Question 1: Free-Response Question — 15 points

(a) For correctly determining the charge on the outer surface of the shell — **1 point**

Example Response

$q_{net} = q_{inner} + q_{outer}$
$q_{outer} = q_{net} - q_{inner}$
$q_{outer} = +3Q - (+Q)$
$q_{outer} = +2Q$

Scoring Note: A correct response may earn a point even if no work is shown.

Total for part (a) 1 points

(b) For using Gauss's law by substituting for either the area or the enclosed charge — **1 point**

Example Response

$$\frac{q_{enc}}{\varepsilon_0} = \oint E \bullet dA = E\left(4\pi r^2\right)$$

For using a correct expression for q_{enc} as a function of r — **1 point**

Example Response

$$q_{enc} = \rho V = \left(\frac{-Q}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) = -Q\frac{r^3}{R^3}$$

Scoring Note: The response may earn this point regardless of the sign of q_{enc}.

For using the correct area — **1 point**

Example Response

$$E = q_{enc}\frac{1}{\varepsilon_0\left(4\pi r^2\right)} = \left(-Q\frac{r^3}{R^3}\right)\frac{1}{\varepsilon_0\left(4\pi r^2\right)}$$
$$E = -Q\frac{r}{4\pi\varepsilon_0 R^3}$$

Scoring Note: The response may earn this point regardless of the sign of E.

Total for part (b) 3 points

(c) For recognizing the electric field varies as an inverse square of the separation distance from the center of the nonconducting sphere **1 point**

Example Response

$$E \propto \frac{1}{r^2}$$

For the correct magnitude, including units, of the electric field at $2R$ **1 point**

Example Response

$$E_{new} = \frac{E_{old}}{4} = \frac{8 \text{ N/C}}{4}$$
$$E_{new} = 2 \text{ N/C}$$

Total for part (c) **2 points**

(d) For using the equation relating potential difference to the electric field with an attempt at either integration limits or evaluating the integral **1 point**

Example Response

$$V_f - V_i = -\int_{s_i}^{s_f} E dr$$
$$\Delta V = -\int_{r=R}^{r=4R} E dr$$

For substituting the correct expression for the electric field or an expression for E that is consistent with the explicit functional dependence of $E(r)$ from part (c) **1 point**

Example Response

$$\Delta V = -\int_{r=R}^{r=4R} -\frac{Q}{4\pi\varepsilon_0 r^2} dr$$

$$\Delta V = \frac{Q}{4\pi\varepsilon_0} \int_{r=R}^{r=4R} \frac{1}{r^2} dr$$

	For correctly integrating the electric field expression that was substituted in the previous point with correct limits of integration	**1 point**

Example Response

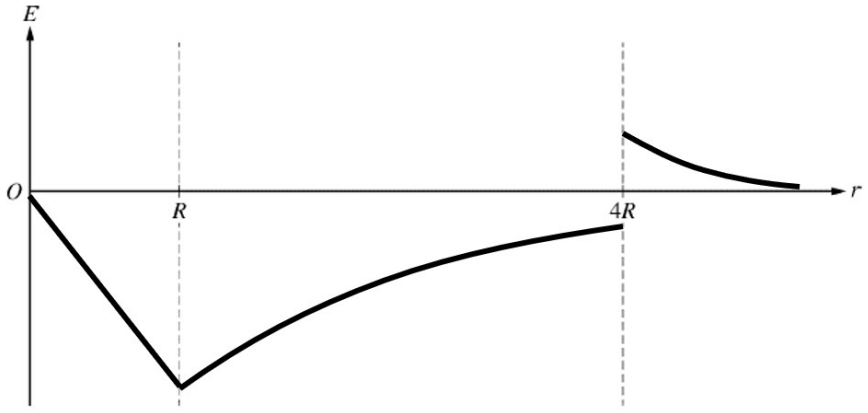
$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{r=R}^{r=4R} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{r=4R}^{r=R} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{4R}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{4-1}{4R}\right)$$

$$\Delta V = \frac{3Q}{16\pi\varepsilon_0 R}$$

Alternate Solution

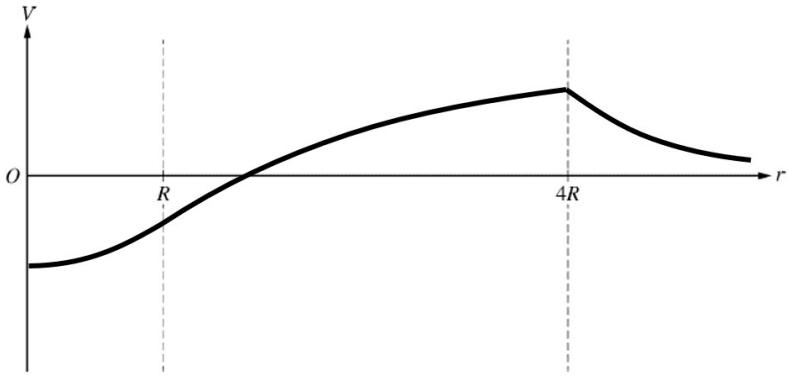
	For using a difference in potentials treating the inner sphere as a point charge by symmetry $\Delta V = \frac{Q}{4\pi\varepsilon_0 r_f} - \frac{Q}{4\pi\varepsilon_0 r_i}$	**1 Point**
	For using the correct charge on both of the terms; Q *not* Q *and* $3Q$	**1 Point**
	For using the correct values for r $\Delta V = \frac{Q}{4\pi\varepsilon_0 R} - \frac{Q}{4\pi\varepsilon_0 4R}$ $\Delta V = \frac{3Q}{16\pi\varepsilon_0 R}$	**1 Point**
		Total for part (d) 3 points
(e)(i)	For indicating that E is negative and the magnitude of E increases linearly from 0 to R	**1 point**
	For a curve that is continuous at R and asymptotically approaches zero in the region $R < r < 4R$	**1 point**
	For a positive, concave-up, and decreasing curve for $r > 4R$	**1 point**

Example Response



(e)(ii)	For a potential curve that is always increasing from 0 to $4R$	**1 point**
	For a potential curve that is decreasing for $r > 4R$	**1 point**
	For a continuous graph across all three regions	**1 point**

Example Response



Scoring Note: The intercept of the curve on the vertical axis is irrelevant. The intercept of the curve on the horizontal axis is irrelevant. The curve can, hence, cross the horizontal axis at any location or even be entirely on the negative side of the horizontal axis as long as the other criteria are met.

Total for part (e) 6 points

Total for question 1 15 points