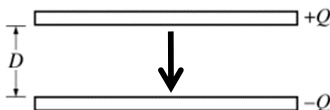
2015 SCORING GUIDELINES

Question 1

15 points total

Distribution
of points

- (a)
i. 1 point



For at least one arrow between the plates pointing downward from the positive plates toward the negative plate and no extraneous arrows pointing in any other direction

1 point

- ii. 1 point



For drawing an appropriate Gaussian surface (enclosing at least the inner edge of one of the plates) that can be used to determine the electric field between the plates

1 point

- iii. 3 points

For using a correct statement of Gauss's law

1 point

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

For applying Gauss's law, using an enclosed charge and surface area consistent with the surface drawn in part (a)-ii

1 point

$$E(A_{GS}) = \frac{q_{enc}}{\epsilon_0} \quad (A_{GS} \text{ is the area of the end of the Gaussian surface between the plates})$$

$$E = \frac{q_{enc}}{\epsilon_0 A_{GS}} \quad \left(\text{using } \sigma = \frac{q_{enc}}{A_{GS}} \right)$$

$$E = \frac{\sigma}{\epsilon_0} \quad \left(\text{using } \sigma = \frac{Q}{A} \right)$$

For a correct answer with work shown

1 point

$$E = \frac{Q}{\epsilon_0 A}$$

2015 SCORING GUIDELINES

Question 1 (continued)

Distribution
of points

- (b) 1 point

Comparing the equation for electric field between parallel plates to the given equation:

$$E = \frac{Q}{\kappa \epsilon_0 A} = \frac{Q}{\epsilon_0 \kappa_0 e^{-x/D} A}$$

For an answer consistent with part (a)-iii

$$\kappa = \kappa_0 e^{-x/D}$$

1 point

- (c)
i. 1 point

Using the equation relating the electric field to potential difference

$$E = -\frac{dV}{dx}$$

For a correct differential equation

1 point

$$\frac{dV}{dx} = -\left(-\frac{Q}{\epsilon_0 \kappa_0 e^{-x/D} A} \right)$$

$$\frac{dV}{dx} = \frac{Q}{\epsilon_0 \kappa_0 e^{-x/D} A}$$

Alternate Solution:

Alternate Point

Using the equation relating the electric field to potential difference:

$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

For a correct differential equation

1 point

$$\Delta V = -\int -\frac{Q}{\epsilon_0 \kappa_0 e^{-x/D} A} dx$$

$$\Delta V = \int \frac{Q}{\epsilon_0 \kappa_0 e^{-x/D} A} dx$$

2015 SCORING GUIDELINES

Question 1 (continued)

Distribution
of points

- (c) (continued)
ii. 4 points

Separating the variables in the differential equation from part (c)(i):

$$\frac{dV}{dx} = \frac{Q}{\epsilon_0 \kappa_0 e^{-x/D} A}$$

$$dV = \left(\frac{Q}{\epsilon_0 \kappa_0 A} \right) e^{x/D} dx$$

For using the correct limits of integration in attempting to integrate the equation above 1 point

$$\int_{V_0}^{V_D} dV = \left(\frac{Q}{\epsilon_0 \kappa_0 A} \right) \int_0^D e^{x/D} dx$$

For correctly integrating the equation 1 point

$$[V]_{V_0}^{V_D} = \left(\frac{Q}{\epsilon_0 \kappa_0 A} \right) [De^{x/D}]_0^D$$

$$(V_D - V_0) = \left(\frac{QD}{\epsilon_0 \kappa_0 A} \right) (e^{D/D} - e^0)$$

For an expression that gives the correct absolute value of the potential difference between the plates 1 point

For having the potential difference be positive 1 point

$$\Delta V = \left(\frac{QD}{\epsilon_0 \kappa_0 A} \right) (e - 1)$$

$$\Delta V = \frac{1.72 QD}{\epsilon_0 \kappa_0 A}$$

- (d) 1 point

Using the equation for capacitance:

$$C = \frac{Q}{\Delta V} = \frac{Q}{\left(\frac{QD}{\epsilon_0 \kappa_0 A} \right) (e - 1)}$$

For an answer consistent with part (c)-ii 1 point

$$C = \frac{\epsilon_0 \kappa_0 A}{D(e - 1)}$$

$$C = \frac{\epsilon_0 \kappa_0 A}{1.72 D}$$

2015 SCORING GUIDELINES

Question 1 (continued)

Distribution
of points

- (e) 3 points

For selecting $U_V > U_C$ 1 point

For correctly comparing the capacitance or the potential difference with the varying dielectric constant to the capacitance or the potential difference with the uniform dielectric constant 1 point

For correctly comparing the two stored energies consistent with the comparison of the capacitances or potential differences 1 point

Example: According to the equation from part (d), $C_C > C_V$. Since $U = \frac{Q^2}{2C}$, if the charge stored on the two capacitors is the same, then $U_V > U_C$.